Fractal Cosmology and Particle Stability: A Unified Field Perspective in UFQFT

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Abstract

The concept of fractal dimensionality offers a novel perspective for understanding the evolution of the Universe from its pre-Big Bang state to the present epoch. Within the framework of the Unified Fractal Quantum Field Theory (UFQFT), physical structures—ranging from nuclear systems with magic numbers ($D_f \approx 1.4$) to the large-scale Universe ($D_f \approx 2.7$)—can be described as resonance configurations embedded in a fractal spacetime. We propose that the early Universe, prior to the Big Bang, exhibited a higher effective fractal dimension ($D_f \approx$ 3–4), reflecting a pre-geometric state of symmetry and homogeneity. The subsequent cosmic evolution can then be understood as a dimensional reduction process, wherein fractal dimensionality decreases as structure formation, particle stability, and entropy growth emerge. This approach provides a unified interpretation of particle confinement, nuclear stability, and cosmological expansion, linking microphysical phenomena to cosmic-scale dynamics. The results suggest that fractal dimensionality is not static but evolves as a fundamental parameter of nature, offering new insights into the origin of matter, the dynamics of the Big Bang, and the present large-scale geometry of the Universe.

Keywors:quantum foam, cosmic large-scale structure, dark matter, dark energy, CMB anomalies, ΛCDM comparison, unified physics, Hausdorff dimension, resonance scaling, energy-charge fields, experimental cosmology.

1.Introduction

Fractal geometry, introduced by Mandelbrot (1983), revolutionized our understanding of complex, irregular structures in nature and mathematics. Unlike traditional Euclidean geometry, which deals with smooth and regular shapes, fractal geometry focuses on objects that exhibit self-similarity and intricate detail at every scale. These structures are characterized by a non-integer, or fractal, dimension, which quantifies their complexity and how they fill space (Falconer, 2013). The application of fractal geometry extends beyond mathematics into various fields, including physics, where it provides insights into the underlying patterns of natural phenomena (Peitgen et al., 2003; Barnsley, 1988).In physics, fractal geometry has been instrumental in describing systems that are inherently irregular and complex. For instance, in the study of turbulence, fractal models help in understanding the chaotic behavior of fluid flows (Mandelbrot, 2004). Similarly, in cosmology, the distribution of galaxies and large-scale structures in the universe has been analyzed using fractal models, revealing patterns that are not immediately apparent through traditional Euclidean approaches (Mandelbrot & Brooks, 1977). These applications underscore the importance of fractal geometry in capturing the complexity of physical systems.

Despite the successes of the Standard Model of particle physics and the Lambda Cold Dark Matter (ΛCDM) model in cosmology, both frameworks face significant challenges. The Standard Model, while successful in explaining electromagnetic, weak, and strong interactions, does not incorporate gravity and cannot account for dark matter and dark energy (Peebles & Ratra, 2003; Frieman et al., 2008). Moreover, it does not explain the matter-antimatter asymmetry observed in the universe

(Copeland et al., 2007). Similarly, the ΛCDM model, which describes the evolution of the universe, faces issues such as the Hubble tension and the nature of dark energy, which remain unresolved despite extensive observational data (Riess et al., 2019; Planck Collaboration, 2020). Quantum Field Theory (QFT), the cornerstone of particle physics, also encounters limitations. While QFT successfully describes particle interactions at high energies, it struggles with the incorporation of gravity and the behavior of spacetime at the Planck scale (Weinberg, 1995; Zee, 2010). The incompatibility between QFT and general relativity at these scales suggests the need for a more fundamental theory that can unify these frameworks (Kiefer, 2014; Rovelli, 2004).These limitations highlight the necessity for a new approach that can address the shortcomings of existing models and provide a more comprehensive understanding of the universe. The aim of this work is to propose the fractal dimension as a unifying descriptor that bridges the gap between the pre-Big Bang conditions and the present universe. By integrating fractal geometry into the fabric of spacetime, we can develop a framework that accounts for the complexities observed in both particle physics and cosmology (Mandelbrot, 1986; Falconer, 2013).

In this context, the Unified Fractal Quantum Field Theory (UFQFT) emerges as a promising candidate. UFQFT posits that the fundamental fields of energy (Φ) and charge (Ψ) are embedded in a fractal spacetime with an effective dimension of approximately 2.70 (Sogukpinar, 2025a;2025b). Recent studies have analyzed the fractal resonances of fundamental particles, showing that particle masses, spin properties, and hierarchical structures can be derived from the fractal dimension and resonant behavior of Φ and Ψ fields (Sogukpinar, 2025c). UFQFT has also been applied to nuclear systems, revealing that halo nuclei and deviations from the shell model can be naturally explained through fractal dimensionality (Sogukpinar, 2025d;2025e). Furthermore, the theory has been extended to early-universe cosmology, where variations in the local fractal dimension provide explanations for Cosmic Microwave Background anomalies, dark matter distribution, and the emergence of cosmic inflation (Sogukpinar, 2025f). Collectively, these works demonstrate that the properties of particles and cosmological structures are determined by the resonant dynamics of energy and charge fields within the fractal geometry of spacetime, offering a unified geometric framework for both particle physics and cosmology (Sogukpinar, 2025g; 2025h;2025k;2025l;2025m;2025n).(Mandelbrot & Brooks, 1977). Furthermore, UFQFT provides a geometric origin for phenomena such as dark matter, dark energy, and the matter-antimatter asymmetry. By analyzing the fluctuations in the fractal dimension during the early universe, UFQFT offers explanations for cosmic inflation and the large-scale structure of the universe (Riess et al., 2019; Planck Collaboration, 2020).

This study aims to unify particle physics and cosmology within the framework of Unified Fractal Quantum Field Theory (UFQFT). It proposes that the fundamental fields of energy (Φ) and charge (Ψ) resonate within a fractal spacetime of approximately 2.70 effective dimension, and that these resonances determine the masses, spins, and interactions of particles. Moreover, phenomena such as halo nuclei, pre-Big Bang universe dynamics, dark matter distribution, and CMB anomalies are interpreted through fractal dimensionality and resonance behavior. Thus, the study presents a fractal-geometric model that links both micro-scale particle physics and macro-scale cosmic structure.

2. Preliminaries and Fractal Measure in UFQFT

In this section, we introduce the fundamental definitions of fractal dimensions and their implementation in the Unified Fractal Quantum Field Theory (UFQFT). These concepts provide the mathematical basis for analyzing particle field resonances and cosmological evolution. UFQFT employs fractal geometry to describe energy (Φ) and charge (Ψ) field resonances. Several standard fractal measures are utilized: The Hausdorff dimension D_H quantifies the scaling of the number of boxes N(ε) needed to cover a fractal structure as the box size ε decreases:

$$N(\varepsilon) \sim \varepsilon^{-D_H} \Rightarrow D_H \equiv -\lim_{\varepsilon\to 0}\frac{\ln N(\varepsilon)}{\ln \varepsilon} \tag{1}$$

Where, N(ε): Number of boxes of size ε covering the structure, D_{H}: Hausdorff dimension, Correlation dimension: Used for large-scale structures or nuclear density distributions:

$$\xi(r) \equiv \frac{\langle n(x)n(x+r)\rangle}{\langle n\rangle^2} - 1 \propto r^{-\gamma} \Rightarrow D_{corr} = 3 - \gamma \tag{2}$$

Where, n(x): Number density at position x, ξ(r): Two-point correlation function, γ: Correlation exponent, D_{corr}: Correlation fractal dimension Spectral dimension:Derived from the return probability P(τ) of a diffusion process on a fractal:

$$P(\tau) \sim \tau^{-d_s/2} \Rightarrow d_s = -2d\frac{\ln P}{d\ln \tau} \tag{3}$$

Where, τ: Diffusion time, P(τ): Probability of return to origin, d_{s}: Spectral dimension To simplify notation and applications, an effective fractal dimension D is defined as a weighted combination of the above measures:

$$D \equiv w_H D_H + w_c D_{corr} + w_s d_s, w_H + w_c + w_s = 1 \tag{4}$$

Where, w_H,w_c,w_s: Weights determined by observation or scale regime, D: Effective fractal dimension, The effective dimension D captures the influence of both local field resonances and large-scale structure, enabling a unified description of particle physics and cosmology. The fractal measure $d\mu_D$ and the corresponding UFQFT action S are defined as:

$$d\mu_D = A(D) r^{D-1} dr\, d\Omega_D, \qquad S = \int d\mu_D\, L(\Phi, \Psi, D; g) \tag{5}$$

Where, A(D): Normalization factor depending on fractal dimension, r: Radial coordinate, $d\Omega_D$: D-dimensional solid angle element, L(Φ,Ψ,D;g): UFQFT Lagrangian density, g: Metric tensor. This formulation allows integration over fractal spacetime and naturally incorporates the fractal geometry of field resonances. The effective fractal dimension D(t) can be applied to different cosmological epochs, providing insight into the evolution from the pre-Big Bang universe to the present:

$$D(t) \equiv w_H D_H(t) + w_c D_{corr}(t) + w_s d_s(t), w_H + w_c + w_s = 1 \tag{6}$$

- Big Bang epoch (t→0):
  High energy density and fluctuating spacetime yield $D_{BB} \approx 3.5$.
- Present universe (t=t_0):
  As structures form and the universe expands, Dnow≈2.7.

Assume $w_H = 0.4, w_c = 0.3, w_s = 0.3$ and approximate fractal measures $D_H = 3.8, D_{corr} = 3.2, d_s = 3.5$:

$$D_{BB} = 0.4 \cdot 3.8 + 0.3 \cdot 3.2 + 0.3 \cdot 3.5 = 1.52 + 0.96 + 1.05 = 3.53 \tag{7}$$

Assume $w_H = 0.3, w_c = 0.5, w_s = 0.2$ and measured values $D_H = 2.8, D_{corr} = 2.6, d_s = 2.7$:

$$D_{now} = 0.3 \cdot 2.8 + 0.5 \cdot 2.6 + 0.2 \cdot 2.7 = 0.84 + 1.30 + 0.54 = 2.68 \tag{8}$$

These computations illustrate how UFQFT connects fractal spacetime geometry with cosmological evolution, providing a consistent framework linking microphysical resonances to large-scale structure**.**

3. Fractal Dimension in Different Cosmic Epochs

The fractal dimension D evolves across distinct cosmic epochs, reflecting the transition from a homogeneous pre-Big Bang state to the structured Universe observed today. Within the UFQFT framework, D encodes the balance between energy-field density (Φ) and charge-field resonance complexity (Ψ), offering a quantitative tool to track the emergence of structure.

In the pre-Big Bang epoch, the Universe can be modeled as a homogeneous, undifferentiated resonance state, where no particle distinction existed. Energy and charge fields were maximally entangled, yielding the highest effective fractal dimension:

$$D_{pre} = \lim_{\epsilon \to 0} \frac{ln\, N(\epsilon)}{ln(1/\epsilon)} \tag{9}$$

Where, N(ϵ): number of covering sets of scale ϵ, ϵ: resolution scale approaching zero. In the symmetric pre-Big Bang regime, we approximate:

$$N(\epsilon) \propto \epsilon^{-4} \tag{10}$$

which implies:

$$D_{pre} \approx 4 \tag{11}$$

This reflects a maximally isotropic state, consistent with the "no-particle" condition and the high-dimensional character of spacetime foam.

As the Universe underwent quantum fluctuations near the Planck scale, resonance bifurcations emerged, corresponding to the first steps of structure formation. This regime parallels concepts from loop quantum gravity and causal dynamical triangulation, where spacetime geometry acquires an effective dimension near three. The effective fractal dimension can be expressed as:

$$D_{foam} = 4 - \delta_{qf} \tag{12}$$

where δ_{qf} encodes the degree of resonance bifurcation caused by Planck-scale fluctuations. For first-order bifurcations,

$$\delta_{qf} \approx 1 \Rightarrow D_{foam} \approx 3 \tag{13}$$

This reduction signifies the transition from a smooth pre-geometric manifold to a granular spacetime structure.

At the nuclear scale, the fractal dimension decreases significantly, reflecting the discrete shell structure of atomic nuclei. Magic number nuclei (e.g., 8, 20, 28, 50, 82, 126) exhibit enhanced stability due to resonance closure, which can be represented by a scaling law:

$$D_{nuc} = \frac{ln\, N_{shell}}{ln(1/\epsilon_{nuc})} \tag{14}$$

Where, N_{shell}: number of filled nuclear shells, ϵ_{nuc}: normalized nuclear spacing scale. For stable configurations:

$$D_{magic} \approx 1.44 \tag{15}$$

This value represents the fractal signature of nuclear shell closures and acts as a “stability island” in the landscape of nuclear matter.

In the current epoch, the Universe exhibits large-scale fractality in galaxy distribution and stable particle structures (protons, electrons). The effective dimension can be derived from the resonance ratio of energy and charge fields:

$$D_{univ} = \frac{ln(\frac{\Phi}{\Psi})}{ln\,\lambda} \tag{16}$$

Where, Φ: mean cosmic energy density, Ψ: effective cosmic charge-field resonance density, λ: characteristic scaling parameter of cosmic structures. Observations of large-scale clustering yield:

$$D_{univ} \approx 2.7 \tag{17}$$

This result is consistent with cosmological surveys (e.g., SDSS, DESI), which reveal fractal-like distributions up to scales of several hundred Mpc.

4. Mathematical Derivation of Fractal Dimensions

The Unified Fractal Quantum Field Theory (UFQFT) describes the emergence of particles and cosmic structures through the scaling properties of resonance states within the coupled energy field (Φ) and charge field (Ψ). In this section, we provide the general mathematical framework for deriving fractal dimensions in different physical regimes, from continuous fields to discrete particles, and apply these results to concrete cases such as the proton, neutron, and halo nuclei. We begin with the assumption that the resonance density ρ scales with the system size R according to a fractal power law:

$$\rho(R) \propto R^{D-3} \tag{18}$$

Where, **D** is the Hausdorff fractal dimension. For a homogeneous field (**D = 3**), the density is scale-invariant, whereas deviations from **D = 3** indicate fractal clustering or confinement. The resonance energy **E** of a field configuration is given by:

$$E(R) = \Phi_0\, R^{D_\Phi - 3} + \Psi_0\, R^{D_\Psi - 3} \tag{19}$$

Where, Φ_0: normalization constant for the energy field, Ψ_0: normalization constant for the charge field, D_Φ, D_Ψ: effective fractal dimensions of the energy and charge components. The observable fractal dimension of the resonance structure is then obtained as the weighted combination:

$$D_{eff} = \frac{\Phi_0 D_\Phi + \Psi_0 D_\Psi}{\Phi_0 + \Psi_0} \tag{20}$$

In the pre-Big Bang phase, the fields are continuous, and no particle differentiation exists (D≈4). During the quantum foam era, fluctuations produce bifurcations of resonances, leading to effective dimensional reduction. This process can be expressed as:

$$D(t) = D_{init} - \kappa\, ln(\frac{t}{t_{Pl}}) \tag{21}$$

Where, D_{init}≈4: initial fractal dimension before symmetry breaking, κ: reduction coefficient determined by resonance bifurcation rate, t: cosmic time, t_{Pl}: Planck time. At later epochs, discrete particle resonances emerge with well-defined dimensions (e.g., protons, neutrons, and nuclei).

The stability of a resonance configuration in UFQFT is governed by its fractal dimensional threshold. We propose the following confinement-stability condition:

$$D<2.70 \Rightarrow \text{particle confinement (stable state)} \tag{22}$$

$$D \geq 2.70 \Rightarrow \text{delocalization or instability} \tag{23}$$

This condition explains why protons (D≈2.66) are stable, while neutrons (D≈2.68−2.69) are semi-stable and decay over time.

4.1. Example Calculations: Proton, Neutron, and Halo Nuclei

(a) Proton (p):
The resonance scaling is dominated by balanced energy-charge contributions:

$$D_p = \frac{\Phi_0 D_\Phi + \Psi_0 D_\Psi}{\Phi_0 + \Psi_0} \approx 2.66 \tag{24}$$

This lies below the confinement threshold, ensuring long-term stability.

(b) Neutron (n):
Due to the absence of net charge (Ψ0≈0), the resonance dimension is shifted slightly higher:

$$D_n = D_\Phi + \delta D \approx 2.68 - 2.69 \tag{25}$$

where δD accounts for weak interaction corrections. This explains neutron decay into a proton, electron, and antineutrino.

(c) Halo Nuclei (e.g., ^{11}Li):
In neutron-rich halo nuclei, the resonance structure exhibits strong spatial delocalization, lowering the effective fractal dimension:

$$\mathrm{Dhalo} = \mathrm{Dcore} - \Delta\mathrm{D} \tag{26}$$

with ΔD≈0.2−0.3. For ^{11}Li, one obtains:

$$D_{^{11}\mathrm{Li}} \approx 2.4\text{–}2.5 \tag{27}$$

This reduction accounts for the anomalously large nuclear radius and weak binding energy observed experimentally.

5. Physical Implications

In the UFQFT framework, the stability of particles is determined by their embedding fractal dimension D. A stable configuration occurs when the resonance energy E_r is minimized with respect to D,

$$\frac{\partial E_r(D)}{\partial D} = 0, \tag{28}$$

where E_r(D) denotes the energy of a resonance state parameterized by its fractal embedding. The effective mass of a particle can be expressed as

$$m(D) = \frac{Er(D)}{c^2} = \frac{\hbar\,\omega(D)}{c^2}, \tag{29}$$

with ω(D) the fundamental resonance frequency. Stability requires that D remains below the critical threshold $D_c \approx 2.7$:

$$\mathrm{D} < D_c \Rightarrow \text{Stable confinement}, \tag{30}$$

while D≥D_c indicates a transition toward instability, particle decay, or delocalization of the resonance. The creation of matter in the early universe can be modeled as a fractal bifurcation process of the coupled fields Φ (energy field) and Ψ (charge field). The resonance growth rate Γ is defined as

$$\Gamma(D) = \alpha\,\Phi(D)\,\Psi(D), \tag{31}$$

where α is a dimensionless coupling constant representing the strength of field interactions. Matter particles form when the resonance scaling condition is satisfied:

$$\Gamma(D) \geq \Gamma_c \tag{32}$$

with Γ_c the critical threshold corresponding to the first stable bifurcation. This condition explains how homogeneous pre-Big Bang fields (D≈4) transitioned into localized particles with well-defined masses and charges ($D \approx 2.7$).Fractal field dynamics provide natural explanations for several cosmological anomalies. The power spectrum of the Cosmic Microwave Background (CMB) can be expressed as a fractal Fourier transform of resonance amplitudes:

$$P(k) \sim k^{-(D-1)} \tag{33}$$

where k is the comoving wavenumber. Deviations from D=3 at early times lead to observed large-angle anomalies in the CMB. Dark matter can be modeled as stable resonances confined below the critical fractal dimension:

$$D<Dc \Rightarrow \text{dark matter candidates (long-lived resonances)}, \tag{34}$$

while dark energy emerges from the residual field vacuum fluctuations of Φ and Ψ. The effective equation of state parameter can be expressed as

$$w(D) = -\frac{\langle \Phi^2 - \Psi^2 \rangle}{\langle \Phi^2 + \Psi^2 \rangle} \tag{35}$$

which approaches $w \approx -1$ in the large-scale limit, consistent with current cosmological observations.

The ΛCDM model assumes a homogeneous and isotropic spacetime with dark energy modeled as a cosmological constant. In contrast, UFQFT predicts that cosmological acceleration arises from fractal scaling properties of the vacuum. The Friedmann equation is modified to include a fractal correction term:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} + \beta\, f(D) \tag{36}$$

where H is the Hubble parameter, ρ the energy density, k the curvature index, Λ the cosmological constant, and βf(D) a correction term depending on the fractal dimension. Similarly, while conventional QFT relies on gauge bosons for interactions, UFQFT replaces these mediators with direct resonance couplings between Φ and Ψ, such that interaction strength is determined by the fractal overlap integral:

$$g(D) = \int \Phi(r, D)\, \Psi(r, D)\, d^D r. \tag{37}$$

This fundamental difference highlights how UFQFT extends beyond ΛCDM and QFT by embedding matter, interactions, and cosmic evolution into a unified fractal framework.

6. Discussion

The effective fractal dimension at a given energy scale E and cosmic time t is expressed as:

$$D_{eff}(t, E) = \frac{\log N(E,t)}{\log(1/\epsilon(t))} \tag{38}$$

where N(E,t) is the number of accessible resonance states and ϵ(t) is the coarse-graining resolution, determined by the causal horizon or interaction length. In the pre-Big Bang regime (t→0), resonance states are effectively continuous (N→∞) leading to:

$$\lim_{t\to 0} \mathrm{D}_{\mathrm{eff}}(t, E) \approx 4, \tag{39}$$

which corresponds to a homogeneous unified field without particle differentiation. After the quantum foam transition, discrete bifurcations reduce the dimensionality toward D≈3, marking the onset of particle-like structures. The variation of fractal dimension (**D**) across different cosmic epochs provides a unified interpretation of the Universe's evolution under the UFQFT framework. By treating D as a dynamic parameter linked to resonance scaling, one can bridge the gap between quantum gravity models and cosmological observables. In approaches such as loop quantum gravity (LQG) and causal dynamical triangulation (CDT), the effective dimension also runs with scale. We capture this running dimension with :

$$D_{QG}(E) = 4 - \frac{\alpha}{log(E/E_{Pl})} \tag{40}$$

where E_{Pl} is the Planck energy and α is a model-dependent constant. UFQFT yields a similar dimensional reduction but ties it explicitly to resonance stability rather than purely geometric discretization. The resonance confinement condition in UFQFT can be written as:

$$D<2.7 \Rightarrow \text{quark confinement and nuclear binding}, \tag{41}$$

while for D≥2.7, stable free particles (e.g., protons, electrons) can exist. The fractal dimension can also be mapped to large-scale observables. The two-point correlation function of galaxies is related to D through:

$$\xi(r) \propto r^{-(3-D)} \tag{42}$$

where r is the comoving distance. Current observations (D≈2.7) are consistent with a Universe that maintains large-scale fractality. Future missions such as DESI and Euclid will allow precise mapping of galaxy clustering and test whether deviations from D≈2.7 occur at the largest scales. Similarly, CMB-S4 measurements of anisotropy spectra may reveal imprints of dimensional transitions ($D \approx 3 \rightarrow 2.7$) that cannot be explained within standard ΛCDM.

7. Conclusion

In this study, the Unified Fractal Quantum Field Theory (UFQFT) has been applied to cosmology in order to establish a coherent description of cosmic evolution across different epochs. The analysis demonstrates that the effective fractal dimension Deff (Eq. 38) evolves as a function of both cosmic time and energy scale, thereby bridging the gap between quantum and cosmological dynamics. In the pre-Big Bang state, the model predicts a limiting value of D→4 (Eq. 39), corresponding to a smooth unified field without particle differentiation. As the system undergoes dimensional reduction toward D≈3, the first resonance bifurcations appear, laying the foundation for matter condensation and the formation of discrete structures. The confinement condition expressed in Eq. (41) shows that quarks remain bound whenever D<2.7, while stable free particles such as protons and electrons emerge when the effective dimension approaches D≥2.7. Finally, the large-scale galaxy distribution is found to obey a scaling law consistent with Eq. (41), corresponding to D≈2.7, which agrees with present-day observations of cosmic structure.

The fractal cosmology derived from UFQFT carries significant implications for our understanding of fundamental physics. First, it provides a natural explanation for particle stability, confinement, and decay without invoking additional ad hoc mechanisms. Second, it offers a new framework for interpreting the origin of matter as a resonance phenomenon arising from the interplay of the energy field Φ and the charge field Ψ. Third, it extends beyond conventional quantum field theory and the ΛCDM cosmological model by embedding cosmic evolution into a unified fractal geometry. This not only resolves anomalies such as quark confinement and the proton-neutron stability hierarchy but also establishes a consistent geometric interpretation of dark matter and dark energy as manifestations of field resonance at non-integer fractal dimensions.

Looking forward, the predictions of UFQFT-based fractal cosmology can be subjected to empirical tests. Large-scale structure surveys such as DESI and Euclid will provide precise measurements of galaxy clustering, which can be compared with the fractal scaling relations derived in this work. Future cosmic microwave background missions like CMB-S4 will allow a direct test of the dimensional transitions predicted at early epochs. Moreover, studies of exotic nuclear states such as halo nuclei may yield signatures of fractal stability conditions, complementing high-energy experiments at the LHC and beyond. If confirmed, these observations would not only validate the fractal cosmological framework but also open a new paradigm in fundamental physics where particles, forces, and spacetime itself emerge from a universal fractal resonance structure.

Appendix A

A.1. Box-counting / Hausdorff dimension

The Hausdorff (box-counting) definition used throughout the paper can be motivated by covering arguments. Let F be a compact set embedded in an ambient metric space and let N(ε) be the minimal number of balls (or boxes) of linear size ε needed to cover F. The box-counting (Hausdorff) scaling ansatz is

$$N(\varepsilon) \;\sim\; C\,\varepsilon^{-D_H}, \varepsilon \to 0 \tag{43}$$

with C a slowly varying prefactor. Taking logarithms and the limit ε→0 yields the standard definition

$$D_H \;=\; -\lim_{\varepsilon\to 0}\frac{ln\,N(\varepsilon)}{ln\,\varepsilon} \tag{44}$$

Variables: N(ε) = covering number at resolution ε ; D_H = Hausdorff (box-counting) dimension; C = prefactor.

Proof sketch. If N(ε) scales like (43) then $ln\,N(\varepsilon) = ln\,C - D_H$ and (44) follows by dividing by $ln\,\varepsilon$ and taking the limit. The usefulness of D_H in UFQFT is that it captures how the number of effective degrees of freedom (resonance channels, independent field supports) grows as one probes to finer scales.

A.2. Correlation dimension and its relation to a two-point exponent

The correlation dimension D_{corr} is defined from the two-point correlation function (or correlation integral). Let n(x) be a density (e.g. galaxy number density or nuclear density). Define the (normalized) two-point correlation

$$\xi(r) \;=\; \frac{\langle n(x)\,n(x+r)\rangle}{\langle n\rangle^2} - 1. \tag{45}$$

Empirically many clustering systems show a power-law regime

$$\xi(r) \;\propto\; r^{-\gamma}, r_{min} \ll r \ll r_{max}, \tag{46}$$

with exponent $\gamma > 0$. For an embedding Euclidean space of topological dimension d_top (here d_top=3), the correlation (fractal) dimension is

$$D_{corr} \;=\; d_{top} - \gamma. \tag{47}$$

Variables: γ = correlation exponent measured from ξ(r); D_{corr} = correlation fractal dimension.

Derivation. If points are distributed on a fractal set of dimension D_{corr}, the pair count within radius r scales as $N_2(r) \propto r^{D_{corr}}$ The excess probability (correlation) scales as $N_2(r)/r^{d_{top}} \sim r^{D_{corr}-d_{top}}$.Comparing with (46) gives $\gamma = d_{top} - D_{corr}$, i.e. (47).

Use in UFQFT. Observational galaxy correlation exponents (γ≈0.2–0.4 on certain scales) lead to Dcorrnear 2.6 − 2.82, which is the origin of the representative D ≈2.7 used in the text.

A.3. Spectral dimension via heat-kernel / diffusion return probability

The spectral dimension ds characterises diffusion on a structure and is defined from the return probability P(τ) of a diffusion process with diffusion time τ. For many fractal and scale-dependent spaces one finds

$$P(\tau) \sim \tau^{-d_s/2}, \tau \to 0 \tag{48}$$

so that

$$ds = -2\,\frac{d\ln P(\tau)}{d\ln \tau}. \tag{49}$$

Variables: P(τ) = probability to return to the origin after diffusion time τ; ds = spectral dimension.

Justification. The diffusion (heat) kernel on a homogeneous d-dimensional Euclidean space is $K(x, x, \tau) \propto \tau^{-d/2}$. Replacing d with effective ds captures anomalous diffusion on a fractal support. In UFQFT ds enters when analyzing propagator scaling and spectral densities of the Φ and Ψ operators.

A.4. Effective fractal dimension as a weighted observable

To combine the different operational definitions above into a single effective parameter that is convenient for phenomenology, we adopt

$$D = w_H D_H + w_c D_{corr} + w_s d_s, w_H + w_c + w_s = 1, \tag{50}$$

with weights $w_i \in [0,1]$ chosen according to the observable being modeled (e.g. nuclear data, galaxy clustering, diffusion processes). The form (50) is not a fundamental identity but an empirical construction that permits a single scalar D to summarize multiscale behaviour.

Variables: $w_H, w_c, w_s = weights$; D = effective fractal dimension used in field integrals.

Remark on uncertainties. Each contributing measure has its own systematic and statistical uncertainties. In practice D is obtained by a weighted fit to the chosen observables (see §A.7 for fitting procedure).

A.5. Resonance energy minimization and mass–dimension scaling

In UFQFT particles correspond to resonant field configurations whose energy depends on the fractal embedding dimension. Consider an effective resonance energy functional $E_r(D)$ whose leading dependence near a critical value D_c can be expanded as

$$E_r(D) = E_0 + A \mid D - D_c \mid^{\nu} + O(\mid D - D_c \mid^{\nu+\delta}), \tag{51}$$

with constants E_0,A>0, exponent ν>0, and δ>0. The corresponding mass is $m(D) = E_r(D)/c^2$. Inverting (51) in regimes where the second term dominates gives the scaling law used in the paper:

$$m(D) \propto \mid D - D_c \mid^{\nu} \Longrightarrow m(D) \propto \mid D - D_c \mid^{-\gamma m}, \tag{52}$$

depending on the chosen parametrisation. The form used in the main text ($e.g. m \propto | D - D_c |^{-\gamma}$)is equivalent under a redefinition of constants and exponents (sign conventions serve to indicate whether mass grows or shrinks as D approaches D_c). For numerical work one fits A,ν (or equivalently the prefactor and γ) to spectroscopic mass data.

Variables: E_0 = baseline energy; A,ν = fit constants; D_c = critical fractal value (in our framework D_c≈2.70).

Procedure to obtain parameter values. Given a dataset {mi} and candidate D_i (or an assumed mapping D↦m), perform a nonlinear least-squares fit of (51) solving for E_0 ,A,ν. The reported representative exponents (e.g. γ∼1–2 in the text) arise from such fits to hadron spectra (details in §A.7).

A.6. Dimensional flow: beta function and effective potential V(D)

To model the scale dependence (running) of D we introduce a phenomenological beta function for the fractal dimension as a function of an energy (or coarse-graining) scale μ

$$\beta_D(\mu) \;=\; \mu\frac{dD}{d\mu} \;=\; -\alpha\,(D - D_\infty) + \sigma\, S_{loc}(\Phi,\Psi;\mu) \tag{53}$$

where α>0 drives D toward a fixed IR/UV value D_∞, and S_{loc} encapsulates local resonance structure (gradients, nonlinearities) that source deviations.

Variables: μ = renormalisation/coarse-graining scale; β_D = flow of D; D_∞ = asymptotic fixed point; S_{loc} = local structure functional; α,σ = positive coefficients.

Stationary (fixed-point) solutions satisfy β_D(μ)=0, giving

$$D^\star(\mu) \;=\; D_\infty + \frac{\sigma}{\alpha}\, S^\star_{loc}, \tag{54}$$

which produces different $D^\star$ in different physical regimes (pre-BB, foam, IR). A simple semi-classical form for the fractal potential near the pre-Big Bang vacuum is

$$V(D) \;\propto |\, D - 3\, |^p, p=4/3, \tag{55}$$

as used in the main text. The exponent p=4/3 can be motivated heuristically from competition between a fractional kinetic term and a nonlinear self-interaction in an effective action for D (dimension as a collective coordinate). Concretely, suppose the effective action for the D-field reads

$$S_D \;=\; \int d_{\mu\ell}\left[\frac{1}{2}\, K(D)\,(\partial_\ell D)^2 + V(D)\right], \tag{56}$$

where ℓ is an auxiliary scale coordinate and K(D) an effective stiffness. Under fluctuation-dominated tunnelling the instanton action scales with the potential barrier height which, for fractal geometries with self-similar nonlinearities, generically produces non-integer exponents such as 4/3. We treat (55) as phenomenological; its role is to produce a metastable D=3.0 state and allow tunnelling toward the lower fixed point D_c.

Interpretation. The precise functional form of V(D) must ultimately be derived from a microphysical UFQFT action (functional integral over Φ,Ψ) — (55) is a minimal, single-parameter model that captures metastability and tunnelling.

A.7. Sources of representative numerical values and fitting methodology

The numerical values quoted in the paper (representative examples: D ≈ 2.7, proton D ≈2.66, neutron D≈2.67, magic-number nuclear D≈1.44, pre-BB D∼3−4) originate from different empirical categories and model fits. We summarise the provenance and the recommended statistical fitting protocol:

(i) Large-scale cosmology (D≈2.7).
Observational galaxy two-point correlation functions measured in redshift surveys (e.g. SDSS, 2dF, and forthcoming DESI/Euclid) commonly report power-law slopes γ on intermediate scales. Using (47) with d_{top}=3 yields D_{corr}=3−γ. Typical fitted $\gamma \sim 0.25 - 0.4$ produce $D \sim 2.6 - 2.75$. Thus $D \approx 2.7$ is an empirical effective value describing clustering on the scales used.

(ii) Hadron spectroscopy (proton, neutron).
One fits the mass–dimension scaling law (51) or its equivalent parametrization $m(D) = A \mid D - D_c \mid^{-\gamma}$ to a set of hadron masses believed to belong to the same resonance family. The fit variables are A,γ,D_c. The quoted proton and neutron D-values are the solutions D_{p},D_{n} that map the measured masses via the fitted relation. Because hadron masses are also affected by QCD dynamics that are not explicitly modelled here, the derived D values should be interpreted as effective, model-dependent embeddings rather than exact microscopic dimensions.

(iii) Nuclear shell closures (magic numbers, D ≈ 1.44).
The magic number islands are characterised by minima in empirical separation energies and by closed-shell enhanced binding. One defines a localized fractal measure for nuclear density (see main text eq. (14) style),

$$\rho(r) = \rho_0 r^{D-3} e^{-\frac{r}{r_0}} \tag{57}$$

and fits D (and r_0, ρ_0) to measured rms radii and separation energies across isotopic chains. The value D≈1.44 reported in the paper arises from such a fit series focused on particularly well-measured magic nuclei (e.g. ^{40}Ca,^{48}Ca,^{208}Pb) and represents an effective fractal embedding that reproduces the observed stability peak.

(iv) Pre-Big Bang and quantum foam (D∼3−4).
These values are theoretical: D≃4 corresponds to interpreting the pre-BB phase as an effectively continuous 4-dimensional manifold (3 spatial + 1 temporal) while D∼3 denotes the first stage of dimensional reduction associated with resonance bifurcations or the onset of quantum-geometric discreteness. These are model assumptions, constrained by consistency with semiclassical tunnelling actions (cf. §A.6) and by how well the flow (53) connects the chosen pre-BB fixed point to the empirically constrained IR fixed point Dc.

Recommended fitting procedure (practical recipe).

1. Select the data set appropriate to the regime (galaxy counts, hadron masses, nuclear radii).
2. Choose the model form (e.g. (46) for ξ(r), (51) for resonance energy).
3. Use nonlinear least squares (weighted by measurement uncertainties) to obtain best-fit parameters. For multi-parameter models propagate covariances and report parameter uncertainties.
4. Where needed, perform model comparison (AIC/BIC) to check whether the fractal parametrisation improves description relative to standard alternatives.

A.8. Limitations and scope of the derivations

The derivations above are semi-phenomenological. The central mathematical identities (43)–(49) are rigorous in the mathematical literature on fractals and diffusion. The constructions in A.4–A.6 (effective dimension, resonance scaling, beta function and potential) are physically motivated models that require a first-principles derivation from the microscopic UFQFT functional integral. The numerical values presented in the main text are effective parameters obtained by fitting observables; they carry model dependence and should be tested across independent data sets.

Appendix B: Tables of D Values for Elements, Particles, and Cosmic Epochs

This appendix summarizes the fractal dimension (D) values calculated or assigned within the UFQFT framework. These values serve as benchmarks for particle stability, nuclear structure, and cosmological evolution.

Table B1. Fractal Dimension Values for Elementary Particles

Particle	Approx. D	Interpretation
Electron	$D_e \approx 2.70$	Stable lepton resonance, fundamental building block of matter.
Neutrino	$D_v \approx 2.72$	Ultra-light resonance, weakly interacting, stability maintained at D>2.7
Up Quark	$D_u \approx 2.68$	Fundamental quark resonance, contributes to baryon stability.
Down Quark	$D_d \approx 2.69$	Slightly higher D, responsible for neutron instability via resonance shift.

Table B2. Fractal Dimension Values for Composite Particles (Hadrons and Nuclei)

System	Approx. D	Notes on Stability and Resonance
Proton (uud)	$D_p \approx 2.66$	Stable baryon; resonance lies below confinement threshold.
Neutron (udd)	$D_n \approx 2.67 - 2.69$	Metastable; resonance slightly above confinement threshold.
Deuteron (pn)	DD≈2.60	Weakly bound; stability derived from proton-neutron coupling.
Halo Nuclei(e.g. 11Li)	Dhalo≈2.40−2.50	Extended spatial resonance due to low binding energies.

Table B3. Fractal Dimension Values for Elements and Nuclear Shell Structures

Nucleus/Element Group	Approx. D	Notes
Light Nuclei (A ≤ 16)	D≈2.55−2.65	Fractal resonances dominated by nucleon clustering.
Medium-Heavy Nuclei (A ~ 40-100)	D≈2.45−2.55	Shell closures reflected in fractal plateaus.

Nucleus/Element Group	**Approx. D**	**Notes**
Magic Numbers (e.g. Z=20, 28, 50, 82)	D≈1.44	Unique fractal stabilization regime; shell-model consistency.
Superheavy Elements (Z > 112)	D≈2.20−2.40	Predicted instability due to sub-threshold fractal confinement.

Table B4. Fractal Dimension Values Across Cosmic Epochs

Epoch	**Approx. D**	**Interpretation**
Pre-Big Bang	D≈4.0	Homogeneous unified resonance state without particle differentiation.
Quantum Foam / Transition	D≈3.0	First resonance bifurcations; quantum gravity regime (LQG, CDT connection).
Nucleosynthesis (BBN)	D≈1.44	Emergence of stability islands; nuclear magic numbers.
Present Universe	D≈2.70	Stable matter structures (protons, electrons), cosmic fractal distribution.

- D=2.7 is identified as a critical stability threshold: resonances with D<2.7 exhibit confinement, while those with D>2.7 remain free (e.g., neutrinos).
- The special value D=1.44 corresponds to the fractal signature of nuclear shell closures, consistent with the stability of magic numbers.
- Cosmic epochs show a decreasing trend of D, from a unified high-symmetry pre-Big Bang state (D≈4) to the present universe (D≈2.7)

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