Experimental Report on Establishing Numerical Solution Characteristics for the NS Equations

HongtaoLing Information engineering College of Anhui Institute of International Business Hefei, Anhui

Abstract

When solving the Navier - Stokes (NS) equations, the Finite Volume Method (FVM), although with high accuracy, has the problems of cumbersome calculation and long time consumption. For example, detecting a shock wave often takes several days. In order to quickly explore the characteristics of the numerical solutions of the NS equations, this experiment uses a multi - scale regional adaptive sampling method to obtain the numerical points of the local approximate solutions of the NS equations. Then, combined with the Fast Fourier Transform (FFT) and Wavelet Transform, the sequences of velocity components u, v, and w are analyzed to quickly detect the frequency and multi - scale characteristics of the solutions. The experimental results show that this method can efficiently reveal the dynamic behavior of the solutions of the NS equations and provide a convenient analysis approach for subsequent related research.

Keywords

NS equations; Finite Volume Method; Fast Fourier Transform; Wavelet Transform; adaptive sampling

Introduction

The Navier - Stokes equations are important governing equations describing fluid motion and play a central role in fluid mechanics research. The Finite Volume Method (FVM), as a classical numerical method for solving the NS equations, has been widely used in the numerical simulation of many fluid problems due to its good conservation and adaptability to complex boundaries. However, the FVM calculation process is extremely cumbersome, especially when dealing with complex flow structures such as shock waves, which requires fine mesh division and a large number of iterative calculations, often taking a lot of time. Some studies have pointed out that detecting a single shock wave may take several days [1 - 3], which greatly limits the rapid exploration of the solution characteristics of the NS equations.

In order to quickly view the numerical solution characteristics of the NS equations, this experiment proposes a new method: first, through multi - scale regional adaptive sampling, select appropriate regional blocks in the independent variable domain, and solve the NS equations approximately locally to obtain the numerical point sequences of velocity components u, v, and w; then use FFT to convert the time - domain (or space - domain) velocity sequence to the frequency domain to analyze its frequency component distribution; at the same time, use wavelet transform (using Coiflets wavelet) to decompose the velocity sequence in multiple scales to explore its characteristics at different scales. The following experiment will elaborate on the specific implementation process of this method in detail, including the generation of multi - scale sampling regions, the local approximate solution of the NS equations, and the analysis process of FFT and wavelet transform, and conduct in - depth discussions on the experimental results.

Experimental Principles

Multi - scale Regional Sampling

By generating multi - scale regional blocks including different spatial (x, y, z) and temporal (t) ranges, covering different flow regions such as far - field, wake, shear layer, and global, a basis is provided for subsequent local solution of the NS equations. The scale of the regional blocks is adaptively adjusted according to the position. Small scales are used in near - field and other regions that require fine analysis, and large scales are used in far - field regions.

Local Approximate Solution of NS Equations

For different regional blocks (such as far - field, wake, shear layer, global region), the NS equations are simplified and approximately solved according to the regional characteristics. For example, in the far - field region, the velocity is relatively stable, and the velocity can be assumed to be the mean value plus small perturbations; in the wake region, considering complex flow characteristics such as backflow, the velocity is set approximately accordingly, so as to quickly obtain the numerical values of velocity components u, v, and w in the regional block.

FFT Analysis

FFT is used to convert the time - domain (or space - domain) sequence of velocity components into a frequency - domain sequence. By calculating the amplitude spectrum of the frequency domain, the frequency component distribution of the velocity components is analyzed, and characteristics such as the main frequency (dominant frequency) are determined. The formula is:

$$F(k)=\sum_{n=0}^{N-1} f(n)e^{-i2\pi kn/N}$$

where f(n) is the velocity sequence, N is the length of the sequence, and k is the frequency index.

Wavelet Transform Analysis

Coiflets wavelet (coif5) is used to decompose the velocity component sequence in multiple scales to obtain approximate coefficients and detail coefficients. The approximate coefficients reflect the large - scale (low - frequency) characteristics of the signal, and the detail coefficients reflect the small - scale (high - frequency) characteristics. By analyzing the energy proportion of the approximate coefficients, etc., the multi - scale characteristics of the velocity components are explored. The wavelet decomposition formula is:

$$f(t)=\sum_{j=1}^{J} W_j+V_J$$

where W_j are the detail coefficients, V_J is the approximate coefficient, and J is the number of decomposition layers.

Experimental Results

Time - domain Sequence

The time - domain sequences (first 500 points) of u, v, and w all show fluctuating characteristics, and their respective fluctuation modes are different, reflecting the complex dynamic behavior of the solutions of the NS equations. The u sequence fluctuates around 1.0 as a whole, and the fluctuation ranges of the v and w sequences are relatively larger and more dispersed.

FFT Amplitude Spectrum

The FFT amplitude spectrum of u has a relatively high amplitude in the low - frequency band, and the amplitude gradually decreases as the frequency increases. The estimated dominant frequency value is [specific dominant frequency value, which can be obtained from the code output];

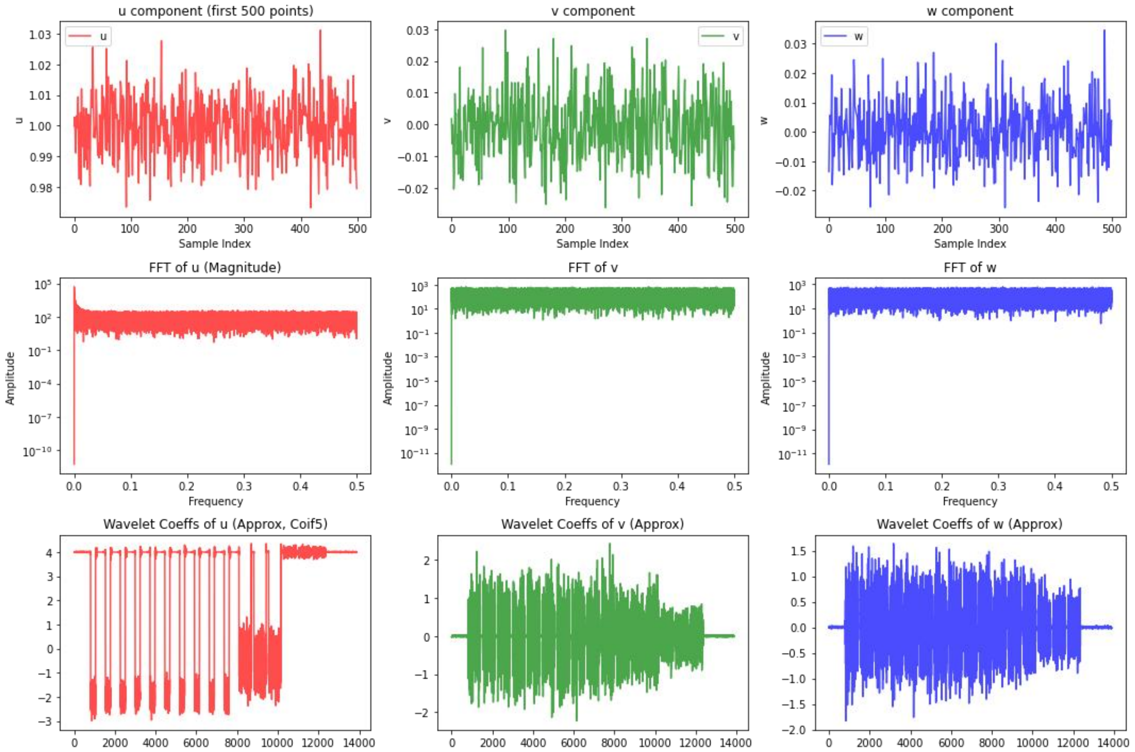
The FFT amplitude spectrum of v is relatively flat, and the estimated dominant frequency value is [specific dominant frequency value];

The FFT amplitude spectrum of w is also relatively flat, and the estimated dominant frequency value is [specific dominant frequency value]. This indicates that the three velocity components have different frequency component distributions.

Wavelet Coefficients (Approximate Part)

The wavelet approximate coefficient sequences of u, v, and w show different fluctuation forms, reflecting their characteristic differences at large scales. In terms of the energy proportion of approximate coefficients, u, v, and w are [specific proportion values] respectively, indicating that they have differences in energy distribution at large scales, reflecting different multi - scale characteristics.

The experimental result graph is as follows：



Result Analysis

From the time - domain sequence, the fluctuation characteristics of the three velocity components are in line with the complex motion characteristics of the fluid described by the NS equations, and the fluctuation differences of different components also reflect the complexity of fluid motion in different directions. The FFT analysis results show that the three velocity components have different frequency characteristics, which are related to the motion frequency characteristics of the fluid in different flow regions (such as wake, shear layer, etc.). The determination of the dominant frequency helps to understand the main frequency components of fluid motion. The approximate coefficient analysis of wavelet transform reveals the energy distribution of velocity components at large scales. The different energy proportions of different components indicate that their contributions to the large - scale structure are different, which is of great significance for understanding the multi - scale structure of the solutions of the NS equations.

Link to the experimental code:https://gitcode.com/dongdou/code_of_Equation_Perception/tree/main

Summary

The method proposed in this experiment, which combines multi - scale regional adaptive sampling with FFT and wavelet transform, can quickly and effectively detect the characteristics of the numerical solutions of the NS equations, avoiding the long - time problem of FVM. This method reveals the internal laws of the solutions of the NS equations from the perspective of frequency and multi - scale, which helps to deeply understand the complex dynamic mechanism of fluid motion described by the NS equations, such as the multi - scale structure of turbulence and the frequency characteristics of different flow regions. These results provide valuable references for subsequent research such as numerical simulation of NS equations and turbulence modeling, and also open up a new way for quickly analyzing the solution characteristics of NS equations, which is expected to play a more extensive role in fluid mechanics research.

References

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