

Experimental Report on Establishing Solution Space Fingerprints for Observed Numerical Solutions Without Equations

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Abstract

In scenarios where only observed numerical solutions are available and there are no explicit governing equations, direct equation fitting is prone to overfitting and is a cumbersome process. Therefore, this study proposes avoiding equation fitting. Instead, numerical solutions are collected, and their features are analyzed using methods such as solution space range estimation, Fast Fourier Transform (FFT), Discrete Cosine Transform (DCT), wavelet transform, and Hilbert-Huang Transform (HHT) to construct "fingerprint features" of the solutions. In the future, the recognition of unknown solutions can be achieved by checking the consistency between new solutions and these fingerprint features.

Keywords

No equation; Numerical solution; Solution space fingerprint; Feature analysis

Introduction

In the fields of science and engineering, there are often situations where only observed numerical solutions can be obtained, but the underlying governing equations are unavailable. Direct equation fitting is not only complex but also highly susceptible to overfitting due to factors such as data characteristics. For a long time, there has been a lack of efficient and universal processing methods for this problem. As mentioned in *Numerical Analysis* (3rd edition) [1], numerical fitting for complex systems requires consideration of multiple factors, making it quite challenging. *Fundamentals of Signal Processing Using MATLAB* [2] also indicates that signal analysis and feature extraction are difficult when no prior equations exist. In view of this, this paper proposes an approach of constructing solution space fingerprints by analyzing various features of numerical solutions instead of fitting equations, and conducts two experiments to verify this idea.

Experiment 1: Solution Space Range Estimation Experiment

Experimental Idea

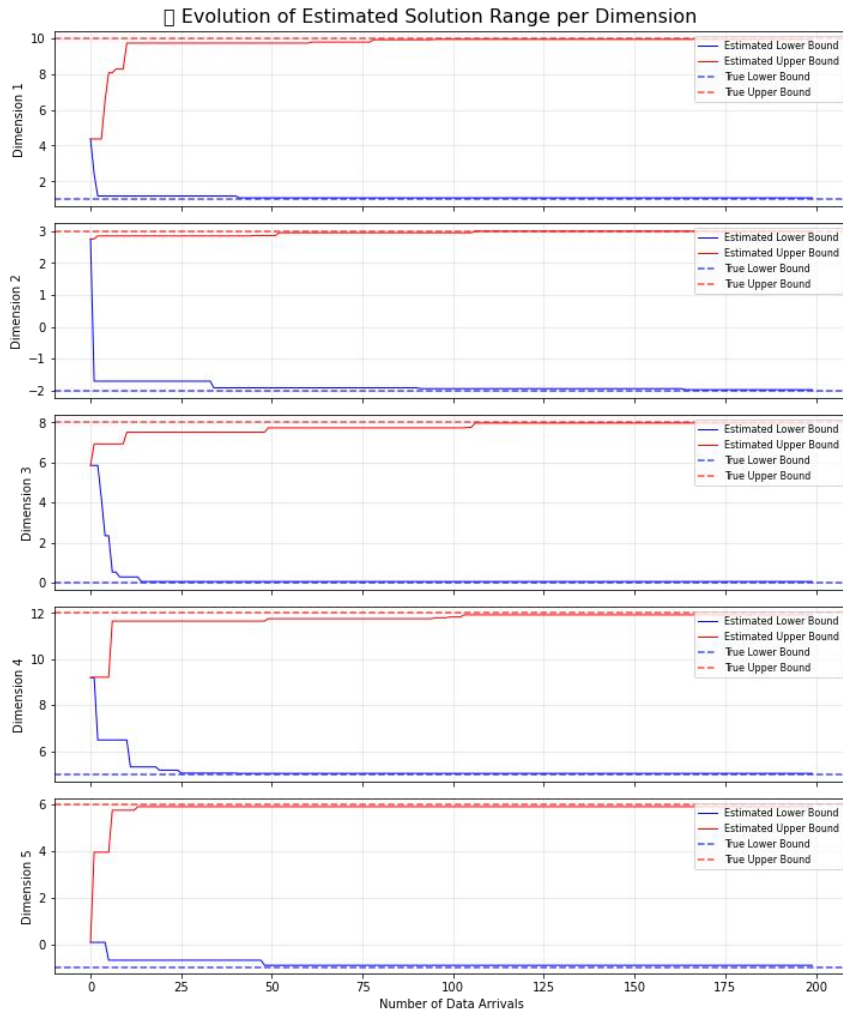
Facing phenomena with no equations and only scattered observed numerical solutions, the minimum update rule is adopted to dynamically explore the solution space range. Without relying on complex models or training, the estimates of the minimum and maximum values of solutions in each dimension are updated by continuously receiving new data, gradually approaching the true solution space range, thereby achieving a general understanding of the solutions.

Experimental Steps

1. **Set the true solution space range:** Predefine the true minimum (`true_min`) and maximum (`true_max`) values for each of the 5 dimensions to simulate the solution space of a real system.
2. **Simulate data generation:** Randomly generate 5-dimensional data points within the true solution space to simulate the observation of scattered numerical solutions of a real system.
3. **Implement the minimum update rule:** Initialize the estimation range, perform 200 rounds of data reception and updates. Each time a new data point is received, update the estimated minimum and maximum values for each dimension, and record the historical estimation ranges.

- Output results and calculate errors:** Output the final estimated solution space range, compare it with the true range, and calculate the total overestimation error.
- Visualization and conclusion summary:** Plot the evolution diagram of the estimated range for each dimension, and summarize the characteristics and applicable scenarios of the method.

The experimental result graph is as follows :



Result Analysis

After 200 updates, the estimated minimum and maximum values of each dimension are close to the true values, and the total overestimation error is small. This method is simple and robust, requires no training, and can be effectively applied to scenarios such as sparse data and high-dimensional systems. It gradually grasps the solution space range by continuously updating its understanding.

Experiment 2: Experiment on Extracting Solution Fingerprint Features via Multiple Transforms

Experimental Idea

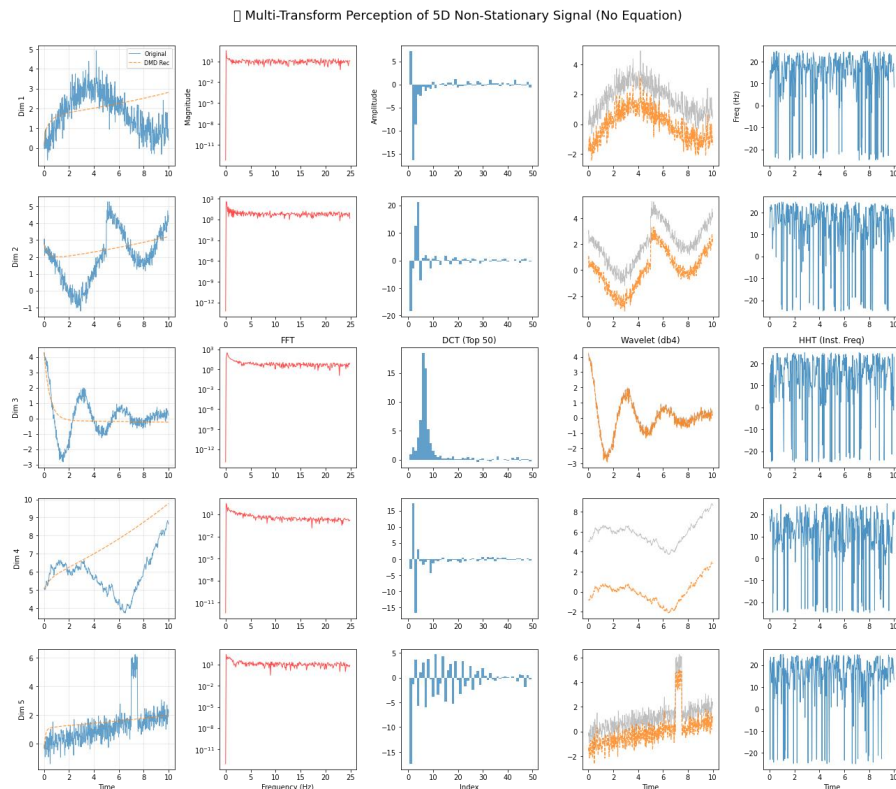
For 5-dimensional non-stationary signals without equations, the traditional approach of "analyzing after establishing equations first" is abandoned. Instead, a multi-transform perception method is adopted. Through various signal processing transforms including FFT, wavelet transform, DCT, Dynamic Mode Decomposition (DMD), and HHT, "fingerprint features" are extracted from the observed signals to

perceive and understand the behavioral characteristics of the signals. This verifies that unknown system signals can also be effectively analyzed without equations.

Experimental Steps

1. **Generate non-stationary random signals:** Design different non-stationary characteristics (such as trend + noise, sudden jump + oscillation, etc.) for each of the 5 dimensions, generate 5-dimensional non-stationary random signals, and simulate the observed signals of unknown systems.
2. **Apply multiple signal transforms:**
 - FFT: Perform FFT on each dimension of the signal after removing the mean to analyze the frequency components.
 - Wavelet transform: Use 4 types of wavelet bases to decompose and reconstruct each dimension of the signal after removing the mean, so as to analyze the time-frequency and transient features.
 - DCT: Perform DCT on each dimension of the signal after removing the mean, and extract the first 50 coefficients to analyze the energy concentration.
 - DMD: Perform DMD on the 5-dimensional signal to obtain the reconstructed signal and analyze the low-rank dynamic characteristics.
 - HHT: First, decompose the signal into Intrinsic Mode Functions (IMFs) using Empirical Mode Decomposition (EMD), then perform Hilbert transform on the IMFs to analyze the transient characteristics.
3. **Visualization and conclusion summary:** Create a multi-transform dashboard to display the results of each transform, and summarize the effectiveness and advantages of the method.

The experimental result graph is as follows:



Result Analysis

FFT successfully extracts the frequency components; all 4 types of wavelet bases can robustly capture transient features; DCT reflects the energy concentration; DMD reconstructs the dynamics with low rank; HHT reveals instantaneous frequency jumps (e.g., the jump of Dimension 2 at $t=5$). All transforms take a short time (less than 2 seconds without GPU and training), which verifies that "perception is superior to solving". A signal perception toolkit suitable for unknown systems is constructed.

Link to the experimental code:https://gitcode.com/dongdou/code_of_Equation_Perception/tree/main

Conclusion

The two experiments respectively verify the effectiveness of the idea of establishing solution space fingerprints by analyzing the solution space range and various transform features of solutions (instead of fitting equations) when there are no equations and only observed numerical solutions are available. This idea avoids the cumbersomeness and overfitting problem of equation fitting, provides a new method for the analysis of numerical solutions of unknown systems, and can be widely applied to scenarios of unknown systems such as sparse data and high-dimensional non-stationary signals.

References

- [1] Burden, R. L., & Faires, J. D. (2010). Numerical Analysis (9th ed.). Brooks/Cole, Cengage Learning.
- [2] McClellan, J. H., Schafer, R. W., & Yoder, M. A. (2003). Signal Processing First. Prentice Hall.