Numerical Simulation of Multi-Row Film Cooling on Curved Turbine Blade Surfaces

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Abstract

The performance of modern gas turbines is strongly influenced by turbine inlet temperature, where higher operating temperatures enhance thrust but also impose severe thermal loads on the blades. Since current materials cannot tolerate conditions exceeding approximately 1300 K, advanced cooling strategies are required to ensure durability. Film cooling, which introduces coolant jets through discrete surface holes to form a protective layer against the hot mainstream, remains one of the most widely used approaches. This study numerically investigates the effect of downstream rows of cooling jets on film cooling performance over a curved turbine blade surface. Using the Reynolds-Averaged Navier–Stokes (RANS) framework with the Shear Stress Transport (SST) turbulence model, simulations were conducted for one, two, and three rows of coolant injection. Validation against experimental data confirmed the accuracy of the framework. The results show that additional rows significantly enhance cooling effectiveness by generating a thicker and more uniform thermal barrier that extends protection farther downstream. Analysis of velocity and temperature contours, wall shear stress, and pressure coefficient distributions revealed that aerodynamic penalties remain limited, though local wall shear stresses increase due to intensified jet–mainstream interactions. These findings highlight the trade-off between cooling effectiveness and mechanical loading, underscoring the importance of optimized multi-row configurations. Overall, the study provides design-relevant insights into achieving improved thermal management without compromising aerodynamic integrity, contributing to the advancement of high-temperature turbine technologies.

1 Introduction

The efficiency of jet engines is largely determined by the turbine inlet temperature, a parameter that plays a central role in influencing performance. Although increasing the turbine inlet temperature enhances the thrust produced per unit mass flow, the thermal constraints of existing materials, which are unable to sustain temperatures beyond 1300 K, present a fundamental limitation. To address this issue, advancements in surface cooling techniques have been developed, providing an effective means to mitigate thermal stresses and accommodate higher operating temperatures without compromising material integrity.

The simultaneous implementation of film cooling systems and advancements in material technology offers a promising approach to increasing the turbine inlet temperature. Film cooling has been widely studied over the years, with much of the research focusing on flat plate geometries featuring injection through slots or rows of cylindrical holes. Goldstein Goldstein (1971) provides a comprehensive review of flat plate film cooling studies conducted up to 1971, which serves as a foundational reference in this field. Subsequent investigations have extended these efforts to more complex configurations, including leading-edge geometries, simplified pressure- or suction-side geometries, and specific airfoil designs Dai and Li (2014); Bogard and Thole (2006). The results of these studies demonstrate that surface curvature can significantly influence film cooling performance. Early investigations by Nicolas and Le Meur Nicolas and Le Meur (1974), Folayan and Whitelaw Folayan and Whitelaw (1976), and Mayle et al. Mayle et al. (1977) focused on generic curved walls with slot injection and revealed that, compared to flat surfaces, injection at low blowing rates $(M = 0.5)$ increases the adiabatic film cooling effectiveness on convex surfaces but decreases it on concave surfaces. At moderate blowing rates $(M = 1.0)$ effectiveness is higher on convex surfaces than on flat

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or concave surfaces, whereas at high blowing rates ($M = 2.0$) concave surfaces achieve the greatest effectiveness. More recent studies extended these findings to multi-row configurations on curved geometries. Jung and Hennecke Jung and Hennecke (2001) showed that convex-side effectiveness remains superior at low and moderate blowing ratios, although the influence of curvature becomes negligible at higher ratios, with diminished performance observed downstream of the second row. Zhou et al. Zhou et al. (2020) reported that reduced spacing between consecutive rows on the curved suction side improves downstream coverage, but also noted that curvature-driven film trajectory deflection toward the midspan can alter cooling effectiveness relative to the intended hole placement. Similarly, Shaker and Mujeebu Shaker et al. (2012) demonstrated that increasing the number of rows on curved blades enhances lateral spreading and reduces surface temperature, while Gao et al. Gao et al. (2007) observed that compound-angled multi-row hole designs provide more uniform suction-side coverage despite the presence of secondary vortices. Together, these studies confirm that curvature remains a critical factor in determining film cooling effectiveness, even when multi-row and staggered hole configurations are employed.

Generally, the influence of curvature appears to be more pronounced on the convex (suction) side. In addition to their measurements of film cooling effectiveness, Mayle et al. Mayle et al. (1977) provided boundary-layer velocity and temperature data, attributing the observed differences to curvature-induced changes in the production, diffusion, and dissipation of Reynolds stresses as well as in the turbulent heat flux. Subsequent research confirmed that curvature modifies near-wall turbulence dynamics, thereby altering the spreading and attachment of coolant films on convex versus concave surfaces. Most early investigations into curvature effects were limited to single-row configurations. Ito et al. Ito et al. (1978) compared the cooling effectiveness on the pressure and suction sides of an airfoil with flat plate data, proposing an analytical criterion to predict favorable coolant momentum fluxes as a function of surface curvature. Kruse Kruse (1985), Schwarz and Goldstein Schwarz and Goldstein (1989), and Schwarz et al. Schwarz et al. (1991) further examined film cooling on convex, concave, and flat plates, showing that laterally averaged effectiveness increases with curvature (from $-1/R$ to $+1/R$), particularly at low blowing ratios, until film separation occurs on walls with strong curvature. More recent investigations extended these findings to multi-row and staggered arrangements on curved surfaces. For example, Jung and Hennecke Jung and Hennecke (2001) observed that curvature effects remain significant at low and moderate blowing ratios in two-row configurations, while Zhou et al. Zhou et al. (2020) demonstrated that reduced spacing between rows on the curved suction side enhances downstream effectiveness but is sensitive to curvature-driven trajectory deflection. Similarly, Shaker and Mujeebu Shaker et al. (2012) showed that increasing the number of rows on curved blades promotes lateral spreading and lowers surface temperature. These results collectively demonstrate that while early single-row studies captured the fundamental role of curvature, recent multi-row analyses reveal its continued importance under more engine-representative configurations.

Schwarz et al. Schwarz et al. (1991) observed that the lateral profiles of local effectiveness are significantly flatter on concave surfaces compared to convex surfaces. This behavior was attributed to enhanced lateral mixing caused by the instability of concave boundary-layer flow, which at higher blowing ratios can lead to blockage between adjacent jets and produce slot-like behavior in row-of-holes injection.

Further investigations by Lutum et al. Lutum et al. (2000, 2001) examined the influence of streamwise pressure gradients ($\delta p/\delta x$) on film cooling performance over convex curved surfaces, showing that the effect of pressure gradients is more pronounced on curved configurations than on flat plates. In addition, film injection through multiple rows has consistently been shown to improve coverage and downstream effectiveness. Early experimental work by Jabbari and Goldstein Jabbari and Goldstein (1978) and Martinez-Botas and Yuen Martinez-Botas and Yuen (2000) demonstrated that adding downstream rows enhances film cooling uniformity on flat plates. More recent studies extended this understanding to curved geometries: Jung and Hennecke Jung and Hennecke (2001) confirmed that multi-row configurations on convex and concave surfaces sustain higher effectiveness at low to moderate blowing ratios, while Zhou et al. Zhou et al. (2020) highlighted that optimized row spacing on curved suction sides can improve downstream protection despite curvature-driven trajectory deflection. Collectively, these results underscore that both curvature and row arrangement strongly influence film cooling behavior and should be considered together in turbine blade design.

Depending on the configuration used, multi-row injection can produce slot-like behavior, further enhancing film coverage and cooling effectiveness. Despite its significance and engine-relevant geometry, relatively few studies have addressed film injection through two or more rows of holes. Early work by Lander et al. Lander et al. (1972) reported that staggered double-row injection on turbine airfoils improves coverage in linear cascades, while Drost and Bölcs Drost and Bölcs (1998) showed that suction-side double-row arrangements achieve superior effectiveness compared to single-row cases due to enhanced coverage and delayed jet separation. Goldstein et al. Goldstein et al. (1982) further demonstrated that curvature significantly affects both single- and double-row injection on convex and concave surfaces, although the impact of curvature diminishes under staggered double-row configurations because of increased jet interactions.

More recent investigations have extended these findings with higher fidelity tools and more complex configurations. Yang et al. Yang et al. (2023) demonstrated experimentally and numerically that double-row discrete holes provide more uniform coolant coverage and suppress secondary flows, lowering aerodynamic losses. Hu and An Hu and An (2024) found that dispersed multi-row layouts outperform double-row configurations for film effectiveness, particularly with diffusion slot holes, while Shaker and Mujeebu Shaker et al. (2012) confirmed that increasing the number of rows on curved blades promotes lateral spreading and reduces surface temperature. Similarly, Giridhara Babu et al. Yepuri et al. (2015) showed that five strategically arranged rows at the leading edge significantly improve thermal protection, though the benefits diminish beyond an optimal blowing ratio. Collectively, these studies highlight that while multi-row injection and surface curvature both strongly influence film cooling, systematic analyses of their combined effects on turbine blades remain scarce.

The present study addresses this gap by numerically investigating the influence of adding two additional downstream rows of cooling jets on a convex turbine blade surface. Using validated RANS-SST simulations, the work quantifies the effect of single-, double-, and triple-row arrangements on film cooling effectiveness, coverage distribution, and aerodynamic performance. By explicitly coupling curvature effects with multi-row injection in an engine-representative configuration, this study provides new insight into the interplay between geometry and row arrangement, offering a more comprehensive understanding of film cooling strategies for advanced turbine designs.

2 Numerical Methodology

This study employs a numerical framework to analyze film cooling performance on curved turbine blade surfaces. The methodology integrates the Reynolds-averaged Navier–Stokes (RANS) equations with the Shear Stress Transport (SST) turbulence model, an approach widely adopted in film cooling research due to its balance between computational cost and predictive accuracy Li et al. (2015); Zhang et al. (2020); Dutta et al. (2022). The computational domain was designed to replicate engine-representative curved geometries with multi-row injection, enabling accurate capture of coolant–mainstream jet interactions and curvature-induced flow dynamics Chen et al. (2024); Li et al. (2021); Yang et al. (2023).

2.1 Governing Equations

The flow of Newtonian fluids in film cooling applications is governed by the Reynolds-averaged Navier–Stokes (RANS) equations, which express conservation of mass and momentum Li et al. (2015); Zhang et al. (2020). The continuity equation is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0, \tag{1}$$

and the momentum equations are:

$$\frac{\partial}{\partial t}(\rho u_j) + \frac{\partial}{\partial x_i}(\rho u_i u_j) = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_i}\left[(\mu + \mu_t)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \rho g_i, \tag{2}$$

where ρ is the density, u_i the velocity components, P the static pressure, μ the molecular viscosity, μ_t the turbulent eddy viscosity, and g_i the body force vector.

In the RANS formulation, instantaneous velocities are decomposed into mean and fluctuating components ($u_i = \overline{u_i} + u_i'$), introducing Reynolds stresses $-\rho\overline{u_i' u_j'}$ that require closure. Turbulence modeling is therefore essential to capture jet–crossflow mixing in film cooling flows, where accurate prediction of near-wall structures and lateral coolant spreading is critical Dutta et al. (2022); Chen et al. (2024).

2.2 Turbulence Modeling

The Shear Stress Transport (SST) k-ω model was employed because of its demonstrated ability to predict flows with strong adverse pressure gradients and separation, which are characteristic of film cooling on curved turbine surfaces Li et al. (2015); Yang et al. (2023); Hu and An (2024). The SST formulation blends the near-wall accuracy of the k-ω model with the free-stream robustness of the k-ϵ model Menter (1993, 1994). The transport equations for turbulent kinetic energy (k) and specific dissipation rate (ω) are:

$$\frac{\partial(\rho k)}{\partial t} + \nabla \cdot (\rho \mathbf{u} k) = P_k - \beta^* \rho k \omega + \nabla \cdot \left[(\mu + \sigma_k \mu_t) \nabla k \right], \tag{3}$$

$$\begin{aligned} \frac{\partial(\rho\omega)}{\partial t} + \nabla \cdot (\rho \mathbf{u} \omega) = \frac{\gamma}{\nu_t} P_k - \beta \rho \omega^2 \\ + \nabla \cdot \left[(\mu + \sigma_\omega \mu_t) \nabla \omega \right] \\ + 2(1 - F_1) \rho \sigma_{\omega 2} \frac{1}{\omega} \nabla k \cdot \nabla \omega, \end{aligned} \tag{4}$$

where P_k is the production of turbulent kinetic energy and F_1 is a blending function that ensures a smooth transition between the k-ω and k-ϵ limits. The turbulent eddy viscosity is obtained from

$$\mu_t = \frac{\rho k}{\omega}. \tag{5}$$

The SST model has been shown to give improved predictions of film cooling effectiveness relative to standard k-ϵ models, particularly in capturing near-wall jet interactions and lateral spreading of coolant Zhang et al. (2020); Jeong et al. (2024). While large-eddy simulation (LES) provides higher fidelity, its computational cost for turbine-representative geometries remains prohibitive; thus, the RANS SST model remains widely used in engineering-scale film cooling analyses Dutta et al. (2022); Li et al. (2015).

2.3 Computational Domain and Boundary Conditions

The computational domain represents the suction-side surface of a curved turbine blade with cylindrical film cooling holes. Three configurations were examined, consisting of one, two, and three rows of injection holes. The applied boundary conditions for these cases are illustrated in Fig. **??**, where the mainstream enters from the inlet boundary as a uniform velocity profile with prescribed turbulence intensity, a fixed static pressure is imposed at the outlet, and coolant injection is introduced through mass-flow inlets. The coolant mass flow was calibrated to achieve the target blowing ratio

$$M = \frac{\rho_c U_c}{\rho_\infty U_\infty}, \tag{6}$$

and density ratio

$$DR = \frac{\rho_c}{\rho_\infty}. \tag{7}$$

The jet total temperature T_{jet} was set below the mainstream temperature T_∞, and cooling performance was quantified using the adiabatic effectiveness,

$$\eta = \frac{T_\infty - T_{aw}}{T_\infty - T_{jet}}, \tag{8}$$

where T_{aw} is the adiabatic wall temperature obtained from the wall-adjacent fluid cell.

The computational grid was generated as a structured multi-block mesh with refinement near the cooling holes and the curved wall, as shown in Fig. 2(A). The first-layer spacing was selected to maintain $y^+ < 1$ across the cooled surface, ensuring compatibility with the low-Reynolds-number formulation of the SST model. Figure 2(B) presents the distribution of y^+ along the blade surface, confirming that the near-wall resolution lies within the viscous sublayer throughout.

All simulations were performed under incompressible, steady-state conditions with second-order discretization. Convergence was declared when residuals dropped below 10^{-5} and monitored values of film effectiveness and pressure coefficient became stable. This setup provides a consistent framework for assessing the influence of one, two, and three rows of coolant injection on film cooling performance along a curved turbine blade surface.

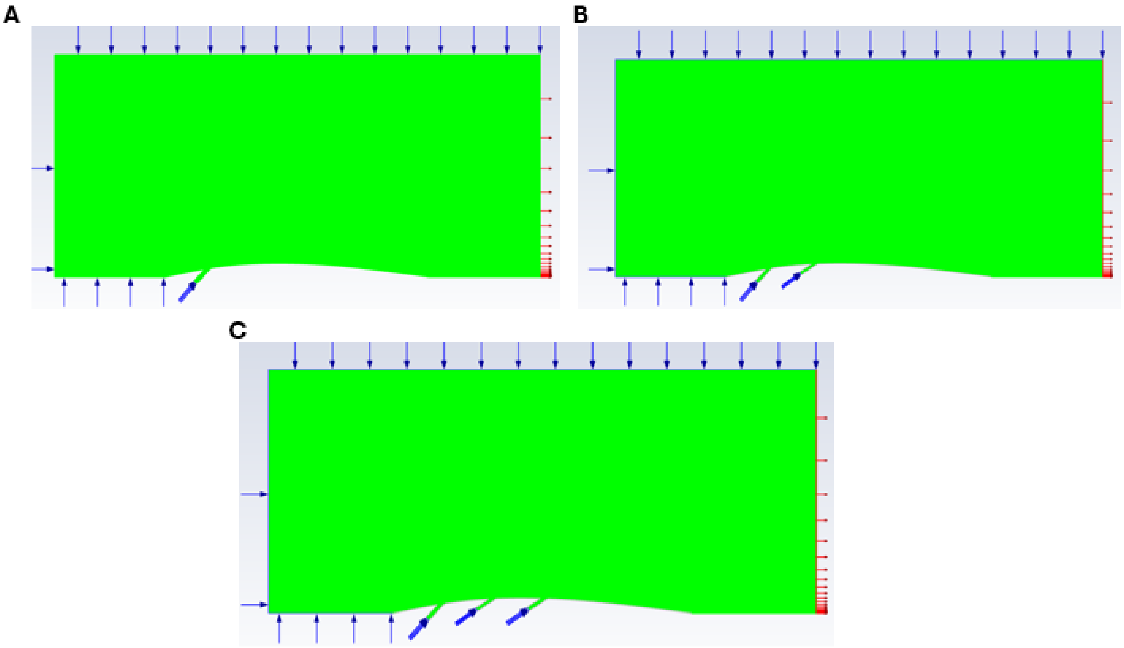

Figure 1: Boundary-condition setups for film cooling on a curved blade: (A) single-row injection, (B) double-row injection, and (C) triple-row injection.

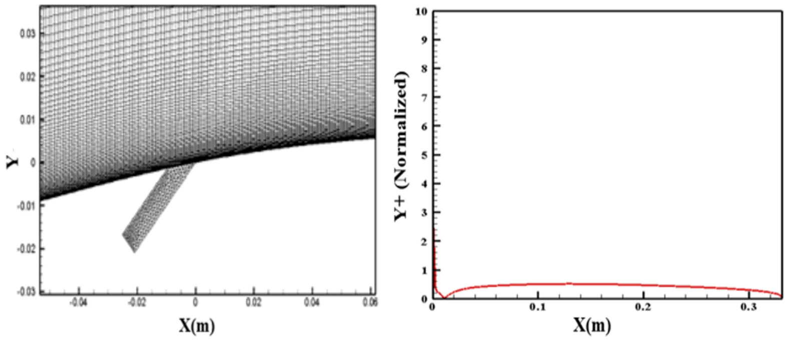

Figure 2: (A) Structured mesh near the blade surface and cooling hole; (B) wall y^+ distribution confirming $y^+ < 1$ across the cooled surface.

2.4 Validation

The numerical framework was validated against the experimental film cooling effectiveness data of Koch et al. Koc et al. (2006). Two aspects were examined: grid independence and turbulence model performance.

Figure 3 presents the grid independence study using meshes of 750,640 and 1,601,280 cells. The nearly identical effectiveness distributions confirm that the coarser grid is sufficient to capture the flow and thermal features while reducing computational cost.

Figure 4 compares predictions from the k–ω SST, k–ϵ RNG, and standard k–ϵ models against the experimental data, plotted using the normalized coordinate X^*. The SST model achieves the closest agreement, especially near the stagnation region and downstream of the jet where adverse pressure gradients dominate. By contrast, the k–ϵ models underpredict effectiveness in these regions, reflecting their limitations in handling shear stresses and flow separation.

These results confirm that the selected mesh resolution and the k–ω SST turbulence model provide a reliable and accurate basis for the present film cooling simulations.

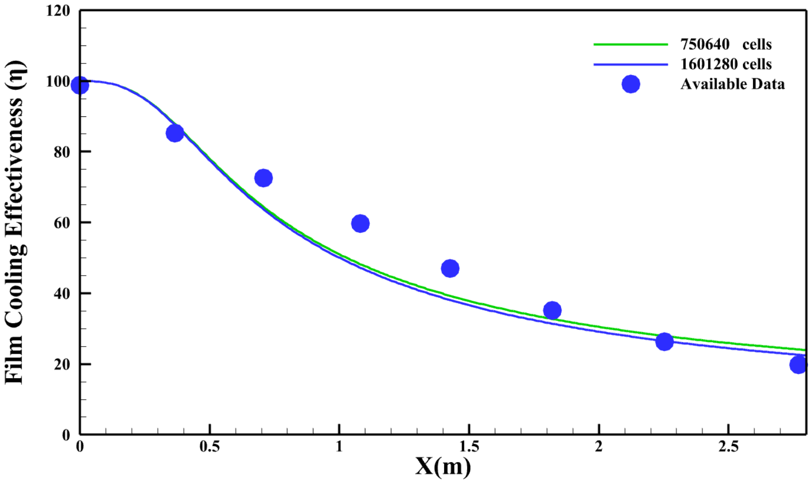

Figure 3: Grid independence study of film cooling effectiveness along the adiabatic blade surface for coarse (750,640 cells) and fine (1,601,280 cells) meshes, compared with experimental data Koc et al. (2006).

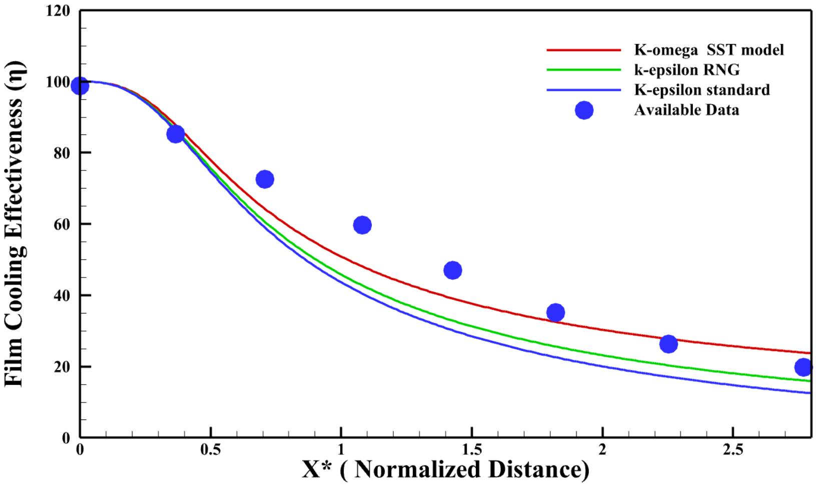

Figure 4: Turbulence model validation: comparison of film cooling effectiveness predicted by the k–ω SST, k–ϵ RNG, and standard k–ϵ models against experimental data Koc et al. (2006), plotted along the normalized distance X^*.

3 Results and Discussion

The cooling performance of the turbine blade was evaluated using the film cooling effectiveness coefficient, a dimensionless metric that quantifies the thermal shielding provided by the coolant jets. A value of unity corresponds to complete protection of the surface, whereas a value of zero represents no protection. The results show that increasing the number of injection rows systematically improves surface coverage, extending the cooling layer further downstream and delaying the thermal decay caused by jet–mainstream mixing. Nevertheless, the effectiveness decreases with distance from the injection holes, reflecting enhanced turbulent mixing and dissipation of jet momentum as the coolant penetrates into the crossflow.

The velocity contours in Fig. 5 highlight the progressive changes in flow structure as additional injection rows are introduced. With a single-row configuration, the jet penetrates locally but fails to sustain downstream momentum, leading to limited lateral coverage. The addition of a second row produces higher velocity magnitudes at successive injection sites, reinforcing the interaction between jets and the mainstream. The three-row configuration develops a stronger and more uniform coolant distribution, with overlapping velocity fields that reduce hot streaks and improve surface shielding. These trends confirm that multiple injection rows enhance jet-to-crossflow momentum transfer, a prerequisite for stable film attachment along curved surfaces.

Figure 6 presents the corresponding temperature contours, which clearly demonstrate the thickening of the coolant blanket with the addition of rows. In the single-row case, the thermal boundary layer is thin, and the mainstream rapidly entrains the coolant, exposing much of the surface to high temperatures. Two rows substantially increase coolant coverage, while three rows generate a continuous, layered cooling film that suppresses hot gas impingement over the majority of the suction surface. This extended thermal barrier reduces wall temperature gradients and lowers local heat fluxes, both of which are critical for blade durability. The observed improvement emphasizes the importance of row multiplicity in delaying coolant decay and sustaining effective thermal protection.

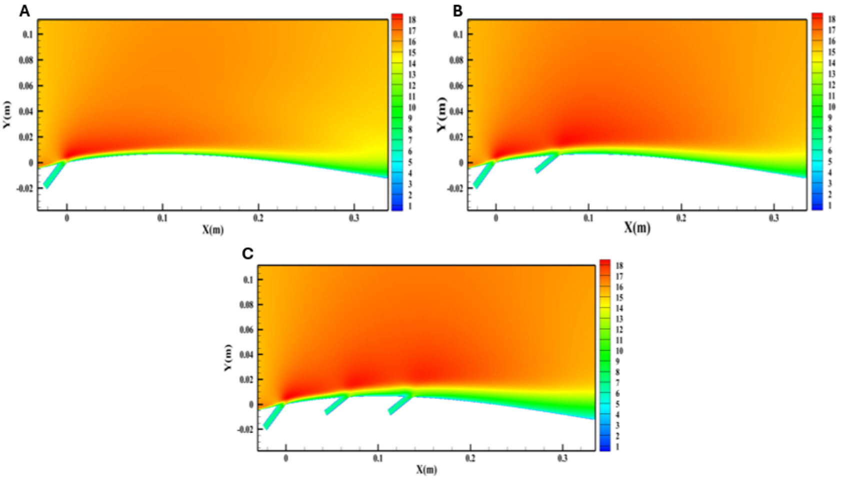

Figure 5: Velocity contours on the curved blade for (A) one row, (B) two rows, and (C) three rows of injection.

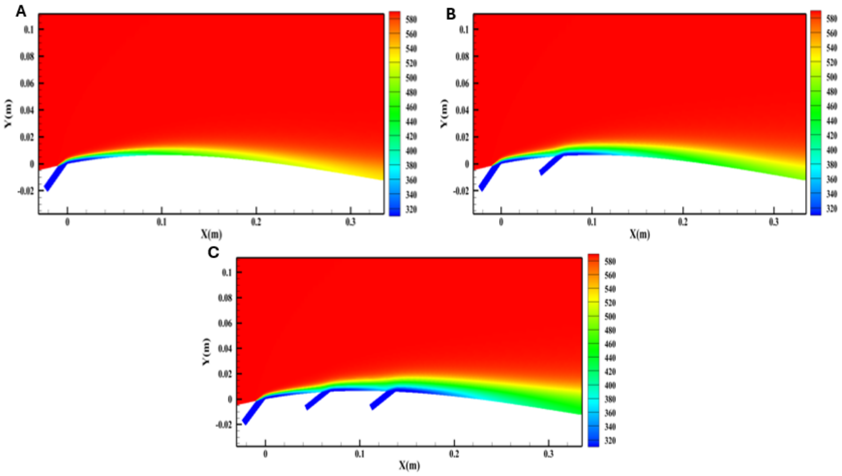

Figure 6: Temperature contours on the curved blade for (A) one row, (B) two rows, and (C) three rows of injection.

The wall shear stress distribution along the blade surface, shown in Fig. 7, increases significantly with the addition of coolant rows. The single-row case exhibits relatively smooth behavior, while the two-row configuration introduces moderate peaks associated with jet–mainstream interaction. In contrast, the three-row configuration produces sharp spikes at each injection location, highlighting intensified turbulence production and momentum exchange near the surface. These elevated stresses promote enhanced mixing and cooling layer attachment but also impose additional mechanical loads on the blade material. Such findings underline a trade-off between improved thermal protection and potential structural challenges, emphasizing the need to balance aerodynamic and thermal design requirements in multi-row cooling strategies.

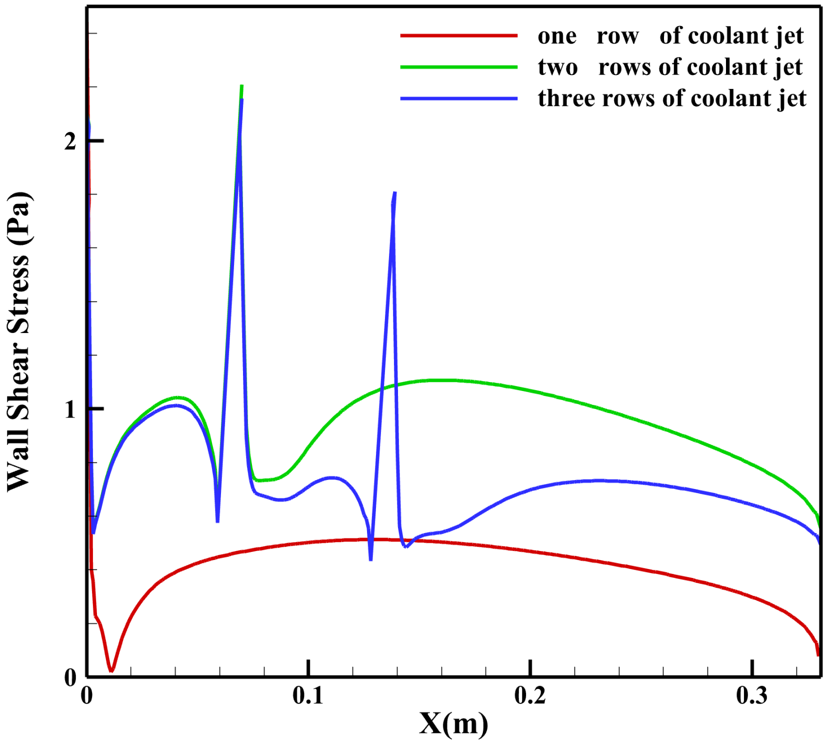

Figure 7: **WSS.** Wall shear stress distribution along the blade surface for one, two, and three rows of coolant injection. Peaks correspond to jet–mainstream interaction zones, where turbulence generation and momentum exchange are strongest.

The distribution of the pressure coefficient (C_p) along the blade surface is presented in Fig. 8. The results indicate that the influence of additional coolant rows is localized primarily near the injection sites. At these locations, sharp gradients are observed due to jet–mainstream interactions, while farther downstream the distributions converge and stabilize. Importantly, the trailing-edge region exhibits minimal sensitivity to the number of coolant rows, suggesting that the overall aerodynamic performance of the blade remains largely unaffected. These results confirm that film cooling design can be optimized for thermal performance without significant penalties in blade aerodynamics.

The film cooling effectiveness (η) for single-, double-, and triple-row injection cases is shown in Fig. 9. Increasing the number of coolant rows leads to a substantial rise in cooling effectiveness, with noticeable improvements in downstream surface coverage. The additional rows provide a more uniform and thicker coolant layer, which effectively insulates the blade surface from the high-temperature mainstream. As a result, effectiveness values remain elevated farther downstream, sustaining thermal protection even as mixing intensifies. This behavior highlights the importance of multi-row injection in extending coolant coverage and maintaining robust thermal barriers.

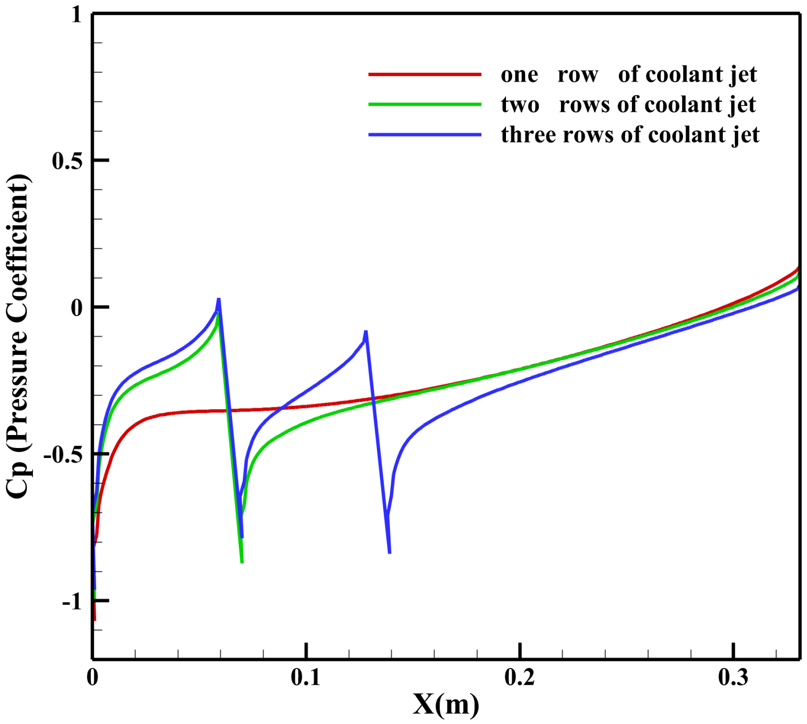

Figure 8: Pressure coefficient distribution along the blade surface for one, two, and three coolant rows. Disturbances are confined to jet injection locations, while the downstream pressure stabilizes across all cases.

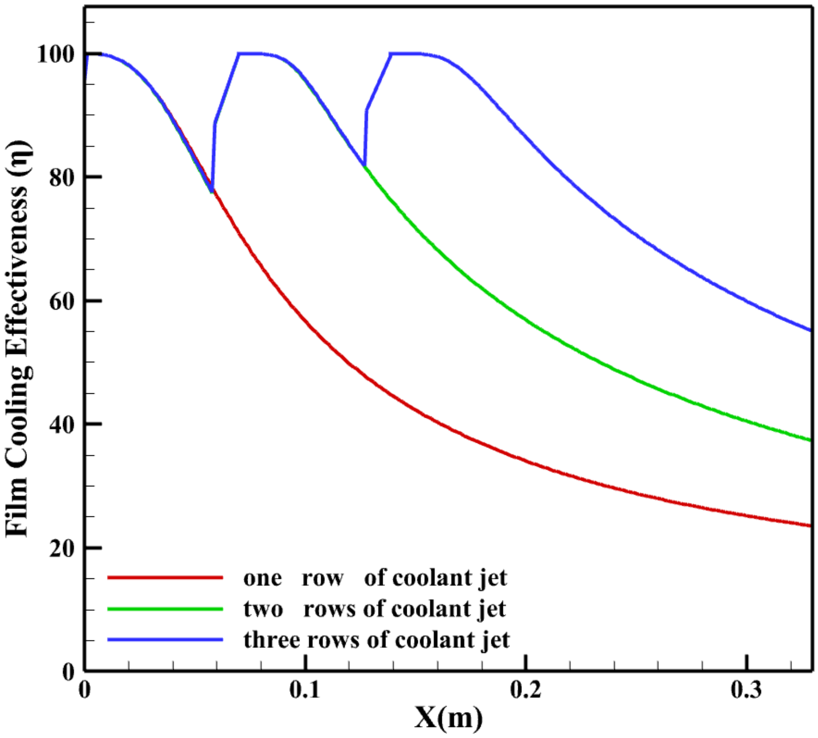

Figure 9: Film cooling effectiveness for one, two, and three coolant rows, showing improved downstream protection and extended coverage with additional injection rows.

In summary, the analysis demonstrates that increasing the number of coolant rows enhances film cooling effectiveness, reduces thermal loads on the blade, and maintains aerodynamic integrity. These findings underscore the importance of balancing jet arrangement, aerodynamic stability, and structural considerations when designing cooling systems for high-temperature turbine environments.

4 Conclusion

This study numerically investigated the influence of downstream rows of film cooling jets on the suction surface of curved turbine blades using the Reynolds-Averaged Navier–Stokes (RANS) framework with the Shear Stress Transport (SST) turbulence model. The results demonstrate that increasing the number of coolant rows substantially improves cooling effectiveness by generating a thicker and more uniform thermal barrier, thereby extending protection farther downstream. Importantly, this improved thermal shielding was achieved without introducing significant penalties to the overall aerodynamic performance, as evidenced by the pressure coefficient distributions.

Beyond confirming earlier findings on single- and double-row injection, this work provides new insights into the cumulative effect of triple-row configurations on curved blade geometries. The combined analysis of velocity and temperature fields, wall shear stress, and pressure coefficients highlights the complex balance between improved thermal protection and increased mechanical loading. These results contribute to the understanding of how multi-row cooling strategies can be optimized to enhance thermal durability while maintaining aerodynamic stability in high-temperature turbine environments.

The originality of this study lies in its systematic evaluation of multi-row injection on curved surfaces, providing design-relevant guidance for balancing film cooling effectiveness with structural integrity. By demonstrating that additional rows can sustain high cooling effectiveness over extended distances without major aerodynamic drawbacks, this research offers a pathway toward more efficient blade cooling configurations.

Future work should focus on experimental validation under engine-representative conditions, as well as higher-fidelity simulations such as Large-Eddy Simulation (LES) to capture transient effects and vortex dynamics. Further exploration of injection geometry, jet inclination, and density ratio will provide additional opportunities to refine cooling strategies and address mechanical trade-offs, enabling more robust turbine blade designs for advanced gas turbine applications.

References

Bogard, D. G. and Thole, K. A. (2006). Gas turbine film cooling. *Journal of propulsion and power*, 22(2):249–270.

Chen, Y., Wan, R., and Ke, Z. (2024). Comparative study of film cooling performance on curved walls with various hole configurations and blowing ratios. *Applied Thermal Engineering*, 238:122195.

Dai, P. and Li, S. X. (2014). Recent advances in the study of film cooling on the gas turbine blade. *Advanced Materials Research*, 971:143–147.

Drost, U. and Bölcs, A. (1998). Investigation of detailed film cooling effectiveness and heat transfer distributions on a gas turbine airfoil. In *Turbo Expo: Power for Land, Sea, and Air*, volume 78651, page V004T09A003. American Society of Mechanical Engineers.

Dutta, S., Kaur, I., and Singh, P. (2022). Review of film cooling in gas turbines with an emphasis on additive manufacturing-based design evolutions. *Energies*, 15(19):6968.

Folayan, C. and Whitelaw, J. (1976). The effectiveness of two-dimensional film-cooling over curved surfaces. *American Society of Mechanical Engineers and American Institute of Chemical Engineers*.

Gao, Z., Narzary, D. P., and Han, J.-C. (2007). Film-cooling on a gas turbine blade pressure side or suction side with compound angle shaped holes. In *Heat Transfer Summer Conference*, volume 42762, pages 569–579.

Goldstein, R., Kornblum, Y., and Eckert, E. R. (1982). Film cooling effectiveness on a turbine blade. *Israel Journal of Technology*, 20(4-5):193–200.

Goldstein, R. J. (1971). Film cooling. In *Advances in heat transfer*, volume 7, pages 321–379. Elsevier.

Hu, J.-J. and An, B.-T. (2024). Film-cooling effectiveness on a turbine blade platform with various hole shapes and layouts. *ASME Journal of Heat and Mass Transfer*, 146(11):113801.

Ito, S., Goldstein, R. J., and Eckert, E. R. (1978). Film cooling of a gas turbine blade.

Jabbari, M. and Goldstein, R. (1978). Adiabatic wall temperature and heat transfer downstream of injection through two rows of holes.

Jeong, J. Y., Jo, Y. R., Kang, M. S., Kang, Y. J., and Kwak, J. S. (2024). Film cooling effectiveness and flow structures of butterfly-shaped film cooling hole configuration. *Journal of Thermal Science and Engineering Applications*, 16(3):031009.

Jung, K. and Hennecke, D. (2001). Curvature effects on film cooling with injection through two rows of holes. In *Proc. RTO/AVT Symp*.

Koc, I., Parmaksızoglu, C., and Cakan, M. (2006). Numerical investigation of film cooling effectiveness on the curved surface. *Energy conversion and Management*, 47(9-10):1231–1246.

Kruse, H. (1985). Effects of hole geometry, wall curvature and pressure gradient on film cooling downstream of a single row. *In AGARD Heat Transfer and Cooling in Gas Turbines 13 p (SEE N86-29823 21-07*.

Lander, R. D., FISH, R. W., and SUO, M. (1972). External heat-transfer distribution on film cooled turbine vanes. *Journal of Aircraft*, 9(10):707–714.

Li, H., Zhou, Z., Xie, G., Xia, S., and Tao, Z. (2021). The cooling performance of multiple rows of film holes on the suction surface of a turbine blade under rotating conditions. *Applied Thermal Engineering*, 188:116125.

Li, S., Zhan, J., Gong, Y., and Hu, W. (2015). Numerical investigation of film cooling using rans and les. *Procedia Engineering*, 126:701–705.

Lutum, E., von Wolfersdorf, J., Semmler, K., Dittmar, J., and Weigand, B. (2001). An experimental investigation of film cooling on a convex surface subjected to favourable pressure gradient flow. *International journal of heat and mass transfer*, 44(5):939–951.

Lutum, E., von Wolfersdorf, J., Weigand, B., and Semmler, K. (2000). Film cooling on a convex surface with zero pressure gradient flow. *International Journal of Heat and Mass Transfer*, 43(16):2973–2987.

Martinez-Botas, R. and Yuen, C. (2000). Measurement of local heat transfer coefficient and film cooling effectiveness through discrete holes. In *Turbo Expo: Power for Land, Sea, and Air*, volume 78569, page V003T01A050. American Society of Mechanical Engineers.

Mayle, R. E., Kopper, F. C., Blair, M. F., and Bailey, D. A. (1977). Effect of streamline curvature on film cooling. *Journal of Engineering for Gas Turbines and Power*, DOI: `10.1115/1.3446255`.

Menter, F. (1993). Zonal two equation kw turbulence models for aerodynamic flows. In *23rd fluid dynamics, plasmadynamics, and lasers conference*, page 2906.

Menter, F. R. (1994). Two-equation eddy-viscosity turbulence models for engineering applications. *AIAA journal*, 32(8):1598–1605.

Nicolas, J. and Le Meur, A. (1974). Curvature effects on a turbine blade cooling film. In *Turbo Expo: Power for Land, Sea, and Air*, volume 79795, page V01BT02A073. American Society of Mechanical Engineers.

Schwarz, S. and Goldstein, R. (1989). The two-dimensional behavior of film cooling jets on concave surfaces.

Schwarz, S., Goldstein, R. J., and Eckert, E. R. (1991). The influence of curvature on film cooling performance.

Shaker, S., Abdullah, M., Mujeebu, M., Ahmad, K., and Abdullah, M. (2012). Study on the effect of number of film cooling rows on the thermal performance of gas turbine blade. *Journal of Thermal Science & Technology/Isı Bilimi ve Tekniği Dergisi*, 32(2).

Yang, X., Zhao, Q., Wu, H., and Feng, Z. (2023). Film cooling on turbine vane endwalls with different inlet cooling configurations: experimental and computational results. *Journal of Turbomachinery*, 145(10):101010.

Yepuri, G. B., Jesuraj, F., Batchu, S., and Venkataraman, K. (2015). Experimental investigation of adiabatic film cooling effectiveness over a circular fan and laidback fan shaped hole flat plate test models. In *Gas Turbine India Conference*, volume 57311, page V001T04A008. American Society of Mechanical Engineers.

Zhang, J., Zhang, S., et al. (2020). Recent advances in film cooling enhancement: A review. *Chinese Journal of Aeronautics*, 33(4):1119–1136.

Zhou, Z., Li, H., Xie, G., and You, R. (2020). Film cooling performance on turbine blade suction side with various film cooling hole arrangements. In *Turbo Expo: Power for Land*, *Sea*, *and Air*, volume 84171, page V07BT12A034. American Society of Mechanical Engineers.