Three-Mode Upside-Down Logic within Plithogenic Neutrosophic Set

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Abstract. In the real world, many reversal phenomena occur, such as statements once considered false later becoming recognized as true. *Upside-Down Logic* provides a framework to formalize such reversals by inverting truth and falsity through contextual transformations, thus capturing ambiguity and dynamic changes in reasoning processes.

A *Plithogenic Set* models elements via attribute-based membership and contradiction functions, extending the established frameworks of fuzzy, intuitionistic, and neutrosophic sets. *De-Plithogenication* is the process of systematically neutralizing contradictions in plithogenic structures, resetting or transforming attribute relationships into consistent, contradiction-free states. A *Plithogenic Neutrosophic Set* further represents truth, indeterminacy, and falsity degrees under contradictions, enriching neutrosophic sets with context-sensitive semantics.

In this paper, we define and study *Three-Mode Upside-Down Logic*, an improved version of Upside-Down Logic, together with *De-Plithogenication*, and investigate their behavior within the framework of Plithogenic Neutrosophic Sets.

Keywords: Upside-down-logic, Plithogenic Set, Three-Mode Upside-Down Logic, De-Plithogenication

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1. **Preliminaries**

This section fixes basic terminology and notation used throughout the paper. Unless explicitly stated otherwise, all sets and structures considered here are finite.

1.1. *Formal Setup for Upside-Down Logic*

We give a precise framework in which *Upside-Down Logic* will be stated and studied. Informally, the idea is to formalize context-dependent reversals of truth and falsity; see [1–6] for related viewpoints.

Definition 1.1 (Logical system)**.** (cf. [7]) A *logical system* is a tuple

$$\mathcal{M} \;=\; (\mathcal{L},\ \mathcal{P},\ \mathcal{V},\ v;\ \mathcal{A},\ \mathcal{I}),$$

where $\mathcal{L}$ is a formal language, $\mathcal{P}$ is the set of well-formed propositions in $\mathcal{L}$, $\mathcal{V}$ is a set of truth-values (e.g. $\{\mathrm{T},\mathrm{F}\}$ or $\{\mathrm{T},\mathrm{F},\mathrm{I}\}$), and $v:\mathcal{P}\to\mathcal{V}$ is a valuation. We also allow an axiom set $\mathcal{A}\subseteq\mathcal{P}$ and a collection $\mathcal{I}$ of inference rules.

Notation 1 (Contexts and contextual valuation)**.** *Let $\mathcal{C}$ be a set of* contexts*. A* contextual valuation *is a map*

$$T:\ \mathcal{P}\times\mathcal{C}\longrightarrow\mathcal{V},\qquad (A,C)\longmapsto T(A,C),$$

which evaluates each proposition A under a context C. The ordinary (context-free) valuation v is recovered by fixing a baseline $C_0\in\mathcal{C}$ and setting $v(A):=T(A,C_0)$.

Definition 1.2 (Upside↓Down Logic)**.** Given a logical system $\mathcal{M}$ together with a contextual valuation T as in Notation 1, an *Upside↓Down Logic* based on $\mathcal{M}$ is any structure

$$\mathcal{M}' \;=\; (\mathcal{L},\ \mathcal{P},\ \mathcal{V},\ T^U;\ \mathcal{A}',\ \mathcal{I}')$$

obtained from $\mathcal{M}$ by a transformation U acting on propositions and/or contexts together with a fixed "flip" permutation $\pi:\mathcal{V}\to\mathcal{V}$ that swaps T and F and leaves I (if present) unchanged:

$$\pi(\mathrm{T})=\mathrm{F},\quad \pi(\mathrm{F})=\mathrm{T},\quad \pi(\mathrm{I})=\mathrm{I}.$$

The new contextual valuation is

$$T^U(A,C)\;:=\;\pi\big(T\big(U(A),\,U(C)\big)\big)\,,\qquad (A\in\mathcal{P},\ C\in\mathcal{C}).$$

We require T^U to be total and that $(\mathcal{A}',\mathcal{I}')$ is chosen so the resulting proof system is consistent.

Example 1.3 (Traffic speed policy under snow emergency)**. Interpretation.** Let contexts be parametrized by a snow-severity index $s\in[0,1]$; s near 1 means blizzard. Define the (effective) legal speed limit

$$L(s)\;:=\;\begin{cases}50 & \text{if } s<0.80 \quad \text{(normal conditions)},\\ 30 & \text{if } s\ge 0.80 \quad \text{(snow emergency)}.\end{cases}$$

Consider the propositions

$$A:\ \text{"Driving at 50 km/h is permitted on segment } S\text{."},$$

$$B:\ \text{"Driving at } \le 30 \text{ km/h is \textit{mandatory} on } S\text{."}.$$

Contextual valuation. For C_s the context "severity s," set

$$T(A,C_s)=\begin{cases}\mathrm{T}, & L(s)=50,\\ \mathrm{F}, & L(s)=30,\end{cases}\qquad T(B,C_s)=\begin{cases}\mathrm{T}, & L(s)=30,\\ \mathrm{F}, & L(s)=50.\end{cases}$$

Upside↓Down transform. Use Definition 1.2 with the flip permutation $\pi(\mathrm{T})=\mathrm{F},\ \pi(\mathrm{F})=\mathrm{T},\ \pi(\mathrm{I})=\mathrm{I}$, and choose U to act as the identity on propositions and context: $U(A)=A,\ U(B)=B,\ U(C_s)=C_s$. Then, for any proposition $X\in\{A,B\}$,

$$T^U(X,C_s)\;=\;\pi\big(T(U(X),U(C_s))\big)\;=\;\pi\big(T(X,C_s)\big).$$

Concrete computation. Pick $s_0=0.20$ (clear roads), so $L(s_0)=50$ and hence

$$T(A,C_{s_0})=\mathrm{T},\qquad T(B,C_{s_0})=\mathrm{F}.$$

Applying the transform gives

$$T^U(A, C_{s_0}) = \pi(\mathrm{T}) = \mathrm{F} \quad \text{(falsification of the true);}$$

$$T^U(B, C_{s_0}) = \pi(\mathrm{F}) = \mathrm{T} \quad \text{(truthification of the false).}$$

Indeterminate values (if present) would remain unchanged because $\pi(\mathrm{I}) = \mathrm{I}$.

Example 1.4 (Hospital visitation policy under outbreak thresholds)**. Interpretation.** Let contexts be triples $C_{(i,o;\tau)}$, where $i \in [0,1]$ is an infection index, $o \in [0,1]$ is ICU occupancy, and $\tau \in (0,1]$ is the visitation risk threshold. Define the policy predicate

$$\mathrm{Allow}(i,o;\tau) \;:=\; \begin{cases} \mathrm{T}, & i \le \tau \text{ and } o \le 0.85, \\ \mathrm{I}, & i \le \tau \text{ and } 0.85 < o < 0.95 \quad \text{(borderline capacity)}, \\ \mathrm{F}, & \text{otherwise.} \end{cases}$$

Propositions:

$$A: \text{ "In-person ICU visit at 18:00 is permitted.",} \qquad B: \text{ "Tele-visit at 18:00 is required."}$$

with contextual valuation

$$T(A, C_{(i,o;\tau)}) = \mathrm{Allow}(i,o;\tau), \qquad T(B, C_{(i,o;\tau)}) = \begin{cases} \mathrm{T}, & \mathrm{Allow}(i,o;\tau) = \mathrm{F}, \\ \mathrm{F}, & \mathrm{Allow}(i,o;\tau) = \mathrm{T}, \\ \mathrm{I}, & \mathrm{Allow}(i,o;\tau) = \mathrm{I}. \end{cases}$$

Upside↓Down transform. As in Definition 1.2, take the same fixed flip π and again let U be the identity (on purpose, to isolate the flip effect): $U(A) = A,\ U(B) = B,\ U(C_{(i,o;\tau)}) = C_{(i,o;\tau)}$. Then $T^U(X, C) = \pi(T(X, C))$ for $X \in \{A, B\}$.

Concrete computation. Choose baseline

$$C_0 = C_{(i_0,o_0;\tau_0)} = (0.15,\ 0.90;\ 0.20).$$

Here $i_0 \le \tau_0$ and $0.85 < o_0 < 0.95$, so

$$T(A, C_0) = \mathrm{I} \quad \text{(borderline capacity)}, \qquad T(B, C_0) = \mathrm{I}.$$

Apply the transform:

$$T^U(A, C_0) = \pi(\mathrm{I}) = \mathrm{I} \quad \text{(indeterminacy preserved)}, \qquad T^U(B, C_0) = \pi(\mathrm{I}) = \mathrm{I}.$$

Now modify only the *context values* (for illustration of flips) while keeping U the identity: set $o_1 = 0.78$ (capacity sufficient) with (i_0, τ_0) unchanged, so

$$T(A, C_{(0.15,0.78;0.20)}) = \mathrm{T}, \qquad T(B, C_{(0.15,0.78;0.20)}) = \mathrm{F}.$$

Hence

$$T^U(A, C_{(0.15,0.78;0.20)}) = \pi(\mathrm{T}) = \mathrm{F} \quad \text{(falsification of the true),}$$

$$T^U(B, C_{(0.15,0.78;0.20)}) = \pi(\mathrm{F}) = \mathrm{T} \quad \text{(truthification of the false)}.$$

Thus, with a fixed flip π and identity U, the transform inverts permissions and requirements, while leaving indeterminate judgments intact.

Definition 1.5 (Context)**.** A *context* $C \in \mathcal{C}$ is any specification of parameters relevant to evaluation (e.g. spatial, temporal, semantic, or interpretive conditions) under which a proposition is judged by $T(\cdot, C)$.

1.2. *Plithogenic Set*

A Plithogenic Set augments membership with *attribute values* and an explicit *contradiction degree* between such values, generalizing fuzzy/intuitionistic/neutrosophic paradigms [8–13].

Definition 1.6 (Plithogenic Set)**.** [9, 14] Let S be a universe and $P \subseteq S$ be nonempty. A *Plithogenic Set* is a quintuple

$$PS = \big(P,\ v,\ Pv,\ pdf,\ pCF\big),$$

where v is an attribute, Pv is the set of admissible values of v,

$$pdf:\ P \times Pv \longrightarrow [0,1]^s$$

is the *degree of appurtenance function (DAF)*, and

$$pCF:\ Pv \times Pv \longrightarrow [0,1]^t$$

is the *degree of contradiction function (DCF)*. We assume for all $a, b \in Pv$:

$$\text{(Reflexivity) } pCF(a,a) = 0, \qquad \text{(Symmetry) } pCF(a,b) = pCF(b,a).$$

Here $s, t \in \mathbb{N}$ are fixed dimensions. *Convention.* For $s > 1$ the codomain $[0,1]^s$ is understood componentwise.

Example 1.7 (E-commerce shipping choice as a Plithogenic Set)**.** Let $P = \{x\}$ with $x =$ "The chosen shipping method for order #12345 is appropriate." Take the attribute $v =$ ShippingMethod with

$$Pv = \{\text{Standard}(:= S),\ \text{Express}(:= E),\ \text{LockerPickup}(:= L)\}.$$

Use a two-dimensional degree of appurtenance ($s = 2$):

$$pdf(x,a) = \big(\text{BudgetFit},\ \text{Convenience}\big) \in [0,1]^2.$$

Other variants in the literature allow powerset-valued (or hyper)-appurtenance. We use the cube $[0,1]^s$ for concreteness; cf. [15].

Specify

a	BudgetFit	Convenience
S	0.90	0.60
E	0.55	0.95
L	0.75	0.70

(each component independently in $[0, 1]$).

Model the (scalar) degree of contradiction $pCF : Pv \times Pv \to [0, 1]$ $(t = 1)$, symmetric with $pCF(a, a) = 0$:

pCF	S	E	L
S	0	0.72	0.30
E	0.72	0	0.55
L	0.30	0.55	0

capturing, e.g., a strong tension between low-cost *Standard* and speed-focused *Express* (0.72), and a mild tension between *Standard* and *LockerPickup* (0.30). Thus $PS = (P, v, Pv, pdf, pCF)$ is a concrete plithogenic set for a typical checkout decision with multi-criteria membership and explicit contradictions among attribute values.

Example 1.8 (Healthcare appointment modality as a Plithogenic Set)**.** Let $P = \{y\}$ with $y =$ "Today's appointment modality for patient C is appropriate." Take the attribute $v =$ VisitType with

$$Pv = \{\text{InPerson}(:= I),\ \text{Telehealth}(:= T),\ \text{HomeVisit}(:= H)\}.$$

Use a three-dimensional degree of appurtenance $(s = 3)$:

$$pdf(y, a) = \big(\text{ClinicalAdequacy},\ \text{Convenience},\ \text{Safety}\big) \in [0, 1]^3.$$

Assign

a	Adequacy	Convenience	Safety
I	0.90	0.50	0.75
T	0.70	0.95	0.95
H	0.80	0.70	0.85

and define the scalar contradiction map $(t = 1)$, symmetric with $pCF(a, a) = 0$:

pCF	I	T	H
I	0	0.68	0.35
T	0.68	0	0.55
H	0.35	0.55	0

reflecting, for instance, a substantial modality conflict between *InPerson* and *Telehealth* (0.68) and a moderate conflict between *Telehealth* and *HomeVisit* (0.55). Hence $PS = (P, v, Pv, pdf, pCF)$ concretely encodes multi-criteria suitability with explicit inter-modality contradictions for real clinical scheduling.

1.3. *De−Plithogenication: contradiction reset via Upside↓Down transforms*

We make precise a simple, finitely terminating procedure that eliminates pairwise contradictions among attribute values by iterated Upside ↓ Down transforms with thresholded resets [16].

Definition 1.9 (Single-step transform with reset)**.** [16] Fix $b \in Pv$ (anchor) and $\tau \in [0,1]$ (threshold). Define the *activation set*

$$\mathrm{Act}_{b,\tau} \;:=\; \{\, a \in Pv \mid pCF(a,b) \;\geq\; \tau \,\}.$$

The transform $U_{b,\tau}$ maps $PS = (P, v, Pv, pdf, pCF)$ to

$$PS^{U_{b,\tau}} = (P, v, Pv, pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}})$$

by the following rules, for all $x \in P$ and $a, u, w \in Pv$:

$$\text{(DAF flip)} \quad pdf^{U_{b,\tau}}(x,a) \;=\; \begin{cases} \mathbf{1} - pdf(x,a), & a \in \mathrm{Act}_{b,\tau}, \\ pdf(x,a), & a \notin \mathrm{Act}_{b,\tau}, \end{cases}$$

$$\text{(DCF reset)} \quad pCF^{U_{b,\tau}}(u,w) \;=\; \begin{cases} 0, & \{u,w\} \cap \{b\} \neq \varnothing \text{ and } u \in \mathrm{Act}_{b,\tau} \text{ or } w \in \mathrm{Act}_{b,\tau}, \\ pCF(u,w), & \text{otherwise}, \end{cases}$$

where $\mathbf{1} \in [0,1]^s$ is the all-ones vector and the subtraction is componentwise when $s > 1$.

Definition 1.10 (De-plithogenication)**.** A *de-plithogenication* of PS is a finite composition

$$PS^{\mathrm{dep}} \;:=\; U_{b_k,\tau_k} \circ \cdots \circ U_{b_2,\tau_2} \circ U_{b_1,\tau_1}(PS)$$

such that the resulting contradiction function is identically zero:

$$pCF^{\mathrm{dep}}(u,w) \equiv 0 \qquad \text{for all } u, w \in Pv.$$

We call PS^{dep} the *contradiction-free normal form* of PS.

Example 1.11 (Supplier selection: de-plithogenication to neutralize policy contradictions)**.**
Setup. Let $P = \{x\}$ with $x =$ "Supplier S is appropriate now." Take $v = \mathrm{PolicyFocus}$ and

$$Pv = \{\mathrm{CostEffective}(:= C),\ \mathrm{HighQuality}(:= Q),\ \mathrm{FastDelivery}(:= F)\}.$$

Use neutrosophic triples $pdf(x,a) = (T, I, F) \in [0,1]^3$:

a	T	I	F
C	0.70	0.10	0.20
Q	0.55	0.25	0.35
F	0.40	0.20	0.60

and a symmetric contradiction map pCF (zero diagonal):

pCF	C	Q	F
C	0	0.62	0.80
Q	0.62	0	0.58
F	0.80	0.58	0

Step 1 (anchor $b_1 = C$, threshold $\tau_1 = 0.60$). Activation set

$$\mathsf{Act}_{C,0.60} = \{\, a \in Pv \mid pCF(a, C) \geq 0.60 \,\} = \{Q, F\}.$$

Apply DAF flip $pdf^{(1)} = U_{C,0.60}(pdf)$ on activated a componentwise via $\mathbf{1} - pdf$:

$$pdf^{(1)}(x, Q) = \mathbf{1} - (0.55, 0.25, 0.35) = (0.45, 0.75, 0.65),$$

$$pdf^{(1)}(x, F) = \mathbf{1} - (0.40, 0.20, 0.60) = (0.60, 0.80, 0.40),$$

$$pdf^{(1)}(x, C) = (0.70, 0.10, 0.20) \quad \text{(unchanged)}.$$

Reset DCF on processed pairs (Definition 1.9):

$$pCF^{(1)}(C, Q) = pCF^{(1)}(Q, C) = 0, \quad pCF^{(1)}(C, F) = pCF^{(1)}(F, C) = 0, \quad pCF^{(1)}(Q, F) = 0.58.$$

Step 2 (anchor $b_2 = F$, threshold $\tau_2 = 0.58$). Activation set under $pCF^{(1)}$:

$$\mathsf{Act}_{F,0.58} = \{\, a \mid pCF^{(1)}(a, F) \geq 0.58 \,\} = \{Q\}.$$

Flip Q again:

$$pdf^{(2)}(x, Q) = \mathbf{1} - (0.45, 0.75, 0.65) = (0.55, 0.25, 0.35).$$

Reset $pCF^{(2)}(F, Q) = pCF^{(2)}(Q, F) = 0$. Now all contradictions vanish:

$$pCF^{(2)}(u, w) = 0 \quad \forall\, u, w \in \{C, Q, F\}.$$

Contradiction-free normal form.

$$pdf^{\text{dep}}(x, C) = (0.70, 0.10, 0.20),$$

$$pdf^{\text{dep}}(x, Q) = (0.55, 0.25, 0.35),$$

$$pdf^{\text{dep}}(x, F) = (0.60, 0.80, 0.40), \qquad pCF^{\text{dep}} \equiv 0.$$

This two-step de-plithogenication neutralizes C–Q and C–F in step 1, then Q–F in step 2, yielding a contradiction-free representation for supplier policy focus.

Example 1.12 (IT change management: de-plithogenication across action choices)**. Setup.** Let $P = \{y\}$ with $y =$ "Today's action on the production change is appropriate." Take $v =$ Action and

$$Pv = \{\text{ProceedChange}(:= P),\ \text{ScheduleMaintenance}(:= S),\ \text{Rollback}(:= R)\}.$$

Neutrosophic triples $pdf(y,a) = (T, I, F)$:

a	T	I	F
P	0.50	0.20	0.45
S	0.60	0.15	0.35
R	0.30	0.25	0.70

Contradiction map pCF (symmetric, zero diagonal):

pCF	P	S	R
P	0	0.55	0.90
S	0.55	0	0.72
R	0.90	0.72	0

Step 1 (anchor $b_1 = P$, threshold $\tau_1 = 0.70$). Activation set:

$$\mathsf{Act}_{P,0.70} = \{\, a \mid pCF(a,P) \geq 0.70 \,\} = \{R\}.$$

Flip R:

$$pdf^{(1)}(y,R) = \mathbf{1} - (0.30, 0.25, 0.70) = (0.70, 0.75, 0.30),$$

and reset $pCF^{(1)}(P,R) = pCF^{(1)}(R,P) = 0$ (others unchanged).

Step 2 (anchor $b_2 = S$, threshold $\tau_2 = 0.70$). Activation set under $pCF^{(1)}$:

$$\mathsf{Act}_{S,0.70} = \{\, a \mid pCF^{(1)}(a,S) \geq 0.70 \,\} = \{R\}.$$

Flip R again:

$$pdf^{(2)}(y,R) = \mathbf{1} - (0.70, 0.75, 0.30) = (0.30, 0.25, 0.70),$$

and reset $pCF^{(2)}(S,R) = pCF^{(2)}(R,S) = 0$. Remaining contradiction: $pCF^{(2)}(P,S) = 0.55$.

Step 3 (anchor $b_3 = P$, threshold $\tau_3 = 0.55$). Activation set:

$$\mathsf{Act}_{P,0.55} = \{\, a \mid pCF^{(2)}(a,P) \geq 0.55 \,\} = \{S\}.$$

Flip S:

$$pdf^{(3)}(y,S) = \mathbf{1} - (0.60, 0.15, 0.35) = (0.40, 0.85, 0.65),$$

and reset $pCF^{(3)}(P,S) = pCF^{(3)}(S,P) = 0$. Therefore $pCF^{(3)} \equiv 0$.

Contradiction-free normal form.

$$pdf^{\text{dep}}(y,P) = (0.50, 0.20, 0.45),$$

$$pdf^{\text{dep}}(y,S) = (0.40, 0.85, 0.65),$$

$$pdf^{\text{dep}}(y,R) = (0.30, 0.25, 0.70), \qquad pCF^{\text{dep}} \equiv 0.$$

The three-step de-plithogenication resolves strong antagonisms P–R and S–R first, then neutralizes the moderate P–S tension, producing a stable, contradiction-free assessment for change management actions.

Proposition 1.13 (Finite elimination of contradictions)**.** *Let* $Pv = \{c_1, \ldots, c_m\}$*. Choose anchors* $b_1, \ldots, b_m$ *as a permutation of* Pv *and take thresholds* $\tau_i := 0$ *for* $1 \le i \le m$*. Then the composition* $U_{b_m,0} \circ \cdots \circ U_{b_1,0}$ *is a de-plithogenication: for every unordered pair* $\{u, w\} \subseteq Pv$ *one has* $pCF^{\text{dep}}(u, w) = 0$*.*

Proof. Write $pCF^{(0)} := pCF$ and $pCF^{(i)}$ for the DCF after i transforms. By Definition 1.9 with $\tau_i = 0$, at step i we reset to 0 every pair involving the anchor b_i:

$$pCF^{(i)}(b_i, a) = 0 \ \text{ for all } a \in Pv, \qquad pCF^{(i)}(u, w) = pCF^{(i-1)}(u, w) \text{ otherwise.}$$

Fix distinct $u, w \in Pv$ and let i be the index with $b_i = u$ (exists by construction). Then $pCF^{(i)}(u, w) = 0$, and subsequent steps do not change zeros, hence $pCF^{(m)}(u, w) = 0$. As $\{u, w\}$ was arbitrary, $pCF^{(m)} \equiv 0$. □

Remark 1.14 (Idempotence of the normal form)**.** If $pCF^{\text{dep}} \equiv 0$, then for any b, τ one has $pCF^{U_{b,\tau}} = pCF^{\text{dep}}$ by Definition 1.9; thus further applications of $U_{b,\tau}$ leave the contradiction-free normal form unchanged.

1.4. *Plithogenic Neutrosophic Set*

A Neutrosophic Set represents truth, indeterminacy, and falsity degrees independently in [0,1], extending classical and fuzzy sets [17–20]. A Plithogenic Neutrosophic Set represents truth, indeterminacy, and falsity degrees under contradictions, extending neutrosophic sets with contextual contradiction-sensitive semantics [21–24]. We now examine the methods of applying Upside-Down Logic.

Definition 1.15 (Plithogenic Neutrosophic Set ($s = 3,\ t = 1$))**.** [9] A *Plithogenic Neutrosophic Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \longrightarrow [0,1]^3, \qquad pdf(x, a) = \big(T_P(x \mid a),\, I_P(x \mid a),\, F_P(x \mid a)\big),$$

and

$$pCF : Pv \times Pv \longrightarrow [0,1],$$

where, for each $(x, a) \in P \times Pv$,

$$T_P(x \mid a),\ I_P(x \mid a),\ F_P(x \mid a) \in [0,1].$$

No global normalization is required; one may optionally impose $0 \le T_P + I_P + F_P \le 3$ (single-valued setting). Here T_P, I_P, F_P denote, respectively, the degrees of truth, indeterminacy, and falsity, each evaluated with respect to the attribute value a. The DCF is scalar, symmetric, and null on the diagonal.

Example 1.16 (E-commerce return resolution for a single order)**.** Let $P = \{x\}$ with $x =$ "The chosen resolution for order #12345 is appropriate." Take the attribute $v = \textit{Resolution}$ with value set

$$Pv = \{\text{Refund},\ \text{Replace},\ \text{Repair}\}.$$

Define the plithogenic neutrosophic degrees $pdf(x, a) = (T, I, F) \in [0, 1]^3$ as:

a	T	I	F
Refund	0.68	0.12	0.30
Replace	0.55	0.20	0.35
Repair	0.35	0.25	0.55

(independent components; no normalization required)

Model the contradiction degrees $pCF : Pv \times Pv \to [0, 1]$ (symmetric, zero diagonal) by

pCF	Refund	Replace	Repair
Refund	0	0.35	0.78
Replace	0.35	0	0.60
Repair	0.78	0.60	0

so that Refund and Repair are highly contradictory (0.78), while Refund and Replace are only mildly contradictory (0.35).

Concrete computation (bounded-sum t-conorm preview). With $S_{\text{bs}}(u, v) = \min\{1, u+v\}$, the uncertainty mass that could be absorbed from truth/falsity for Repair is

$$S_{\text{bs}}(T, F) \;=\; \min\{1,\, 0.35 + 0.55\} \;=\; 0.90,$$

which, if needed in a downstream "Absorb" step, would yield $I' = \min\{1,\, 0.25 + 0.90\} = 1.00$ while setting $T' = F' = 0$ (Definition 2.1). This example provides a fully specified PNS (P, v, Pv, pdf, pCF) grounded in a routine returns workflow.

Example 1.17 (Hiring decision for a candidate)**.** Let $P = \{y\}$ with $y =$ "Today's action for candidate C is appropriate." Take the attribute $v = \textit{Action}$ with value set

$$Pv = \{\text{HireNow},\ \text{InterviewAgain},\ \text{Reject}\}.$$

Assign neutrosophic triples (independent components in $[0, 1]$):

a	T	I	F
HireNow	0.52	0.18	0.44
InterviewAgain	0.60	0.25	0.30
Reject	0.40	0.15	0.58

and a contradiction map (symmetric, $pCF(a,a)=0$):

pCF	HireNow	InterviewAgain	Reject
HireNow	0	0.50	0.90
InterviewAgain	0.50	0	0.65
Reject	0.90	0.65	0

indicating that HireNow and Reject are strongly contradictory (0.90), whereas HireNow vs. InterviewAgain shows moderate tension (0.50).

Concrete computation (sanity checks). For $a=$ InterviewAgain one may verify potential uncertainty consolidation via

$$S_{\mathrm{bs}}\big(T,F\big)=\min\{1,\,0.60+0.30\}=0.90,\qquad \text{and}\qquad T+I+F=0.60+0.25+0.30=1.15,$$

which is permitted in neutrosophic modeling (no global normalization constraint). This yields a practical PNS (P,v,Pv,pdf,pCF) for day-to-day hiring triage, explicitly encoding both support levels and pairwise contradictions among actions.

Definition 1.18 (Upside-Down Logic in Plithogenic Neutrosophic Set (truth–falsity swap))**.** Let

$$PS=(P,v,Pv,pdf,pCF)$$

be a Plithogenic Neutrosophic Set with

$$pdf(x,a)=\big(T_P(x\mid a),\,I_P(x\mid a),\,F_P(x\mid a)\big)\in[0,1]^3,\qquad c(a,b):=pCF(a,b)\in[0,1],$$

where $c(a,a)=0$ and $c(a,b)=c(b,a)$. Fix an *anchor* $b\in Pv$ and a *threshold* $\tau\in[0,1]$. Declare the flip to *activate* when

$$\mathrm{Act}(a;b,\tau)\;:\Longleftrightarrow\;c(a,b)\;\geq\;\tau.$$

Define the Upside-Down transform $U_{b,\tau}$ on pdf by

$$pdf^{U_{b,\tau}}(x,a)\;:=\;\begin{cases}\big(F_P(x\mid a),\;I_P(x\mid a),\;T_P(x\mid a)\big), & \text{if }\mathrm{Act}(a;b,\tau),\\ \big(T_P(x\mid a),\;I_P(x\mid a),\;F_P(x\mid a)\big), & \text{otherwise.}\end{cases}$$

That is, under high contradiction with the anchor b, the *truth* and *falsity* degrees are swapped, while the *indeterminacy* degree is preserved.

Example 1.19 (School event under weather conditions: truth–falsity swap)**.** Let $P=\{x\}$ with $x=$ "Holding the outdoor ceremony now is appropriate." Let the attribute be $v=$ Weather with

$$Pv=\{\text{Clear},\ \text{LightRain},\ \text{Thunderstorm}\}.$$

Give neutrosophic degrees $pdf(x, a) = (T, I, F) \in [0, 1]^3$ by

a	T	I	F
Clear	0.75	0.10	0.20
LightRain	0.45	0.25	0.50
Thunderstorm	0.15	0.20	0.85

and a symmetric contradiction map $c(\cdot, \cdot) = pCF(\cdot, \cdot)$ (zero on the diagonal) by

$$c(\text{LightRain}, \text{Clear}) = 0.58, \qquad c(\text{Thunderstorm}, \text{Clear}) = 0.93.$$

Fix anchor $b = \text{Clear}$ and threshold $\tau = 0.80$. Activation:

$$\mathsf{Act}(\text{LightRain}; b, \tau) = \mathbf{1}[\,0.58 \geq 0.80\,] = 0, \qquad \mathsf{Act}(\text{Thunderstorm}; b, \tau) = \mathbf{1}[\,0.93 \geq 0.80\,] = 1.$$

By the Upside-Down transform $U_{b,\tau}$ (swap T and F, keep I on activated a),

$$\begin{aligned} pdf^U(x, \text{Clear}) &= (0.75,\ 0.10,\ 0.20) \quad \text{(not activated)}, \\ pdf^U(x, \text{LightRain}) &= (0.45,\ 0.25,\ 0.50) \quad \text{(not activated)}, \\ pdf^U(x, \text{Thunderstorm}) &= (0.85,\ 0.20,\ 0.15) \quad \text{(activated; } (T, F) \text{ swapped)}. \end{aligned}$$

Thus, under high contradiction with the anchor condition "Clear," the assessment for *Thunderstorm* flips from mostly-false to mostly-true for the proposition x.

Example 1.20 (Same-day delivery policy under inventory states: truth–falsity swap)**.** Let $P = \{y\}$ with $y =$ "Offering same-day delivery is appropriate." Let the attribute be $v =$ InventoryState with

$$Pv = \{\text{InStock},\ \text{LowStock},\ \text{Outage}\}.$$

Specify $pdf(y, a) = (T, I, F)$ as

a	T	I	F
InStock	0.82	0.06	0.12
LowStock	0.58	0.22	0.33
Outage	0.28	0.30	0.72

and contradiction degrees

$$c(\text{LowStock}, \text{InStock}) = 0.65, \qquad c(\text{Outage}, \text{InStock}) = 0.88.$$

Fix anchor $b = \text{InStock}$ and threshold $\tau = 0.70$. Then

$$\mathsf{Act}(\text{LowStock}; b, \tau) = \mathbf{1}[\,0.65 \geq 0.70\,] = 0, \qquad \mathsf{Act}(\text{Outage}; b, \tau) = \mathbf{1}[\,0.88 \geq 0.70\,] = 1.$$

Applying $U_{b,\tau}$ (swap T and F only on activated values),

$$pdf^U(y, \text{InStock}) = (0.82,\ 0.06,\ 0.12) \quad \text{(not activated)},$$

$$pdf^U(y, \text{LowStock}) = (0.58,\ 0.22,\ 0.33) \quad \text{(not activated)},$$

$$pdf^U(y, \text{Outage}) = (0.72,\ 0.30,\ 0.28) \quad \text{(activated; } (T, F) \text{ swapped)}.$$

Hence, when *Outage* is highly contradictory to the anchor *InStock*, the evaluation of same-day delivery reverses its truth/falsity while preserving indeterminacy.

2. **Three-Mode Upside-down logic in Plithogenic Neutrosophic Set**

Context-triggered operator on plithogenic neutrosophic memberships: for activated attributes, Keep preserves (T,I,F), Swap exchanges truth/falsity, Absorb aggregates uncertainty into indeterminacy.

Definition 2.1 (Three-Mode Upside-Down Logic on a Plithogenic Neutrosophic Set)**.** Fix an *anchor* $b \in Pv$, a *threshold* $\tau \in [0, 1]$, and a *mode selector*

$$\mathcal{M} : Pv \longrightarrow \{\text{Keep}, \text{Swap}, \text{Absorb}\}.$$

Activation is controlled by the contradiction with the anchor:

$$\mathsf{Act}(a \mid b, \tau) \ := \ \mathbf{1}\,[\,c(a, b) \geq \tau\,] \ \in \ \{0, 1\}.$$

Let $S : [0, 1]^2 \to [0, 1]$ be any *t-conorm* (s-norm) used to aggregate degrees into indeterminacy; in the single-valued setting the *bounded sum*

$$S_{\text{bs}}(x, y) := \min\{1,\ x + y\}$$

is a canonical choice.

The *Three-Mode Upside-Down transform*

$$U^{(3)}_{b,\tau,\mathcal{M},S} :\ PS \ \longmapsto\ PS^U$$

acts only on activated attribute-values and leaves the contradiction map either unchanged (*no-reset*) or optionally reset on the processed pairs (*reset* choice, see below). Concretely, for each $(x, a) \in P \times Pv$,

$$pdf^U(x, a) = \big(T'_P(x \mid a),\ I'_P(x \mid a),\ F'_P(x \mid a)\big) \quad \text{is given by}$$

$$(T'_P,\ I'_P,\ F'_P) = \begin{cases} \big(T_P,\ I_P,\ F_P\big), & \mathsf{Act}(a \mid b, \tau) = 0 \text{ (not activated)}, \\ \big(T_P,\ I_P,\ F_P\big), & \mathsf{Act}(a \mid b, \tau) = 1 \text{ and } \mathcal{M}(a) = \text{Keep}, \\ \big(F_P,\ I_P,\ T_P\big), & \mathsf{Act}(a \mid b, \tau) = 1 \text{ and } \mathcal{M}(a) = \text{Swap}, \\ \big(0,\ S\big(I_P,\ S(T_P, F_P)\big),\ 0\big), & \mathsf{Act}(a \mid b, \tau) = 1 \text{ and } \mathcal{M}(a) = \text{Absorb}, \end{cases}$$

where $T_P = T_P(x \mid a)$, $I_P = I_P(x \mid a)$, $F_P = F_P(x \mid a)$. Thus the three modes are:

- *Keep*: leave (T, I, F) unchanged on activated a;
- *Swap*: interchange truth and falsity, $(T, I, F) \mapsto (F, I, T)$;
- *Absorb*: *neither true nor false*; move all support into indeterminacy by setting $T' = F' = 0$ and $I' = S\big(I, S(T, F)\big)$.

Contradiction map update (two standard choices).

- *No-reset (involutive on Keep/Swap):* set $pCF^U = pCF$.
- *Reset (idempotent on processed pairs):* for each activated a,

$$pCF^U(a, b) = pCF^U(b, a) = 0, \qquad pCF^U(u, w) = pCF(u, w) \text{ otherwise.}$$

Both choices are compatible with the transform on *pdf*; one may select either depending on whether contradictions should be retained as context or neutralized after the update.

Example 2.2 (Airport security alert levels: Keep & Swap modes)**. Interpretation.** Let $P = \{x\}$ contain a single proposition $x =$ "Secondary screening at gate G is required." Let the attribute alphabet (policy alert level) be

$$Pv = \{\text{Green}, \ \text{Orange}, \ \text{Red}\}.$$

We model the plithogenic neutrosophic degrees for x under each alert level by

$$pdf(x, a) = \big(T_P(x \mid a), \, I_P(x \mid a), \, F_P(x \mid a)\big) \in [0, 1]^3.$$

Contradiction degrees $c(a, b) = pCF(a, b)$ are symmetric with $c(a, a) = 0$.

Numerical setup (before transform). Choose anchor $b =$ Green and set the threshold $\tau = 0.60$. Let the contradiction map satisfy

$$c(\text{Orange}, \text{Green}) = 0.62, \qquad c(\text{Red}, \text{Green}) = 0.85,$$

so both Orange and Red are *activated* since $c(\cdot, \text{Green}) \geq \tau$. Take initial neutrosophic triples:

$$\begin{aligned} pdf(x, \text{Green}) &= (0.20, \ 0.10, \ 0.70), \\ pdf(x, \text{Orange}) &= (0.55, \ 0.15, \ 0.35), \\ pdf(x, \text{Red}) &= (0.30, \ 0.20, \ 0.60). \end{aligned}$$

Mode selector. We choose

$$\mathcal{M}(\text{Orange}) = \text{Keep}, \qquad \mathcal{M}(\text{Red}) = \text{Swap}.$$

("Keep" = leave (T, I, F) unchanged; "Swap" = interchange truth and falsity.)

Three-Mode UD transform (no-reset option). Applying Definition 2.1 with the no-reset choice for pCF:

$$pdf^U(x,a) = \begin{cases} (T,I,F), & a \text{ not activated}, \\ (T,I,F), & a \text{ activated and } \mathcal{M}(a) = \text{Keep}, \\ (F,I,T), & a \text{ activated and } \mathcal{M}(a) = \text{Swap}. \end{cases}$$

Hence,

$$\begin{aligned} pdf^U(x,\text{Green}) &= (0.20,\ 0.10,\ 0.70) \quad (\text{not activated}), \\ pdf^U(x,\text{Orange}) &= (0.55,\ 0.15,\ 0.35) \quad (\text{Keep}), \\ pdf^U(x,\text{Red}) &= (0.60,\ 0.20,\ 0.30) \quad (\text{Swap of } (0.30, 0.20, 0.60)). \end{aligned}$$

In words, under a *Red* alert the previously mostly-false statement becomes mostly-true by swapping T and F, while the *Orange* setting is kept unchanged despite activation (e.g., policy decides not to overturn it). Since we selected the no-reset option, pCF remains the same after the transform.

Example 2.3 (Loan approval triage: Absorb mode with bounded-sum and reset)**. Interpretation.** Consider a single application x and the decision attribute alphabet

$$Pv = \{\text{Approve},\ \text{Deny},\ \text{ManualReview}\}.$$

Let $pdf(x,a) = (T,I,F)$ encode the neutrosophic support that the decision a is appropriate. We regard Approve as anchor b and measure contradiction $c(a,b)$ between a and b.

Numerical setup (before transform). Take

$$\begin{aligned} pdf(x,\text{Approve}) &= (0.65,\ 0.10,\ 0.25), \\ pdf(x,\text{Deny}) &= (0.40,\ 0.20,\ 0.45), \\ pdf(x,\text{ManualReview}) &= (0.35,\ 0.30,\ 0.40), \end{aligned}$$

and contradiction degrees

$$c(\text{Deny},\text{Approve}) = 0.88, \qquad c(\text{ManualReview},\text{Approve}) = 0.50.$$

Set the threshold $\tau = 0.80$ so that only Deny is *activated.* Choose the bounded-sum t-conorm $S_{\text{bs}}(x,y) = \min\{1, x+y\}$ and select

$$\mathcal{M}(\text{Deny}) = \text{Absorb}.$$

Absorb mode (reset option). By Definition 2.1, for Absorb we set $T' = F' = 0$ and move all support into I' via the t-conorm:

$$I' = S_{\text{bs}}\Big(I,\ S_{\text{bs}}(T,F)\Big).$$

For $a = \text{Deny}$ we compute

$$S_{\text{bs}}(T,F) = \min\{1,\ 0.40 + 0.45\} = 0.85, \qquad I' = \min\{1,\ 0.20 + 0.85\} = 1.00,$$

hence

$$pdf^U(x, \text{Deny}) = (0,\ 1.00,\ 0).$$

Non-activated values remain unchanged:

$$pdf^U(x, \text{Approve}) = (0.65, 0.10, 0.25), \quad pdf^U(x, \text{ManualReview}) = (0.35, 0.30, 0.40).$$

With the *reset* choice for the contradiction map, the processed pair is neutralized:

$$pCF^U(\text{Deny}, \text{Approve}) = pCF^U(\text{Approve}, \text{Deny}) = 0,$$

while all other pCF-entries are left unchanged.

Interpretation of the update. Because crucial documents are missing (high contradiction with "Approve"), the system refuses to assert "Approve" or "Deny" as true/false for the Deny decision and *absorbs* the support into indeterminacy, signaling a state that is neither true nor false but requires resolution. The reset erases the immediate antagonism between Deny and Approve for this case, avoiding downstream oscillations in subsequent evaluations.

Proposition 2.4 (Well-posedness and range preservation)**.** *Let S be any t-conorm on $[0,1]$ (commutative, associative, monotone, with $S(x,0) = x$ and $S(x,1) = 1$). Then, for every (x,a), the image triple $pdf^U(x,a)$ lies in $[0,1]^3$. In particular, with the bounded-sum S_{bs}, one has*

$$0 \ \le\ I'_P(x \mid a) \ =\ \min\{1,\ I_P + T_P + F_P\} \ \le\ 1,$$

and $T'_P, F'_P \in [0,1]$ by construction. Hence $U^{(3)}_{b,\tau,\mathcal{M},S}$ is a well-defined endomorphism on the class of plithogenic neutrosophic sets.

Proof. Each branch maps $[0,1]^3$ into $[0,1]^3$: Keep is the identity; Swap is a permutation of coordinates; Absorb uses a t-conorm S that is range-preserving on $[0,1]$, so $I' = S\big(I, S(T,F)\big) \in [0,1]$ and $T' = F' = 0$. The optional reset replaces some pCF-entries by 0, preserving the codomain $[0,1]$. □

3. Three-Mode De−Plithogenication in Plithogenic Neutrosophic Set

Procedure neutralizing contradictions by sequential Keep, Swap, and Absorb modes on activated attributes, aggregating uncertainty and optionally resetting contradiction degrees.

Definition 3.1 (Three-Mode De-Plithogenication in a Plithogenic Neutrosophic Set)**.** Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Neutrosophic Set with

$$pdf(x, a) = \big(T_P(x \mid a),\, I_P(x \mid a),\, F_P(x \mid a)\big) \in [0, 1]^3, \quad pCF : Pv \times Pv \to [0, 1]$$

(symmetric, $pCF(a, a) = 0$). Fix:

- an *anchor* $b \in Pv$ and a *threshold* $\tau \in [0, 1]$;
- a *mode selector* $\mathcal{M} : Pv \to \{\text{Keep}, \text{Swap}, \text{Absorb}\}$;
- a t-conorm $S : [0, 1]^2 \to [0, 1]$ (canonical choice: bounded sum $S_{\text{bs}}(x, y) = \min\{1, x + y\}$).

Define activation by

$$\mathsf{Act}(a \mid b, \tau) := \mathbf{1}[\, pCF(a, b) \geq \tau \,].$$

The *single-step three-mode de-plithogenication operator*

$$\mathcal{D}^{(3)}_{b,\tau,\mathcal{M},S} :\ (pdf, pCF) \ \longmapsto\ (pdf^\star, pCF^\star)$$

is given pointwise in $(x, a) \in P \times Pv$ by

$$pdf^\star(x, a) = \begin{cases} (T_P,\, I_P,\, F_P), & \mathsf{Act}(a \mid b, \tau) = 0, \\ (T_P,\, I_P,\, F_P), & \mathsf{Act}(a \mid b, \tau) = 1 \ \wedge\ \mathcal{M}(a) = \text{Keep}, \\ (F_P,\, I_P,\, T_P), & \mathsf{Act}(a \mid b, \tau) = 1 \ \wedge\ \mathcal{M}(a) = \text{Swap}, \\ \big(0,\, S(I_P,\, S(T_P, F_P)),\, 0\big), & \mathsf{Act}(a \mid b, \tau) = 1 \ \wedge\ \mathcal{M}(a) = \text{Absorb}, \end{cases}$$

where $T_P = T_P(x \mid a)$, $I_P = I_P(x \mid a)$, $F_P = F_P(x \mid a)$ and S is applied as a t-conorm. The *contradiction reset* is

$$pCF^\star(u, w) \ = \ \begin{cases} 0, & \big(\{u, w\} = \{a, b\} \text{ with } \mathsf{Act}(a \mid b, \tau) = 1\big), \\ pCF(u, w), & \text{otherwise.} \end{cases}$$

A *Three-Mode De-Plithogenication (sequence)* is a finite composition

$$PS^{\text{dep3}} \ :=\ \mathcal{D}^{(3)}_{b_k,\tau_k,\mathcal{M}_k,S} \circ \cdots \circ \mathcal{D}^{(3)}_{b_2,\tau_2,\mathcal{M}_2,S} \circ \mathcal{D}^{(3)}_{b_1,\tau_1,\mathcal{M}_1,S}(PS),$$

such that the final contradiction map is identically zero, i.e. $pCF^{\text{dep3}}(u, w) = 0$ for all $u, w \in Pv$. We call PS^{dep3} the *three-mode de-plithogenicated normal form.*

Example 3.2 (Three-step de-plithogenication on three attribute values)**. Setup.** Let $P = \{x\}$ and $Pv = \{A, B, C\}$. Initial neutrosophic triples (for decision/proposition x) are

$$pdf(x, A) = (0.70,\, 0.10,\, 0.20), \quad pdf(x, B) = (0.30,\, 0.20,\, 0.60), \quad pdf(x, C) = (0.40,\, 0.30,\, 0.50).$$

Initial contradictions:

$$pCF(A, B) = 0.85, \quad pCF(A, C) = 0.55, \quad pCF(B, C) = 0.65$$

(symmetry and zero diagonal understood). Use the bounded-sum t-conorm S_{bs}.

Step 1 (anchor A, threshold $\tau_1 = 0.60$). Activation against A:

$$\mathsf{Act}(B \mid A, 0.60) = \mathbf{1}[0.85 \geq 0.60] = 1, \qquad \mathsf{Act}(C \mid A, 0.60) = \mathbf{1}[0.55 \geq 0.60] = 0.$$

Choose $\mathcal{M}_1(B) =$ Swap (treat B as a genuine reversal), any choice for C is irrelevant (not activated). Then

$$\begin{aligned} pdf^{(1)}(x, B) &= (F, I, T) = (0.60,\ 0.20,\ 0.30), \\ pdf^{(1)}(x, A) &= (0.70,\ 0.10,\ 0.20), \\ pdf^{(1)}(x, C) &= (0.40,\ 0.30,\ 0.50). \end{aligned}$$

Reset processed pair: $pCF^{(1)}(A, B) = 0$, with other entries unchanged:

$$pCF^{(1)}(A, C) = 0.55, \qquad pCF^{(1)}(B, C) = 0.65.$$

Step 2 (anchor C, threshold $\tau_2 = 0.60$). Activation against C under $pCF^{(1)}$:

$$\mathsf{Act}(B \mid C, 0.60) = \mathbf{1}[0.65 \geq 0.60] = 1, \qquad \mathsf{Act}(A \mid C, 0.60) = \mathbf{1}[0.55 \geq 0.60] = 0.$$

Choose $\mathcal{M}_2(B) =$ Absorb. Compute for B with S_{bs}:

$$S_{\mathrm{bs}}\big(T_B^{(1)}, F_B^{(1)}\big) = \min\{1,\ 0.60 + 0.30\} = 0.90, \quad I'_B = \min\{1,\ 0.20 + 0.90\} = 1.00,$$

so

$$pdf^{(2)}(x, B) = (0,\ 1.00,\ 0),$$

while A and C remain as in step 1. Reset the processed pair: $pCF^{(2)}(B, C) = 0$. At this point,

$$pCF^{(2)}(A, B) = 0, \qquad pCF^{(2)}(B, C) = 0, \qquad pCF^{(2)}(A, C) = 0.55.$$

Step 3 (anchor A, threshold $\tau_3 = 0.55$). Activation against A: $\mathsf{Act}(C \mid A, 0.55) = \mathbf{1}[0.55 \geq 0.55] = 1$. Choose $\mathcal{M}_3(C) =$ Keep (we only wish to neutralize the contradiction). Thus $pdf^{(3)}(x, C) = pdf^{(2)}(x, C) = (0.40,\ 0.30,\ 0.50)$ and we reset

$$pCF^{(3)}(A, C) = 0.$$

Resulting normal form. All pairwise contradictions are now zero:

	A	B	C
A	0	0	0
B	0	0	0
C	0	0	0

with the final neutrosophic triples

$$\begin{aligned} pdf^{\mathrm{dep3}}(x, A) &= (0.70,\ 0.10,\ 0.20), \\ pdf^{\mathrm{dep3}}(x, B) &= (0,\ 1.00,\ 0), \\ pdf^{\mathrm{dep3}}(x, C) &= (0.40,\ 0.30,\ 0.50). \end{aligned}$$

The sequence demonstrates all three modes: *Swap* (Step 1) for a genuine reversal, *Absorb* (Step 2) to encode "neither true nor false" by moving support into indeterminacy, and *Keep* (Step 3) to neutralize a residual contradiction without altering memberships.

Example 3.3 (Severe-weather school operations: Keep + Absorb to neutralize conflicts)**.** **Interpretation.** Let $P = \{x\}$ with the proposition $x =$ "Today's chosen operating mode is appropriate." Let the attribute alphabet be

$$Pv = \{\text{OpenCampus},\ \text{OnlineOnly},\ \text{CloseCampus}\}.$$

For each $a \in Pv$, $pdf(x, a) = (T, I, F) \in [0, 1]^3$ is the neutrosophic support that option a is appropriate. The contradiction degrees pCF satisfy $pCF(a, b) = pCF(b, a)$, $pCF(a, a) = 0$. Use the bounded-sum t-conorm $S_{\text{bs}}(u, v) = \min\{1, u + v\}$.

Initial data.

$$pdf(x, \text{OpenCampus}) = (0.55,\ 0.15,\ 0.40), \qquad pCF(\text{CloseCampus}, \text{OpenCampus}) = 0.82,$$
$$pdf(x, \text{OnlineOnly}) = (0.45,\ 0.25,\ 0.35), \qquad pCF(\text{OnlineOnly}, \text{OpenCampus}) = 0.65,$$
$$pdf(x, \text{CloseCampus}) = (0.30,\ 0.35,\ 0.60), \qquad pCF(\text{OnlineOnly}, \text{CloseCampus}) = 0.50.$$

Step 1 (anchor $b =$ OpenCampus, threshold $\tau_1 = 0.75$). Activation:

$$\mathsf{Act}(\text{CloseCampus} \mid b, \tau_1) = \mathbf{1}[0.82 \geq 0.75] = 1, \quad \mathsf{Act}(\text{OnlineOnly} \mid b, \tau_1) = \mathbf{1}[0.65 \geq 0.75] = 0.$$

Choose the mode *Absorb* for CloseCampus. Then

$$S_{\text{bs}}(T, F) = \min\{1,\ 0.30 + 0.60\} = 0.90, \qquad I' = \min\{1,\ 0.35 + 0.90\} = 1.00,$$

so

$$pdf^{(1)}(x, \text{CloseCampus}) = (0,\ 1.00,\ 0),$$
$$pdf^{(1)}(x, \text{OpenCampus}) = (0.55, 0.15, 0.40),$$
$$pdf^{(1)}(x, \text{OnlineOnly}) = (0.45, 0.25, 0.35).$$

Reset the processed pair:

$$pCF^{(1)}(\text{CloseCampus}, \text{OpenCampus}) = 0,$$
$$pCF^{(1)}(\text{OnlineOnly}, \text{OpenCampus}) = 0.65,$$
$$pCF^{(1)}(\text{OnlineOnly}, \text{CloseCampus}) = 0.50.$$

Step 2 (same anchor $b =$ OpenCampus, threshold $\tau_2 = 0.60$). Now $\mathsf{Act}(\text{OnlineOnly} \mid b, \tau_2) = \mathbf{1}[0.65 \geq 0.60] = 1$. Choose *Keep* for OnlineOnly; hence *pdf* unchanged and reset

$$pCF^{(2)}(\text{OnlineOnly}, \text{OpenCampus}) = 0.$$

Remaining contradiction is $pCF^{(2)}(\text{OnlineOnly}, \text{CloseCampus}) = 0.50$.

Step 3 (anchor b = OnlineOnly, threshold $\tau_3 = 0.50$). $\mathsf{Act}(\text{CloseCampus} \mid b, \tau_3) = \mathbf{1}[0.50 \geq 0.50] = 1$. Choose *Keep* (we only wish to neutralize). Thus *pdf* remains and reset

$$pCF^{(3)}(\text{OnlineOnly}, \text{CloseCampus}) = 0.$$

Outcome (three-mode de-plithogenicated normal form). All pairwise contradictions are zero; the final triples are

$$pdf^{\text{dep3}}(x, \text{OpenCampus}) = (0.55,\ 0.15,\ 0.40),$$
$$pdf^{\text{dep3}}(x, \text{OnlineOnly}) = (0.45,\ 0.25,\ 0.35),$$
$$pdf^{\text{dep3}}(x, \text{CloseCampus}) = (0,\ 1.00,\ 0).$$

This sequence uses *Absorb* to encode "neither true nor false yet" for closure, and *Keep* steps to neutralize the remaining contradictions without altering memberships.

Example 3.4 (Clinical triage plan: Swap + Absorb + Keep to stabilize recommendations). **Interpretation.** Let $P = \{x\}$ with x = "The plan is appropriate now." Take

$$Pv = \{\text{StartAntibiotics},\ \text{WatchfulWaiting},\ \text{SendToER}\}.$$

As before $pdf(x, a) = (T, I, F)$ and pCF is symmetric with zero diagonal. Use S_{bs} as the t-conorm.

Initial data.

$$pdf(x, \text{StartAntibiotics}) = (0.40,\ 0.30,\ 0.50), \qquad pCF(\text{SendToER}, \text{StartAntibiotics}) = 0.88,$$
$$pdf(x, \text{WatchfulWaiting}) = (0.55,\ 0.25,\ 0.35), \qquad pCF(\text{SendToER}, \text{WatchfulWaiting}) = 0.72,$$
$$pdf(x, \text{SendToER}) = (0.35,\ 0.20,\ 0.65), \qquad pCF(\text{StartAntibiotics}, \text{WatchfulWaiting}) = 0.60.$$

Step 1 (anchor b = StartAntibiotics, threshold $\tau_1 = 0.80$). Activation:

$$\mathsf{Act}(\text{SendToER} \mid b, \tau_1) = \mathbf{1}[0.88 \geq 0.80] = 1.$$

Choose *Swap* for SendToER to reflect a genuine reversal under red flags:

$$pdf^{(1)}(x, \text{SendToER}) = (F, I, T) = (0.65,\ 0.20,\ 0.35).$$

Reset the processed pair:

$$pCF^{(1)}(\text{SendToER}, \text{StartAntibiotics}) = 0,$$

other entries unchanged.

Step 2 (anchor b = WatchfulWaiting, threshold $\tau_2 = 0.70$). $\mathsf{Act}(\text{SendToER} \mid b, \tau_2) = \mathbf{1}[0.72 \geq 0.70] = 1$. Choose *Absorb* for SendToER to encode "neither true nor false until diagnostics":

$$S_{\text{bs}}(T^{(1)}, F^{(1)}) = \min\{1, 0.65 + 0.35\} = 1.00, \quad I' = \min\{1, 0.20 + 1.00\} = 1.00,$$

hence

$$pdf^{(2)}(x, \text{SendToER}) = (0,\ 1.00,\ 0).$$

Reset the processed pair:

$$pCF^{(2)}(\text{SendToER}, \text{WatchfulWaiting}) = 0.$$

Step 3 (anchor b = StartAntibiotics, threshold $\tau_3 = 0.60$). $\mathsf{Act}(\text{WatchfulWaiting} \mid b, \tau_3) = \mathbf{1}[0.60 \geq 0.60] = 1$. Choose *Keep* for WatchfulWaiting—we only neutralize:

$$pdf^{(3)}(x, \text{WatchfulWaiting}) = (0.55, 0.25, 0.35), \qquad pCF^{(3)}(\text{StartAntibiotics}, \text{WatchfulWaiting}) = 0.$$

Outcome (three-mode de-plithogenicated normal form). All pairwise contradictions are zero, with final triples

$$pdf^{\text{dep3}}(x, \text{StartAntibiotics}) = (0.40,\ 0.30,\ 0.50),$$
$$pdf^{\text{dep3}}(x, \text{WatchfulWaiting}) = (0.55,\ 0.25,\ 0.35),$$
$$pdf^{\text{dep3}}(x, \text{SendToER}) = (0,\ 1.00,\ 0).$$

This sequence demonstrates *Swap* (genuine reversal), *Absorb* (uncertainty consolidation), and *Keep* (neutralization without reweighting), culminating in a stable, contradiction-free representation.

4. Conclusion

In this paper, we defined and studied *Three-Mode Upside-Down Logic*, an improved version of Upside-Down Logic, together with *De-Plithogenication*, and investigated their behavior within the framework of Plithogenic Neutrosophic Sets. Looking ahead, we hope that future research will further examine the behavior of Plithogenic Neutrosophic Sets and Three-Mode Upside-Down Logic in the contexts of HyperGraphs [25–29] and SuperHyperGraphs [30–35].

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Code Availability

No code or software was developed for this study.

Clinical Trial

This study did not involve any clinical trials.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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