Linear Programming Problems Representing Dorfman's Model of Monopoly

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Abstract

In a model similar to the one discussed in chapter 3 of Robert Dorfman's dissertation on production theory we formulate the problem faced by a profit maximizing multi-product firm that is a uniform-price monopolist in the market for a single product, as a linear programming problem. Subsequently we formulate the problem faced by the same profit maximizing firm if it is a price-discriminating monopolist in the market for a single product, as a linear programming problem. We require the Marshallian market demand curve faced by the firm to be a non-increasing step function with a finite number of points of discontinuity, for the product that the firm is a monopolist.

Keywords: profit maximizing producer, monopolist, linear programming, natural monopoly

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1. Introduction: In chapter 3 of his doctoral dissertation, Robert Dorfman (in Dorfman (1951)) formulated the profit maximizing problem of a uniform-price monopolist producer, as a quadratic programming problem under the assumption that the producer faces a downward sloping continuous linear Marshallian market demand curve in each market. Our purpose here, is to see if the same or a similar problem can be represented as a linear programming problem. In order to do so, we will make the simplifying assumption that the producer is a monopolist in "only one" market and is a parametric price-taking producer in the other markets. In principle, the method we adopt in this note, can be easily (conceptually) generalized to the situation where the producer is a monopolist in more than one market. However, this would require paying an enormous price by way of notational complexity. We choose to avoid such clutter of notations here.

To formulate the profit maximizing problem as a linear programming problem, we need to assume that the Marshallian market demand curve for the product in which the producer is a monopolist, is a non-increasing "step function" that allows for a finite number of points of discontinuity. The maximum quantity that can be sold at any price, is "strictly" inversely related to the price. We do not require the ratio of the changes to be constant along the demand curve.

The interesting feature of the model is that unless the producer faces a "budget constraint", an optimal solution cannot exist if there is any scope for earning profits in the markets where the producer- as a seller- is a "price taker". Thus, if the producer is financially unconstrained, this could be one more version of a "natural monopoly" thereby extending the scope of what is discussed in considerable detail in Sharkey (1982) and more recently in Greer (2011).

The formulation of the linear programming problem for a uniform-price monopolist can almost immediately be recognised from the profit maximizing problem faced by the monopolist. We also study the problem faced by a profit maximizing price-discriminating monopolist, and in this case proving the equivalence of the economic problem with the corresponding linear programming problem, requires a detailed proof, which we provide in this note.

An important reason for formulating decision-making problems as linear programming problems is that linear programming problems can be solved using "Excel" solver.

This note is not meant to contribute anything new to the enormous literature that already exists on imperfect competition or monopoly pricing, which to an extent is also true for Dorfman (1951) relative to the seminal contribution of Joan Robinson (in Robinson (1933)). A brief history of imperfect completion is available in Dixon (2021). The work in Dorfman (1951) is a valuable methodological contribution to the theory of firm under perfect competition and when the firm is a monopolist. There is related work in the more general context of oligopoly that can be found in Goodman (1967), where the production process of each firm in the market conforms to linear activity analysis. However, the decision making problem of each firm in the oligopolistic market does not seem to be formally stated in the paper. Considering that a firm in an oligopoly is a "monopolist facing a downward sloping *residual* Marshallian demand curve", the difference between a monopolist and a firm in an oligopoly is restricted to the conjectures- or what the firm anticipates- about the behaviour of other firms in the market, that is required of a firm in oligopoly. The sole purpose of this note is to formulate the problem faced by a monopolist as a linear programming problem, and to the best of our knowledge, there is no precedent for such a formulation. With some embellishment (and a story if necessary), this note could be a continuation of the inimitable classic- Gass (1970).

2. The Model: Consider a producer who, for some positive integer 'n', produces 'n' products jointly, using a finite number of processes. The number of processes is denoted by K, where K is positive integer.

For $k \in \{1, \ldots, K\}$, let A^k be a n-dimensional column vector whose j^{th} coordinate for $j \in \{1, \ldots, n\}$ denoted by $a_{jk} \geq 0$, is the amount of the j^{th} good produced if "1 unit of money" is spent on operating the k^{th} process.

Let A be the $n \times K$ matrix whose k^{th} column is A^k.

A **production plan** is a K-dimensional column vector x all whose coordinates are non-negative, whose k^{th} coordinate denoted by x_k, is the amount of money spent on operating the k^{th} process.

An **output vector** is a n-dimensional column vector y, whose j^{th} coordinate denoted by y_j is the amount of the j^{th} output produced.

The essential idea in Robert Dorfman's model of production in a non-competitive setting for the "producer as the seller" which is discussed in chapter 3 of his doctoral dissertation (i.e. Dorfman (1951)), is that an output vector y must satisfy y = Ax for some production plan x.

Let us assume that the producer is a monopolist only in the market for the first good and is a perfectly competitive price taker in the market for the remaining products. An immediate consequence of such an assumption is that given any output vector y, all coordinates y_j for j > 1 can now be measured in "money units".

In order to model the producer's problem as a linear programming problem we assume that for some integer M ≥ 2, there is an array $<(P_m, Q_m)| m = 1, 2, \ldots, M>$ in $\mathbb{R}^2_{++}$ such that $P_m > P_{m+1}$ and $Q_m < Q_{m+1}$ for all $m \in \{1, \ldots, m-1\}$ such for all $y_1 \in (0, Q_1]$ the maximum price per unit the consumers are willing to pay for the first product is P_1,for $m \in \{2, \ldots, M\}$ and $y_1 \in (Q_{m-1}, Q_m)$ the maximum price per unit the consumers are willing to pay for the first product is P_m. For $y_1 > Q_M$ the maximum price per unit the consumers are willing to pay for the first product is a "non-negative" price per unit P_{M+1}. It is quite possible that $P_{M+1} = 0$, in which case the producer would have no incentive to produce any quantity of the first good that exceeds Q_M.

For $j \in \{1, \ldots, n\}$, let A_j denote the j^{th} row of A. Thus, for any production plan x and $j \in \{1, \ldots, n\}$, A_jx is the output of the j^{th} product.

For any production plan x, the producer's profit is at least $P_{M+1}A_1x + (\sum_{j=2}^{n} A_j)x - \sum_{k=1}^{K} x_k = (P_{M+1}A_1 + \sum_{j=2}^{n} A_j)x - \sum_{k=1}^{K} x_k$.

3. Profit Maximization Problem of an Unconstrained Producer: There are two alternative versions of the production model, the first having a budget constraint for the producer and the second having no such constraint. If the producer faces a budget constraint, then there is a positive amount of money μ, such that a production plan x must satisfy $\sum_{k=1}^{K} x_k \leq \mu$. If the producer does not face a budget constraint, then no such inequality is required to be satisfied by the production plan.

Towards a contradiction suppose that for some production plan x, $(P_{M+1}A_1 + \sum_{j=2}^{n} A_j)x - \sum_{k=1}^{K} x_k > 0$.

Then, for an **unconstrained producer** (i.e., for a producer who does not face any budget constraint), $(P_{M+1}A_1 + \sum_{j=2}^{n} A_j)tx - t\sum_{k=1}^{K} x_k > 0$ for all t > 0 and the function $t \mapsto (P_{M+1}A_1 + \sum_{j=2}^{n} A_j)tx - t\sum_{k=1}^{K} x_k$ on $(0, +\infty)$ is unbounded above.

Thus, if there exists a production plan x such that $(P_{M+1}A_1 + \sum_{j=2}^{n} A_j)x - \sum_{k=1}^{K} x_k > 0$, then no profit maximization problem of the unconstrained producer can have an optimal solution.

Thus, in order that a profit maximizing problem of an unconstrained producer has a solution, it is necessary that $(P_{M+1}A_1 + \sum_{j=2}^{n} A_j)x - \sum_{k=1}^{K} x_k \leq 0$ for all $x \in \mathbb{R}^K_+$.

This condition is equivalent to and can be formally expressed as follows.

Assumption for Unconstrained Production: For all $k \in \{1, \ldots, K\}$, $P_{M+1}a_{1k} + \sum_{j=2}^{n} a_{jk} \leq 1$.

Under the above assumption, it must be the case that for all $k \in \{1, \ldots, K\}$, $P_{M+1}a_{1k} \leq 1$ so that $P_{M+1}A_1x \leq \sum_{k=1}^{K} x_k$ for all $x \in \mathbb{R}_+^K$.

Hence, under no circumstances can the producer when producing an amount $y_1 \leq Q_M$ of the first product, increase his profit by producing a quantity of the first product greater than Q_M.

Thus, in what follows, we can reasonably assume the following:

Quantity Constraint: $y_1 \leq Q_M$.

4. The producer as a "uniform-price monopolist" in the market for the first product: As a "uniform-price monopolist" the producer sells every unit of the first product at the same price. This is the kind of monopolist that chapter 3 of Dorman (1951) is concerned about.

In this case, for each $m \in \{1, \ldots, M\}$, the producer solves the following linear programming problem denoted LP (m).

Maximize $P_m y_1 + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$ subject to $y_1 = A_1x$, $y_1 \leq Q_m$, $x \in \mathbb{R}_+^K$.

Let V_m be the maximum value of the objective function for LP (m).

The producer chooses $m^* \in \underset{m \in \{1,\ldots,M\}}{\operatorname{argmax}} V_m$ and sells the first product at a price of P_{m^*} per unit.

The optimum production plan of the producer is any solution to the following version of LP(m^*).

Maximize $P_{m^*}A_1x + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$ subject to $A_1x \leq Q_{m^*}$, $x \in \mathbb{R}_+^K$.

Consider the following linear programming problem denoted by LP-UP:

Maximize $\sum_{m=1}^{M} [P_m A_1 x^{(m)} + \sum_{j=2}^{n} A_j x^{(m)} - \sum_{k=1}^{K} x_k^{(m)}]$, subject to $A_1x^{(m)} \leq Q_m$, $x^{(m)} \in \mathbb{R}_+^K$ for all $m = 1, \ldots, M$.

It is easy to see that the solution procedure outlined above, can be represented as the linear programming problem LP-UP.

Proposition 1: The array $\langle x^{(m)*} \mid m = 1, \ldots, M \rangle$ solves the linear programming problem LP-UP <u>if and only if</u> for all $m \in \{1, 2, \ldots, M\}$, $x^{(m)*}$ solves LP(m).

5. The producer as a "price-discriminating monopolist" in the market for the first product: We will now consider the profit maximization problem of a type of monopolist, that is not discussed in Dorfman (1951) - a "price-discriminating monopolist". Unlike the case of a uniform-price monopolist, a price-discriminating monopolist can sell the first Q_1 units of the first product at a constant unit price of P_1, the next Q_2- Q_1 units at a constant unit price of P_2, and so on.

The profit maximizing problem of the price-discriminating monopolist reduces to solving the following optimization problem denoted OPT-PD.

Maximize $P_1y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$ subject to $y_1 = A_1x$, $y_1 \leq Q_M$, $z_1 = \max\{y_1 - Q_1, 0\}$, $z_m = \max\{z_{m-1} - Q_m, 0\}$, $m \in \{2, \ldots, M\}$, $x \in \mathbb{R}_+^K$.

Note 1: If for $m < M$, $z_m = 0$, then it must be the case that $z_{m+1} = 0$.

Consider the following linear programming problem denoted LP-PD.

Maximize $P_1y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$ subject to $y_1 = A_1x$, $y_1 \leq Q_M$, $z_1 \geq y_1 - Q_1$, $z_m \geq z_{m-1} - Q_m$, $m \in \{2, \ldots, M\}$, $z_m \geq 0$, $m \in \{1, 2, \ldots, M\}$, $x \in \mathbb{R}_+^K$.

Since $P_{m+1} - P_m < 0$ for all $m \in \{1, 2, \ldots, M\}$, the solution set of OPT-PD is the same the solution set of LP-PD. This is what the next proposition is all about.

Proposition 2: x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ solve OPT-PD <u>if and only if</u> they solve LP-PD.

Proof: (I) Suppose x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ solve OPT-PD and suppose x, y_1, $<z_m| m \in \{1, 2, \ldots, M\}>$ satisfyy$_1 = A_1x$, $y_1 \leq Q_M$, $z_1 \geq y_1 - Q_1$, $z_m \geq z_{m-1} - Q_m$, $m \in \{2, \ldots, M\}$, $z_m \geq 0$, $m \in \{1, 2, \ldots, M\}$, $x \in \mathbb{R}_+^K$.

We define the array $<\xi_m| m \in \{1, 2, \ldots, M\}>$ as follows:

$\xi_1 = \max\{y_1 - Q_1, 0\}$ and having defined ξ_m for $m \in \{1, \ldots, M-1\}$ let $\xi_{m+1} = \max\{\xi_m - Q_{m+1}, 0\}$.

Thus, $z_1 \geq \max\{y_1 - Q_1, 0\} = \xi_1$.

$z_1 \geq \xi_1$ implies $z_2 \geq \max\{z_1 - Q_2, 0\} \geq \max\{\xi_1 - Q_2, 0\} = \xi_2$.

If $z_m \geq \xi_m$ for some $m \in \{1, \ldots, M-1\}$, $z_{m+1} \geq \max\{z_m - Q_{m+1}, 0\} \geq \max\{\xi_m - Q_{m+1}, 0\} = \xi_{m+1}$.

Thus, $z_m \geq \xi_m$ for all $m \in \{1, \ldots, M\}$.

Clearly x, y_1, $<\xi_m| m \in \{1, 2, \ldots, M\}>$ satisfy the constraints of OPT-PD and hence $P_1y_1^* + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m^* + \sum_{j=2}^{n} A_j x^*$ - $\sum_{k=1}^{K} x_k^* \geq P_1y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)\xi_m + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$.

Since $P_{m+1} - P_m < 0$ and $z_m \geq \xi_m$ for all $m \in \{1, 2, \ldots, M\}$ it must be the case that $(P_{m+1} - P_m)z_m \leq (P_{m+1} - P_m)\xi_m$ for all $m \in \{1, 2, \ldots, M\}$.

Thus, $\sum_{m=1}^{M}(P_{m+1} - P_m)\xi_m \geq \sum_{m=1}^{M}(P_{m+1} - P_m)z_m$.

Hence, $P_1y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)\xi_m + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k \geq P_1y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$.

Hence, $P_1y_1^* + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m^* + \sum_{j=2}^{n} A_j x^*$ - $\sum_{k=1}^{K} x_k^* \geq P_1y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m + \sum_{j=2}^{n} A_j x$ - $\sum_{k=1}^{K} x_k$.

Further, [$y_1^* = A_1x^*$, $y_1^* \leq Q_M$, $z_1^* = \max\{y_1^* - Q_1, 0\}$, $z_m^* = \max\{z_{m-1}^* - Q_m, 0\}$, $m \in \{2, \ldots, M\}$, $x^* \in \mathbb{R}_+^K$] <u>implies</u> [$y_1^* = A_1x^*$, $y_1^* \leq Q_M$, $z_1^* \geq y_1^* - Q_1$, $z_m^* \geq z_{m-1}^* - Q_m$, $m \in \{2, \ldots, M\}$, $z_m^* \geq 0$, $m \in \{1, \ldots, M\}$, $x^* \in \mathbb{R}_+^K$].

Thus, x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ satisfy the constraints of LP-PD.

Hence, x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ solve LP-PD.

(II) To prove the converse, suppose x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ solve LP-PD.

Thus, $y_1^* = A_1x^*$, $y_1^* \leq Q_M$, $z_1^* \geq y_1^* - Q_1$, $z_m^* \geq z_{m-1}^* - Q_m$, $m \in \{2, \ldots, M\}$, $z_m^* \geq 0$, $m \in \{1, \ldots, M\}$, $x \in \mathbb{R}_+^K$.

An equivalent representation of the constraints would be $y_1^* = A_1x^*$, $y_1^* \leq Q_M$, $z_1^* \geq \max\{y_1^* - Q_1, 0\}$, $z_m^* \geq \max\{z_{m-1}^* - Q_m, 0\}$, $m \in \{2, \ldots, M\}$, $x^* \in \mathbb{R}_+^K$.

Towards a contradiction suppose, <u>either</u> $z_1^* > \max\{y_1^* - Q_1, 0\}$ <u>or</u> $\{m| z_m^* > \max\{z_{m-1}^* - Q_m, 0\}, m \in \{2, \ldots, M\}\} \neq \phi$.

Let $\xi_1 = \max\{y_1^* - Q_1, 0\}$ and for $m \in \{2, \ldots, M\}$, let $\xi_m = \max\{\xi_{m-1} - Q_m, 0\}$.

Clearly $\xi_1 \leq z_1^*$ and for $m \in \{1, \ldots, M-1\}$, $\xi_m \leq z_m^*$ implies $\xi_{m+1} = \max\{\xi_m - Q_{m+1}, 0\} \leq \max\{z_m^* - Q_{m+1}, 0\} \leq z_{m+1}^*$.

Thus, $\xi_m \leq z_m^*$ for all $m \in \{1, \ldots, M\}$.

Let $m^0 = 1$ if $z_1^* > \max\{y_1^* - Q_1, 0\}$ and if $z_1^* = \max\{y_1^* - Q_1, 0\}$, let $m^0 = \min\ \{m| z_m^* > \max\{z_{m-1}^* - Q_m, 0\}, m \in \{2, \ldots, M\}\}$.

If $m^0 = 1$, then $z_1^* > \xi_1$ and if $m^0 > 1$, then $z_m^* = \xi_m$ for $m = 1, \ldots, m^0-1$, $z_{m^0}^* > \xi_{m^0}$. If $m^0 < M$, then $z_m^* \geq \xi_m$ for all $m = m^0 + 1, \ldots, M$.

$[y_1^* = A_1x^*, y_1^* \leq Q_M, \xi_1 = \max\{y_1^* - Q_1, 0\}, \xi_{m+1} = \max\{\xi_m - Q_{m+1}, 0\}, m \in \{2, \ldots, M\}, x^* \in \mathbb{R}_+^K]$ <u>implies</u> $[y_1^* = A_1x^*, y_1^* \leq Q_M, \xi_1 \geq y_1^* - Q_1, \xi_{m+1} \geq \xi_m - Q_{m+1}, m \in \{2, \ldots, M\}, \xi_m \geq 0, m \in \{1, \ldots, M\}, x^* \in \mathbb{R}_+^K]$.

Thus, x^*, y_1^*, $<\xi_m| m \in \{1, 2, \ldots, M\}>$ satisfies the constraints of LP-PD.

Since $P_{m+1} - P_m < 0$ and $z_m^* \geq \xi_m$ for all $m \in \{1, 2, \ldots, M\}$, $z_{m^0}^* > \xi_{m^0}$ it must be the case that $(P_{m+1} - P_m)z_m^* \leq (P_{m+1} - P_m)\xi_m$ for all $m \in \{1, 2, \ldots, M\}$ and $(P_{m^0+1} - P_{m^0})z_{m^0}^* < (P_{m^0+1} - P_{m^0})\xi_{m^0}$.

Thus, $\sum_{m=1}^{M}(P_{m+1} - P_m)\xi_m > \sum_{m=1}^{M}(P_{m+1} - P_m)z_m^*$.

Hence, $P_1y_1^* + \sum_{m=1}^{M}(P_{m+1} - P_m)\xi_m + \sum_{j=2}^{n} A_jx^* - \sum_{k=1}^{K} x_k^* > P_1y_1^* + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m^* + \sum_{j=2}^{n} A_jx^* - \sum_{k=1}^{K} x_k^*$.

This contradicts the hypothesis that x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ solve LP-PD.

Thus, it must be the case that $z_1^* = \max\{y_1^* - Q_1, 0\}$, $z_m^* = \max\{z_{m-1}^* - Q_m, 0\}$, $m \in \{2, \ldots, M\}$.

Hence, x^*, y_1^*, $<z_m^*| m = 1, 2, \ldots, M>$ satisfies all the constraints of OPT-PD.

Let x, y_1, $<z_m| m \in \{1, 2, \ldots, M\}>$ satisfy $y_1 = A_1x$, $y_1 \leq Q_M$, $z_1 = \max\ \{y_1 - Q_1, 0\}$, $z_m = \max\{ z_{m-1} - Q_m, 0\}$, $m \in \{2, \ldots, M\}$, $x \in \mathbb{R}_+^K$.

Then, x, y_1, $\langle z_m \mid m \in \{1, 2, \ldots, M\}\rangle$ satisfies $y_1 = A_1x$, $y_1 \leq Q_M$, $z_1 \geq y_1 - Q_1$, $z_m \geq z_{m-1} - Q_m$, $m \in \{2, \ldots, M\}$, $z_m \geq 0$, $m \in \{1, 2, \ldots, M\}$, $x \in \mathbb{R}_+^K$.

Thus, x, y_1, $\langle z_m \mid m \in \{1, 2, \ldots, M\}\rangle$ satisfies all the constraints of LP-PD.

Since, x^*, y_1^*, $\langle z_m^* \mid m = 1, 2, \ldots, M\rangle$ solve LP-PD it must be that $P_1 y_1^* + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m^* + \sum_{j=2}^{n} A_j x^* - \sum_{k=1}^{K} x_k^* \geq P_1 y_1 + \sum_{m=1}^{M}(P_{m+1} - P_m)z_m + \sum_{j=2}^{n} A_j x - \sum_{k=1}^{K} x_k$.

Thus, x^*, y_1^*, $\langle z_m^* \mid m = 1, 2, \ldots, M\rangle$ solve OPT-PD. Q.E.D.

To give a simple illustration of how a price-discriminating monopolist chooses the price for the first product, we provide an example.

Example 1: Suppose n = 1.

In this case, the producer might as well use any one process, which yields at least as much output for one unit of money spent on operating the process than any other output. Without loss of generality process 1 is such a process and $a_{11} > 0$.

If $P_1 a_{11} < 1$, then the producer produces nothing at all.

If $P_1 a_{11} = 1$, then for any $x_1 \in [0, \frac{Q_1}{a_{11}}]$, the profit earned by the producer is zero and for any $x_1 > \frac{Q_1}{a_{11}}$, the profit earned by the producer.

If $P_1 a_{11} > 1$, then the profit-maximizing output of the producer $q_1 = \max\{Q_m \mid P_m a_{11} \geq 1, m \in \{1, \ldots, M\}\}$.

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