Upside-Down Logic in Double-Valued and Multi-Valued Plithogenic Fuzzy Sets: Toward Handling Two or More Contradiction Values

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Abstract

In the real world, many reversal phenomena occur, such as cases where a statement once regarded as false is later recognized as true. *Upside-Down Logic* is a framework designed to formalize such reversal phenomena within a logical system. It inverts the truth and falsity of propositions through contextual transformations, thereby capturing ambiguity and reversals in reasoning processes. A *Plithogenic Set* models elements by means of attribute-based membership and contradiction functions, extending the classical frameworks of fuzzy, intuitionistic, and neutrosophic sets. The essential difference between Plithogenic Sets and traditional Fuzzy Sets lies in their ability to incorporate contradiction parameters, which naturally arise when uncertainty is transformed into certainty.

While most studies assume that only a single contradiction parameter exists, this paper investigates how to handle situations where two or more contradiction values are present, in conjunction with the principles of Upside-Down Logic. Furthermore, we also examine methods for eliminating contradictions altogether.

Keywords: Upside-down-logic, Plithogenic Set, De-Plithogenication, Double-Valued Plithiogenic Fuzzy Set

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1 Preliminaries

In this section we gather the basic notions and notation used throughout the paper. Unless explicitly stated otherwise, all sets and structures are finite. All graphs under consideration are simple and undirected.

1.1 Formal Definition of Upside-Down Logic

We now formalize *Upside-Down Logic*, a framework that models reversal phenomena—situations where context switches turn formerly false statements into true ones and vice versa [1–6]. The required terminology is introduced below.

Definition 1.1 (Context). [7, 8] A *context* C is a collection of parameters that determine how propositions are evaluated (e.g., spatial, temporal, semantic, or interpretive conditions).

Notation 1.2. *Let $\mathcal{P}$ be a set of propositions and C a set of contexts. The* truth valuation

$$T : \mathcal{P} \times C \longrightarrow \{\text{True, False, Indeterminate}\}$$

assigns to each pair (A, C) a truth value.

Notation 1.3. *Let $\mathcal{L}$ be a formal language, and let $\mathcal{M} = (\mathcal{P}, \mathcal{V}, v)$ be a logical system whose propositions are $\mathcal{P}$, truth-value set is $\mathcal{V}$, and valuation is $v : \mathcal{P} \to \mathcal{V}$.*

Definition 1.4 (Upside-Down Logic)**.** [7,8] An *Upside-Down Logic* is a logical system $\mathcal{M}'$ obtained from $\mathcal{M}$ by introducing a transformation U acting on propositions and/or contexts, with the following properties:

1. For every proposition $A \in \mathcal{P}$ and context C:
   - (*True→False switch*) If $v(A) = \text{True}$ in C, then $v\big(U(A)\big) = \text{False}$ in $U(C)$.
   - (*False→True switch*) If $v(A) = \text{False}$ in C, then $v\big(U(A)\big) = \text{True}$ in $U(C)$.
2. The operator U is well defined and yields a consistent system $\mathcal{M}'$.

Intuitively, U captures how changing the ambient conditions can invert truth values, thereby formalizing ambiguity and context-driven reversals in reasoning.

Example 1.5 (Public-health policy reversal under changing epidemiological context)**.** Let the proposition

$$A := \text{“Mass events with} > 1000 \text{ attendees are permitted.”}, \qquad B := \text{“An indoor mask mandate is in effect.”}$$

Contexts encode epidemiological conditions $C = (R_t, \text{ICU})$, where R_t is the effective reproduction number and $\text{ICU} \in [0, 1]$ is the ICU occupancy ratio. Define the truth valuation T by the threshold rules

$$T(A, (R_t, \text{ICU})) = \mathbf{1}\big[\, R_t < 1.0 \ \wedge\ \text{ICU} < 0.50 \,\big], \qquad T(B, (R_t, \text{ICU})) = \mathbf{1}\big[\, R_t \geq 1.2 \ \vee\ \text{ICU} \geq 0.80 \,\big].$$

Consider a “stable” context $C_0 = (0.90, 0.40)$ and a “surge” context $C_1 = (1.30, 0.85)$. Define the Upside-Down transform to act on contexts by $U(C_0) = C_1$ (leaving the propositions unchanged).

Evaluation.

$$T(A, C_0) = \mathbf{1}[\, 0.90 < 1.0 \ \wedge\ 0.40 < 0.50 \,] = 1, \quad T(A, U(C_0)) = T(A, C_1) = \mathbf{1}[\, 1.30 < 1.0 \ \wedge\ 0.85 < 0.50 \,] = 0,$$
$$T(B, C_0) = \mathbf{1}[\, 0.90 \geq 1.2 \ \vee\ 0.40 \geq 0.80 \,] = 0, \quad T(B, U(C_0)) = T(B, C_1) = \mathbf{1}[\, 1.30 \geq 1.2 \ \vee\ 0.85 \geq 0.80 \,] = 1.$$

Thus A exhibits a *True→False* switch and B a *False→True* switch under the context transform U, illustrating how policy truth values invert when epidemiological conditions change.

Example 1.6 (Credit underwriting reversal under a macroeconomic shift)**.** Let the propositions

$$A := \text{“The loan is auto-approved.”}, \qquad B := \text{“Manual risk review is required.”}$$

Contexts encode applicant/state variables $C = (d, s)$, where $d \in [0, 1]$ is the debt-to-income ratio (DTI) and $s \in \mathbb{R}$ is a credit score. Define

$$T(A, (d, s)) = \mathbf{1}\big[\, d \leq 0.35 \ \wedge\ s \geq 680 \,\big], \qquad T(B, (d, s)) = \mathbf{1}\big[\, d > 0.40 \ \vee\ s < 650 \,\big].$$

Consider a baseline context $C_0 = (0.32, 700)$ and a stressed context $C_1 = (0.42, 660)$ (e.g., interest-rate increase raising DTI and a temporary score dip). Define the Upside-Down transform on contexts by $U(C_0) = C_1$.

Evaluation.

$$T(A, C_0) = \mathbf{1}[\, 0.32 \leq 0.35 \ \wedge\ 700 \geq 680 \,] = 1, \quad T(A, U(C_0)) = T(A, C_1) = \mathbf{1}[\, 0.42 \leq 0.35 \ \wedge\ 660 \geq 680 \,] = 0,$$
$$T(B, C_0) = \mathbf{1}[\, 0.32 > 0.40 \ \vee\ 700 < 650 \,] = 0, \quad T(B, U(C_0)) = T(B, C_1) = \mathbf{1}[\, 0.42 > 0.40 \ \vee\ 660 < 650 \,] = 1.$$

Hence A flips *True→False* and B flips *False→True* under U, capturing a practical underwriting reversal induced by a macroeconomic context change.

1.2 Plithogenic Set

A Plithogenic Set [9–12] models elements with attribute-based membership and contradiction functions, extending fuzzy [13, 14], intuitionistic [15, 16], and neutrosophic sets [17, 18]. Plithogenic Fuzzy Set models elements via attribute-based memberships and contradiction degrees, generalizing fuzzy, intuitionistic, neutrosophic sets to handle multi-attribute conflict [19–21].

Definition 1.7 (Plithogenic Set)**.** [9, 22] Let S be a universal set and $P \subseteq S$ a nonempty subset. A *Plithogenic Set* is a quintuple

$$PS = (P, v, Pv, pdf, pCF),$$

where

- v is an attribute,
- Pv is the set of possible values of the attribute v,
- $pdf : P \times Pv \to [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)*,[1]
- $pCF : Pv \times Pv \to [0, 1]^t$ is the *Degree of Contradiction Function (DCF)*.

The DCF satisfies, for all $a, b \in Pv$,

$$\text{Reflexivity: } pCF(a, a) = 0, \qquad \text{Symmetry: } pCF(a, b) = pCF(b, a).$$

Here $s \in \mathbb{N}$ is the appurtenance dimension and $t \in \mathbb{N}$ the contradiction dimension.

Definition 1.8 (Plithogenic Fuzzy Set ($s = 1, t = 1$))**.** [9, 24] A *Plithogenic Fuzzy Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \longrightarrow [0, 1], \qquad pCF : Pv \times Pv \longrightarrow [0, 1].$$

For $x \in P$ and attribute value $a \in Pv$, write

$$\mu_P(x \mid a) := pdf(x, a) \in [0, 1],$$

the (single) fuzzy membership degree of x w.r.t. a. The contradiction between two attribute values is the scalar $c(a, b) := pCF(a, b) \in [0, 1]$ with $c(a, a) = 0$ and $c(a, b) = c(b, a)$.

Example 1.9 (Plithogenic Fuzzy Set in grocery choices: plant-based friendly milk (anchor = Plant))**.** Let items (products) be $P = \{\mathrm{CM}, \mathrm{LM}, \mathrm{SM}, \mathrm{AM}\}$ with CM = cow's milk, LM = lactose-free cow's milk, SM = soy milk, AM = almond milk. Take the attribute v = "dietary style" with values $Pv = \{\mathrm{D}, \mathrm{L}, \mathrm{P}\}$ for Dairy / Lactose-free / Plant. For $x \in P$ and $a \in Pv$ write $\mu(x \mid a) := pdf(x, a) \in [0, 1]$. Choose the anchor $b = \mathrm{P}$ (we care about plant compatibility). The (symmetric) contradiction is $c(a, b) := pCF(a, b) \in [0, 1]$ with

$$c(\mathrm{D}, \mathrm{P}) = 0.95, \qquad c(\mathrm{L}, \mathrm{P}) = 0.40, \qquad c(\mathrm{P}, \mathrm{P}) = 0.$$

Memberships.

	$\mu(\cdot \mid \mathrm{D})$	$\mu(\cdot \mid \mathrm{L})$	$\mu(\cdot \mid \mathrm{P})$
CM	0.98	0.05	0.00
LM	0.70	0.95	0.00
SM	0.05	0.95	0.98
AM	0.02	0.90	0.98

Anchor-relative acceptance (flip–blend average). For each product x, define

$$D_{\mathrm{P}}(x) = \frac{1}{3} \sum_{a \in \{\mathrm{D}, \mathrm{L}, \mathrm{P}\}} \Big((1 - c(a, \mathrm{P}))\, \mu(x \mid a) + c(a, \mathrm{P})\, (1 - \mu(x \mid a)) \Big) \in [0, 1].$$

[1] In the literature, DAF is defined in slightly different ways: some variants use powerset–valued constructions, others the simple cube $[0, 1]^s$. We adopt the latter (classical) form here; cf. [23].

Let $T_a(x)$ denote each summand.

CM: $T_{\mathrm{D}} = 0.05 \cdot 0.98 + 0.95 \cdot 0.02 = 0.068$, $T_{\mathrm{L}} = 0.60 \cdot 0.05 + 0.40 \cdot 0.95 = 0.410$, $T_{\mathrm{P}} = 1 \cdot 0.00 = 0.00$; so $D_{\mathrm{P}}(\mathrm{CM}) = (0.068 + 0.410 + 0.000)/3 \approx 0.1593$.

LM: $T_{\mathrm{D}} = 0.05 \cdot 0.70 + 0.95 \cdot 0.30 = 0.320$, $T_{\mathrm{L}} = 0.60 \cdot 0.95 + 0.40 \cdot 0.05 = 0.590$, $T_{\mathrm{P}} = 0$; so $D_{\mathrm{P}}(\mathrm{LM}) = (0.320 + 0.590 + 0)/3 \approx 0.3033$.

SM: $T_{\mathrm{D}} = 0.05 \cdot 0.05 + 0.95 \cdot 0.95 = 0.905$, $T_{\mathrm{L}} = 0.60 \cdot 0.95 + 0.40 \cdot 0.05 = 0.590$, $T_{\mathrm{P}} = 0.98$; so $D_{\mathrm{P}}(\mathrm{SM}) = (0.905 + 0.590 + 0.980)/3 = 0.8250$.

AM: $T_{\mathrm{D}} = 0.05 \cdot 0.02 + 0.95 \cdot 0.98 = 0.932$, $T_{\mathrm{L}} = 0.60 \cdot 0.90 + 0.40 \cdot 0.10 = 0.580$, $T_{\mathrm{P}} = 0.98$; so $D_{\mathrm{P}}(\mathrm{AM}) = (0.932 + 0.580 + 0.980)/3 \approx 0.8307$.

With plant as the anchor, the plithogenic acceptance ranks AM (0.831) $\approx$ SM (0.825) $\gg$ LM (0.303) $>$ CM (0.159). The contradiction term $c(a, \mathrm{P})$ strongly penalizes Dairy against the Plant anchor, while allowing Lactose-free to contribute partially.

1.3 Upside-Down Logic in Plithogenic Fuzzy Set with contradiction reset

An Upside-Down Logic in PFS with contradiction reset is a context-triggered flip in plithogenic fuzzy memberships; activated attribute–anchor pairs invert degrees, then set all contradiction coefficients to zero afterward [25].

Definition 1.10 (Upside-Down Logic in Plithogenic Fuzzy Set with contradiction reset)**.** [25] Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Fuzzy Set with

$$\mu_P(x \mid a) := pdf(x, a) \in [0,1], \qquad c(a,b) := pCF(a,b) \in [0,1],$$

and $c(a,a) = 0,\ c(a,b) = c(b,a)$. Fix a reference (anchor) attribute $b \in Pv$ and a threshold $\tau \in [0,1]$. Define the *activation set*

$$A_\tau(b) := \{ a \in Pv : c(a,b) \geq \tau \}.$$

The *Upside-Down transform with contradiction reset* produces a new Plithogenic Fuzzy Set

$$PS^{U_{b,\tau}} = (P,\ v,\ Pv,\ pdf^{U_{b,\tau}},\ pCF^{U_{b,\tau}})$$

by

$$pdf^{U_{b,\tau}}(x,a) := \begin{cases} 1 - \mu_P(x \mid a), & a \in A_\tau(b), \\ \mu_P(x \mid a), & a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}}$ defined for all $u, v \in Pv$ by

$$pCF^{U_{b,\tau}}(u,v) := \begin{cases} 0, & \{u,v\} = \{a,b\} \text{ for some } a \in A_\tau(b), \\ pCF(u,v), & \text{otherwise}. \end{cases}$$

That is, whenever the flip is triggered for the pair (a,b) (i.e. $a \in A_\tau(b)$), the post-transform contradiction between a and b is *reset to zero*.

Example 1.11 (Real-life scenarios of the Upside-Down transform with contradiction reset)**.** We instantiate the definition with concrete numbers and compute the activation set $A_\tau(b)$, the flipped membership $pdf^{U_{b,\tau}}$, and the reset contradiction $pCF^{U_{b,\tau}}$.

(a) Workplace policy reversal: remote-first → on-site emphasis (context flip). Let $P = \{\mathrm{E1}, \mathrm{E2}, \mathrm{E3}\}$ be employees, v = "work mode", and $Pv = \{\mathrm{O}, \mathrm{H}, \mathrm{R}\}$ for On-site, Hybrid, Remote. Choose anchor $b = \mathrm{R}$ (remote-first policy) and threshold $\tau = 0.7$.

Initial memberships (fuzzy suitability by mode):

	$\mu(\cdot \mid \mathrm{O})$	$\mu(\cdot \mid \mathrm{H})$	$\mu(\cdot \mid \mathrm{R})$
E1	0.20	0.60	0.90
E2	0.70	0.50	0.40
E3	0.80	0.30	0.20

Contradiction matrix (symmetric; zeros on diagonal):

$$c(\cdot,\cdot) = \begin{array}{c|ccc} & \text{O} & \text{H} & \text{R} \\ \hline \text{O} & 0 & 0.50 & \mathbf{0.85} \\ \text{H} & 0.50 & 0 & 0.40 \\ \text{R} & \mathbf{0.85} & 0.40 & 0 \end{array}$$

Activation set for $b = \text{R}$ at $\tau = 0.7$:

$$A_\tau(\text{R}) = \{\text{O}\} \quad \big(\text{since } c(\text{O},\text{R}) = 0.85 \geq 0.7,\ c(\text{H},\text{R}) = 0.40 < 0.7\big).$$

Apply the transform $pdf^{U_{b,\tau}}(x,a) = \begin{cases} 1-\mu(x \mid a), & a \in A_\tau(b), \\ \mu(x \mid a), & a \notin A_\tau(b). \end{cases}$ Only the O-column flips:

	$\mu^U(\cdot \mid \text{O})$	$\mu^U(\cdot \mid \text{H})$	$\mu^U(\cdot \mid \text{R})$
E1	$1-0.20 = 0.80$	0.60	0.90
E2	$1-0.70 = 0.30$	0.50	0.40
E3	$1-0.80 = 0.20$	0.30	0.20

Contradiction reset on the activated pair $\{\text{O},\text{R}\}$:

$$pCF^{U_{b,\tau}}(\text{O},\text{R}) = pCF^{U_{b,\tau}}(\text{R},\text{O}) = 0, \quad \text{all other entries unchanged.}$$

Interpretation: in the new (Upside-Down) context, modes most contradictory to the anchor are flipped (on-site "suitability" becomes its complement), and the policy resolves the O–R conflict by resetting their contradiction to 0.

(b) Food-safety reclassification after improved testing (context flip). Let $P = \{\text{Le},\text{Be},\text{Me}\}$ denote lettuce, beans (canned), and meat, v = "contamination risk", $Pv = \{\text{L},\text{M},\text{H}\}$ for Low/Medium/High. Choose anchor $b = \text{L}$ (company prioritizes low risk) and $\tau = 0.9$.

Initial memberships (risk assessments):

	$\mu(\cdot \mid \text{L})$	$\mu(\cdot \mid \text{M})$	$\mu(\cdot \mid \text{H})$
Le	0.70	0.30	0.10
Be	0.60	0.40	0.20
Me	0.20	0.50	0.80

Contradictions (symmetric):

$$c(\cdot,\cdot) = \begin{array}{c|ccc} & \text{L} & \text{M} & \text{H} \\ \hline \text{L} & 0 & 0.50 & \mathbf{0.95} \\ \text{M} & 0.50 & 0 & 0.60 \\ \text{H} & \mathbf{0.95} & 0.60 & 0 \end{array}$$

Activation set:

$$A_\tau(\text{L}) = \{\text{H}\} \quad \big(\text{since } c(\text{H},\text{L}) = 0.95 \geq 0.9,\ c(\text{M},\text{L}) = 0.50 < 0.9\big).$$

Flip only the H-column:

	$\mu^U(\cdot \mid \text{L})$	$\mu^U(\cdot \mid \text{M})$	$\mu^U(\cdot \mid \text{H})$
Le	0.70	0.30	$1-0.10 = 0.90$
Be	0.60	0.40	$1-0.20 = 0.80$
Me	0.20	0.50	$1-0.80 = 0.20$

Reset the contradiction on the activated pair:

$$pCF^{U_{b,\tau}}(\text{H},\text{L}) = pCF^{U_{b,\tau}}(\text{L},\text{H}) = 0, \quad \text{others unchanged.}$$

Interpretation: in the Upside-Down context (e.g. after revised lab protocols), the assessment attached to "High" reverses numerically, and the strongest conflict (H vs L) is declared resolved by setting its contradiction to 0.

1.4 Two-mode De-Plithogenication in Plithogenic Fuzzy Set

We define Two-mode De-Plithogenication as follows. This method is designed to handle both cases: when a contradiction is a genuine conflict, and when the contradiction was not in fact real.

Definition 1.12 (Improved De-Plithogenication (two-mode, general plithogenic set))**.** Let $PS = (P, v, Pv, \mathrm{pdf}, \mathrm{pCF})$ be a plithogenic set, where $\mathrm{pdf} : P \times Pv \to [0,1]$ is the Degree of Appurtenance Function (DAF) and $\mathrm{pCF} : Pv \times Pv \to [0,1]$ is the Degree of Contradiction Function (DCF), symmetric with $\mathrm{pCF}(a,a) = 0$. Fix an *anchor* $b \in Pv$ and a *threshold* $\tau \in [0,1]$.

Activation. An attribute value $a \in Pv$ is said to *activate (w.r.t.* b, τ*)* if

$$\mathsf{Act}(a \mid b, \tau) \ := \ \mathbf{1}\,[\,\mathrm{pCF}(a,b) \ \geq \ \tau\,] = 1.$$

Mode selection. Let $\mathcal{M} : Pv \times Pv \to \{0,1\}$ be a (user/policy) *mode selector* on unordered pairs: $\mathcal{M}(\{a,b\}) = \mathcal{M}(\{b,a\})$ with the following semantics for an activated pair $\{a,b\}$:

- Mode 0 (Neutralize-only): $\mathcal{M}(\{a,b\}) = 0$ means "the apparent contradiction was not a real conflict"; we *do not flip* any appurtenance coordinates tied to a (or b) and we simply reset their contradiction to 0.
- Mode 1 (Invert+Neutralize): $\mathcal{M}(\{a,b\}) = 1$ means "the contradiction is genuine"; we *invert* the appurtenance coordinates tied to a (or b) and then reset their contradiction to 0.

Two-mode Upside-Down operator. Define $U^{\mathrm{imp}}_{b,\tau,\mathcal{M}}(PS) =: (P, v, Pv, \mathrm{pdf}^U, \mathrm{pCF}^U)$ by

$$\mathrm{pdf}^U(x,a) := \begin{cases} 1 - \mathrm{pdf}(x,a), & \mathsf{Act}(a \mid b,\tau) = 1 \text{ and } \mathcal{M}(\{a,b\}) = 1, \\ \mathrm{pdf}(x,a), & \text{otherwise,} \end{cases} \qquad (x \in P, a \in Pv),$$

$$\mathrm{pCF}^U(u,w) := \begin{cases} 0, & \mathsf{Act}(u \mid b,\tau) = 1 \text{ or } \mathsf{Act}(w \mid b,\tau) = 1, \\ \mathrm{pCF}(u,w), & \text{otherwise,} \end{cases} \qquad (u, w \in Pv, \ \text{with symmetry preserved}).$$

Improved De-Plithogenication sequence. A finite composition

$$PS^{\text{imp-dep}} \ := \ U^{\mathrm{imp}}_{b_k,\tau_k,\mathcal{M}_k} \circ \cdots \circ$$
$$U^{\mathrm{imp}}_{b_2,\tau_2,\mathcal{M}_2} \circ U^{\mathrm{imp}}_{b_1,\tau_1,\mathcal{M}_1}(PS)$$

is called an *Improved De-Plithogenication* if, for every pair $\{u,w\} \subseteq Pv$ that was ever activated in the sequence, the final contradiction is zero: $\mathrm{pCF}^{\text{imp-dep}}(u,w) = 0$. If activations cover all pairs, then $\mathrm{pCF}^{\text{imp-dep}} \equiv 0$. The operator is idempotent on the stabilized state.

Example 1.13 (Two-mode De-Plithogenication in retail packaging policy)**.** Consider a plithogenic set

$$PS = (P, v, Pv, \mathrm{pdf}, \mathrm{pCF}), \qquad \mu(x \mid a) := \mathrm{pdf}(x,a) \in [0,1], \quad c(a,b) := \mathrm{pCF}(a,b) \in [0,1],$$

modelling one product's *packaging choice* under a corporate sustainability policy.

Items (candidate packages): $P = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ where A = single-use *virgin plastic* pouch, B = *bioplastic* pouch, C = *refillable glass* jar.

Attribute and values: v = "sustainability sourcing", $Pv = \{\mathrm{VP}, \mathrm{BP}, \mathrm{RP}, \mathrm{RG}\}$, with VP = Virgin Plastic, BP = BioPlastic, RP = Recycled Paper, RG = Refillable Glass.

Anchor and threshold: choose b = RG (the corporate target) and $\tau = 0.80$.

Initial fuzzy appurtenances $\mu(x \mid a)$ (how well each item fits each value):

	$\mu(\cdot \mid \text{VP})$	$\mu(\cdot \mid \text{BP})$	$\mu(\cdot \mid \text{RP})$	$\mu(\cdot \mid \text{RG})$
A	0.90	0.40	0.20	0.10
B	0.30	0.80	0.50	0.40
C	0.05	0.30	0.60	0.95

Contradictions $c(\cdot,\cdot)$ (symmetric, zero diagonal):

	VP	BP	RP	RG
VP	0	0.40	0.75	**0.95**
BP	0.40	0	0.35	**0.85**
RP	0.75	0.35	0	0.50
RG	**0.95**	**0.85**	0.50	0

Activation set (with $b = \text{RG}$, $\tau = 0.80$):

$$A_\tau(\text{RG}) = \{\text{VP}, \text{BP}\} \quad \text{since } c(\text{VP}, \text{RG}) = 0.95,\ c(\text{BP}, \text{RG}) = 0.85\ (\geq 0.80).$$

Two-mode selection for the activated pairs:

$$\mathcal{M}(\{\text{VP}, \text{RG}\}) = 1 \quad (\text{genuine conflict: } \textit{Invert+Neutralize}),$$
$$\mathcal{M}(\{\text{BP}, \text{RG}\}) = 0 \quad (\text{false conflict: } \textit{Neutralize-only}).$$

Apply $U^{\text{imp}}_{b,\tau,\mathcal{M}}$. Memberships transform by

$$\mu^U(x \mid a) = \begin{cases} 1 - \mu(x \mid a), & a \in \{\text{VP}, \text{BP}\} \text{ and } \mathcal{M}(\{a, \text{RG}\}) = 1, \\ \mu(x \mid a), & \text{otherwise.} \end{cases}$$

Here, only the VP column flips (because $\mathcal{M}(\{\text{VP}, \text{RG}\}) = 1$ and $\mathcal{M}(\{\text{BP}, \text{RG}\}) = 0$):

$$\mu^U(\text{A} \mid \text{VP}) = 1 - 0.90 = 0.10, \quad \mu^U(\text{B} \mid \text{VP}) = 1 - 0.30 = 0.70, \quad \mu^U(\text{C} \mid \text{VP}) = 1 - 0.05 = 0.95,$$
$$\mu^U(x \mid \text{BP}) = \mu(x \mid \text{BP}), \quad \mu^U(x \mid \text{RP}) = \mu(x \mid \text{RP}), \quad \mu^U(x \mid \text{RG}) = \mu(x \mid \text{RG}).$$

Thus the post-transform table is

	$\mu^U(\cdot \mid \text{VP})$	$\mu^U(\cdot \mid \text{BP})$	$\mu^U(\cdot \mid \text{RP})$	$\mu^U(\cdot \mid \text{RG})$
A	0.10	0.40	0.20	0.10
B	0.70	0.80	0.50	0.40
C	0.95	0.30	0.60	0.95

Contradictions reset by the rule (for this two-mode operator)

$$\text{pCF}^U(u, w) = \begin{cases} 0, & u \in A_\tau(\text{RG}) \text{ or } w \in A_\tau(\text{RG}) \quad (\text{i.e. any pair involving VP or BP}), \\ \text{pCF}(u, w), & \text{otherwise,} \end{cases}$$

so every entry in the VP and BP rows/columns becomes 0:

	VP	BP	RP	RG
VP	0	0	0	0
BP	0	0	0	0
RP	0	0	0	0.50
RG	0	0	0.50	0

Lifecycle analysis identified virgin plastic (VP) as genuinely at odds with the refillable-glass target (RG), hence Mode 1 flips its memberships and neutralizes its contradictions. Subsequent regulatory guidance clarified that bioplastic (BP) was not in real conflict with the target, so Mode 0 leaves its memberships unchanged while neutralizing its contradictions. The resulting state is "de-plithogenicated" along the activated dimensions and remains stable under further applications of $U^{\text{imp}}_{b,\tau,\mathcal{M}}$ (idempotence on the stabilized state).

2 Main Result of this paper

In this section, we present the principal results of this study. We introduce and formally define both the *Double-Valued Plithogenic Fuzzy Set* and the *Multi-Valued Plithogenic Fuzzy Set*, thereby extending the framework of classical Plithogenic Fuzzy Sets. Furthermore, we examine their applications, including the integration of *Upside-Down Logic*, to demonstrate how these generalized structures can capture and resolve multiple forms of contradiction in real-world contexts.

2.1 Double-Valued Plithiogenic Fuzzy Set

A Double-Valued Plithogenic Fuzzy Set assigns each element a fuzzy membership degree and two independent contradiction values, modeling dual conflict dimensions.

Definition 2.1 (Double-Valued Plithogenic Fuzzy Set)**.** Let S be a universe and $P \subseteq S$ a nonempty set. Fix an attribute v with value-set $Pv \neq \emptyset$. A *Double-Valued Plithogenic Fuzzy Set* (*DV-PFS*) on P (with parameters $s = 1,\ t = 2$) is a quintuple

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

where

- $pdf : P \times Pv \to [0,1]$ is the (scalar) *Degree of Appurtenance Function* assigning $\mu(x \mid a) := pdf(x,a) \in [0,1]$ to each $(x,a) \in P \times Pv$;
- $pCF : Pv \times Pv \to [0,1]^2$ is the *Double-Valued Degree of Contradiction Function* assigning to each pair (a,b) the vector

  $$pCF(a,b) = \big(c_1(a,b),\ c_2(a,b)\big) \in [0,1]^2,$$

  whose two components model two (independent) kinds of contradiction.

The map pCF satisfies, for all $a, b \in Pv$ and for each $k \in \{1,2\}$,

$$c_k(a,a) = 0 \qquad \text{and} \qquad c_k(a,b) = c_k(b,a).$$

We write $\mathbf{c}(a,b) := (c_1(a,b), c_2(a,b))$ for brevity.

Remark 2.2 (Effective (scalar) contradiction from two components)**.** In applications one often needs a single coefficient in $[0,1]$ to modulate fuzzy operations. Fix a weight vector $\omega = (\omega_1, \omega_2)$ with $\omega_1, \omega_2 \in [0,1]$ and $\omega_1 + \omega_2 = 1$. Define the *effective contradiction* (a convex scalarization)

$$c_\omega(a,b) \ :=\ \omega_1\, c_1(a,b)\ +\ \omega_2\, c_2(a,b)\ \in [0,1].$$

Then $c_\omega(a,a) = 0$ and $c_\omega(a,b) = c_\omega(b,a)$ follow immediately from Definition 2.1.

Definition 2.3 (DV-plithogenic logical operators (relative to an anchor))**.** Fix a *dominant (anchor) value* $b \in Pv$ and a continuous t-norm T with its dual t-conorm S (e.g. product and probabilistic sum, or minimum and maximum). For two fuzzy degrees $x, y \in [0,1]$ attached to the same attribute value $a \in Pv$, define the *DV-plithogenic intersection* and *union* relative to b by

$$x \ \wedge^{\mathrm{DV}}_{(a|b,\omega)}\ y := \ (1 - c_\omega(a,b))\, T(x,y)\ +\ c_\omega(a,b)\, S(x,y),$$
$$x \ \vee^{\mathrm{DV}}_{(a|b,\omega)}\ y := \ (1 - c_\omega(a,b))\, S(x,y)\ +\ c_\omega(a,b)\, T(x,y),$$

where $c_\omega(a,b)$ is the scalar effective contradiction from Remark 2.2. (When $t = 1$ these reduce to the standard plithogenic operators.)

Proposition 2.4 (Well-definedness and reductions)**.** *Let PS be a DV-PFS and let $b \in Pv$, $\omega \in [0,1]^2$ with $\omega_1 + \omega_2 = 1$. Then for all $x, y \in [0,1]$ and $a \in Pv$:*

1. $x \wedge^{\mathrm{DV}}_{(a|b,\omega)} y \in [0,1]$ *and* $x \vee^{\mathrm{DV}}_{(a|b,\omega)} y \in [0,1]$.
2. *If* $\mathbf{c}(a,b) = (0,0)$ *then* $x \wedge^{\mathrm{DV}}_{(a|b,\omega)} y = T(x,y)$ *and* $x \vee^{\mathrm{DV}}_{(a|b,\omega)} y = S(x,y)$ *(classical fuzzy case).*

3. *If* $\mathbf{c}(a,b) = (1,1)$ *then the roles of* T *and* S *are swapped:* $x \wedge^{\mathrm{DV}}_{(a|b,\omega)} y = S(x,y)$ *and* $x \vee^{\mathrm{DV}}_{(a|b,\omega)} y = T(x,y)$.

Proof. (1) Since $T, S : [0,1]^2 \to [0,1]$ and $c_\omega(a,b) \in [0,1]$, each expression is a convex combination of values in $[0,1]$, hence is in $[0,1]$. (2) If $\mathbf{c}(a,b) = (0,0)$ then $c_\omega(a,b) = 0$, so the weights become $(1,0)$ and the formulas reduce to T, S. (3) If $\mathbf{c}(a,b) = (1,1)$ then $c_\omega(a,b) = 1$, so the weights become $(0,1)$ and T, S are interchanged. □

Remark 2.5 (Thresholded activation (optional))**.** When decisions depend on whether a contradiction is "strong" in at least one component, one may use either the max-activation $\max\{c_1(a,b), c_2(a,b)\} \geq \tau$ or a weighted rule $c_\omega(a,b) \geq \tau$ with $\tau \in [0,1]$. Both respect the symmetry and zero-diagonal properties.

Example 2.6 (DV-PFS in hiring: two kinds of contradiction (expertise vs. leadership))**.** We model a hiring decision with a Double-Valued Plithogenic Fuzzy Set (DV-PFS). Let the items be candidates $P = \{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$. The attribute is v = "role level" with values $Pv = \{\mathrm{J}, \mathrm{M}, \mathrm{S}\}$ (Junior/Mid/Senior). Memberships $\mu(x \mid a) := pdf(x,a) \in [0,1]$ encode how well a candidate x fits level a.

Memberships.

	$\mu(\cdot \mid \mathrm{J})$	$\mu(\cdot \mid \mathrm{M})$	$\mu(\cdot \mid \mathrm{S})$
X	0.80	0.50	0.20
Y	0.40	0.70	0.50
Z	0.10	0.50	0.90

Double contradiction. We use two independent contradiction components:

$$\mathbf{c}(a,b) = (c_1(a,b), c_2(a,b)) \in [0,1]^2,$$

where c_1 = expertise gap, c_2 = leadership gap (both symmetric, zero on the diagonal). Relative to the anchor $b = \mathrm{S}$ (company prefers Senior):

(a,b)	(J, S)	(M, S)	(S, S)
$c_1(a,b)$	0.90	0.50	0
$c_2(a,b)$	0.80	0.40	0

To obtain a single modulation coefficient, fix a weight vector $\omega = (\omega_1, \omega_2) = (0.6, 0.4)$ and define the convex scalarization

$$c_\omega(a,b) = \omega_1\, c_1(a,b) + \omega_2\, c_2(a,b).$$

Hence

$$c_\omega(\mathrm{J},\mathrm{S}) = 0.6 \cdot 0.90 + 0.4 \cdot 0.80 = 0.86, \quad c_\omega(\mathrm{M},\mathrm{S}) = 0.6 \cdot 0.50 + 0.4 \cdot 0.40 = 0.46, \quad c_\omega(\mathrm{S},\mathrm{S}) = 0.$$

Aggregation (flip–blend). For each candidate x, the anchor-relative acceptance is

$$D_{\mathrm{S}}(x) = \frac{1}{|Pv|} \sum_{a \in Pv} \Big((1 - c_\omega(a,\mathrm{S}))\, \mu(x \mid a) + c_\omega(a,\mathrm{S})\, (1 - \mu(x \mid a)) \Big) \in [0,1].$$

Calculations. Write $T_a(x) := (1 - c_\omega(a, \mathrm{S}))\, \mu(x \mid a) + c_\omega(a, \mathrm{S})\, (1 - \mu(x \mid a))$.

$$\begin{aligned}
\textbf{X:}\quad T_{\mathrm{J}} &= 0.14 \cdot 0.80 + 0.86 \cdot 0.20 = 0.112 + 0.172 = 0.284,\\
T_{\mathrm{M}} &= 0.54 \cdot 0.50 + 0.46 \cdot 0.50 = 0.27 + 0.23 = 0.50,\\
T_{\mathrm{S}} &= 0.20,\\
D_{\mathrm{S}}(\mathrm{X}) &= \tfrac{0.284+0.50+0.20}{3} = 0.328.\\
\textbf{Y:}\quad T_{\mathrm{J}} &= 0.14 \cdot 0.40 + 0.86 \cdot 0.60 = 0.056 + 0.516 = 0.572,\\
T_{\mathrm{M}} &= 0.54 \cdot 0.70 + 0.46 \cdot 0.30 = 0.378 + 0.138 = 0.516,\\
T_{\mathrm{S}} &= 0.50,\\
D_{\mathrm{S}}(\mathrm{Y}) &= \tfrac{0.572+0.516+0.50}{3} \approx 0.5293.\\
\textbf{Z:}\quad T_{\mathrm{J}} &= 0.14 \cdot 0.10 + 0.86 \cdot 0.90 = 0.014 + 0.774 = 0.788,\\
T_{\mathrm{M}} &= 0.50,\\
T_{\mathrm{S}} &= 0.90,\\
D_{\mathrm{S}}(\mathrm{Z}) &= \tfrac{0.788+0.50+0.90}{3} \approx 0.7293.
\end{aligned}$$

Result. Ranking under anchor $b = \mathrm{S}$: $\mathrm{Z} \succ \mathrm{Y} \succ \mathrm{X}$. The two contradiction channels (expertise/leadership) jointly steer the aggregation.

Example 2.7 (DV-PFS in packaging policy: environmental vs. regulatory contradiction)**.** Let the products be $P = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ with packages: A = virgin plastic pouch, B = bioplastic pouch, C = refillable glass jar. Attribute v = "sustainability sourcing" with $Pv = \{\mathrm{VP}, \mathrm{BP}, \mathrm{RP}, \mathrm{RG}\}$ (VirginPlastic/BioPlastic/RecycledPaper/RefillableGlass). Choose anchor $b = \mathrm{RG}$.

Memberships.

	$\mu(\cdot \mid \mathrm{VP})$	$\mu(\cdot \mid \mathrm{BP})$	$\mu(\cdot \mid \mathrm{RP})$	$\mu(\cdot \mid \mathrm{RG})$
A	0.90	0.20	0.10	0.05
B	0.20	0.80	0.40	0.30
C	0.05	0.30	0.60	0.95

Double contradiction. Two components are used: c_1 = environmental conflict (emissions/waste), c_2 = regulatory/safety risk conflict. Relative to $b = \mathrm{RG}$:

a	VP	BP	RP	RG
$c_1(a, \mathrm{RG})$	0.95	0.60	0.20	0
$c_2(a, \mathrm{RG})$	0.85	0.30	0.20	0

Fix $\omega = (0.7, 0.3)$; then the effective contradiction is

$$c_\omega(a, \mathrm{RG}) = 0.7\, c_1(a, \mathrm{RG}) + 0.3\, c_2(a, \mathrm{RG}),$$

hence

$$c_\omega(\mathrm{VP}, \mathrm{RG}) = 0.92, \quad c_\omega(\mathrm{BP}, \mathrm{RG}) = 0.51, \quad c_\omega(\mathrm{RP}, \mathrm{RG}) = 0.20, \quad c_\omega(\mathrm{RG}, \mathrm{RG}) = 0.$$

Aggregation (flip–blend).

$$D_{\mathrm{RG}}(x) = \frac{1}{4} \sum_{a \in Pv} \Big((1 - c_\omega(a, \mathrm{RG}))\, \mu(x \mid a) + c_\omega(a, \mathrm{RG})\, (1 - \mu(x \mid a)) \Big).$$

Calculations. Let $T_a(x)$ denote each summand.

$$\begin{aligned}
\textbf{A (VP pouch):}\quad & T_{\mathrm{VP}} = 0.08\cdot 0.90 + 0.92\cdot 0.10 = 0.072 + 0.092 = 0.164,\\
& T_{\mathrm{BP}} = 0.49\cdot 0.20 + 0.51\cdot 0.80 = 0.098 + 0.408 = 0.506,\\
& T_{\mathrm{RP}} = 0.80\cdot 0.10 + 0.20\cdot 0.90 = 0.08 + 0.18 = 0.26,\\
& T_{\mathrm{RG}} = 0.05,\\
D_{\mathrm{RG}}(\mathrm{A}) =\ & \tfrac{0.164+0.506+0.26+0.05}{4} = 0.245.\\
\textbf{B (BioPlastic):}\quad & T_{\mathrm{VP}} = 0.08\cdot 0.20 + 0.92\cdot 0.80 = 0.016 + 0.736 = 0.752,\\
& T_{\mathrm{BP}} = 0.49\cdot 0.80 + 0.51\cdot 0.20 = 0.392 + 0.102 = 0.494,\\
& T_{\mathrm{RP}} = 0.80\cdot 0.40 + 0.20\cdot 0.60 = 0.32 + 0.12 = 0.44,\\
& T_{\mathrm{RG}} = 0.30,\\
D_{\mathrm{RG}}(\mathrm{B}) =\ & \tfrac{0.752+0.494+0.44+0.30}{4} = 0.4965.\\
\textbf{C (Refillable Glass):}\quad & T_{\mathrm{VP}} = 0.08\cdot 0.05 + 0.92\cdot 0.95 = 0.004 + 0.874 = 0.878,\\
& T_{\mathrm{BP}} = 0.49\cdot 0.30 + 0.51\cdot 0.70 = 0.147 + 0.357 = 0.504,\\
& T_{\mathrm{RP}} = 0.80\cdot 0.60 + 0.20\cdot 0.40 = 0.48 + 0.08 = 0.56,\\
& T_{\mathrm{RG}} = 0.95,\\
D_{\mathrm{RG}}(\mathrm{C}) =\ & \tfrac{0.878+0.504+0.56+0.95}{4} = 0.723.
\end{aligned}$$

Result. Ranking under anchor $b = \mathrm{RG}$: $\mathrm{C} \succ \mathrm{B} \succ \mathrm{A}$. The two contradiction channels (environmental vs. regulatory) jointly penalize packages that conflict with the refillable-glass target.

2.2 Two-mode De-Plithogenication in Double-valued Plithogenic Fuzzy Set

Two-mode De-Plithogenication in a Double-Valued Plithogenic Fuzzy Set updates memberships by flipping or preserving them, while neutralizing both independent contradiction values.

Definition 2.8 (Two-mode De-Plithogenication for DV-PFS)**.** Let $PS = (P,\ v,\ Pv,\ pdf,\ pCF)$ be a *Double-Valued Plithogenic Fuzzy Set* with $pdf : P \times Pv \to [0,1]$ and

$$pCF : Pv \times Pv \longrightarrow [0,1]^2, \qquad pCF(a,b) = \big(c_1(a,b),\ c_2(a,b)\big),$$

where, for each $k \in \{1,2\}$, $c_k(a,a) = 0$ and $c_k(a,b) = c_k(b,a)$. Fix an *anchor* $b \in Pv$ and a (componentwise) threshold vector $\boldsymbol{\tau} = (\tau_1, \tau_2) \in [0,1]^2$.

Per-component activation. For $k \in \{1,2\}$ and $a \in Pv$, define

$$\mathsf{Act}_k(a \mid b, \tau_k) \ :=\ \mathbf{1}[\, c_k(a,b)\ \geq\ \tau_k \,], \qquad A^{(k)}_{\tau_k}(b)\ :=\ \{\, a \in Pv :\ \mathsf{Act}_k(a \mid b, \tau_k) = 1 \,\}.$$

Per-component mode selection. Let $\mathcal{M}_k : \{\{a,b\} : a \in Pv\} \to \{0,1\}$ be symmetric in its arguments, with the semantics (for an activated pair $\{a,b\}$):

- $\mathcal{M}_k(\{a,b\}) = 0$ **(Neutralize-only in component** k): keep memberships; set the k-th contradiction on $\{a,b\}$ to 0.
- $\mathcal{M}_k(\{a,b\}) = 1$ **(Invert+Neutralize in component** k): invert the memberships tied to value a (see below) and set the k-th contradiction on $\{a,b\}$ to 0.

Flip indicator (OR across components). For $a \in Pv$, define the *flip flag*

$$1 - (a \mid b, \boldsymbol{\tau}, \mathcal{M}_1, \mathcal{M}_2)\ :=\ \max_{k \in \{1,2\}} \Big(\mathsf{Act}_k(a \mid b, \tau_k)\cdot \mathcal{M}_k(\{a,b\})\Big)\ \in\ \{0,1\}.$$

Two-mode DV operator. The *Two-mode De-Plithogenication* of PS at $(b, \boldsymbol{\tau}, \mathcal{M}_1, \mathcal{M}_2)$ is the updated structure

$$U^{\text{imp},2}_{b,\tau,\mathcal{M}_1,\mathcal{M}_2}(PS) \;:=\; \big(P,\ v,\ Pv,\ pdf^U,\ pCF^U\big),$$

defined componentwise by, for all $x \in P$, $a, u, v \in Pv$, and $k \in \{1, 2\}$,

$$pdf^U(x, a) := \;(1 - 1 - (a \mid b, \boldsymbol{\tau}, \mathcal{M}_1, \mathcal{M}_2)) \cdot pdf(x, a) \;+\; 1 - (a \mid b, \boldsymbol{\tau}, \mathcal{M}_1, \mathcal{M}_2) \cdot (1 - pdf(x, a)),$$

$$c^U_k(u, v) := \begin{cases} 0, & \text{if } \{u, v\} = \{a, b\} \text{ for some } a \in A^{(k)}_{\tau_k}(b), \\ c_k(u, v), & \text{otherwise.} \end{cases}$$

Equivalently, at the pair level $pCF^U(u, v) = \big(c^U_1(u, v),\ c^U_2(u, v)\big)$. Thus, each contradiction component is neutralized exactly on its own activated pairs, while membership inversion (if any) occurs once per value a whenever at least one component is activated in Mode 1.

Remark 2.9 (Using DV contradictions inside scalar aggregations)**.** When a scalar modulation is required (e.g. in flip–blend aggregation), fix a weight vector $\omega = (\omega_1, \omega_2)$ with $\omega_1, \omega_2 \geq 0$, $\omega_1 + \omega_2 = 1$, and define the (post-update) effective contradiction

$$c^U_\omega(a, b) \;:=\; \omega_1\, c^U_1(a, b) \;+\; \omega_2\, c^U_2(a, b) \;\in [0, 1].$$

Substituting c^U_ω for c yields the usual plithogenic operators or flip–blend acceptances while preserving the DV structure in pCF^U.

Proposition 2.10 (Basic properties)**.** *Let* $U := U^{\text{imp},2}_{b,\tau,\mathcal{M}_1,\mathcal{M}_2}$. *For all DV-PFS PS and parameters as above:*

(i) ***Idempotence.*** $U(U(PS)) = U(PS)$.

(ii) ***Componentwise contradiction monotonicity.*** $c^U_k(u, v) \leq c_k(u, v)$ *for all* $u, v \in Pv$ *and each* $k = 1, 2$.

(iii) ***Membership range preservation.*** $pdf^U(x, a) \in [0, 1]$ *for all* $x \in P$, $a \in Pv$.

(iv) ***Reductions.*** *If* $c_2 \equiv 0$ *(or* $\tau_2 > 1$, *or* $\mathcal{M}_2 \equiv 0$*), then U reduces to the single-valued two-mode operator (with* $t = 1$*). If, moreover, all modes are* 1*, U coincides with the Upside-Down transform with contradiction reset.*

Proof. (i) After one application, for each component k and each activated pair $\{a, b\}$, $c^U_k(a, b) = 0$. Hence $\mathsf{Act}_k(a \mid b, \tau_k) = 0$ for every a at the second application, so neither further inversions nor further resets occur; thus $U \circ U = U$.

(ii) By definition, each c^U_k is obtained from c_k by setting some entries to 0 and keeping all others; therefore $c^U_k \leq c_k$ componentwise.

(iii) Each update of pdf is either the identity $x \mapsto x$ or the flip $x \mapsto 1 - x$; both map $[0, 1]$ into $[0, 1]$.

(iv) If component $k = 2$ is inert, activation and mode effects occur only in component $k = 1$, reproducing the ordinary two-mode scheme; if all active modes are 1, every activated value is inverted once and all corresponding contradictions are reset, which is exactly the Upside-Down-with-reset behavior. □

Example 2.11 (Two-mode De-Plithogenication (DV-PFS): Work-mode policy with two contradictions)**.** We model staff suitability for work modes with a Double-Valued Plithogenic Fuzzy Set (DV-PFS). Items (employees): $P = \{\text{E1}, \text{E2}\}$. Attribute: v = "work mode" with values $Pv = \{\text{On}, \text{Hy}, \text{Re}\}$ (On-site / Hybrid / Remote). Anchor: b = On. Memberships $\mu(x \mid a) := pdf(x, a) \in [0, 1]$:

	$\mu(\cdot \mid \text{On})$	$\mu(\cdot \mid \text{Hy})$	$\mu(\cdot \mid \text{Re})$
E1	0.30	0.70	0.80
E2	0.85	0.50	0.20

Double contradictions. Two components $pCF(a,b) = (c_1(a,b), c_2(a,b))$: c_1 = policy/process conflict; c_2 = security/compliance risk. Against b = On:

a	$(c_1(a,\text{On}))$	$(c_2(a,\text{On}))$
Hy	0.75	0.85
Re	0.90	0.60
On	0	0

Activation and modes. Thresholds $\boldsymbol{\tau} = (\tau_1, \tau_2) = (0.80, 0.70)$. Then $A^{(1)}_{\tau_1}(\text{On}) = \{\text{Re}\}$ (since $0.90 \geq 0.80$), $A^{(2)}_{\tau_2}(\text{On}) = \{\text{Hy}\}$ (since $0.85 \geq 0.70$). Choose per-component modes:

$$\mathcal{M}_1(\{\text{Re},\text{On}\}) = 1 \;\; \text{(Invert+Neutralize in } c_1\text{)}, \quad \mathcal{M}_2(\{\text{Hy},\text{On}\}) = 0 \;\; \text{(Neutralize-only in } c_2\text{)}.$$

Hence the value Re *flips* its memberships (due to component 1) and Hy does *not* flip (component 2 is neutralize-only). Resets: $c^U_1(\text{Re},\text{On}) = 0$, $c^U_2(\text{Hy},\text{On}) = 0$; all other non-activated components remain unchanged.

Scalarization for aggregation. Use weights $\omega = (0.6, 0.4)$ and $c_\omega(a,b) = 0.6\,c_1(a,b) + 0.4\,c_2(a,b)$. Before update:

$$c_\omega(\text{Hy},\text{On}) = 0.6 \cdot 0.75 + 0.4 \cdot 0.85 = 0.79, \quad c_\omega(\text{Re},\text{On}) = 0.6 \cdot 0.90 + 0.4 \cdot 0.60 = 0.78.$$

After update:

$$c^U_\omega(\text{Hy},\text{On}) = 0.6 \cdot 0.75 + 0.4 \cdot 0 = 0.45, \quad c^U_\omega(\text{Re},\text{On}) = 0.6 \cdot 0 + 0.4 \cdot 0.60 = 0.24.$$

Membership flip (only Re): for each $x \in P$, $\mu^U(x \mid \text{Re}) = 1 - \mu(x \mid \text{Re})$; other columns unchanged.

Flip–blend acceptance (anchor b = On).

$$D_{\text{On}}(x) = \frac{1}{3} \sum_{a \in Pv} \Big((1 - c(a,\text{On}))\,\mu(x \mid a) + c(a,\text{On})\,(1 - \mu(x \mid a)) \Big),$$

using $c = c_\omega$ (pre) or $c = c^U_\omega$ (post) and μ (pre) or μ^U (post).

<u>E1, before:</u> $T_{\text{On}} = 0.30$, $T_{\text{Hy}} = 0.21 \cdot 0.70 + 0.79 \cdot 0.30 = 0.147 + 0.237 = 0.384$, $T_{\text{Re}} = 0.22 \cdot 0.80 + 0.78 \cdot 0.20 = 0.176 + 0.156 = 0.332$. So $D_{\text{On}}(\text{E1}) = (0.30 + 0.384 + 0.332)/3 \approx 0.3387$.

<u>E1, after:</u> $\mu^U(\text{E1} \mid \text{Re}) = 1 - 0.80 = 0.20$. $T^U_{\text{On}} = 0.30$, $T^U_{\text{Hy}} = 0.55 \cdot 0.70 + 0.45 \cdot 0.30 = 0.385 + 0.135 = 0.520$, $T^U_{\text{Re}} = 0.76 \cdot 0.20 + 0.24 \cdot 0.80 = 0.152 + 0.192 = 0.344$. So $D^U_{\text{On}}(\text{E1}) = (0.30 + 0.520 + 0.344)/3 \approx 0.3880$.

<u>E2, before:</u> $T_{\text{On}} = 0.85$, $T_{\text{Hy}} = 0.21 \cdot 0.50 + 0.79 \cdot 0.50 = 0.105 + 0.395 = 0.500$, $T_{\text{Re}} = 0.22 \cdot 0.20 + 0.78 \cdot 0.80 = 0.044 + 0.624 = 0.668$. So $D_{\text{On}}(\text{E2}) = (0.85 + 0.500 + 0.668)/3 \approx 0.6727$.

<u>E2, after:</u> $\mu^U(\text{E2} \mid \text{Re}) = 1 - 0.20 = 0.80$. $T^U_{\text{On}} = 0.85$, $T^U_{\text{Hy}} = 0.500$ (unchanged), $T^U_{\text{Re}} = 0.76 \cdot 0.80 + 0.24 \cdot 0.20 = 0.608 + 0.048 = 0.656$. So $D^U_{\text{On}}(\text{E2}) = (0.85 + 0.500 + 0.656)/3 \approx 0.6687$.

Effect. The genuine policy conflict with Remote is inverted and neutralized (component 1), while the security conflict with Hybrid is neutralized only (component 2), yielding higher acceptance for E1 and a slight adjustment for E2.

Example 2.12 (Two-mode De-Plithogenication (DV-PFS): Loan underwriting with credit vs. fraud contradictions)**.** We model applicant acceptability by risk level. Items (applicants): $P = \{\text{A}, \text{B}\}$. Attribute: v = "risk tier" with $Pv = \{\text{L}, \text{M}, \text{H}\}$ (Low/Medium/High). Anchor: b = L. Memberships:

	$\mu(\cdot \mid \text{L})$	$\mu(\cdot \mid \text{M})$	$\mu(\cdot \mid \text{H})$
A	0.40	0.60	0.20
B	0.85	0.35	0.10

Double contradictions. c_1 = credit risk gap, c_2 = fraud/anomaly risk gap (symmetric, zero diagonal):

a	$(c_1(a,\mathrm{L}))$	$(c_2(a,\mathrm{L}))$
M	0.60	0.30
H	0.95	0.85
L	0	0

Activation and modes. Thresholds $\boldsymbol{\tau} = (0.80, 0.80)$ give $A^{(1)}_{\tau_1}(\mathrm{L}) = \{\mathrm{H}\}$, $A^{(2)}_{\tau_2}(\mathrm{L}) = \{\mathrm{H}\}$. Choose modes

$$\mathcal{M}_1(\{\mathrm{H},\mathrm{L}\}) = 1 \;\; \text{(Invert+Neutralize in } c_1\text{)}, \qquad \mathcal{M}_2(\{\mathrm{H},\mathrm{L}\}) = 0 \;\; \text{(Neutralize-only in } c_2\text{)}.$$

Thus value H flips; both components on $\{\mathrm{H},\mathrm{L}\}$ are reset: $c^U_1(\mathrm{H},\mathrm{L}) = 0$, $c^U_2(\mathrm{H},\mathrm{L}) = 0$. Value M is not activated and remains unchanged.

Scalarization. Use $\omega = (0.5, 0.5)$. Before: $c_\omega(\mathrm{M},\mathrm{L}) = 0.5(0.60)+0.5(0.30) = 0.45$, $c_\omega(\mathrm{H},\mathrm{L}) = 0.5(0.95)+0.5(0.85) = 0.90$. After: $c^U_\omega(\mathrm{H},\mathrm{L}) = 0$, $c^U_\omega(\mathrm{M},\mathrm{L}) = 0.45$. Membership flip: $\mu^U(x \mid \mathrm{H}) = 1 - \mu(x \mid \mathrm{H})$.

Flip–blend acceptance (anchor $b = \mathrm{L}$).

$$D_\mathrm{L}(x) = \frac{1}{3} \sum_{a\in\{\mathrm{L},\mathrm{M},\mathrm{H}\}} \Big((1 - c(a,\mathrm{L}))\, \mu(x \mid a) + c(a,\mathrm{L})\, (1 - \mu(x \mid a))\Big).$$

Applicant A, before: $T_\mathrm{L} = 0.40$, $T_\mathrm{M} = 0.55{\cdot}0.60+0.45{\cdot}0.40 = 0.33+0.18 = 0.51$, $T_\mathrm{H} = 0.10{\cdot}0.20+0.90{\cdot}0.80 = 0.02 + 0.72 = 0.74$. So $D_\mathrm{L}(\mathrm{A}) = (0.40 + 0.51 + 0.74)/3 = 0.55$.

Applicant A, after: $\mu^U(\mathrm{A} \mid \mathrm{H}) = 1 - 0.20 = 0.80$, $T^U_\mathrm{L} = 0.40$, $T^U_\mathrm{M} = 0.51$ (unchanged), $T^U_\mathrm{H} = (1 - 0) \cdot 0.80 + 0 \cdot 0.20 = 0.80$. So $D^U_\mathrm{L}(\mathrm{A}) = (0.40 + 0.51 + 0.80)/3 = 0.57$.

Applicant B, before: $T_\mathrm{L} = 0.85$, $T_\mathrm{M} = 0.55 \cdot 0.35 + 0.45 \cdot 0.65 = 0.1925 + 0.2925 = 0.485$, $T_\mathrm{H} = 0.10 \cdot 0.10 + 0.90 \cdot 0.90 = 0.01 + 0.81 = 0.82$. So $D_\mathrm{L}(\mathrm{B}) = (0.85 + 0.485 + 0.82)/3 \approx 0.7183$.

Applicant B, after: $\mu^U(\mathrm{B} \mid \mathrm{H}) = 1 - 0.10 = 0.90$, $T^U_\mathrm{L} = 0.85$, $T^U_\mathrm{M} = 0.485$, $T^U_\mathrm{H} = 0.90$. So $D^U_\mathrm{L}(\mathrm{B}) = (0.85 + 0.485 + 0.90)/3 = 0.745$.

Effect. A genuine credit-risk conflict (component 1) at the High tier is inverted and neutralized, while the fraud-risk channel (component 2) is neutralized-only. This raises anchor-relative acceptance for both applicants, more notably for B.

2.3 Multi-valued Plithogenic Fuzzy Set

A Multi-Valued Plithogenic Fuzzy Set assigns each element a fuzzy membership degree and multiple independent contradiction values, modeling complex multi-dimensional conflicts.

Definition 2.13 (Multi-Valued Plithogenic Fuzzy Set (MV-PFS))**.** Let S be a universe and $P \subseteq S$ a nonempty set. Fix an attribute v with nonempty value-set Pv. For a fixed integer $m \geq 1$, a *Multi-Valued Plithogenic Fuzzy Set* with parameters $(s, t) = (1, m)$ is a quintuple

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF_m),$$

where

- $pdf : P{\times}Pv \to [0, 1]$ is the (scalar) *Degree of Appurtenance Function* assigning $\mu(x \mid a) := pdf(x, a) \in [0, 1]$ to each $(x, a) \in P \times Pv$;
- $pCF_m : Pv \times Pv \to [0, 1]^m$ is the *Multi-Valued Degree of Contradiction Function* assigning, for each pair (a, b), an m-vector

  $$pCF_m(a, b) = \mathbf{c}(a, b) = \big(c_1(a, b), \ldots, c_m(a, b)\big) \in [0, 1]^m,$$

  whose coordinates model m independent kinds of contradiction.

For every $a, b \in Pv$ and each $k \in \{1, \dots, m\}$,

$$c_k(a,a) = 0, \qquad c_k(a,b) = c_k(b,a).$$

Remark 2.14 (Effective scalar contradiction and basic bounds)**.** When a single contradiction coefficient is required (e.g., for aggregation or logical connectives), fix a weight vector $\omega = (\omega_1, \dots, \omega_m)$ in the probability simplex

$$\Delta^{m-1} := \{\omega \in [0,1]^m : \sum_{k=1}^{m} \omega_k = 1\}.$$

Define the *effective (scalar) contradiction* by convex scalarization

$$c_\omega(a,b) \;:=\; \sum_{k=1}^{m} \omega_k \, c_k(a,b) \;\in [0,1].$$

Indeed, $0 \le c_\omega(a,b) \le \sum_k \omega_k = 1$, and symmetry/zero-diagonal follow componentwise: $c_\omega(a,a) = 0$, $c_\omega(a,b) = c_\omega(b,a)$.

Example 2.15 (Urban mobility planning (anchor = LowCarbon; $m = 3$))**. Items (modes):** $P = \{\text{Car, Bus, Bike, Metro}\}$. **Attribute values:** $Pv = \{\text{T, C, L, A}\}$ for (Time Efficiency, Cost Saving, Low Carbon, Accessibility). Anchor $b = \text{L}$.

Memberships $\mu(x \mid a)$:

	T	C	L	A
Car	0.80	0.20	0.10	0.60
Bus	0.50	0.70	0.60	0.80
Bike	0.60	0.90	0.95	0.50
Metro	0.85	0.60	0.80	0.90

Three contradiction components (w.r.t. the anchor L):

$(a, b{=}\text{L})$	T	C	A	L
c_1 (emissions)	0.70	0.30	0.20	0
c_2 (budget)	0.40	0.50	0.30	0
c_3 (safety/comfort)	0.20	0.20	0.40	0

Choose $\omega = (0.5, 0.3, 0.2)$. Then $c_\omega(\text{T,L}) = 0.51,\; c_\omega(\text{C,L}) = 0.34,\; c_\omega(\text{A,L}) = 0.27,\; c_\omega(\text{L,L}) = 0$.

Anchor–relative acceptances $D_{\text{L}}^{(\omega)}(x)$:

x	T	C	L	A	$D_{\text{L}}^{(\omega)}(x)$
Car	0.318	0.468	0.100	0.378	**0.386**
Bus	0.745	0.398	0.600	0.254	**0.576**
Bike	0.588	0.306	0.950	0.635	**0.644**
Metro	0.234	0.412	0.800	0.205	**0.627**

(Rightmost column is the average of the four entries.) *Ranking:* Bike $\succ$ Metro $\succ$ Bus $\succ$ Car. Multiple contradiction channels jointly penalize attributes that conflict with the LowCarbon anchor, yielding a policy-aware ordering of modes.

Example 2.16 (Laptop procurement under a repairability/sustainability policy ($m = 3$))**. Items:** $P = \{\text{L1, L2, L3}\}$ (gaming, ultrabook, modular/repairable). **Attribute values:** $Pv = \{\text{P, B, R, S}\}$ for (Performance, Battery life, low pRice, Repairability/Sustainability). Anchor $b = \text{S}$.

Memberships $\mu(x \mid a)$:

	P	B	R	S
L1	0.95	0.40	0.20	0.30
L2	0.75	0.90	0.50	0.60
L3	0.70	0.80	0.40	0.95

Three contradiction components (vs. S):

$(a, b{=}\mathrm{S})$	P	B	R	S
c_1 (environmental)	0.80	0.30	0.60	0
c_2 (TCO cost)	0.50	0.20	0.70	0
c_3 (repairability)	0.60	0.50	0.70	0

With weights $\omega = (0.6, 0.2, 0.2)$, $c_\omega(\mathrm{P}, \mathrm{S}) = 0.70,\ c_\omega(\mathrm{B}, \mathrm{S}) = 0.32,\ c_\omega(\mathrm{R}, \mathrm{S}) = 0.64,\ c_\omega(\mathrm{S}, \mathrm{S}) = 0.$

Anchor–relative acceptances $D_{\mathrm{S}}^{(\omega)}(x)$:

x	P	B	R	S	$D_{\mathrm{S}}^{(\omega)}(x)$
L1	0.385	0.568	0.712	0.300	**0.417**
L2	0.425	0.268	0.570	0.600	**0.536**
L3	0.510	0.336	0.624	0.950	**0.627**

Ranking: L3 ≻ L2 ≻ L1. High performance and very low price face multi-dimensional contradictions against the repairability/sustainability anchor, whereas modular design yields the strongest anchor-relative acceptance.

Definition 2.17 (MV-plithogenic connectives relative to an anchor)**.** Fix a *dominant (anchor) value* $b \in Pv$, a continuous t-norm T and its dual t-conorm S. For $x, y \in [0, 1]$ associated with the same value $a \in Pv$, define

$$x \wedge^{\mathrm{MV}}_{(a|b,\omega)} y := (1 - c_\omega(a, b))\, T(x, y) + c_\omega(a, b)\, S(x, y),$$

$$x \vee^{\mathrm{MV}}_{(a|b,\omega)} y := (1 - c_\omega(a, b))\, S(x, y) + c_\omega(a, b)\, T(x, y),$$

with c_ω from Remark 2.14. These are convex blends of T and S, hence map $[0, 1]^2$ into $[0, 1]$.

Example 2.18 (High-contradiction case: product t-norm and probabilistic-sum t-conorm)**.** Let the anchor be $b \in Pv$ and fix value $a \in Pv$. Take $T(x, y) = xy$ and $S(x, y) = x + y - xy$. Suppose two degrees $x = 0.70$, $y = 0.50$ are attached to a. Assume $m = 3$ contradiction components (relative to b):

$$(c_1, c_2, c_3) = (0.80,\ 0.60,\ 0.20), \qquad \omega = (0.5,\ 0.3,\ 0.2),$$

so the effective contradiction is

$$c_\omega(a, b) = 0.5 \cdot 0.80 + 0.3 \cdot 0.60 + 0.2 \cdot 0.20 = 0.62.$$

Compute the classical connectives:

$$T(x, y) = 0.70 \cdot 0.50 = 0.35, \qquad S(x, y) = 0.70 + 0.50 - 0.35 = 0.85.$$

Then the MV-plithogenic connectives (Def. 2.17) are

$$x \wedge^{\mathrm{MV}}_{(a|b,\omega)} y = (1 - 0.62) \cdot 0.35 + 0.62 \cdot 0.85 = 0.38 \cdot 0.35 + 0.62 \cdot 0.85 = 0.133 + 0.527 = 0.660,$$

$$x \vee^{\mathrm{MV}}_{(a|b,\omega)} y = (1 - 0.62) \cdot 0.85 + 0.62 \cdot 0.35 = 0.38 \cdot 0.85 + 0.62 \cdot 0.35 = 0.323 + 0.217 = 0.540.$$

Interpretation. A high contradiction ($c_\omega = 0.62$) pushes $\wedge$ toward the t-conorm and $\vee$ toward the t-norm.

Example 2.19 (Low-contradiction case: minimum/maximum pair)**.** Let the anchor be b and value $a \in Pv$. Choose $T(x, y) = \min\{x, y\}$, $S(x, y) = \max\{x, y\}$. Let $x = 0.40$, $y = 0.90$. Assume $m = 2$ components with

$$(c_1, c_2) = (0.10,\ 0.20), \qquad \omega = (0.6,\ 0.4),$$

so $c_\omega(a, b) = 0.6 \cdot 0.10 + 0.4 \cdot 0.20 = 0.14$. Classical connectives:

$$T(x, y) = 0.40, \qquad S(x, y) = 0.90.$$

MV-plithogenic results:

$$x \wedge^{\mathrm{MV}}_{(a|b,\omega)} y = (1 - 0.14) \cdot 0.40 + 0.14 \cdot 0.90 = 0.86 \cdot 0.40 + 0.14 \cdot 0.90 = 0.344 + 0.126 = 0.470,$$

$$x \vee^{\mathrm{MV}}_{(a|b,\omega)} y = (1 - 0.14) \cdot 0.90 + 0.14 \cdot 0.40 = 0.86 \cdot 0.90 + 0.14 \cdot 0.40 = 0.774 + 0.056 = 0.830.$$

Interpretation. With small contradiction ($c_\omega = 0.14$), the results stay close to the classical min/max (0.40, 0.90) while allowing a modest blend.

Proposition 2.20 (Well-definedness)**.** *Let PS be an MV-PFS, $b \in Pv$, and $\omega \in \Delta^{m-1}$. Then for all $x, y \in [0,1]$ and $a \in Pv$, the connectives in Definition 2.17 satisfy*

$$x \wedge^{\mathrm{MV}}_{(a|b,\omega)} y \in [0,1], \qquad x \vee^{\mathrm{MV}}_{(a|b,\omega)} y \in [0,1].$$

Proof. Since $T, S : [0,1]^2 \to [0,1]$ and $c_\omega(a,b) \in [0,1]$, each expression is a convex combination of values in $[0,1]$. □

Theorem 2.21 (MV-PFS generalizes PFS (t=1) and DV-PFS (t=2))**.** *Let $PS = (P, v, Pv, pdf, pCF_m)$ be an MV-PFS with $m \geq 1$.*

1. *If $m = 1$, then $pCF_1 : Pv \times Pv \to [0,1]$ and PS is exactly a (single-valued) Plithogenic Fuzzy Set (PFS) with $(s,t) = (1,1)$.*
2. *If $m = 2$, then $pCF_2 : Pv \times Pv \to [0,1]^2$ and PS is exactly a Double-Valued Plithogenic Fuzzy Set (DV-PFS) with $(s,t) = (1,2)$.*
3. *For arbitrary $m \geq 2$, any PFS $PS^{(1)}$ and any DV-PFS $PS^{(2)}$ embed into some MV-PFS via canonical injections that preserve memberships and contradictions; moreover, suitable choices of ω recover the original scalar contradiction used by their operators.*

Proof. (1) Taking $m = 1$ gives $pCF_1(a,b) = (c_1(a,b)) \in [0,1]$ with $c_1(a,a) = 0$ and symmetry, which is precisely the standard PFS contradiction map. With $\omega = (1)$, $c_\omega = c_1$, hence the MV-connectives reduce to the usual plithogenic connectives; thus MV-PFS coincides with PFS.

(2) Taking $m = 2$ gives $pCF_2(a,b) = (c_1(a,b), c_2(a,b)) \in [0,1]^2$ with the same axioms componentwise; this is exactly the DV-PFS structure. Any DV scalarization $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2) \in \Delta^1$ is obtained by choosing $\omega = \tilde{\omega}$, hence the MV-connectives coincide with the DV ones.

(3) *Embeddings.*

- PFS $\hookrightarrow$ MV-PFS. Given a PFS with contradiction $c : Pv \times Pv \to [0,1]$, define $\iota_1(c) := (c, 0, \ldots, 0) \in [0,1]^m$. Set $pCF_m(a,b) := \iota_1(c(a,b))$. Then for $\omega = (1, 0, \ldots, 0)$, $c_\omega(a,b) = c(a,b)$. Memberships pdf are shared. All axioms are preserved componentwise; hence this is a structure-preserving injection.
- DV-PFS $\hookrightarrow$ MV-PFS. Given a DV-PFS with $(c_1, c_2) : Pv \times Pv \to [0,1]^2$, define $\iota_2(c_1, c_2) := (c_1, c_2, 0, \ldots, 0) \in [0,1]^m$. For any DV weight $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2)$, choose $\omega = (\tilde{\omega}_1, \tilde{\omega}_2, 0, \ldots, 0) \in \Delta^{m-1}$; then $c_\omega(a,b) = \tilde{\omega}_1 c_1(a,b) + \tilde{\omega}_2 c_2(a,b)$, reproducing the DV effective contradiction. Again, all axioms are preserved.

Thus PFS and DV-PFS are special (degenerate) cases of MV-PFS, and both embed faithfully for any $m \geq 2$. □

Remark 2.22 (Flip–blend acceptance as a special MV aggregator)**.** Fix $b \in Pv$ and $\omega \in \Delta^{m-1}$. For an item $x \in P$, an anchor-relative "acceptance" can be computed by the flip–blend average

$$D_b^{(\omega)}(x) := \frac{1}{|Pv|} \sum_{a \in Pv} \Big((1 - c_\omega(a,b))\, \mu(x \mid a) \;+\; c_\omega(a,b)\, (1 - \mu(x \mid a)) \Big) \in [0,1],$$

where c_ω is the MV effective contradiction. When $m = 1$ (resp. $m = 2$) this reduces exactly to the PFS (resp. DV-PFS) formula by choosing ω as in the proof above.

2.4 Two-mode De-Plithogenication in Multi-valued Plithogenic Fuzzy Set

Two-mode De-Plithogenication in a Multi-Valued Plithogenic Fuzzy Set adjusts memberships by flipping or retaining values while neutralizing multiple contradiction components simultaneously.

Definition 2.23 (Two-mode De-Plithogenication for MV-PFS)**.** Let $PS = (P,\ v,\ Pv,\ pdf,\ pCF_m)$ be a *Multi-Valued Plithogenic Fuzzy Set* with parameters $(s,t) = (1,m)$, where

$$pdf : P \times Pv \to [0,1], \qquad pCF_m : Pv \times Pv \to [0,1]^m, \quad pCF_m(a,b) = \big(c_1(a,b), \dots, c_m(a,b)\big),$$

and for every $k \in \{1,\dots,m\}$: $c_k(a,a) = 0$ and $c_k(a,b) = c_k(b,a)$. Fix an *anchor* $b \in Pv$ and a (componentwise) threshold vector $\boldsymbol{\tau} = (\tau_1,\dots,\tau_m) \in [0,1]^m$.

Per-component activation. For $k \in \{1,\dots,m\}$ and $a \in Pv$, set

$$\mathsf{Act}_k(a \mid b,\tau_k) \ :=\ \mathbf{1}[\, c_k(a,b) \ \geq\ \tau_k \,] \in \{0,1\}, \qquad A^{(k)}_{\tau_k}(b) \ :=\ \{\, a \in Pv :\ \mathsf{Act}_k(a \mid b,\tau_k) = 1 \,\}.$$

Per-component mode selector. For each component k, let $\mathcal{M}_k : \{\{a,b\} : a \in Pv\} \to \{0,1\}$ be symmetric in its arguments, with the following semantics on any *activated* pair $\{a,b\}$:

- $\mathcal{M}_k(\{a,b\}) = 0$ <u>(Neutralize-only in component k)</u>: keep memberships; set $c_k(a,b)$ to 0.
- $\mathcal{M}_k(\{a,b\}) = 1$ <u>(Invert+Neutralize in component k)</u>: invert the memberships tied to value a (see below) and set $c_k(a,b)$ to 0.

Flip indicator (OR across components). For $a \in Pv$ define the flip flag

$$\mathrm{flip}\big(a \mid b,\boldsymbol{\tau},\{\mathcal{M}_k\}_{k=1}^m\big) \ :=\ \max_{1\leq k\leq m} \Big(\mathsf{Act}_k(a \mid b,\tau_k) \cdot \mathcal{M}_k(\{a,b\})\Big) \ \in\ \{0,1\}.$$

Two-mode MV operator. The *Two-mode De-Plithogenication* of PS at $\big(b,\boldsymbol{\tau},\{\mathcal{M}_k\}_{k=1}^m\big)$ is the updated structure

$$U^{\mathrm{imp},m}_{b,\boldsymbol{\tau},\{\mathcal{M}_k\}}(PS) \ :=\ \big(P,\ v,\ Pv,\ pdf^U,\ pCF^U_m\big),$$

given, for all $x \in P$, $a,u,v \in Pv$, and $k \in \{1,\dots,m\}$, by

$$pdf^U(x,a) := \big(1 - \mathrm{flip}(a \mid b,\boldsymbol{\tau},\{\mathcal{M}_k\})\big)\, pdf(x,a) \ +\ \mathrm{flip}(a \mid b,\boldsymbol{\tau},\{\mathcal{M}_k\})\,(1 - pdf(x,a)),$$

$$c^U_k(u,v) := \begin{cases} 0, & \text{if } \{u,v\} = \{a,b\} \text{ for some } a \in A^{(k)}_{\tau_k}(b), \\ c_k(u,v), & \text{otherwise.} \end{cases}$$

Equivalently, $pCF^U_m(u,v) = \big(c^U_1(u,v),\dots,c^U_m(u,v)\big)$. Thus, each contradiction component k is neutralized exactly on its own activated pairs, while a membership inversion for a value a occurs once whenever $\mathrm{flip}(a \mid b,\cdot) = 1$ for at least one activated component in Mode 1.

Remark 2.24 (Post-update scalarization and aggregation)**.** When a single modulation coefficient is needed, choose $\omega = (\omega_1,\dots,\omega_m)$ in the simplex Δ^{m-1} and define the effective contradiction

$$c^U_\omega(a,b) \ :=\ \sum_{k=1}^m \omega_k\, c^U_k(a,b) \ \in [0,1].$$

Substituting c^U_ω for c in plithogenic operators (e.g. the flip–blend aggregator Agg) yields scalar acceptances consistent with the multi-valued update.

Proposition 2.25 (Basic properties)**.** *Let* $U := U^{\mathrm{imp},m}_{b,\boldsymbol{\tau},\{\mathcal{M}_k\}}$*. For every MV-PFS PS and parameters as above:*

(i) ***Idempotence.*** $U(U(PS)) = U(PS)$.

(ii) ***Componentwise contradiction monotonicity.*** *$c_k^U(u,v) \le c_k(u,v)$ for all $u, v \in Pv$ and each $k = 1, \dots, m$.*

(iii) ***Membership range preservation.*** *$pdf^U(x,a) \in [0,1]$ for all $x \in P$, $a \in Pv$.*

(iv) ***Reductions.*** *If $m = 1$ (resp. $m = 2$) the operator reduces to the single-valued (resp. double-valued) two-mode scheme. More generally, if some component k is inert ($c_k \equiv 0$ or $\tau_k > 1$ or $\mathcal{M}_k \equiv 0$), it has no effect on U.*

Proof. (i) After one application, for every component k and activated pair $\{a,b\}$, $c_k^U(a,b) = 0$. Hence $\mathsf{Act}_k(a \mid b, \tau_k) = 0$ at the next step and no further resets or flips occur, so $U \circ U = U$. (ii) Each c_k^U is obtained by setting some entries of c_k to 0 and keeping others unchanged; thus $c_k^U \le c_k$ componentwise. (iii) The update of pdf is either the identity $x \mapsto x$ or a complement $x \mapsto 1 - x$, both mapping $[0,1]$ into $[0,1]$. (iv) The stated reductions are immediate from the definitions when the number of components is 1 or 2, or when a component is inactive. □

Example 2.26 (Urban mobility planning with three contradiction channels)**.** *Setting.* Items (projects) $P = \{X_1, X_2, X_3\}$: road widening (X_1), BRT (X_2), bike-lane network (X_3). Attribute v = "travel mode" with values $Pv = \{\mathrm{C}, \mathrm{T}, \mathrm{B}\}$ (Car/Transit/Bike). Anchor $b = \mathrm{T}$ (transit-centric plan). Memberships $\mu(x \mid a) = pdf(x,a)$:

	$\mu(\cdot \mid \mathrm{C})$	$\mu(\cdot \mid \mathrm{T})$	$\mu(\cdot \mid \mathrm{B})$
X_1	0.95	0.20	0.10
X_2	0.50	0.90	0.40
X_3	0.10	0.50	0.95

Three contradictions against T (symmetric, zero diagonal): c_1 =environment, c_2 =cost, c_3 =equity:

	$c_1(\cdot, \mathrm{T})$	$c_2(\cdot, \mathrm{T})$	$c_3(\cdot, \mathrm{T})$
C	0.90	0.60	0.85
B	0.10	0.40	0.30
T	0	0	0

Thresholds $\boldsymbol{\tau} = (0.8, 0.5, 0.8)$. Activations: C activates all three components; B activates none. Modes: $\mathcal{M}_1(\{\mathrm{C}, \mathrm{T}\}) = 1$ (genuine env. conflict), $\mathcal{M}_2(\{\mathrm{C}, \mathrm{T}\}) = 0$ (cost deemed non-genuine), $\mathcal{M}_3(\{\mathrm{C}, \mathrm{T}\}) = 1$ (genuine equity conflict). Thus the C-column flips; all $c_k(\mathrm{C}, \mathrm{T})$ reset to 0.

Scalarization. With $\omega = (0.5, 0.2, 0.3)$,

$$c_\omega(\mathrm{C}, \mathrm{T}) = 0.825, \quad c_\omega(\mathrm{B}, \mathrm{T}) = 0.22, \quad c_\omega^U(\mathrm{C}, \mathrm{T}) = 0,\ c_\omega^U(\mathrm{B}, \mathrm{T}) = 0.22.$$

Flip–blend acceptance (after update). For X_1: $\mu^U(X_1 \mid \mathrm{C}) = 1 - 0.95 = 0.05$. Terms: $\mathrm{C} : 0.05$, $\mathrm{B} : 0.78 \cdot 0.10 + 0.22 \cdot 0.90 = 0.276$, $\mathrm{T} : 0.20$. So $D_{\mathrm{T}}^{(\omega)}(X_1) = (0.05 + 0.276 + 0.20)/3 \approx 0.175$.

For X_2: $\mu^U(X_2 \mid \mathrm{C}) = 0.50$ (unchanged since $1 - 0.50 = 0.50$). Terms: $\mathrm{C} : 0.50$, $\mathrm{B} : 0.78 \cdot 0.40 + 0.22 \cdot 0.60 = 0.444$, $\mathrm{T} : 0.90$. Hence $D_{\mathrm{T}}^{(\omega)}(X_2) = (0.50 + 0.444 + 0.90)/3 \approx 0.615$.

For X_3: $\mu^U(X_3 \mid \mathrm{C}) = 1 - 0.10 = 0.90$. Terms: $\mathrm{C} : 0.90$, $\mathrm{B} : 0.78 \cdot 0.95 + 0.22 \cdot 0.05 = 0.752$, $\mathrm{T} : 0.50$. Thus $D_{\mathrm{T}}^{(\omega)}(X_3) = (0.90 + 0.752 + 0.50)/3 \approx 0.717$.

Post-update ranking: $X_3(0.717) > X_2(0.615) \gg X_1(0.175)$. Mode 1 on the activated car value flips memberships and, together with resets, boosts transit-/bike-oriented projects while penalizing car-centric widening.

Example 2.27 (Personalized diet planning with three contradictions)**.** *Setting.* Items $P = \{M_1, M_2, M_3\}$: Keto box, Balanced bento, Vegan salad kit. Attribute v = "diet style" with $Pv = \{\mathrm{K}, \mathrm{B}, \mathrm{V}\}$ (Keto/Balanced/Vegan). Anchor $b = \mathrm{B}$. Memberships:

	$\mu(\cdot \mid \mathrm{K})$	$\mu(\cdot \mid \mathrm{B})$	$\mu(\cdot \mid \mathrm{V})$
M_1	0.95	0.30	0.05
M_2	0.40	0.90	0.30
M_3	0.10	0.60	0.95

Contradictions against B: c_1 =medical, c_2 =budget, c_3 =availability:

	$c_1(\cdot, \mathrm{B})$	$c_2(\cdot, \mathrm{B})$	$c_3(\cdot, \mathrm{B})$
K	0.90	0.70	0.40
V	0.20	0.60	0.30
B	0	0	0

Thresholds $\boldsymbol{\tau} = (0.8, 0.65, 0.5)$. Activations: K activates c_1, c_2; V activates none. Modes: $\mathcal{M}_1(\{\mathrm{K}, \mathrm{B}\}) = 1$ (invert+neutralize), $\mathcal{M}_2(\{\mathrm{K}, \mathrm{B}\}) = 0$ (neutralize-only). Thus $\mu^U(\cdot \mid \mathrm{K}) = 1 - \mu(\cdot \mid \mathrm{K})$ and $c_1^U(\mathrm{K}, \mathrm{B}) = c_2^U(\mathrm{K}, \mathrm{B}) = 0$.

Scalarization. With $\omega = (0.5, 0.3, 0.2)$,

$$c_\omega(\mathrm{K}, \mathrm{B}) = 0.74, \quad c_\omega(\mathrm{V}, \mathrm{B}) = 0.34, \qquad c_\omega^U(\mathrm{K}, \mathrm{B}) = 0.08 \text{ (only } c_3 \text{ remains)}.$$

Flip–blend acceptance (after update). For M_1: $\mu^U(M_1 \mid \mathrm{K}) = 0.05$. Terms: $\mathrm{K} : 0.92{\cdot}0.05{+}0.08{\cdot}0.95 = 0.122$, $\mathrm{V} : 0.66 \cdot 0.05 + 0.34 \cdot 0.95 = 0.356$, $\mathrm{B} : 0.30$. Hence $D_{\mathrm{B}}^{(\omega)}(M_1) = (0.122 + 0.356 + 0.30)/3 \approx 0.259$.

For M_2: $\mu^U(M_2 \mid \mathrm{K}) = 1{-}0.40 = 0.60$. Terms: $\mathrm{K} : 0.92{\cdot}0.60{+}0.08{\cdot}0.40 = 0.584$, $\mathrm{V} : 0.66{\cdot}0.30{+}0.34{\cdot}0.70 = 0.436$, $\mathrm{B} : 0.90$. So $D_{\mathrm{B}}^{(\omega)}(M_2) = (0.584 + 0.436 + 0.90)/3 \approx 0.640$.

For M_3: $\mu^U(M_3 \mid \mathrm{K}) = 1{-}0.10 = 0.90$. Terms: $\mathrm{K} : 0.92{\cdot}0.90{+}0.08{\cdot}0.10 = 0.836$, $\mathrm{V} : 0.66{\cdot}0.95{+}0.34{\cdot}0.05 = 0.644$, $\mathrm{B} : 0.60$. Thus $D_{\mathrm{B}}^{(\omega)}(M_3) = (0.836 + 0.644 + 0.60)/3 \approx 0.693$.

Post-update ranking: $M_3(0.693) > M_2(0.640) \gg M_1(0.259)$. A genuine medical contradiction (component c_1) triggers inversion of Keto; budget conflict is neutralized-only, yielding higher balanced-anchor acceptability for Vegan/Balanced plans.

3 Discussion: Classical Fuzzy vs Plithogenic Fuzzy vs Double-valued Plithogenic Fuzzy vs Multi-valued Plithogenic Fuzzy

The decisive distinction between a Plithogenic Fuzzy Set and a Classical Fuzzy Set lies in the presence of contradiction parameters. While Classical Fuzzy Sets only capture graded membership, Plithogenic Fuzzy Sets explicitly incorporate contradiction values, enabling them to represent conflicts within attribute values. In comparison with a Plithogenic Fuzzy Set, the Double-valued and Multi-valued Plithogenic Fuzzy Sets provide a major advantage: they can address multiple forms of contradiction that arise in everyday life. This enhancement allows for richer modeling of complex decision-making scenarios in which several independent sources of conflict coexist. For clarity, Table 1 summarizes the comparison among Classical Fuzzy Sets, Plithogenic Fuzzy Sets (t=1), Double-valued Plithogenic Fuzzy Sets (t=2), and Multi-valued Plithogenic Fuzzy Sets (t=m). Likewise, Table 2 outlines the two-mode De-plithogenication procedure across these frameworks. From these observations, it is evident that further research on Plithogenic Fuzzy Sets, particularly in their double- and multi-valued variants, is both promising and necessary. We expect that these enriched frameworks will inspire more active and diverse investigations in the future.

4 Conclusion

This paper investigated how to handle situations where two or more contradiction values are present, in conjunction with the principles of Upside-Down Logic. Furthermore, we also examined methods for eliminating contradictions altogether. In the future, we hope to explore the behavior of Double-valued Plithogenic Sets and Multi-valued Plithogenic Sets within Graphs [26, 27], HyperGraphs [28–30], Bidirected Graphs [31–33], and SuperHyperGraphs [12, 34, 35]. We also anticipate that experiments using various datasets will be conducted to validate and further develop these theoretical frameworks.

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Framework	s	t	Contradiction structure	Operator modulation (example)	Typical scenario
Classical Fuzzy Set	1	0	none	use a t-norm T and t-conorm S only; $x \wedge y = T(x, y), x \vee y = S(x, y)$	Simple vagueness (e.g., "warm")
Plithogenic Fuzzy Set (PFS)	1	1	$c(a, b) \in [0, 1]$, symmetric, $c(a, a) = 0$	blend by contradiction: $(1 - c(a, b))\, T(x, y) + c(a, b)\, S(x, y)$ (dual for $\vee$)	One conflict channel across attribute values
Double-valued PFS (DV-PFS)	1	2	$(c_1(a, b),\ c_2(a, b)) \in [0, 1]^2$ (two independent contradictions)	scalarize $c_\omega = \omega_1 c_1 + \omega_2 c_2$; then $(1 - c_\omega)T(x, y) + c_\omega S(x, y)$	Two conflicts (e.g., environmental vs regulatory)
Multi-valued PFS (MV-PFS)	1	$m \geq 2$	$(c_1, \ldots, c_m) \in [0, 1]^m$ (multiple independent contradictions)	$c_\omega = \sum_{k=1}^{m} \omega_k c_k$ with $\omega \in \Delta^{m-1}$; then $(1 - c_\omega)T + c_\omega S$	Multi-criteria conflicts (environment, cost, safety, equity, . . .)

Table 1: Comparison of Classical Fuzzy, Plithogenic Fuzzy (t=1), Double-valued (t=2), and Multi-valued (t=m) Plithogenic Fuzzy Sets.

Framework	Activation rule	Membership update (two-mode)	Contradiction update	Scalarization / notes
Plithogenic Fuzzy Set (PFS) (t=1)	Activate value a w.r.t. anchor b if $c(a, b) \geq \tau$.	Mode 0: keep $pdf(x, a)$; Mode 1: $pdf^U(x, a) = 1 - pdf(x, a)$ (only for activated a).	$pCF^U(u, v) = 0$ if $\{u, v\} = \{a, b\}$ with $c(a, b) \geq \tau$; otherwise unchanged.	No scalarization needed (t=1). Example aggregator (flip–blend): $(1 - c)\mu + c(1 - \mu)$; idempotent after one update.
Double-valued PFS (DV-PFS) (t=2)	Per component $k \in \{1, 2\}$: activate if $c_k(a, b) \geq \tau_k$. Value a flips iff $\max_k\{\mathbf{1}[c_k \geq \tau_k] \cdot \mathcal{M}_k(\{a, b\})\} = 1$.	Mode 0 in component k: keep pdf; Mode 1 in some activated k: set $pdf^U(x, a) = 1 - pdf(x, a)$ (flip once if any such k).	Componentwise reset: $c_k^U(u, v) = 0$ when $\{u, v\} = \{a, b\}$ is activated in component k; otherwise $c_k^U = c_k$.	Use weights $\omega = (\omega_1, \omega_2)$; effective contradiction $c_\omega = \omega_1 c_1 + \omega_2 c_2 \in [0, 1]$ for operators/aggregation. Idempotent; strictly generalizes PFS.
Multi-valued PFS (MV-PFS) (t=$m \geq 2$)	Per component $k = 1, \ldots, m$: activate if $c_k(a, b) \geq \tau_k$. Flip rule: OR across components with Mode 1.	Mode 0 in any activated k: no flip from that k; Mode 1 in at least one activated k: $pdf^U(x, a) = 1 - pdf(x, a)$ (single flip per a).	For each component k: $c_k^U(u, v) = 0$ on its activated pair(s) $\{u, v\} = \{a, b\}$; other entries unchanged.	Choose $\omega \in \Delta^{m-1}$; $c_\omega = \sum_{k=1}^{m} \omega_k c_k$. Reduces to DV ($m$=2) and PFS ($m$=1); idempotent after one update.

Table 2: De-plithogenication (two-mode) across Plithogenic Fuzzy (t=1), Double-valued (t=2), and Multi-valued (t=m) frameworks. Mode 0 = Neutralize-only; Mode 1 = Invert+Neutralize.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Code Availability

No code or software was developed for this study.

Clinical Trial

This study did not involve any clinical trials.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

References

[1] Antonios Paraskevas, Michael Madas, Florentin Smarandache, and Takaaki Fujita. Utility-based upside-down logic: A neutrosophic decision-making framework under uncertainty. *Journal of Decisions and Operations Research*, 2025.

[2] Lihua Gu and Ming Lei. A neutrosophic framework for nursing education quality analysis using upside-down logics and narrative factor. *Neutrosophic Sets and Systems*, 87:362–378, 2025.

[3] Qingde Li and Xiangheng Kong. A mathematical neutrosophic offset framework with upside down logics for quality evaluation of multi-sensor intelligent vehicle environment perception systems. *Neutrosophic Sets and Systems*, 87:1014–1023, 2025.

[4] Takaaki Fujita. Exploring upside-down logic within neutrosophic graphs: A novel framework for reversal-based reasoning. 2025.

[5] Junqiao Lou, Daoping Zhang, and Youyin Mo. Upside-down neutrosophic multi-fuzzy ideals for it-enhanced college dance teaching quality assessment. *Neutrosophic Sets and Systems*, 93:370–380, 2025.

[6] Yantao Bu. An upside-down logic-based neutrosophic methodology: Assessment of online marketing effectiveness in e-commerce enterprises. *Neutrosophic Sets and Systems*, 85:801–813, 2025.

[7] Takaaki Fujita and Florentin Smarandache. Introduction to upside-down logic: Its deep relation to neutrosophic logic and applications. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond (Third Volume)*, 2024.

[8] Florentin Smarandache. *Upside-Down Logics: Falsification of the Truth and Truthification of the False*. Infinite Study, 2024.

[9] Florentin Smarandache. *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*. Infinite study, 2018.

[10] Weiyi Li. A plithogenic upside-down neutrosophic framework for psychological assessment: A case study on college vocal music majors. *Neutrosophic Sets and Systems*, 93:77–87, 2025.

[11] Fazeelat Sultana, Muhammad Gulistan, Mumtaz Ali, Naveed Yaqoob, Muhammad Khan, Tabasam Rashid, and Tauseef Ahmed. A study of plithogenic graphs: applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, 14(10):13139–13159, 2023.

[12] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.

[13] Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.

[14] Talal Al-Hawary. Complete fuzzy graphs. *International Journal of Mathematical Combinatorics*, 4:26, 2011.

[15] Krassimir T Atanassov. Circular intuitionistic fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, 39(5):5981–5986, 2020.

[16] Muhammad Akram, Bijan Davvaz, and Feng Feng. Intuitionistic fuzzy soft k-algebras. *Mathematics in Computer Science*, 7:353–365, 2013.

[17] Haibin Wang, Florentin Smarandache, Yanqing Zhang, and Rajshekhar Sunderraman. *Single valued neutrosophic sets*. Infinite study, 2010.

[18] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Single valued neutrosophic graphs. *Journal of New theory*, (10):86–101, 2016.

[19] Nivetha Martin, Seyyed Ahmad Edalpathnath, Florentin Smarandache, et al. Extended plithogenic hypersoft sets in decision making on electric vehicle charging infrastructure. *Journal of Fuzzy Extension and Applications*, page e226517, 2025.

[20] Manuel Michael Beraún-Espíritu, Ketty Marilú Moscoso-Paucarchuco, Humberto Dax Bonilla-Mancilla, Edson Hilmer Julca-Marcelo, Anani Basaldua Galarza, Jean Pierre Espeza-Gavilán, Lizve Vilcapoma Ureta, and Jacqueline Denisse Llacza-Molina. Environmental sustainability and organizational commitment in rainbow trout production in junín, peru: An approach from plithogenic fuzzy soft sets. *Neutrosophic Sets and Systems*, 84(1):33, 2025.

[21] Muhammad Azeem, Humera Rashid, Muhammad Kamran Jamil, Selma Gütmen, and Erfan Babaee Tirkolaee. Plithogenic fuzzy graph: A study of fundamental properties and potential applications. *Journal of Dynamics and Games*, pages 0–0, 2024.

[22] Florentin Smarandache. Plithogeny, plithogenic set, logic, probability, and statistics. *arXiv preprint arXiv:1808.03948*, 2018.

[23] S Sudha, Nivetha Martin, and Florentin Smarandache. *Applications of Extended Plithogenic Sets in Plithogenic Sociogram*. Infinite Study, 2023.

[24] Raúl González Salas, Jhael Mikaela Gómez Ashqui, and Marco Paúl Medina Valencia. Assessing viral susceptibility in tilapia species using plithogenic fuzzy soft set: a multi-expert approach for tilv. *Neutrosophic Sets and Systems*, 89:185–196, 2025.

[25] Takaaki Fujita and Arif Mehmood. A study on the effectiveness of contradiction values in upside-down logic and de-plithogenication within plithogenic sets. *Neutrosophic Computing and Machine Learning*, 40(1):28–65, 2025.

[26] Reinhard Diestel. *Graph theory*. Springer (print edition); Reinhard Diestel (eBooks), 2024.

[27] Jonathan L Gross, Jay Yellen, and Mark Anderson. *Graph theory and its applications*. Chapman and Hall/CRC, 2018.

[28] Yifan Feng, Haoxuan You, Zizhao Zhang, Rongrong Ji, and Yue Gao. Hypergraph neural networks. In *Proceedings of the AAAI conference on artificial intelligence*, volume 33, pages 3558–3565, 2019.

[29] Alain Bretto. Hypergraph theory. *An introduction. Mathematical Engineering. Cham: Springer*, 1, 2013.

[30] Yuxin Wang, Quan Gan, Xipeng Qiu, Xuanjing Huang, and David Wipf. From hypergraph energy functions to hypergraph neural networks. In *International Conference on Machine Learning*, pages 35605–35623. PMLR, 2023.

[31] Erling Wei, Wenliang Tang, and Xiaofeng Wang. Flows in 3-edge-connected bidirected graphs. *Frontiers of Mathematics in China*, 6:339–348, 2011.

[32] Rui Xu and Cun-Quan Zhang. On flows in bidirected graphs. *Discrete mathematics*, 299(1-3):335–343, 2005.

[33] Soumalya Joardar and Atibur Rahaman. Almost complex structure on finite points from bidirected graphs. *Journal of Noncommutative Geometry*, 2023.

[34] Takaaki Fujita. Extensions of multidirected graphs: Fuzzy, neutrosophic, plithogenic, rough, soft, hypergraph, and superhypergraph variants. *International Journal of Topology*, 2(3):11, 2025.

[35] Mohammad Hamidi, Florentin Smarandache, and Elham Davneshvar. Spectrum of superhypergraphs via flows. *Journal of Mathematics*, 2022(1):9158912, 2022.