Plithogenic Fuzzy Natural Language and Its Upside-Down Logic: A Framework for Contradiction-Aware Reasoning

Takaaki Fujita [1] *
[1] Independent Researcher, Tokyo, Japan. Email: Takaaki.fujita060@gmail.com

Abstract

In the real world, many reversal phenomena occur, such as cases where a statement once regarded as false is later recognized as true. *Upside-Down Logic* is a framework designed to formalize such reversal phenomena as a logical system. It inverts the truth and falsity of lemmas through contextual transformations, thereby capturing ambiguity and reversals within reasoning processes. A *Plithogenic Set* models elements by means of attribute-based membership and contradiction functions, extending the classical frameworks of fuzzy, intuitionistic, and neutrosophic sets. In this paper, we investigate *Plithogenic Fuzzy Natural Language*, a concept that augments Fuzzy Natural Language by incorporating the notion of contradiction.

Keywords: Upside-down-logic, Plithogenic Set, De-Plithogenication, Plithogenic Language

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Contents in this paper

The remainder of this paper is organized as follows.

1 Preliminaries

This section presents the fundamental concepts and definitions that underpin the discussions in this paper. Throughout this paper, all structures and sets are assumed to be finite. Moreover, the graph is assumed to be undirected and simple.

1.1 Formal Definition of Upside-Down Logic

This subsection presents the mathematical definition of Upside-Down Logic. In brief, Upside-Down Logic flips the truth and falsity of lemmas under contextual transformations, thereby formalizing ambiguity and reversals in reasoning systems [1–6]. The related definitions and notations are given below.

Definition 1.1 (Context). [2, 3] A *context* C is a collection of parameters or conditions under which lemmas are evaluated. These may include spatial, temporal, semantic, or interpretive settings.

Example 1.2 (Real-life example for *Context*). Let the lemma be

$$A := \text{“Wearing a mask is required.”}$$

and consider two concrete contexts that collect spatial, temporal, and policy parameters:

$$C_{\text{hosp}} := \{\text{location=hospital, date=2021-01-15, policy=mandate\_active}\},$$

$$C_{\text{park}} := \{\text{location=city\_park},\ \text{date=2023-07-01},\ \text{policy=mandate\_lifted}\}.$$

Define a truth evaluation map $T : \mathcal{P} \times C \to \{\text{True}, \text{False}\}$. Then

$$T(A \mid C_{\text{hosp}}) = \text{True} \qquad \text{but} \qquad T(A \mid C_{\text{park}}) = \text{False}.$$

Thus, the same lemma A flips truth across contexts because the underlying conditions (space, time, policy) differ.

Definition 1.3 (Logical System)**.** (cf. [7]) A *logical system* $\mathcal{M}$ is a mathematical structure that formalizes reasoning. It consists of

$$\mathcal{M} = (\mathcal{P}, \mathcal{V}, v),$$

where:

- $\mathcal{P}$ is the set of lemmas (statements) in the formal language $\mathcal{L}$;
- $\mathcal{V}$ is the set of truth values (e.g., $\{\text{True}, \text{False}\}$ in classical logic);
- $v : \mathcal{P} \to \mathcal{V}$ is a *valuation* (interpretation) that assigns a truth value to each lemma in $\mathcal{P}$.

A logical system may additionally include:

- a set of *axioms* $\mathcal{A} \subseteq \mathcal{P}$ assumed true within the system;
- a set of *inference rules* $\mathcal{I}$ specifying valid transformations used to derive new truths.

Example 1.4 (Real-life example for *Logical System*)**.** Consider a building access-control system modeled as $\mathcal{M} = (\mathcal{P}, \mathcal{V}, v)$ with $\mathcal{V} = \{\text{True}, \text{False}\}$. Let the lemmas (atomic statements) be

$$B := \text{“Badge is valid”}, \quad T := \text{“Safety training completed”},$$

$$H := \text{“Current time is within 09:00–18:00”}, \quad A := \text{“Entry to the lab is permitted”}.$$

Axioms $\mathcal{A}$ and inference rules $\mathcal{I}$ are:

$$\mathcal{A} = \Big\{(B \wedge T \wedge H) \to A,\ \neg B \to \neg A\Big\}, \qquad \mathcal{I} = \{\text{modus ponens, conjunction introduction/elimination}\}.$$

Given a day where the valuation satisfies

$$v(B) = \text{True}, \qquad v(T) = \text{True}, \qquad v(H) = \text{True},$$

by conjunction introduction $v(B \wedge T \wedge H) = \text{True}$, and by modus ponens with $(B \wedge T \wedge H) \to A$ we derive $v(A) = \text{True}$ (entry permitted). Conversely, if $v(B) = \text{False}$ (e.g., expired badge), then by the axiom $\neg B \to \neg A$ and modus ponens, $v(A) = \text{False}$ (entry denied). This instantiates a concrete logical system whose axioms and rules govern the truth assignments of access decisions.

Notation 1.5. *Let $\mathcal{P}$ be a set of lemmas and C a set of contexts. Define the truth-valuation*

$$T :\ \mathcal{P} \times C \longrightarrow \{\mathit{True},\ \mathit{False},\ \mathit{Indeterminate}\},$$

which assigns a truth value to each lemma–context pair.

Notation 1.6. *Let $\mathcal{L}$ be a formal language, and let $\mathcal{M}$ be a logical system with lemma set $\mathcal{P}$, truth-value set $\mathcal{V}$, and valuation $v : \mathcal{P} \to \mathcal{V}$.*

Definition 1.7 (Upside-Down Logic)**.** [2,3] An *Upside-Down Logic* is a logical system $\mathcal{M}'$ obtained from $\mathcal{M}$ by introducing a transformation U on lemmas and/or contexts such that:

1. For any lemma $A \in \mathcal{P}$ with truth value $v(A)$ in context C, there exists a transformed lemma $U(A)$ and/or a transformed context $U(C)$ for which:
   - *Falsification of the Truth*: If $v(A) = \text{True}$ in C, then $v\big(U(A)\big) = \text{False}$ in $U(C)$.

- *Truthification of the False*: If $v(A) = \text{False}$ in C, then $v\big(U(A)\big) = \text{True}$ in $U(C)$.

2. The transformation U is well defined and consistent within the resulting system $\mathcal{M}'$.

Example 1.8 (Real-life Example 1: Parking Regulation by Time Window)**.** Let the lemma be

$$A := \text{“Parking on Main St. is allowed.”}$$

and let contexts encode the applicable time-rule:

$$C_{\text{wknd}} := \text{Weekend (no rush-hour restriction)}, \qquad C_{\text{rush}} := \text{Weekday 7–9am (rush-hour no-parking)}.$$

Define an Upside-Down transformation U that maps the context $U(C_{\text{wknd}}) = C_{\text{rush}}$ (while keeping A unchanged). With valuation $v(\cdot \mid \cdot)$:

$$v(A \mid C_{\text{wknd}}) = \text{True} \quad \Longrightarrow \quad v\big(U(A) \mid U(C_{\text{wknd}})\big) = v\big(A \mid C_{\text{rush}}\big) = \text{False}.$$

Hence, the same surface statement flips truth when moving from a “parking-permitted” temporal context to a “rush-hour” context.

Truthification of the False. Consider the complementary lemma $\neg A$:= “Parking on Main St. is *not* allowed.” Then

$$v(\neg A \mid C_{\text{wknd}}) = \text{False} \quad \Longrightarrow \quad v\big(U(\neg A) \mid U(C_{\text{wknd}})\big) = v\big(\neg A \mid C_{\text{rush}}\big) = \text{True}.$$

This illustrates both directions (falsification of the true and truthification of the false) via a context flip.

Example 1.9 (Real-life Example 2: Corporate Remote-Work Policy Reversal)**.** Let the lemma be

$$B := \text{“Employee } e \text{ may work remotely on Fridays.”}$$

and define two organizational-policy contexts:

$$C_{\text{remote-first}} := \text{Remote work encouraged (policy year 1)}, \qquad C_{\text{on-site}} := \text{On-site mandate (policy year 2)}.$$

Let U be the policy-reversal operator that maps the *context* $U(C_{\text{remote-first}}) = C_{\text{on-site}}$. Then

$$v(B \mid C_{\text{remote-first}}) = \text{True} \quad \Longrightarrow \quad v\big(U(B) \mid U(C_{\text{remote-first}})\big) = v\big(B \mid C_{\text{on-site}}\big) = \text{False}.$$

Truthification of the False is seen by reversing the arrow: if the company later restores flexibility (inverse transform U^{-1}), then a previously false permission becomes true again:

$$v(B \mid C_{\text{on-site}}) = \text{False} \implies v\big(U^{-1}(B) \mid U^{-1}(C_{\text{on-site}})\big) = v\big(B \mid C_{\text{remote-first}}\big) = \text{True}.$$

Thus, a policy-induced context transformation inverts the truth status of the same lemma.

1.2 Plithogenic Set

A Plithogenic Set [8–10] models elements with attribute-based membership and contradiction functions, extending fuzzy [11, 12], intuitionistic [13, 14], and neutrosophic sets [15, 16].

Definition 1.10 (Plithogenic Set)**.** [8, 17] Let S be a universal set and $P \subseteq S$ a nonempty subset. A *Plithogenic Set* is a quintuple

$$PS = (P, v, Pv, pdf, pCF),$$

where

- v is an attribute,
- Pv is the set of possible values of the attribute v,
- $pdf : P \times Pv \to [0, 1]^s$ is the *Degree of Appurtenance Function (DAF)*,[1]

[1] In the literature, DAF is defined in slightly different ways: some variants use powerset–valued constructions, others the simple cube $[0, 1]^s$. We adopt the latter (classical) form here; cf. [18].

- $pCF : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function (DCF).*

The DCF satisfies, for all $a, b \in Pv$,

$$\text{Reflexivity: } pCF(a,a) = 0, \qquad \text{Symmetry: } pCF(a,b) = pCF(b,a).$$

Here $s \in \mathbb{N}$ is the appurtenance dimension and $t \in \mathbb{N}$ the contradiction dimension.

Definition 1.11 (Plithogenic Fuzzy Set ($s = 1,\ t = 1$))**.** [8,19,20] A *Plithogenic Fuzzy Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \longrightarrow [0,1], \qquad pCF : Pv \times Pv \longrightarrow [0,1].$$

For $x \in P$ and attribute value $a \in Pv$, write

$$\mu_P(x \mid a) \ := \ pdf(x,a) \in [0,1],$$

the (single) fuzzy membership degree of x w.r.t. a. The contradiction between two attribute values is the scalar $c(a,b) := pCF(a,b) \in [0,1]$ with $c(a,a) = 0$ and $c(a,b) = c(b,a)$.

Example 1.12 (Concrete real–life instances of a Plithogenic Fuzzy Set with explicit calculations)**.** Fix a Plithogenic Fuzzy Set with one–dimensional membership and contradiction

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF), \qquad \mu(x \mid a) := pdf(x,a) \in [0,1], \quad c(a,b) := pCF(a,b) \in [0,1].$$

For decision making, suppose a user specifies a *dominant* attribute value $b \in Pv$. For illustration, we use the following linear plithogenic score

$$D_b(x) \ := \ \frac{1}{|Pv|} \sum_{a \in Pv} \Big((1 - c(a,b))\, \mu(x \mid a) \ + \ c(a,b)\, (1 - \mu(x \mid a)) \Big), \qquad (\star)$$

which keeps μ when a is close to b ($c \approx 0$) and blends toward the complement when a contradicts b ($c \approx 1$).

(i) Smartphone purchase (attribute: price level). Let $P = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ be three phones, v = "price level", $Pv = \{\mathrm{Cheap(Ch)}, \mathrm{Mid(Mi)}, \mathrm{Exp(Ex)}\}$. Dominant value: $b = \mathrm{Ch}$. Membership degrees:

	$\mu(\cdot \mid \mathrm{Ch})$	$\mu(\cdot \mid \mathrm{Mi})$	$\mu(\cdot \mid \mathrm{Ex})$
A	0.8	0.4	0.1
B	0.5	0.7	0.2
C	0.2	0.5	0.9

Contradiction (symmetric; zeros on the diagonal): $c(\mathrm{Ch}, \mathrm{Mi}) = 0.3$, $c(\mathrm{Ch}, \mathrm{Ex}) = 0.9$, $c(\mathrm{Mi}, \mathrm{Ex}) = 0.6$.

Using ($\star$) with $b = \mathrm{Ch}$:

$$D_{\mathrm{Ch}}(\mathrm{A}) = \tfrac{1}{3}\Big[(1-0)\cdot 0.8 + (1-0.3)\cdot 0.4 + 0.3\cdot(1-0.4) + (1-0.9)\cdot 0.1 + 0.9\cdot(1-0.1)\Big]$$
$$= \tfrac{1}{3}\big[0.8 + 0.28 + 0.18 + 0.01 + 0.81\big] = \tfrac{2.08}{3} \approx 0.693,$$

$$D_{\mathrm{Ch}}(\mathrm{B}) = \tfrac{1}{3}\big[0.5 + 0.7\cdot 0.7 + 0.3\cdot 0.3 + 0.1\cdot 0.2 + 0.9\cdot 0.8\big] = \tfrac{1}{3}\big[0.5 + 0.49 + 0.09 + 0.02 + 0.72\big] = \tfrac{1.82}{3} \approx 0.607,$$

$$D_{\mathrm{Ch}}(\mathrm{C}) = \tfrac{1}{3}\big[0.2 + 0.7\cdot 0.5 + 0.3\cdot 0.5 + 0.1\cdot 0.9 + 0.9\cdot 0.1\big] = \tfrac{1}{3}\big[0.2 + 0.35 + 0.15 + 0.09 + 0.09\big] = \tfrac{0.88}{3} \approx 0.293.$$

Ranking: $\mathrm{A} \succ \mathrm{B} \succ \mathrm{C}$ for a user preferring Cheap.

(ii) Hiring decision (attribute: experience level). Let $P = \{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$ be candidates, v = "experience", $Pv = \{\mathrm{Junior(J)}, \mathrm{Mid(M)}, \mathrm{Senior(S)}\}$. Dominant value: $b = \mathrm{S}$. Memberships:

	$\mu(\cdot \mid \mathrm{J})$	$\mu(\cdot \mid \mathrm{M})$	$\mu(\cdot \mid \mathrm{S})$
X	0.7	0.5	0.2
Y	0.4	0.6	0.5
Z	0.1	0.4	0.9

Contradictions: $c(\mathrm{J},\mathrm{S}) = 0.8$, $c(\mathrm{M},\mathrm{S}) = 0.3$, $c(\mathrm{J},\mathrm{M}) = 0.5$ (symmetric).

Computations:

$$\begin{aligned} D_{\mathrm{S}}(\mathrm{X}) &= \tfrac{1}{3}\Big[(1-0.8)\cdot 0.7 + 0.8\cdot(1-0.7) + (1-0.3)\cdot 0.5 + 0.3\cdot(1-0.5) + 0.2\Big] \\ &= \tfrac{1}{3}\big[0.14 + 0.24 + 0.35 + 0.15 + 0.2\big] = \tfrac{1.08}{3} \approx 0.360, \end{aligned}$$

$$D_{\mathrm{S}}(\mathrm{Y}) = \tfrac{1}{3}\big[0.2\cdot 0.4 + 0.8\cdot 0.6 + 0.7\cdot 0.6 + 0.3\cdot 0.4 + 0.5\big] = \tfrac{1}{3}\big[0.08 + 0.48 + 0.42 + 0.12 + 0.5\big] = \tfrac{1.60}{3} \approx 0.533,$$

$$\begin{aligned} D_{\mathrm{S}}(\mathrm{Z}) &= \tfrac{1}{3}\big[0.2\cdot 0.1 + 0.8\cdot 0.9 + 0.7\cdot 0.4 + 0.3\cdot 0.6 + 0.9\big] \\ &= \tfrac{1}{3}\big[0.02 + 0.72 + 0.28 + 0.18 + 0.9\big] = \tfrac{2.10}{3} \approx 0.700. \end{aligned}$$

Ranking: $\mathrm{Z} \succ \mathrm{Y} \succ \mathrm{X}$ for an employer prioritizing Senior.

(iii) Dietary choice for a diabetic (attribute: sugar level). Let $P = \{\mathrm{Oatmeal(O)},\ \mathrm{Smoothie(S)},\ \mathrm{Cake(C)}\}$, $v =$ "sugar", $Pv = \{\mathrm{Low(L)}, \mathrm{Moderate(M)}, \mathrm{High(H)}\}$. Dominant value: $b = \mathrm{L}$. Memberships:

	$\mu(\cdot \mid \mathrm{L})$	$\mu(\cdot \mid \mathrm{M})$	$\mu(\cdot \mid \mathrm{H})$
O	0.9	0.3	0.1
S	0.4	0.6	0.3
C	0.1	0.4	0.95

Contradictions: $c(\mathrm{L},\mathrm{M}) = 0.4$, $c(\mathrm{L},\mathrm{H}) = 0.95$, $c(\mathrm{M},\mathrm{H}) = 0.6$.

Computations:

$$\begin{aligned} D_{\mathrm{L}}(\mathrm{O}) &= \tfrac{1}{3}\big[0.9 + 0.6\cdot 0.3 + 0.4\cdot 0.7 + 0.05\cdot 0.1 + 0.95\cdot 0.9\big] \\ &= \tfrac{1}{3}\big[0.9 + 0.18 + 0.28 + 0.005 + 0.855\big] = \tfrac{2.22}{3} \approx 0.740, \\ D_{\mathrm{L}}(\mathrm{S}) &= \tfrac{1}{3}\big[0.4 + 0.6\cdot 0.6 + 0.4\cdot 0.4 + 0.05\cdot 0.3 + 0.95\cdot 0.7\big] \\ &= \tfrac{1}{3}\big[0.4 + 0.36 + 0.16 + 0.015 + 0.665\big] = \tfrac{1.60}{3} \approx 0.533, \\ D_{\mathrm{L}}(\mathrm{C}) &= \tfrac{1}{3}\big[0.1 + 0.6\cdot 0.4 + 0.4\cdot 0.6 + 0.05\cdot 0.95 + 0.95\cdot 0.05\big] \\ &= \tfrac{1}{3}\big[0.1 + 0.24 + 0.24 + 0.0475 + 0.0475\big] = \tfrac{0.675}{3} \approx 0.225. \end{aligned}$$

Ranking: $\mathrm{O} \succ \mathrm{S} \succ \mathrm{C}$ (low–sugar preference enforced by high contradictions with H).

These three scenarios instantiate $PS = (P, v, Pv, pdf, pCF)$ with concrete numeric pdf and pCF, and show, via ($\star$), how contradiction systematically modulates fuzzy memberships to produce decisions aligned with a chosen dominant value.

1.3 Fuzzy Language

Fuzzy Natural Language models linguistic expressions with graded memberships, representing vagueness and partial truth, enabling nuanced interpretation and context-sensitive acceptability [21–23]. Plithogenic Natural Language assigns multi-attribute membership vectors to words and contradiction degrees, supporting composition under conflicting attributes for semantic reasoning.

Definition 1.13 (Fuzzy Language)**.** (cf. [24–26]) Let Σ be a finite alphabet. Denote by Σ^* (resp. Σ^ω) the set of all finite (resp. right-infinite) words over Σ.

(i) Fuzzy set. A fuzzy set A on a universe X is given by its membership function

$$\mu_A : X \longrightarrow [0,1], \qquad \mu_A(x) \text{ is the degree of } x \text{ belonging to } A.$$

(ii) Fuzzy language on finite words. A (finite-word) fuzzy language is a map

$$r : \Sigma^* \longrightarrow [0,1], \qquad r(w) \text{ is the degree that } w \text{ is accepted.}$$

(iii) Fuzzy language on infinite words. A (infinite-word) fuzzy language is a map

$$r_\omega : \Sigma^\omega \longrightarrow [0,1], \qquad r_\omega(w) \text{ is the degree that } w \text{ is accepted.}$$

(iv) Support. The (finite-word) support and the ω-support are

$$\operatorname{supp}(r) := \{\, w \in \Sigma^* \mid r(w) > 0 \,\}, \qquad \operatorname{supp}(r_\omega) := \{\, w \in \Sigma^\omega \mid r_\omega(w) > 0 \,\}.$$

These notions model uncertainty by assigning to each word a graded acceptance in $[0,1]$.

Definition 1.14 (Fuzzy Natural Language)**.** (cf. [27, 28]) A *Fuzzy Natural Language* formalizes linguistic uncertainty with fuzzy sets. Fix a finite vocabulary Σ. Define

$$\mathcal{L}_F = (\Sigma,\ \mathcal{M}_F,\ \mathcal{P}_F,\ \mathcal{S}_F),$$

where

- $\mathcal{M}_F : \Sigma^* \to [0,1]$ assigns to each string w a degree $\mathcal{M}_F(w)$ measuring how strongly w belongs to the language/domain (e.g., in a temperature domain, $\mathcal{M}_F(\text{"warm"}) = 0.8$).
- $\mathcal{P}_F : \Sigma^* \times \Sigma^* \to [0,1]$ is a fuzzy similarity/proximity (e.g., $\mathcal{P}_F(\text{"warm"}, \text{"cozy"}) = 0.7$).
- $\mathcal{S}_F$ is a set of fuzzy constraints (syntactic/semantic rules); each rule ρ is given a degree $\mathcal{S}_F(\rho) \in [0,1]$ indicating admissibility of a construction.

When context variables (e.g., a measured temperature T, a wait time t) are present, $\mathcal{M}_F$ can be instantiated from fuzzy predicates (membership profiles) over those variables and then mapped to strings in Σ^*.

Example 1.15 (Daily-life instances of Fuzzy Natural Language)**.** We illustrate $\mathcal{L}_F$ with two concrete domains. In both, membership profiles are triangular/trapezoidal and map a real-valued context to degrees for linguistic labels.

(i) Smart thermostat (room temperature descriptions). Let the context be temperature T (in °C). Consider labels {"cold", "cool", "warm", "hot"} $\subset \Sigma$. Define memberships (piecewise linear):

$$\mu_{\text{cold}}(T) = \begin{cases} 1, & T \le 14, \\ \dfrac{20-T}{6}, & 14 < T < 20, \\ 0, & T \ge 20, \end{cases} \qquad \mu_{\text{cool}}(T) = \begin{cases} \dfrac{T-16}{4}, & 16 \le T \le 20, \\ \dfrac{24-T}{4}, & 20 < T \le 24, \\ 0, & \text{else}, \end{cases}$$

$$\mu_{\text{warm}}(T) = \begin{cases} \dfrac{T-20}{4}, & 20 \le T \le 24, \\ \dfrac{28-T}{4}, & 24 < T \le 28, \\ 0, & \text{else}, \end{cases} \qquad \mu_{\text{hot}}(T) = \begin{cases} 0, & T \le 26, \\ \dfrac{T-26}{6}, & 26 < T < 32, \\ 1, & T \ge 32. \end{cases}$$

Given a reading T, define $\mathcal{M}_F^{(T)}(w) := \mu_w(T)$ for each label w. For example,

T (°C)	$\mathcal{M}_F^{(T)}$("cold")	"cool"	"warm"	"hot"
18	0.333	0.500	0	0
24	0	0	1.000	0
27	0	0	0.250	0.167

A voice assistant could select the label with maximum degree, or keep all degrees for graded feedback (e.g., "somewhat warm").

(ii) Restaurant queue (expected wait time). Let the context be wait time t (minutes) with labels

$$\{\text{"short"}, \text{"moderate"}, \text{"long"}\} \subset \Sigma$$

. Define

$$\mu_{\text{short}}(t) = \begin{cases} \dfrac{t}{10}, & 0 \le t \le 10, \\ \dfrac{30-t}{20}, & 10 < t \le 30, \\ 0, & \text{else}, \end{cases}$$

$$\mu_{\text{moderate}}(t) = \begin{cases} \dfrac{t-20}{15}, & 20 \le t \le 35, \\ \dfrac{50-t}{15}, & 35 < t \le 50, \\ 0, & \text{else}, \end{cases}$$

$$\mu_{\text{long}}(t) = \begin{cases} 0, & t \le 40, \\ \dfrac{t-40}{20}, & 40 < t \le 60, \\ 1, & t \ge 60. \end{cases}$$

Then $\mathcal{M}_F^{(t)}(w) := \mu_w(t)$. For instance,

$$t = 25: \ (\text{short}, \text{moderate}, \text{long}) = (0.25,\ 0.333,\ 0.0), \qquad t = 55: \ (0.0,\ 0.0,\ 0.75).$$

A signage system can phrase updates like "moderate wait" at $t = 25$ or "long wait" at $t = 55$ while retaining graded semantics for decision support.

Definition 1.16 (Plithogenic Language)**.** [29] Let $PS = (P,\ v,\ Pv,\ pdf,\ pCF)$ be a plithogenic set (cf. [8,17]), where v is an attribute with value-set Pv, $pdf : P \times Pv \to [0,1]^s$ is the Degree of Appurtenance Function, and $pCF : Pv \times Pv \to [0,1]^t$ is the Degree of Contradiction Function, satisfying $pCF(a,a) = 0$ and $pCF(a,b) = pCF(b,a)$.

A Plithogenic Language over Σ^* (parameters s, t) is a function

$$PL : \Sigma^* \longrightarrow [0,1]^s,$$

assigning to each word w an s-dimensional membership vector $PL(w)$. Each component typically arises from evaluating $pdf(w,a)$ over $a \in Pv$; contradictions $pCF(a,b)$ govern aggregation/interaction among components when comparing or combining words.

Definition 1.17 (Plithogenic Natural Language)**.** A Plithogenic Natural Language integrates multi-attribute membership and explicit attribute-contradictions into language modeling. Fix a vocabulary Σ and a plithogenic structure (P, v, Pv, pdf, pCF). Define

$$\mathcal{L}_{PL} = (\Sigma,\ \mathcal{M}_{PL},\ \mathcal{P}_{PL},\ \mathcal{S}_{PL},\ pdf,\ pCF),$$

with

- $\mathcal{M}_{PL} : \Sigma^* \to [0,1]^s$: an s-vector membership for each word w, whose components are induced by $pdf(w,\cdot)$ along Pv.
- $\mathcal{P}_{PL} : \Sigma^* \times \Sigma^* \to [0,1]^s$: a plithogenic similarity between words, computed componentwise and modulated by pCF to account for contradictory attribute-values.
- $\mathcal{S}_{PL}$: a family of plithogenic constraints (rules) that assign graded well-formedness/acceptability, where rule-composition is performed using the same pdf/pCF-aware calculus.

This framework strictly extends fuzzy (and neutrosophic) language models by supporting multi-attribute memberships together with an explicit, symmetric contradiction structure on attribute-values.

Example 1.18 (Smart-home command interpretation (pragmatics, anchor = Direct))**.** We instantiate a Plithogenic Natural Language in a voice-controlled home setting.

Attribute and values. Let v = "pragmatic mode" with

$$Pv = \{\text{D}, \text{P}, \text{I}, \text{S}\} \quad \text{for Direct, Polite, Indirect, Sarcastic.}$$

For a phrase w, write $\mu(w \mid a) := pdf(w,a) \in [0,1]$ and $c(a,b) := pCF(a,b) \in [0,1]$ (symmetric, with $c(a,a) = 0$). Fix the anchor (dominant value) $b = \mathrm{D}$ (Direct).

Operational scoring. To obtain a scalar "execute-now" acceptability relative to the anchor, we use the convex flip–blend aggregator

$$\mathcal{A}(w;b) \;:=\; \frac{1}{|Pv|}\sum_{a\in Pv}\Big((1-c(a,b))\,\mu(w\mid a) \;+\; c(a,b)\,(1-\mu(w\mid a))\Big) \in [0,1].$$

Contradiction to the anchor. (only the row vs. D is needed here)

$$c(\cdot,\mathrm{D}) \;: \qquad c(\mathrm{D},\mathrm{D}) = 0,\;\; c(\mathrm{P},\mathrm{D}) = 0.30,\;\; c(\mathrm{I},\mathrm{D}) = 0.70,\;\; c(\mathrm{S},\mathrm{D}) = 0.90.$$

Utterances and memberships.

w	$\mu(w \mid \mathrm{D})$	$\mu(w \mid \mathrm{P})$	$\mu(w \mid \mathrm{I})$	$\mu(w \mid \mathrm{S})$
"Turn the heat down now."	0.95	0.40	0.10	0.02
"It's a bit chilly in here."	0.35	0.70	0.80	0.05

Calculations. For brevity let $T_a(w) := (1-c(a,b))\,\mu(w\mid a) + c(a,b)\,(1-\mu(w\mid a))$.

w	T_D	T_P	T_I	T_S	$\mathcal{A}(w;\mathrm{D})$
"Turn the heat down now."	0.950	0.460	0.660	0.884	$\frac{0.950+0.460+0.660+0.884}{4} \approx \mathbf{0.739}$
"It's a bit chilly in here."	0.350	0.580	0.380	0.860	$\frac{0.350+0.580+0.380+0.860}{4} \approx \mathbf{0.543}$

Interpretation. The plithogenic contradiction $c(\cdot,\mathrm{D})$ downweights evidence from values that oppose *Direct* (e.g. Sarcastic, Indirect) by blending toward their complements, yielding a higher anchored acceptability for a direct imperative than for an indirect hint.

Example 1.19 (Clinical messaging modality (certainty, anchor = Certain))**.** We model how wording strength affects *recommendation strength* in informational messages.

Attribute and values. Let v = "certainty modality" with

$$Pv = \{\mathrm{Ce},\mathrm{Pr},\mathrm{Po},\mathrm{Un}\} \quad \text{for Certain, Probable, Possible, Unlikely.}$$

Fix the anchor $b = \mathrm{Ce}$ (Certain). As before, $\mu(w \mid a) = pdf(w,a) \in [0,1]$ and $c(a,b) = pCF(a,b) \in [0,1]$ (symmetric, $c(a,a) = 0$).

Anchor-based aggregator.

$$\mathcal{A}(w;\mathrm{Ce}) \;:=\; \frac{1}{4}\sum_{a\in Pv}\Big((1-c(a,\mathrm{Ce}))\,\mu(w\mid a) + c(a,\mathrm{Ce})\,(1-\mu(w\mid a))\Big).$$

Contradiction to the anchor.

$$c(\cdot,\mathrm{Ce}) \;: \qquad c(\mathrm{Ce},\mathrm{Ce}) = 0,\;\; c(\mathrm{Pr},\mathrm{Ce}) = 0.30,\;\; c(\mathrm{Po},\mathrm{Ce}) = 0.60,\;\; c(\mathrm{Un},\mathrm{Ce}) = 0.90.$$

Sentences and memberships.

w	$\mu(w \mid \mathrm{Ce})$	$\mu(w \mid \mathrm{Pr})$	$\mu(w \mid \mathrm{Po})$	$\mu(w \mid \mathrm{Un})$
"Start the antibiotic today."	0.95	0.30	0.05	0.02
"You might consider starting the antibiotic."	0.40	0.60	0.70	0.10

Calculations ($T_a(w)$ defined as above):

w	T_Ce	T_Pr	T_Po	T_Un	$\mathcal{A}(w;\mathrm{Ce})$
"Start the antibiotic today."	0.950	0.420	0.590	0.884	$\frac{0.950+0.420+0.590+0.884}{4} \approx \mathbf{0.711}$
"You might consider starting the antibiotic."	0.400	0.540	0.460	0.820	$\frac{0.400+0.540+0.460+0.820}{4} \approx \mathbf{0.555}$

Interpretation. Under a *Certain*-anchored policy, hedged modalities (Possible, Unlikely) are contradictory to the anchor and thus contribute via their complements, lowering the final anchored acceptability for the tentative sentence compared with the unequivocal directive. This construction keeps the full plithogenic membership vector while yielding a single, policy-aligned acceptance score for decision-making.

1.4 Upside-Down Logic in Plithogenic Fuzzy Set with contradiction reset

Upside-Down Logic in a Plithogenic Fuzzy Set with contradiction reset flips fuzzy memberships when contradictions exceed a threshold, then neutralizes contradictions, ensuring consistency in contexts where values previously conflicted (cf. [30, 31]).

Definition 1.20 (Upside-Down Logic in Plithogenic Fuzzy Set with contradiction reset)**.** (cf. [30, 31]) Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Fuzzy Set with

$$\mu_P(x \mid a) := pdf(x, a) \in [0, 1], \qquad c(a, b) := pCF(a, b) \in [0, 1],$$

and $c(a, a) = 0,\ c(a, b) = c(b, a)$. Fix a reference (anchor) attribute $b \in Pv$ and a threshold $\tau \in [0, 1]$. Define the *activation set*

$$A_\tau(b) \ := \ \{\, a \in Pv \ : \ c(a, b) \geq \tau \,\}.$$

The *Upside-Down transform with contradiction reset* produces a new Plithogenic Fuzzy Set

$$PS^{U_{b,\tau}} \ = \ (P,\ v,\ Pv,\ pdf^{U_{b,\tau}},\ pCF^{U_{b,\tau}})$$

by

$$pdf^{U_{b,\tau}}(x, a) \ := \ \begin{cases} 1 - \mu_P(x \mid a), & a \in A_\tau(b), \\ \mu_P(x \mid a), & a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}}$ defined for all $u, v \in Pv$ by

$$pCF^{U_{b,\tau}}(u, v) \ := \ \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

That is, whenever the flip is triggered for the pair (a, b) (i.e. $a \in A_\tau(b)$), the post-transform contradiction between a and b is *reset to zero*.

Example 1.21 (Real-life scenarios of the Upside-Down transform with contradiction reset)**.** We instantiate the definition with concrete numbers and compute the activation set $A_\tau(b)$, the flipped membership $pdf^{U_{b,\tau}}$, and the reset contradiction $pCF^{U_{b,\tau}}$.

(a) Workplace policy reversal: remote-first → on-site emphasis (context flip). Let $P = \{\mathrm{E1}, \mathrm{E2}, \mathrm{E3}\}$ be employees, v = "work mode", and $Pv = \{\mathrm{O}, \mathrm{H}, \mathrm{R}\}$ for On-site, Hybrid, Remote. Choose anchor $b = \mathrm{R}$ (remote-first policy) and threshold $\tau = 0.7$.

Initial memberships (fuzzy suitability by mode):

	$\mu(\cdot \mid \mathrm{O})$	$\mu(\cdot \mid \mathrm{H})$	$\mu(\cdot \mid \mathrm{R})$
E1	0.20	0.60	0.90
E2	0.70	0.50	0.40
E3	0.80	0.30	0.20

Contradiction matrix (symmetric; zeros on diagonal):

$c(\cdot, \cdot) =$

	O	H	R
O	0	0.50	**0.85**
H	0.50	0	0.40
R	**0.85**	0.40	0

Activation set for $b = \mathrm{R}$ at $\tau = 0.7$:

$$A_\tau(\mathrm{R}) = \{\mathrm{O}\} \quad \big(\text{since } c(\mathrm{O}, \mathrm{R}) = 0.85 \geq 0.7,\ c(\mathrm{H}, \mathrm{R}) = 0.40 < 0.7\big).$$

Apply the transform $pdf^{U_{b,\tau}}(x,a) = \begin{cases} 1-\mu(x \mid a), & a \in A_\tau(b), \\ \mu(x \mid a), & a \notin A_\tau(b). \end{cases}$ Only the O-column flips:

	$\mu^U(\cdot \mid \text{O})$	$\mu^U(\cdot \mid \text{H})$	$\mu^U(\cdot \mid \text{R})$
E1	1 − 0.20 = 0.80	0.60	0.90
E2	1 − 0.70 = 0.30	0.50	0.40
E3	1 − 0.80 = 0.20	0.30	0.20

Contradiction reset on the activated pair {O, R}:

$$pCF^{U_{b,\tau}}(\text{O},\text{R}) = pCF^{U_{b,\tau}}(\text{R},\text{O}) = 0, \quad \text{all other entries unchanged.}$$

Interpretation: in the new (Upside-Down) context, modes most contradictory to the anchor are flipped (on-site "suitability" becomes its complement), and the policy resolves the O–R conflict by resetting their contradiction to 0.

(b) Food-safety reclassification after improved testing (context flip). Let $P = \{\text{Le}, \text{Be}, \text{Me}\}$ denote lettuce, beans (canned), and meat, v = "contamination risk", $Pv = \{\text{L}, \text{M}, \text{H}\}$ for Low/Medium/High. Choose anchor $b = \text{L}$ (company prioritizes low risk) and $\tau = 0.9$.

Initial memberships (risk assessments):

	$\mu(\cdot \mid \text{L})$	$\mu(\cdot \mid \text{M})$	$\mu(\cdot \mid \text{H})$
Le	0.70	0.30	0.10
Be	0.60	0.40	0.20
Me	0.20	0.50	0.80

Contradictions (symmetric):

$$c(\cdot,\cdot) = \begin{array}{c|ccc} & \text{L} & \text{M} & \text{H} \\ \hline \text{L} & 0 & 0.50 & \mathbf{0.95} \\ \text{M} & 0.50 & 0 & 0.60 \\ \text{H} & \mathbf{0.95} & 0.60 & 0 \end{array}$$

Activation set:

$$A_\tau(\text{L}) = \{\text{H}\} \quad \big(\text{since } c(\text{H},\text{L}) = 0.95 \ge 0.9,\ c(\text{M},\text{L}) = 0.50 < 0.9\big).$$

Flip only the H-column:

	$\mu^U(\cdot \mid \text{L})$	$\mu^U(\cdot \mid \text{M})$	$\mu^U(\cdot \mid \text{H})$
Le	0.70	0.30	1 − 0.10 = 0.90
Be	0.60	0.40	1 − 0.20 = 0.80
Me	0.20	0.50	1 − 0.80 = 0.20

Reset the contradiction on the activated pair:

$$pCF^{U_{b,\tau}}(\text{H},\text{L}) = pCF^{U_{b,\tau}}(\text{L},\text{H}) = 0, \quad \text{others unchanged.}$$

Interpretation: in the Upside-Down context (e.g. after revised lab protocols), the assessment attached to "High" reverses numerically, and the strongest conflict (H vs L) is declared resolved by setting its contradiction to 0.

1.5 Two-mode De-Plithogenication

We define Two-mode De-Plithogenication as follows (cf. [30, 31]). This method is designed to handle both cases: when a contradiction is a genuine conflict, and when the contradiction was not in fact real.

Definition 1.22 (Improved De-Plithogenication (two-mode, general plithogenic set))**.** [30] Let $PS = (P, v, Pv, \mathrm{pdf}, \mathrm{pCF})$ be a plithogenic set, where $\mathrm{pdf} : P \times Pv \to [0,1]$ is the Degree of Appurtenance Function (DAF) and $\mathrm{pCF} : Pv \times Pv \to [0,1]$ is the Degree of Contradiction Function (DCF), symmetric with $\mathrm{pCF}(a,a) = 0$. Fix an *anchor* $b \in Pv$ and a *threshold* $\tau \in [0,1]$.

Activation. An attribute value $a \in Pv$ is said to *activate (w.r.t.* b, τ*)* if

$$\mathsf{Act}(a \mid b, \tau) \ := \ \mathbf{1}[\,\mathrm{pCF}(a,b) \ \geq \ \tau\,] = 1.$$

Mode selection. Let $\mathcal{M} : Pv \times Pv \to \{0,1\}$ be a (user/policy) *mode selector* on unordered pairs: $\mathcal{M}(\{a,b\}) = \mathcal{M}(\{b,a\})$ with the following semantics for an activated pair $\{a,b\}$:

- Mode 0 (Neutralize-only): $\mathcal{M}(\{a,b\}) = 0$ means "the apparent contradiction was not a real conflict"; we *do not flip* any appurtenance coordinates tied to a (or b) and we simply reset their contradiction to 0.
- Mode 1 (Invert+Neutralize): $\mathcal{M}(\{a,b\}) = 1$ means "the contradiction is genuine"; we *invert* the appurtenance coordinates tied to a (or b) and then reset their contradiction to 0.

Two-mode Upside-Down operator. Define $U^{\mathrm{imp}}_{b,\tau,\mathcal{M}}(PS) =: (P, v, Pv, \mathrm{pdf}^U, \mathrm{pCF}^U)$ by

$$\mathrm{pdf}^U(x,a) := \begin{cases} 1 - \mathrm{pdf}(x,a), & \mathsf{Act}(a \mid b,\tau) = 1 \text{ and } \mathcal{M}(\{a,b\}) = 1, \\ \mathrm{pdf}(x,a), & \text{otherwise}, \end{cases} \qquad (x \in P, a \in Pv),$$

$$\mathrm{pCF}^U(u,w) := \begin{cases} 0, & \mathsf{Act}(u \mid b,\tau) = 1 \text{ or } \mathsf{Act}(w \mid b,\tau) = 1, \\ \mathrm{pCF}(u,w), & \text{otherwise}, \end{cases} \qquad (u, w \in Pv, \text{ with symmetry preserved}).$$

Improved De-Plithogenication sequence. A finite composition

$$PS^{\text{imp-dep}} \ := \ U^{\mathrm{imp}}_{b_k,\tau_k,\mathcal{M}_k} \circ \cdots \circ U^{\mathrm{imp}}_{b_2,\tau_2,\mathcal{M}_2} \circ U^{\mathrm{imp}}_{b_1,\tau_1,\mathcal{M}_1}(PS)$$

is called an *Improved De-Plithogenication* if, for every pair $\{u,w\} \subseteq Pv$ that was ever activated in the sequence, the final contradiction is zero: $\mathrm{pCF}^{\text{imp-dep}}(u,w) = 0$. If activations cover all pairs, then $\mathrm{pCF}^{\text{imp-dep}} \equiv 0$. The operator is idempotent on the stabilized state.

Example 1.23 (Two-mode De-Plithogenication in retail packaging policy)**.** Consider a plithogenic set

$$PS = (P, v, Pv, \mathrm{pdf}, \mathrm{pCF}), \qquad \mu(x \mid a) := \mathrm{pdf}(x,a) \in [0,1], \quad c(a,b) := \mathrm{pCF}(a,b) \in [0,1],$$

modelling one product's *packaging choice* under a corporate sustainability policy.

Items (candidate packages): $P = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ where A = single-use *virgin plastic* pouch, B = *bioplastic* pouch, C = *refillable glass* jar.

Attribute and values: v = "sustainability sourcing", $Pv = \{\mathrm{VP}, \mathrm{BP}, \mathrm{RP}, \mathrm{RG}\}$, with VP = Virgin Plastic, BP = BioPlastic, RP = Recycled Paper, RG = Refillable Glass.

Anchor and threshold: choose b = RG (the corporate target) and $\tau = 0.80$.

Initial fuzzy appurtenances $\mu(x \mid a)$ (how well each item fits each value):

	$\mu(\cdot \mid \mathrm{VP})$	$\mu(\cdot \mid \mathrm{BP})$	$\mu(\cdot \mid \mathrm{RP})$	$\mu(\cdot \mid \mathrm{RG})$
A	0.90	0.40	0.20	0.10
B	0.30	0.80	0.50	0.40
C	0.05	0.30	0.60	0.95

Contradictions $c(\cdot,\cdot)$ (symmetric, zero diagonal):

	VP	BP	RP	RG
VP	0	0.40	0.75	**0.95**
BP	0.40	0	0.35	**0.85**
RP	0.75	0.35	0	0.50
RG	**0.95**	**0.85**	0.50	0

Activation set (with $b = \text{RG}$, $\tau = 0.80$):

$$A_\tau(\text{RG}) = \{\text{VP}, \text{BP}\} \quad \text{since } c(\text{VP}, \text{RG}) = 0.95,\ c(\text{BP}, \text{RG}) = 0.85\ (\geq 0.80).$$

Two-mode selection for the activated pairs:

$$\mathcal{M}(\{\text{VP}, \text{RG}\}) = 1 \quad (\text{genuine conflict: } \textit{Invert+Neutralize}),$$

$$\mathcal{M}(\{\text{BP}, \text{RG}\}) = 0 \quad (\text{false conflict: } \textit{Neutralize-only}).$$

Apply $U^{\text{imp}}_{b,\tau,\mathcal{M}}$. Memberships transform by

$$\mu^U(x \mid a) = \begin{cases} 1 - \mu(x \mid a), & a \in \{\text{VP}, \text{BP}\} \text{ and } \mathcal{M}(\{a, \text{RG}\}) = 1, \\ \mu(x \mid a), & \text{otherwise.} \end{cases}$$

Here, only the VP column flips (because $\mathcal{M}(\{\text{VP}, \text{RG}\}) = 1$ and $\mathcal{M}(\{\text{BP}, \text{RG}\}) = 0$):

$$\mu^U(\text{A} \mid \text{VP}) = 1 - 0.90 = 0.10, \quad \mu^U(\text{B} \mid \text{VP}) = 1 - 0.30 = 0.70, \quad \mu^U(\text{C} \mid \text{VP}) = 1 - 0.05 = 0.95,$$
$$\mu^U(x \mid \text{BP}) = \mu(x \mid \text{BP}), \quad \mu^U(x \mid \text{RP}) = \mu(x \mid \text{RP}), \quad \mu^U(x \mid \text{RG}) = \mu(x \mid \text{RG}).$$

Thus the post-transform table is

	$\mu^U(\cdot \mid \text{VP})$	$\mu^U(\cdot \mid \text{BP})$	$\mu^U(\cdot \mid \text{RP})$	$\mu^U(\cdot \mid \text{RG})$
A	0.10	0.40	0.20	0.10
B	0.70	0.80	0.50	0.40
C	0.95	0.30	0.60	0.95

Contradictions reset by the rule (for this two-mode operator)

$$\text{pCF}^U(u, w) = \begin{cases} 0, & u \in A_\tau(\text{RG}) \text{ or } w \in A_\tau(\text{RG}) \quad (\text{i.e. any pair involving VP or BP}), \\ \text{pCF}(u, w), & \text{otherwise,} \end{cases}$$

so every entry in the VP and BP rows/columns becomes 0:

	VP	BP	RP	RG
VP	0	0	0	0
BP	0	0	0	0
RP	0	0	0	0.50
RG	0	0	0.50	0

Lifecycle analysis identified virgin plastic (VP) as genuinely at odds with the refillable-glass target (RG), hence Mode 1 flips its memberships and neutralizes its contradictions. Subsequent regulatory guidance clarified that bioplastic (BP) was not in real conflict with the target, so Mode 0 leaves its memberships unchanged while neutralizing its contradictions. The resulting state is "de-plithogenicated" along the activated dimensions and remains stable under further applications of $U^{\text{imp}}_{b,\tau,\mathcal{M}}$ (idempotence on the stabilized state).

1.6 Fuzzy Natural Language Processing

Fuzzy Natural Language Processing assigns fuzzy membership functions to words and phrases, handling vagueness, hedges, and graded meanings for semantic interpretation, reasoning, and decision-making in real-world linguistic contexts.

Definition 1.24 (Fuzzy Natural Language Processing (FNLP))**.** Let Σ be a finite vocabulary and K a finite set of perceptual/semantic *categories* (e.g., size, temperature). For each $k \in K$, fix a universe of discourse U_k and let $\widetilde{\mathcal{F}}(U_k)$ denote a class of fuzzy sets on U_k (Type-1 or interval Type-2).

An *FNLP system* is a tuple

$$\mathfrak{F} \;=\; \big(\Sigma,\; K,\; \{U_k\}_{k\in K},\; \mathcal{L},\; \mathcal{H},\; T,\; S,\; \mathrm{Agg},\; \Phi,\; \mathcal{A},\; \rho\big),$$

whose components are:

- **Lexicon:** $\mathcal{L} : \Sigma \times K \to \widetilde{\mathcal{F}}(U_k)$ assigns each token $w \in \Sigma$ and category k a fuzzy set (its lexical meaning in k). For interval Type-2, $\mathcal{L}(w,k) = \langle \underline{\mu}_{w,k}, \overline{\mu}_{w,k} \rangle$.
- **Hedges:** $\mathcal{H}$ is a set of operators $h : \widetilde{\mathcal{F}}(U_k) \to \widetilde{\mathcal{F}}(U_k)$ (e.g., *very*, *slightly*), each monotone and normal (i.e., $0 \le \mu \le \nu \Rightarrow h(\mu) \le h(\nu)$; h preserves the zero/one sets). For Type-2, $h\Big(\langle \underline{\mu}, \overline{\mu} \rangle\Big) = \langle h(\underline{\mu}), h(\overline{\mu}) \rangle$.
- **Connectives:** T (a t-norm) and S (its dual t-conorm) combine meanings for conjunction and disjunction, respectively, pointwise on membership functions.
- **Phrase aggregation:** For each k, $\mathrm{Agg}_k : \big(\widetilde{\mathcal{F}}(U_k)\big)^n \to \widetilde{\mathcal{F}}(U_k)$ is an associative, commutative, monotone aggregator (e.g., pointwise T, OWA), used to compose token-level meanings inside a sentence.
- **Scoring (defuzzification):** For each k, $\Phi_k : \widetilde{\mathcal{F}}(U_k) \to [0,1]$ maps a composed fuzzy meaning to a scalar *acceptance* in $[0,1]$ (e.g., height, area, centroid-based score; for Type-2, any well-defined Type-2 reduction).
- **Semantic interpreter:** $\mathcal{A} : \Sigma^* \to \bigcup_k \widetilde{\mathcal{F}}(U_k)$ parses a string $w = t_1 \cdots t_n$ and, for each k, builds

  $$[\![w]\!]_k \;:=\; \mathrm{Agg}_k\big(\tilde{m}_1, \ldots, \tilde{m}_n\big), \quad \tilde{m}_i \;=\; \begin{cases} \mathcal{L}(t_i, k), & t_i \text{ is a content token,} \\ h\big(\mathcal{L}(t_{i+1}, k)\big), & t_i \in \mathcal{H} \text{ modifies } t_{i+1}. \end{cases}$$

  The sentence-level *acceptance vector* is $\mathbf{a}(w) := \big(\Phi_k([\![w]\!]_k)\big)_{k\in K} \in [0,1]^{|K|}$, and a scalar acceptance (if desired) is $\mathrm{Acc}(w) := \sum_{k\in K} \beta_k \, \Phi_k([\![w]\!]_k)$ with weights $\beta_k \ge 0$, $\sum_k \beta_k = 1$.
- **Similarity:** $\rho : \Sigma^* \times \Sigma^* \to [0,1]$ is an FNLP *sentence similarity* defined from token–token *fuzzy* similarities. Write

  $$\mathrm{sim}(u,v) \;:=\; \frac{1}{|K|} \sum_{k\in K} S_{\mathrm{fuzzy}}\big(\mathcal{L}(u,k),\; \mathcal{L}(v,k)\big),$$

  where S_{fuzzy} is any fuzzy-set similarity in $[0,1]$ (for Type-2, a well-defined interval Type-2 set similarity). Then, with $\mathcal{M}(x,y)$ the set of all matchings between tokens of x and y,

  $$\rho(x,y) \;:=\; \lambda \cdot \max_{M\in\mathcal{M}(x,y)} \frac{1}{|M|} \sum_{(u,v)\in M} \mathrm{sim}(u,v) \;+\; (1-\lambda)\cdot \mathrm{Syn}(x,y),$$

  where $\lambda \in [0,1]$ and Syn is any normalized syntactic similarity in $[0,1]$.

Well-posedness. Because $T, S, \mathrm{Agg}_k, \Phi_k, S_{\mathrm{fuzzy}}$ are $[0,1]$-valued and monotone, $\mathbf{a}(w) \in [0,1]^{|K|}$, $\mathrm{Acc}(w) \in [0,1]$, and $\rho(x,y) \in [0,1]$ are well-defined for all inputs.

Example 1.25 (Helpdesk Priority Classification with FNLP)**.** We illustrate FNLP on a real-life IT helpdesk message:

``My laptop won't start at all, need it ASAP.''

We use a single category $k = \mathsf{priority}$ with a discrete universe $U_{\mathsf{priority}} = \{\mathsf{Low}, \mathsf{Med}, \mathsf{High}\}$. The lexicon $\mathcal{L}(\cdot, \mathsf{priority})$ provides fuzzy sets (membership degrees on U_{priority}) for salient tokens:

$$\mathcal{L}(\text{“won't start”}, \mathsf{priority}) = (0.00,\ 0.20,\ 0.90), \qquad \mathcal{L}(\text{“ASAP”}, \mathsf{priority}) = (0.00,\ 0.20,\ 0.95),$$

written as $(\mu_{\mathsf{Low}}, \mu_{\mathsf{Med}}, \mu_{\mathsf{High}})$.

(1) Composition. Using the product t-norm $T(x, y) = xy$, the sentence-level fuzzy meaning (inside this category) is the pointwise product:

$$[\![w]\!]_{\mathsf{priority}}(u) \;=\; T\big(\mathcal{L}(\text{“won't start”}, u),\ \mathcal{L}(\text{“ASAP”}, u)\big), \quad u \in U_{\mathsf{priority}}.$$

Thus

$$[\![w]\!]_{\mathsf{priority}} = (0.00 \cdot 0.00,\ 0.20 \cdot 0.20,\ 0.90 \cdot 0.95) = (0.00,\ 0.04,\ 0.855).$$

(2) Scoring / defuzzification. Map $\{\mathsf{Low}, \mathsf{Med}, \mathsf{High}\}$ to $\{0, \frac{1}{2}, 1\}$ and use a centroid-like score:

$$\Phi_{\mathsf{priority}}([\![w]\!]) = \frac{0 \cdot 0.00 + \frac{1}{2} \cdot 0.04 + 1 \cdot 0.855}{0.00 + 0.04 + 0.855} = \frac{0.020 + 0.855}{0.895} \approx 0.975.$$

(3) Decision. The acceptance vector is $(0.00, 0.04, 0.855)$ and the scalar acceptance is $\mathrm{Acc}(w) = 0.975$, i.e., the request is *overwhelmingly High priority*.

Example 1.26 (Restaurant Reservation Understanding with Hedges (*about*, *ish*, *around*))**.** Consider the utterance:

``A table for about five around 7-ish.''

We use two categories:

$$k_1 = \mathsf{party\_size}, \quad U_{k_1} = \{4, 5, 6\}; \qquad k_2 = \mathsf{time}, \quad U_{k_2} = \{18{:}30,\ 19{:}00,\ 19{:}30\}.$$

Hedges are modeled as fuzzy dilations: $h_{about}(\mu) = \sqrt{\mu}$, $h_{ish}(\mu) = \sqrt{\mu}$, and $h_{around}(\mu) = \sqrt{\mu}$.

(1) Lexicon (base meanings).

- *Size “five” (triangular):* $\mathcal{L}(\text{“five”}, k_1) = (0.5,\ 1.0,\ 0.5)$ on $\{4, 5, 6\}$.
- *Time “7” (triangular around 19:00):* $\mathcal{L}(\text{“7”}, k_2) = (0.5,\ 1.0,\ 0.5)$ on $\{18{:}30,\ 19{:}00,\ 19{:}30\}$.

(2) Apply hedges.

- “about five”: $h_{about}\big(\mathcal{L}(\text{“five”}, k_1)\big) = \big(\sqrt{0.5},\ \sqrt{1.0},\ \sqrt{0.5}\big) = (0.7071,\ 1.0000,\ 0.7071)$.
- “around 7-ish”: two dilations $\Rightarrow$ fourth root of the base profile:

$$h_{around}\big(h_{ish}(\mathcal{L}(\text{“7”}, k_2))\big) = \big(0.5^{1/4},\ 1^{1/4},\ 0.5^{1/4}\big) = (0.8409,\ 1.0000,\ 0.8409).$$

(3) Category-wise acceptances. Use Φ_{k_1} and Φ_{k_2} as normalized centroids on the discrete universes.

Party size (expected value):

$$\Phi_{k_1} = \frac{4(0.7071) + 5(1.0000) + 6(0.7071)}{0.7071 + 1.0000 + 0.7071} = \frac{2.8284 + 5.0000 + 4.2426}{2.4142} \approx \frac{12.0710}{2.4142} \approx 5.00.$$

Time (expected clock hour; encode 18:30=18.5, 19:00=19.0, 19:30=19.5):

$$\Phi_{k_2} = \frac{18.5(0.8409) + 19.0(1.0000) + 19.5(0.8409)}{0.8409 + 1.0000 + 0.8409} = \frac{15.5667 + 19.0000 + 16.3976}{2.6818} \approx \frac{50.9643}{2.6818} \approx 19.00.$$

(4) Interpretation. FNLP yields a *party size* near 5 and a *time* near 19:00 (7 pm), faithfully capturing the imprecision introduced by *about*, *ish*, and *around*. The category-wise fuzzy outputs can be combined into actionable database queries (e.g., search windows $[4, 6]$ and $[18{:}45, 19{:}15]$ with graded scores).

2 Main Results

This section presents the main findings and contributions of this paper.

2.1 Plithogenic Fuzzy Natural Language

A Plithogenic Fuzzy Natural Language models linguistic expressions with attribute-based memberships and explicit contradictions, capturing ambiguity, conflicts, and nuanced contextual meanings beyond classical fuzzy or neutrosophic approaches.

Definition 2.1 (Plithogenic Fuzzy Natural Language (PFNL))**.** Fix a finite vocabulary Σ and a plithogenic structure

$$(P,\ v,\ Pv,\ pdf,\ pCF),$$

where v is an attribute with value-set Pv, the Degree of Appurtenance Function is $pdf : \Sigma^* \times Pv \to [0,1]$, and the Degree of Contradiction Function is $pCF : Pv \times Pv \to [0,1]$ with $pCF(a,a) = 0$ and $pCF(a,b) = pCF(b,a)$. A *Plithogenic Fuzzy Natural Language* is the pair

$$\mathcal{L}_{\mathrm{PF}} = (\Sigma,\ \mathcal{M}_{\mathrm{PF}}),$$

where for a chosen *anchor* (reference) value $b \in Pv$ the (scalar) acceptance of $w \in \Sigma^*$ is obtained from the plithogenic memberships $\mu(w \mid a) := pdf(w,a) \in [0,1]$ and contradictions $c(a,b) := pCF(a,b) \in [0,1]$ via a contradiction-aware aggregator Agg:

$$\mathcal{M}_{\mathrm{PF}}(w;b)\ :=\ \mathsf{Agg}\big(\{\ (\mu(w \mid a),\ c(a,b))\ :\ a \in Pv\ \}\big)\ \in [0,1].$$

For explicit calculations we adopt the linear “flip–blend” aggregator

$$\mathcal{M}_{\mathrm{PF}}(w;b)\ :=\ \frac{1}{|Pv|}\sum_{a\in Pv}\Big((1-c(a,b))\,\mu(w \mid a)\ +\ c(a,b)\,(1-\mu(w \mid a))\Big), \tag{1}$$

which preserves $\mu(w \mid a)$ when a is close to b ($c(a,b) \approx 0$) and blends toward the complement when a strongly contradicts b ($c(a,b) \approx 1$).

Example 2.2 (PFNL in practice: sentiment–pragmatics for customer reviews (anchor = Positive))**.** We illustrate Definition 2.1 with a sentiment–pragmatics attribute. Fix the vocabulary fragment

$$\Sigma \supset \{\text{“great”},\ \text{“okay”},\ \text{“awful”},\ \text{“thanks a lot”}\}$$

. Let the attribute be v = “sentiment/pragmatics” with value-set

$$Pv = \{\text{Pos},\ \text{Neu},\ \text{Neg},\ \text{Sarc}\}.$$

Choose the anchor $b = \text{Pos}$. Write $\mu(w \mid a) := pdf(w,a) \in [0,1]$ and $c(a,b) := pCF(a,b) \in [0,1]$. We compute the scalar acceptance $\mathcal{M}_{\mathrm{PF}}(w;\text{Pos})$ with the flip–blend aggregator (1):

$$\mathcal{M}_{\mathrm{PF}}(w;\text{Pos}) = \frac{1}{4}\sum_{a\in Pv}\Big((1-c(a,\text{Pos}))\,\mu(w \mid a) + c(a,\text{Pos})\,(1-\mu(w \mid a))\Big).$$

Memberships (by expert/lexical prior) and anchor-contradictions:

word w	$\mu(w \mid \text{Pos})$	$\mu(w \mid \text{Neu})$	$\mu(w \mid \text{Neg})$	$\mu(w \mid \text{Sarc})$
“great”	0.95	0.15	0.02	0.05
“okay”	0.60	0.70	0.10	0.10
“awful”	0.03	0.10	0.97	0.20
“thanks a lot”	0.30	0.20	0.10	0.85

a	Pos	Neu	Neg	Sarc
$c(a,\text{Pos})$	0.00	0.35	0.95	0.80

Abbreviate $T_a(w) := (1 - c(a,\text{Pos}))\,\mu(w \mid a) + c(a,\text{Pos})\,(1 - \mu(w \mid a))$.

1) w = "great".

$$\begin{aligned}
T_{\text{Pos}} &= (1-0)\cdot 0.95 + 0\cdot(0.05) = 0.95,\\
T_{\text{Neu}} &= (1-0.35)\cdot 0.15 + 0.35\cdot 0.85 = 0.0975 + 0.2975 = 0.395,\\
T_{\text{Neg}} &= (1-0.95)\cdot 0.02 + 0.95\cdot 0.98 = 0.001 + 0.931 = 0.932,\\
T_{\text{Sarc}} &= (1-0.80)\cdot 0.05 + 0.80\cdot 0.95 = 0.010 + 0.760 = 0.770,\\
\mathcal{M}_{\text{PF}}(\text{“great”};\text{Pos}) &= \tfrac{0.95+0.395+0.932+0.770}{4} = 0.76175.
\end{aligned}$$

2) w = "okay".

$$\begin{aligned}
T_{\text{Pos}} &= 0.60,\\
T_{\text{Neu}} &= 0.65\cdot 0.70 + 0.35\cdot 0.30 = 0.455 + 0.105 = 0.560,\\
T_{\text{Neg}} &= 0.05\cdot 0.10 + 0.95\cdot 0.90 = 0.005 + 0.855 = 0.860,\\
T_{\text{Sarc}} &= 0.20\cdot 0.10 + 0.80\cdot 0.90 = 0.020 + 0.720 = 0.740,\\
\mathcal{M}_{\text{PF}}(\text{“okay”};\text{Pos}) &= \tfrac{0.60+0.560+0.860+0.740}{4} = 0.69000.
\end{aligned}$$

3) w = "awful".

$$\begin{aligned}
T_{\text{Pos}} &= 0.03,\\
T_{\text{Neu}} &= 0.65\cdot 0.10 + 0.35\cdot 0.90 = 0.065 + 0.315 = 0.380,\\
T_{\text{Neg}} &= 0.05\cdot 0.97 + 0.95\cdot 0.03 = 0.0485 + 0.0285 = 0.0770,\\
T_{\text{Sarc}} &= 0.20\cdot 0.20 + 0.80\cdot 0.80 = 0.040 + 0.640 = 0.680,\\
\mathcal{M}_{\text{PF}}(\text{“awful”};\text{Pos}) &= \tfrac{0.03+0.380+0.0770+0.680}{4} = 0.29175.
\end{aligned}$$

4) w = "thanks a lot" (often sarcastic).

$$\begin{aligned}
T_{\text{Pos}} &= 0.30,\\
T_{\text{Neu}} &= 0.65\cdot 0.20 + 0.35\cdot 0.80 = 0.130 + 0.280 = 0.410,\\
T_{\text{Neg}} &= 0.05\cdot 0.10 + 0.95\cdot 0.90 = 0.005 + 0.855 = 0.860,\\
T_{\text{Sarc}} &= 0.20\cdot 0.85 + 0.80\cdot 0.15 = 0.170 + 0.120 = 0.290,\\
\mathcal{M}_{\text{PF}}(\text{“thanks a lot”};\text{Pos}) &= \tfrac{0.30+0.410+0.860+0.290}{4} = 0.46500.
\end{aligned}$$

Conclusion (anchor = Pos):

$\mathcal{M}_{\text{PF}}(\text{“great”}) \approx 0.762 > \mathcal{M}_{\text{PF}}(\text{“okay”}) = 0.690 > \mathcal{M}_{\text{PF}}(\text{“thanks a lot”}) = 0.465 > \mathcal{M}_{\text{PF}}(\text{“awful”}) \approx 0.292.$

The plithogenic contradiction terms $c(a, \text{Pos})$ downweight contradictory values (Neg, Sarc) by blending their complements, thereby capturing pragmatic effects (e.g. sarcasm) while remaining a scalar fuzzy acceptance in $[0, 1]$.

Example 2.3 (PFNL for Home Energy Scheduling (anchor = Eco))**.** Consider a smart-home assistant that schedules appliances. Let the attribute be v = "energy mode" with values

$$Pv = \{\text{Ec}, \text{No}, \text{Ru}, \text{Jo}\} \quad (\text{Eco, Normal, Rush, Joking}).$$

Choose the anchor b = Ec. Denote $\mu(w \mid a) := pdf(w, a) \in [0, 1]$ and $c(a, b) := pCF(a, b) \in [0, 1]$. The scalar acceptance uses the flip–blend aggregator (1). For brevity write

$$T_a(w) := (1 - c(a, b))\,\mu(w \mid a) + c(a, b)\,(1 - \mu(w \mid a)).$$

Contradictions to the anchor (Eco):

$$c(\cdot, \text{Ec}): \quad c(\text{Ec}, \text{Ec}) = 0, \;\; c(\text{No}, \text{Ec}) = 0.30, \;\; c(\text{Ru}, \text{Ec}) = 0.90, \;\; c(\text{Jo}, \text{Ec}) = 0.70.$$

Utterances and memberships.

w	$\mu(\cdot \mid \text{Ec})$	$\mu(\cdot \mid \text{No})$	$\mu(\cdot \mid \text{Ru})$	$\mu(\cdot \mid \text{Jo})$
"Run the dishwasher tonight off-peak."	0.90	0.50	0.05	0.02
"Run the dishwasher now, I'm late!"	0.10	0.30	0.95	0.05

Calculations.

w	T_{Ec}	T_{No}	T_{Ru}	T_{Jo}	$\mathcal{M}_{\text{PF}}(w;\text{Ec})$
off-peak	0.900	0.500	0.860	0.692	$\frac{0.900+0.500+0.860+0.692}{4} \approx \mathbf{0.738}$
now	0.100	0.420	0.140	0.680	$\frac{0.100+0.420+0.140+0.680}{4} \approx \mathbf{0.335}$

Interpretation. With Eco as anchor, evidence from highly contradictory values (Rush, Joking) is blended toward its complement, yielding high acceptance for the off-peak request and low acceptance for the rush request.

Example 2.4 (PFNL for E-commerce Review Helpfulness (anchor = Helpful))**.** We triage product reviews by *helpfulness*. Let v = "review pragmatics" with

$$Pv = \{\text{He}, \text{Ne}, \text{Of}, \text{Sa}\} \quad \text{(Helpful, Neutral, Off-topic, Sarcastic).}$$

Anchor $b = \text{He}$. As above, use (1) and $T_a(w) := (1 - c(a,b))\,\mu(w \mid a) + c(a,b)\,(1 - \mu(w \mid a))$.

Contradictions to Helpful:

$$c(\cdot,\text{He}): \quad c(\text{He},\text{He}) = 0, \;\; c(\text{Ne},\text{He}) = 0.30, \;\; c(\text{Of},\text{He}) = 0.85, \;\; c(\text{Sa},\text{He}) = 0.75.$$

Reviews and memberships.

w	$\mu(\cdot \mid \text{He})$	$\mu(\cdot \mid \text{Ne})$	$\mu(\cdot \mid \text{Of})$	$\mu(\cdot \mid \text{Sa})$
"Size chart helped me pick medium; fits perfectly."	0.92	0.25	0.05	0.02
"Five stars because the courier smiled :)"	0.20	0.40	0.70	0.30

Calculations.

w	T_{He}	T_{Ne}	T_{Of}	T_{Sa}	$\mathcal{M}_{\text{PF}}(w;\text{He})$
size chart	0.920	0.400	0.815	0.740	$\frac{0.920+0.400+0.815+0.740}{4} \approx \mathbf{0.719}$
courier	0.200	0.460	0.360	0.600	$\frac{0.200+0.460+0.360+0.600}{4} \approx \mathbf{0.405}$

Interpretation. Off-topic and sarcastic cues are contradictory to the Helpful anchor and thus contribute via their complements, ensuring concretely informative reviews rank above off-topic praise.

Theorem 2.5 (PFNL is exactly the $s{=}1,\ t{=}1$ case of PNL)**.** *Let $\mathcal{L}_{\text{PL}}$ be a Plithogenic Natural Language with parameters s, t. If $s = 1$ and $t = 1$, then every $\mathcal{M}_{\text{PL}}(w) \in [0,1]$ is scalar and every $pCF(a,b) \in [0,1]$ is scalar; consequently $\mathcal{L}_{\text{PL}}$ specializes to a Plithogenic Fuzzy Natural Language $\mathcal{L}_{\text{PF}}$ as in Definition 2.1. Conversely, any PFNL determines a PNL with $s = t = 1$.*

Proof. By Definition **??**, $\mathcal{M}_{\text{PL}} : \Sigma^* \to [0,1]^s$. Setting $s = 1$ yields $\mathcal{M}_{\text{PL}} : \Sigma^* \to [0,1]$, i.e. scalar membership. Similarly, $t = 1$ makes $pCF : Pv \times Pv \to [0,1]$ scalar with the same reflexive and symmetric axioms as in Definition 2.1. Choosing an anchor $b \in Pv$ and an aggregation scheme (e.g. (1)) produces the scalar acceptance $\mathcal{M}_{\text{PF}}(w;b) \in [0,1]$. Hence we obtain precisely PFNL. Conversely, a PFNL already has scalar memberships and contradictions, hence it is a PNL with $s = t = 1$. □

Theorem 2.6 (PFNL strictly generalizes Fuzzy Natural Language)**.** *Let $r : \Sigma^* \to [0,1]$ be any fuzzy language (Definition 1.13). There exists a PFNL $\mathcal{L}_{\text{PF}}$ such that, for a suitable anchor b, $\mathcal{M}_{\text{PF}}(w;b) = r(w)$ for all $w \in \Sigma^*$.*

Proof. Construct a PFNL as follows. Take $Pv = \{b\}$ a singleton, define

$$\mu(w \mid b) := pdf(w, b) \ := \ r(w) \in [0, 1], \qquad c(b, b) := pCF(b, b) \ := \ 0.$$

With $|Pv| = 1$, the aggregator (1) reduces to

$$\mathcal{M}_{\mathrm{PF}}(w; b) = \ (1 - c(b, b))\, \mu(w \mid b) \ + \ c(b, b)\, (1 - \mu(w \mid b)) = \ 1 \cdot r(w) + 0 \cdot (1 - r(w)) = \ r(w).$$

Thus every fuzzy language is realized as a PFNL (choose the trivial one-attribute, zero-contradiction instance). Since PFNL allows $|Pv| \geq 1$ and nonzero contradictions to modulate membership by (1), PFNL has strictly more expressive power than ordinary fuzzy languages. □

2.2 Upside-down logic in Plithogenic Fuzzy Natural Language

Upside-Down Logic in Plithogenic Fuzzy Natural Language flips word membership acceptances under contextual contradictions, capturing reversals and ambiguity while resetting contradiction parameters for consistent semantic reasoning.

Definition 2.7 (Upside-Down transform with contradiction reset for PFNL)**.** Let a Plithogenic Fuzzy Natural Language (PFNL) be given by

$$\mathcal{L}_{\mathrm{PF}} = (\Sigma,\ \mathcal{M}_{\mathrm{PF}}), \qquad \mu(w \mid a) := pdf(w, a) \in [0, 1], \quad c(a, b) := pCF(a, b) \in [0, 1],$$

where v is a fixed attribute with value set Pv, Σ is a finite vocabulary, $pdf : \Sigma^* \times Pv \to [0, 1]$ is the Degree of Appurtenance Function, and $pCF : Pv \times Pv \to [0, 1]$ is the Degree of Contradiction Function with $c(a, a) = 0$ and $c(a, b) = c(b, a)$.

Fix an *anchor* $b \in Pv$ and a threshold $\tau \in [0, 1]$. Define the activation set

$$A_\tau(b) := \{\, a \in Pv \mid c(a, b) \geq \tau \,\}.$$

The *Upside-Down (UD) transform with contradiction reset* produces a new PFNL

$$U^{\mathrm{PF}}_{b,\tau}(\mathcal{L}_{\mathrm{PF}}) = \big(\Sigma,\ \mathcal{M}^{U}_{\mathrm{PF}}\big)$$

together with updated primitives pdf^U, pCF^U by

$$pdf^U(w, a) := \begin{cases} 1 - \mu(w \mid a), & a \in A_\tau(b), \\ \mu(w \mid a), & a \notin A_\tau(b), \end{cases}$$

$$pCF^U(u, v) := \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise}, \end{cases}$$

and the (scalar) acceptance in the UD context is computed by the flip–blend aggregator

$$\mathcal{M}^{U}_{\mathrm{PF}}(w; b) = \frac{1}{|Pv|} \sum_{a \in Pv} \Big((1 - c^U(a, b))\, pdf^U(w, a) \ + \ c^U(a, b)\, (1 - pdf^U(w, a)) \Big), \tag{2}$$

where $c^U(a, b) := pCF^U(a, b)$.

Lemma 2.8 (Idempotence and range preservation)**.** *For any $b \in Pv$ and $\tau \in [0, 1]$,*

$$U^{\mathrm{PF}}_{b,\tau} \circ U^{\mathrm{PF}}_{b,\tau} = U^{\mathrm{PF}}_{b,\tau}.$$

Moreover, for every $w \in \Sigma^*$, $\mathcal{M}^{U}_{\mathrm{PF}}(w; b) \in [0, 1]$.

Proof. Idempotence: After the first transform, for every $a \in A_\tau(b)$ we have $pdf^U(w, a) = 1 - \mu(w \mid a)$ and $c^U(a, b) = 0$. Hence the new activation set is $A^U_\tau(b) = \{\, a \in Pv \mid c^U(a, b) \geq \tau \,\} = \varnothing$, so a second application makes no further changes: $pdf^{UU} = pdf^U$ and $pCF^{UU} = pCF^U$.

Range: Each summand in (2) is of the form $(1 - \alpha)x + \alpha(1 - x)$ with $\alpha, x \in [0, 1]$. This equals $(1 - 2\alpha)x + \alpha \in [0, 1]$ because it is a convex combination of x and $1 - x$. Averaging preserves $[0, 1]$. □

Remark 2.9 (Extreme case)**.** If $c(a,b)=1$ for all $a \in Pv$ and no reset is applied (pure flip–blend),

$$\mathcal{M}_{\mathrm{PF}}(w;b) = \frac{1}{|Pv|}\sum_a (1-\mu(w \mid a)) = 1 - \frac{1}{|Pv|}\sum_a \mu(w \mid a).$$

Under the UD transform with reset, activated pairs $\{a,b\}$ subsequently satisfy $c^U(a,b)=0$, eliminating further flips w.r.t. b.

Example 2.10 (Concrete PFNL Upside-Down calculation: positive–negative–sarcasm)**.** Let $Pv = \{\mathrm{Pos}, \mathrm{Neg}, \mathrm{Sarc}\}$ and anchor $b = \mathrm{Pos}$. Contradictions to the anchor:

$$c(\mathrm{Pos},\mathrm{Pos}) = 0, \quad c(\mathrm{Neg},\mathrm{Pos}) = 0.95, \quad c(\mathrm{Sarc},\mathrm{Pos}) = 0.75.$$

Choose threshold $\tau = 0.8$, hence $A_\tau(\mathrm{Pos}) = \{\mathrm{Neg}\}$. Consider two words w_1 = "great" and w_2 = "awful" with memberships

w	$\mu(w \mid \mathrm{Pos})$	$\mu(w \mid \mathrm{Neg})$	$\mu(w \mid \mathrm{Sarc})$
"great"	0.90	0.02	0.10
"awful"	0.05	0.95	0.20

Before UD (use (2) *with pdf, c).* For brevity, set $T_a(w) := (1-c(a,\mathrm{Pos}))\,\mu(w \mid a) + c(a,\mathrm{Pos})\,(1-\mu(w \mid a))$.

For w = "great":

$$\begin{aligned}
T_{\mathrm{Pos}} &= (1-0)\cdot 0.90 + 0\cdot 0.10 = 0.90,\\
T_{\mathrm{Neg}} &= (1-0.95)\cdot 0.02 + 0.95\cdot 0.98 = 0.001 + 0.931 = 0.932,\\
T_{\mathrm{Sarc}} &= (1-0.75)\cdot 0.10 + 0.75\cdot 0.90 = 0.025 + 0.675 = 0.700,\\
\mathcal{M}_{\mathrm{PF}}(\text{“great”};\mathrm{Pos}) &= \tfrac{0.90+0.932+0.700}{3} = \tfrac{2.532}{3} \approx 0.844.
\end{aligned}$$

For w = "awful":

$$\begin{aligned}
T_{\mathrm{Pos}} &= 0.05,\\
T_{\mathrm{Neg}} &= (1-0.95)\cdot 0.95 + 0.95\cdot 0.05 = 0.0475 + 0.0475 = 0.095,\\
T_{\mathrm{Sarc}} &= (1-0.75)\cdot 0.20 + 0.75\cdot 0.80 = 0.05 + 0.60 = 0.65,\\
\mathcal{M}_{\mathrm{PF}}(\text{“awful”};\mathrm{Pos}) &= \tfrac{0.05+0.095+0.65}{3} = \tfrac{0.795}{3} \approx 0.265.
\end{aligned}$$

Apply UD with reset (Definition 2.7). Activated value: Neg. Then

$$pdf^U(w,\mathrm{Neg}) = 1-\mu(w \mid \mathrm{Neg}), \qquad c^U(\mathrm{Neg},\mathrm{Pos}) = 0,$$

and all other pdf, c entries remain unchanged.

Recompute with $T_a^U(w) := (1-c^U(a,\mathrm{Pos}))\,pdf^U(w,a) + c^U(a,\mathrm{Pos})\,(1-pdf^U(w,a))$.

For w = "great":

$$\begin{aligned}
T^U_{\mathrm{Pos}} &= 0.90,\\
T^U_{\mathrm{Neg}} &= (1-0)\cdot(1-0.02) = 0.98,\\
T^U_{\mathrm{Sarc}} &= 0.700 \quad \text{(unchanged)},\\
\mathcal{M}^U_{\mathrm{PF}}(\text{“great”};\mathrm{Pos}) &= \tfrac{0.90+0.98+0.700}{3} = \tfrac{2.58}{3} \approx 0.860.
\end{aligned}$$

For w = "awful":

$$\begin{aligned}
T^U_{\mathrm{Pos}} &= 0.05,\\
T^U_{\mathrm{Neg}} &= (1-0)\cdot(1-0.95) = 0.05,\\
T^U_{\mathrm{Sarc}} &= 0.65,\\
\mathcal{M}^U_{\mathrm{PF}}(\text{“awful”};\mathrm{Pos}) &= \tfrac{0.05+0.05+0.65}{3} = \tfrac{0.75}{3} = 0.250.
\end{aligned}$$

Thus the UD transform increases positive acceptance for "great" ($0.844 \to 0.860$) and decreases it for "awful" ($0.265 \to 0.250$), while neutralizing the strongest contradiction pair by setting $c^U(\mathrm{Neg},\mathrm{Pos}) = 0$. By lemma 2.8, reapplying the same UD operation makes no further change.

2.3 Two-mode De-Plithogenication in Plithogenic Fuzzy Natural Language

Two-mode De-Plithogenication in Plithogenic Fuzzy Natural Language selects between neutralizing or inverting contradictory memberships, then resets contradictions, producing context-consistent acceptability within natural language reasoning.

Definition 2.11 (Two-mode De-Plithogenication for PFNL)**.** Let a Plithogenic Fuzzy Natural Language (PFNL) be given by

$$\mathcal{L}_{\mathrm{PF}} = (\Sigma,\ \mathcal{M}_{\mathrm{PF}}), \qquad \mu(w \mid a) := pdf(w, a) \in [0,1], \quad c(a, b) := pCF(a, b) \in [0,1],$$

where v is a fixed attribute with value set Pv, $pdf : \Sigma^* \times Pv \to [0,1]$ is the Degree of Appurtenance Function, and $pCF : Pv \times Pv \to [0,1]$ is the Degree of Contradiction Function with $c(a, a) = 0$ and $c(a, b) = c(b, a)$. Fix an *anchor* $b \in Pv$ and a threshold $\tau \in [0,1]$. Define the activation predicate and set

$$\mathsf{Act}(a \mid b, \tau) := \mathbf{1}[\, c(a, b) \geq \tau \,], \qquad A_\tau(b) := \{\, a \in Pv \ :\ \mathsf{Act}(a \mid b, \tau) = 1 \,\}.$$

A *mode selector* is a map $\mathcal{M} : \{\{a, b\} : a \in Pv\} \to \{0,1\}$, symmetric in its arguments, with the following semantics for any activated pair $\{a, b\}$:

- $\mathcal{M}(\{a, b\}) = 0$ (Neutralize-only): the apparent contradiction is deemed not genuine; we keep memberships and only neutralize contradictions.
- $\mathcal{M}(\{a, b\}) = 1$ (Invert+Neutralize): the contradiction is genuine; we invert memberships for a and then neutralize contradictions.

The *Two-mode De-Plithogenication operator* produces

$$U^{\mathrm{imp,PF}}_{b,\tau,\mathcal{M}}(\mathcal{L}_{\mathrm{PF}}) = \big(\Sigma,\ \mathcal{M}^{U}_{\mathrm{PF}}\big),$$

with updated primitives pdf^U, pCF^U given by, for all $w \in \Sigma^*$, $a, u, v \in Pv$,

$$pdf^U(w, a) := \begin{cases} 1 - \mu(w \mid a), & \mathsf{Act}(a \mid b, \tau) = 1 \ \wedge\ \mathcal{M}(\{a, b\}) = 1, \\ \mu(w \mid a), & \text{otherwise,} \end{cases}$$

$$pCF^U(u, v) := \begin{cases} 0, & \mathsf{Act}(u \mid b, \tau) = 1 \ \text{ or } \ \mathsf{Act}(v \mid b, \tau) = 1, \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

The scalar acceptance in the transformed context (with respect to anchor b) is computed by the same flip–blend aggregator as in (1), but applied to pdf^U and $c^U := pCF^U$:

$$\mathcal{M}^{U}_{\mathrm{PF}}(w; b) = \frac{1}{|Pv|} \sum_{a \in Pv} \Big((1 - c^U(a, b))\, pdf^U(w, a) \ +\ c^U(a, b)\, (1 - pdf^U(w, a)) \Big) \in [0,1]. \tag{3}$$

Lemma 2.12 (Basic properties)**.** *For any $b \in Pv$, $\tau \in [0,1]$, and mode selector $\mathcal{M}$:*

(i) ***Idempotence.*** $U^{\mathrm{imp,PF}}_{b,\tau,\mathcal{M}} \circ U^{\mathrm{imp,PF}}_{b,\tau,\mathcal{M}} = U^{\mathrm{imp,PF}}_{b,\tau,\mathcal{M}}$.

(ii) ***Range preservation.*** *For all $w \in \Sigma^*$, $\mathcal{M}^{U}_{\mathrm{PF}}(w; b) \in [0,1]$.*

(iii) ***Contradiction monotonicity.*** *$pCF^U(u, v) \leq pCF(u, v)$ pointwise on $[0,1]$.*

(iv) ***Reductions.*** *If $\mathcal{M}(\{a, b\}) \equiv 1$ for all activated a, the operator coincides with the Upside-Down transform with contradiction reset; if $\mathcal{M}(\{a, b\}) \equiv 0$, it is a neutralize-only (no-flip) reset.*

Proof. (i) After one application, every activated value has all its incident contradictions reset to 0. Hence no value can activate again (the activation condition $c^U(\cdot, b) \geq \tau$ fails), so a second application performs no change.

(ii) Each summand in (3) is $(1-\alpha)x + \alpha(1-x)$ with $\alpha, x \in [0,1]$, a convex combination of x and $1-x$, thus in $[0,1]$; the average stays in $[0,1]$.

(iii) By construction, some entries are set to 0 and all others are left unchanged.

(iv) Immediate from the definitions of the two modes. □

Example 2.13 (Worked PFNL two-mode update: positive vs neutral vs negative)**.** Let $Pv = \{\text{Pos}, \text{Neu}, \text{Neg}\}$ with anchor $b = \text{Pos}$. Contradictions to the anchor:

$$c(\text{Pos}, \text{Pos}) = 0, \qquad c(\text{Neu}, \text{Pos}) = 0.85, \qquad c(\text{Neg}, \text{Pos}) = 0.95.$$

Choose threshold $\tau = 0.80$, so $A_\tau(\text{Pos}) = \{\text{Neu}, \text{Neg}\}$. Select modes to illustrate both cases:

$$\mathcal{M}(\{\text{Neg}, \text{Pos}\}) = 1 \quad (\text{Invert+Neutralize}), \qquad \mathcal{M}(\{\text{Neu}, \text{Pos}\}) = 0 \quad (\text{Neutralize-only}).$$

Two words with memberships $\mu(w \mid a)$:

w	$\mu(w \mid \text{Pos})$	$\mu(w \mid \text{Neu})$	$\mu(w \mid \text{Neg})$
"okay"	0.60	0.70	0.10
"awful"	0.05	0.20	0.95

Before the update (flip–blend w.r.t. $b = \text{Pos}$):

$$\begin{aligned}\mathcal{M}_{\text{PF}}(\text{"okay"}; \text{Pos}) &= \tfrac{1}{3}\Big((1-0)\cdot 0.60 + 0\cdot 0.40 + (1-0.85)\cdot 0.70 + 0.85\cdot 0.30 \\ &\qquad + (1-0.95)\cdot 0.10 + 0.95\cdot 0.90\Big) \\ &= \tfrac{1}{3}\big(0.60 + 0.105 + 0.255 + 0.005 + 0.855\big) = \tfrac{1.82}{3} \approx 0.6067, \\ \mathcal{M}_{\text{PF}}(\text{"awful"}; \text{Pos}) &= \tfrac{1}{3}\Big(0.05 + (1-0.85)\cdot 0.20 + 0.85\cdot 0.80 + (1-0.95)\cdot 0.95 + 0.95\cdot 0.05\Big) \\ &= \tfrac{1}{3}\big(0.05 + 0.03 + 0.68 + 0.0475 + 0.0475\big) = \tfrac{0.855}{3} = 0.285.\end{aligned}$$

Apply two-mode operator. Activated values are Neu, Neg.

- Mode 1 on Neg: $pdf^U(w, \text{Neg}) = 1 - \mu(w \mid \text{Neg})$.
- Mode 0 on Neu: $pdf^U(w, \text{Neu}) = \mu(w \mid \text{Neu})$.
- Neutralization: $c^U(a, \text{Pos}) = 0$ for $a \in \{\text{Neu}, \text{Neg}\}$ (and, more generally, all pairs involving an activated value are set to 0).

Thus in (3) the terms for Neu, Neg reduce to their (post-)memberships (no blending).

For w = "okay":

$$\mathcal{M}^U_{\text{PF}}(\text{"okay"}; \text{Pos}) = \tfrac{1}{3}\Big(\underbrace{0.60}_{\text{Pos}} + \underbrace{0.70}_{\text{Neu}} + \underbrace{1-0.10}_{\text{Neg}}\Big) = \tfrac{0.60+0.70+0.90}{3} = \tfrac{2.20}{3} \approx 0.7333.$$

For w = "awful":

$$\mathcal{M}^U_{\text{PF}}(\text{"awful"}; \text{Pos}) = \tfrac{1}{3}\Big(0.05 + 0.20 + 1 - 0.95\Big) = \tfrac{0.05+0.20+0.05}{3} = \tfrac{0.30}{3} = 0.100.$$

Effect. The neutralize-only decision for Neu retains its evidence, while the invert+neutralize decision for Neg flips its contribution and removes the conflict. Overall acceptance increases for "okay" (0.6067 → 0.7333) and decreases for "awful" (0.285 → 0.100), as desired.

Example 2.14 (Two-mode De-Plithogenication for PFNL: IT helpdesk priority (anchor = Urgent))**.** We model short ticket phrases with attribute v = "priority intent" and $Pv = \{\text{Urg}, \text{Norm}, \text{Defer}\}$ (urgent / normal / can-defer). Choose anchor $b = \text{Urg}$ and threshold $\tau = 0.80$. Let $\mu(w \mid a) := pdf(w, a)$ and $c(a, b) := pCF(a, b)$.

Primitive data.

$$c(\text{Urg}, \text{Urg}) = 0, \quad c(\text{Norm}, \text{Urg}) = 0.85, \quad c(\text{Defer}, \text{Urg}) = 0.95.$$

Thus $A_\tau(\text{Urg}) = \{\text{Norm}, \text{Defer}\}$. Mode selector:

$$\mathcal{M}(\{\text{Norm}, \text{Urg}\}) = 0 \;\; (\text{Neutralize-only}), \qquad \mathcal{M}(\{\text{Defer}, \text{Urg}\}) = 1 \;\; (\text{Invert+Neutralize}).$$

Two phrases:

w	$\mu(w \mid \text{Urg})$	$\mu(w \mid \text{Norm})$	$\mu(w \mid \text{Defer})$
"system down"	0.95	0.30	0.05
"when convenient"	0.10	0.80	0.70

Before update (flip–blend w.r.t. b). For $T_a(w) := (1 - c(a, b))\,\mu(w \mid a) + c(a, b)\,(1 - \mu(w \mid a))$,

$$\mathcal{M}_{\text{PF}}(w; b) = \frac{T_{\text{Urg}}(w) + T_{\text{Norm}}(w) + T_{\text{Defer}}(w)}{3}.$$

"system down":

$$\begin{aligned}
T_{\text{Urg}} &= 0.95,\\
T_{\text{Norm}} &= 0.15 \cdot 0.30 + 0.85 \cdot 0.70 = 0.045 + 0.595 = 0.640,\\
T_{\text{Defer}} &= 0.05 \cdot 0.05 + 0.95 \cdot 0.95 = 0.0025 + 0.9025 = 0.905,\\
\Rightarrow \;\; \mathcal{M}_{\text{PF}} &= \frac{0.95 + 0.640 + 0.905}{3} = \frac{2.495}{3} \approx 0.8317.
\end{aligned}$$

"when convenient":

$$\begin{aligned}
T_{\text{Urg}} &= 0.10,\\
T_{\text{Norm}} &= 0.15 \cdot 0.80 + 0.85 \cdot 0.20 = 0.120 + 0.170 = 0.290,\\
T_{\text{Defer}} &= 0.05 \cdot 0.70 + 0.95 \cdot 0.30 = 0.035 + 0.285 = 0.320,\\
\Rightarrow \;\; \mathcal{M}_{\text{PF}} &= \frac{0.10 + 0.290 + 0.320}{3} = \frac{0.710}{3} \approx 0.2367.
\end{aligned}$$

Apply two-mode operator (Definition 2.11). Activated values are Norm, Defer. Update

$$pdf^U(w, \text{Norm}) = \mu(w \mid \text{Norm}), \quad pdf^U(w, \text{Defer}) = 1 - \mu(w \mid \text{Defer}),$$

and $c^U(a, \text{Urg}) = 0$ for $a \in \{\text{Norm}, \text{Defer}\}$ (others unchanged). Hence in (3) the terms for Norm, Defer reduce to their (post-)memberships. Therefore

$$\mathcal{M}^U_{\text{PF}}(w; \text{Urg}) = \tfrac{1}{3}\big(\mu(w \mid \text{Urg}) + \mu(w \mid \text{Norm}) + \big(1 - \mu(w \mid \text{Defer})\big)\big).$$

"system down": $(0.95 + 0.30 + 0.95)/3 = 2.20/3 \approx 0.7333$. "when convenient": $(0.10 + 0.80 + 0.30)/3 = 1.20/3 = 0.4000$.

Effect. The invert+neutralize on Defer rewards evidence that is anti-defer (i.e., supports urgency), while neutralize-only on Norm removes its conflict without flipping it.

Example 2.15 (Two-mode De-Plithogenication for PFNL: Tone control in emails (anchor = Formal))**.** Let v = "pragmatic tone" with $Pv = \{\text{For}, \text{Neu}, \text{Cas}\}$ (formal / neutral / casual), anchor $b = \text{For}$, threshold $\tau = 0.80$. Contradictions and modes:

$$c(\text{For}, \text{For}) = 0, \quad c(\text{Neu}, \text{For}) = 0.80, \quad c(\text{Cas}, \text{For}) = 0.95,$$

so $A_\tau(\text{For}) = \{\text{Neu}, \text{Cas}\}$ and

$$\mathcal{M}(\{\text{Neu}, \text{For}\}) = 0 \;\; (\text{Neutralize-only}), \qquad \mathcal{M}(\{\text{Cas}, \text{For}\}) = 1 \;\; (\text{Invert+Neutralize}).$$

Two sentences:

w	$\mu(w \mid \text{For})$	$\mu(w \mid \text{Neu})$	$\mu(w \mid \text{Cas})$
"Please provide the data at your earliest convenience."	0.90	0.60	0.10
"hey, send the data when free"	0.10	0.60	0.85

Before update. With $T_a(w)$ as above:

$$\mathcal{M}_{\mathrm{PF}}(w;\mathrm{For}) = \tfrac{1}{3}\big(T_{\mathrm{For}}(w) + T_{\mathrm{Neu}}(w) + T_{\mathrm{Cas}}(w)\big).$$

First sentence:

$$\begin{aligned} T_{\mathrm{For}} &= 0.90, \\ T_{\mathrm{Neu}} &= (1-0.80)\cdot 0.60 + 0.80\cdot 0.40 = 0.12 + 0.32 = 0.44, \\ T_{\mathrm{Cas}} &= (1-0.95)\cdot 0.10 + 0.95\cdot 0.90 = 0.005 + 0.855 = 0.860, \\ \Rightarrow\ \mathcal{M}_{\mathrm{PF}} &= \frac{0.90+0.44+0.860}{3} = \frac{2.200}{3} \approx 0.7333. \end{aligned}$$

Second sentence:

$$\begin{aligned} T_{\mathrm{For}} &= 0.10, \\ T_{\mathrm{Neu}} &= 0.12 + 0.32 = 0.44, \\ T_{\mathrm{Cas}} &= 0.05\cdot 0.85 + 0.95\cdot 0.15 = 0.0425 + 0.1425 = 0.185, \\ \Rightarrow\ \mathcal{M}_{\mathrm{PF}} &= \frac{0.10+0.44+0.185}{3} = \frac{0.725}{3} \approx 0.2417. \end{aligned}$$

Apply two-mode operator. For activated values, set $c^U(\cdot,\mathrm{For}) = 0$; invert Cas only:

$$pdf^U(w,\mathrm{Cas}) = 1 - \mu(w \mid \mathrm{Cas}), \qquad pdf^U(w,\mathrm{Neu}) = \mu(w \mid \mathrm{Neu}).$$

Hence

$$\mathcal{M}^U_{\mathrm{PF}}(w;\mathrm{For}) = \tfrac{1}{3}\Big(\mu(w \mid \mathrm{For}) + \mu(w \mid \mathrm{Neu}) + \big(1 - \mu(w \mid \mathrm{Cas})\big)\Big).$$

First sentence: $(0.90+0.60+0.90)/3 = 2.40/3 = 0.8000$. Second sentence: $(0.10+0.60+0.15)/3 = 0.85/3 \approx 0.2833$.

Effect. Contradictions involving casual tone are neutralized after inversion (so that strong casual evidence contributes weakly to formal acceptance), while neutral tone evidence is preserved without flip but with conflict removed.

2.4 Plithogenic Fuzzy Natural Language Processing

Plithogenic Fuzzy Natural Language Processing extends FNLP by integrating attribute-based contradictions, enabling nuanced reasoning with multiple conflicting values, ambiguity resolution, and richer semantic interpretation for real-world complex linguistic contexts.

Definition 2.16 (Plithogenic Fuzzy Natural Language Processing (PFNLP))**.** Let Σ be a finite vocabulary and let v be a fixed linguistic attribute (e.g. *sentiment*, *tone*, *formality*) with a nonempty, finite value-set Pv. A *Plithogenic Fuzzy Natural Language Processing system* is a tuple

$$\mathfrak{P} = \big(\Sigma,\ v,\ Pv,\ pdf,\ pCF,\ \mathcal{H},\ T,\ S,\ \mathsf{Compose},\ \mathsf{Agg}\big),$$

whose components are:

- **Degree of Appurtenance (lexicon):** $pdf : \Sigma \times Pv \to [0,1]$, where $pdf(w,a) =: \mu(w \mid a)$ is the fuzzy degree that token $w \in \Sigma$ bears value $a \in Pv$ of the attribute v.
- **Plithogenic contradiction:** $pCF : Pv \times Pv \to [0,1]$, written $c(a,b)$, with $c(a,a) = 0$ and $c(a,b) = c(b,a)$ (reflexive zero and symmetry).
- **Hedges:** a family $\mathcal{H}$ of monotone, normal operators $h : [0,1] \to [0,1]$ (e.g. *very*, *slightly*), acting pointwise on memberships: $\mu \mapsto h(\mu)$.
- **Connectives:** a continuous t-norm T and its dual t-conorm S to combine token-level meanings (e.g. $T = \min$, $S = \max$, or product/sum).

- **Intra-sentence composition:** a deterministic map $\mathsf{Compose}$ that, given a parsed string $w = t_1 \cdots t_n \in \Sigma^*$ and a value $a \in Pv$, returns a sentence-level membership $\mu(w \mid a) \in [0,1]$ by recursively combining the token memberships $\mu(t_i \mid a)$ and hedge-modified memberships using T, S (and standard fuzzy negation $1 - x$ where applicable). This yields the vector

$$\boldsymbol{\mu}(w) \;:=\; \big(\mu(w \mid a)\big)_{a \in Pv} \;\in\; [0,1]^{|Pv|}.$$

- **Plithogenic aggregation across values (anchor-based):** for a chosen *anchor* $b \in Pv$, define the scalar acceptance

$$\mathsf{Agg}(w; b) \;:=\; \frac{1}{|Pv|} \sum_{a \in Pv} \Big((1 - c(a,b))\, \mu(w \mid a) \;+\; c(a,b)\, \big(1 - \mu(w \mid a)\big) \Big) \;\in [0,1]. \tag{4}$$

We call the map $w \mapsto \mathsf{Agg}(w; b)$ the *PFNLP acceptance* of w relative to the anchor b.

Remark 2.17 (Why (4) is plithogenic)**.** The term $(1 - c(a,b))\, \mu(w \mid a)$ preserves evidence from values a that are compatible with the anchor b ($c(a,b) \approx 0$), while the term $c(a,b)\,(1 - \mu(w \mid a))$ softly *inverts* evidence emanating from values that *contradict* b ($c(a,b) \approx 1$). Hence (4) is a contradiction-aware, convex blend of a membership and its complement, summed over attribute-values.

Example 2.18 (PFNLP for curbside parking compliance (anchor = Compliant))**.** Let the attribute be v = *compliance intent* with

$$Pv = \{\mathrm{Co}, \mathrm{Ne}, \mathrm{Vi}, \mathrm{Ir}\} \quad \text{(Compliant, Neutral, Violating, Ironic).}$$

Choose the anchor $b = \mathrm{Co}$. Write $\mu(w \mid a) = pdf(w,a) \in [0,1]$ and $c(a,b) = pCF(a,b) \in [0,1]$ with

$$c(\cdot, \mathrm{Co}): \qquad c(\mathrm{Co},\mathrm{Co}) = 0, \;\; c(\mathrm{Ne},\mathrm{Co}) = 0.35, \;\; c(\mathrm{Vi},\mathrm{Co}) = 0.95, \;\; c(\mathrm{Ir},\mathrm{Co}) = 0.80.$$

Consider two curbside messages:

w	$\mu(w \mid \mathrm{Co})$	$\mu(w \mid \mathrm{Ne})$	$\mu(w \mid \mathrm{Vi})$	$\mu(w \mid \mathrm{Ir})$
(A) "Loading zone, permit A123, 15 min left."	0.90	0.40	0.05	0.02
(B) "Just five minutes with hazards, I'll be quick."	0.20	0.50	0.80	0.10

Using (4), abbreviate $T_a(w) := (1 - c(a,\mathrm{Co}))\, \mu(w \mid a) + c(a,\mathrm{Co})\,(1 - \mu(w \mid a))$ and $\mathsf{Agg}(w;\mathrm{Co}) = \frac{1}{4} \sum_{a \in Pv} T_a(w)$. Then

w	T_{Co}	T_{Ne}	T_{Vi}	T_{Ir}	$\mathsf{Agg}(w;\mathrm{Co})$
(A)	0.900	0.470	0.905	0.788	**0.766**
(B)	0.200	0.500	0.230	0.740	**0.418**

Interpretation. Anchored at *Compliant*, contradictory values (Violating, Ironic) contribute via complementing, yielding a high acceptance for (A) (clearly compliant) and a low acceptance for (B) (a typical non-compliant excuse).

Example 2.19 (PFNLP for healthcare triage messages (anchor = Act-Now))**.** Let the attribute be v = *actionability* with

$$Pv = \{\mathrm{Nw}, \mathrm{So}, \mathrm{SC}, \mathrm{Un}\} \quad \text{(Act-Now, Soon, Self-Care, Uncertain).}$$

Choose the anchor $b = \mathrm{Nw}$. Let the contradictions be

$$c(\cdot, \mathrm{Nw}): \qquad c(\mathrm{Nw},\mathrm{Nw}) = 0, \;\; c(\mathrm{So},\mathrm{Nw}) = 0.30, \;\; c(\mathrm{SC},\mathrm{Nw}) = 0.90, \;\; c(\mathrm{Un},\mathrm{Nw}) = 0.70.$$

Two patient messages:

w	$\mu(w \mid \mathrm{Nw})$	$\mu(w \mid \mathrm{So})$	$\mu(w \mid \mathrm{SC})$	$\mu(w \mid \mathrm{Un})$
(C) "Crushing chest pain and shortness of breath."	0.95	0.30	0.02	0.05
(D) "Mild sore throat, no fever, eating okay."	0.05	0.40	0.85	0.20

With $T_a(w)$ as above and $\mathsf{Agg}(w;\mathrm{Nw}) = \frac{1}{4} \sum_{a \in Pv} T_a(w)$,

w	T_{Nw}	T_{So}	T_{SC}	T_{Un}	$\mathsf{Agg}(w;\mathrm{Nw})$
(C)	0.950	0.420	0.884	0.680	**0.734**
(D)	0.050	0.460	0.220	0.620	**0.338**

Interpretation. Under an *Act-Now* anchor, *Self-Care* and *Uncertain* are highly contradictory and thus blended toward their complements. Message (C) is strongly accepted as *Act-Now*, while (D) is downweighted toward non-urgent handling.

Example 2.20 (PFNLP in an in-car voice assistant: safe speed commands (anchor = Safe)). We model how an in-car assistant decides whether to *execute* a speed-related utterance under a safety policy.

Attribute and values. Let v = "driving pragmatics" with

$$Pv = \{\mathrm{Sa},\ \mathrm{Ca},\ \mathrm{Ag},\ \mathrm{Jo}\} \quad \text{for Safe, Cautious, Aggressive, Joking.}$$

Choose the anchor $b = \mathrm{Sa}$. Write $\mu(w \mid a) := pdf(w, a) \in [0, 1]$ and $c(a, b) := pCF(a, b) \in [0, 1]$ (symmetric, $c(a, a) = 0$).

Contradiction to the anchor.

$$c(\cdot, \mathrm{Sa}): \qquad c(\mathrm{Sa}, \mathrm{Sa}) = 0,\ \ c(\mathrm{Ca}, \mathrm{Sa}) = 0.20,\ \ c(\mathrm{Ag}, \mathrm{Sa}) = 0.90,\ \ c(\mathrm{Jo}, \mathrm{Sa}) = 0.70.$$

Utterances and memberships.

w	$\mu(\cdot \mid \mathrm{Sa})$	$\mu(\cdot \mid \mathrm{Ca})$	$\mu(\cdot \mid \mathrm{Ag})$	$\mu(\cdot \mid \mathrm{Jo})$
"Please set cruise to 60."	0.85	0.60	0.10	0.05
"Floor it!"	0.05	0.05	0.95	0.20

Flip–blend terms. Set $T_a(w) := (1 - c(a, b))\, \mu(w \mid a) + c(a, b)\, (1 - \mu(w \mid a))$.

For w_1 = "Please set cruise to 60.":

$$T_{\mathrm{Sa}} = 0.85, \qquad T_{\mathrm{Ca}} = 0.80 \cdot 0.60 + 0.20 \cdot 0.40 = 0.56,$$
$$T_{\mathrm{Ag}} = 0.10 \cdot 0.10 + 0.90 \cdot 0.90 = 0.82, \quad T_{\mathrm{Jo}} = 0.30 \cdot 0.05 + 0.70 \cdot 0.95 = 0.680.$$

Hence

$$\mathcal{M}_{\mathrm{PF}}(w_1; \mathrm{Sa}) = \frac{0.85 + 0.56 + 0.82 + 0.680}{4} = 0.7275.$$

For w_2 = "Floor it!":

$$T_{\mathrm{Sa}} = 0.05, \qquad T_{\mathrm{Ca}} = 0.80 \cdot 0.05 + 0.20 \cdot 0.95 = 0.23,$$
$$T_{\mathrm{Ag}} = 0.10 \cdot 0.95 + 0.90 \cdot 0.05 = 0.14, \quad T_{\mathrm{Jo}} = 0.30 \cdot 0.20 + 0.70 \cdot 0.80 = 0.62.$$

Thus

$$\mathcal{M}_{\mathrm{PF}}(w_2; \mathrm{Sa}) = \frac{0.05 + 0.23 + 0.14 + 0.62}{4} = 0.2600.$$

Decision. With an execution threshold $\theta = 0.60$, the assistant *executes* w_1 ($0.728 \geq \theta$) but *rejects* w_2 ($0.260 < \theta$). High contradictions (Aggressive vs. Safe) flip/blend evidence toward complements, penalizing unsafe intents while tolerating cautious phrasing.

Example 2.21 (PFNLP in a clinic scheduler: booking intent from phrasing (anchor = Confirmed)). We model how a scheduling agent decides whether to *book* an appointment based on wording strength.

Attribute and values. Let v = "intent modality" with

$$Pv = \{\mathrm{Cf},\ \mathrm{Tn},\ \mathrm{Ex},\ \mathrm{Sa}\} \quad \text{for Confirmed, Tentative, Exploratory, Sarcastic.}$$

Anchor $b = \mathrm{Cf}$. Contradictions to the anchor:

$$c(\cdot, \mathrm{Cf}): \qquad c(\mathrm{Cf}, \mathrm{Cf}) = 0,\ \ c(\mathrm{Tn}, \mathrm{Cf}) = 0.30,\ \ c(\mathrm{Ex}, \mathrm{Cf}) = 0.60,\ \ c(\mathrm{Sa}, \mathrm{Cf}) = 0.85.$$

Sentences and memberships.

w	$\mu(\cdot \mid \mathrm{Cf})$	$\mu(\cdot \mid \mathrm{Tn})$	$\mu(\cdot \mid \mathrm{Ex})$	$\mu(\cdot \mid \mathrm{Sa})$
"Book me with Dr. Lee tomorrow morning."	0.92	0.25	0.05	0.02
"I guess I might see a doctor someday, sure."	0.20	0.70	0.80	0.10

Flip–blend calculations. Using $T_a(w)$ as above:

For w_1:

$$T_{\mathrm{Cf}} = 0.92, \quad T_{\mathrm{Tn}} = 0.70 \cdot 0.25 + 0.30 \cdot 0.75 = 0.400,$$
$$T_{\mathrm{Ex}} = 0.40 \cdot 0.05 + 0.60 \cdot 0.95 = 0.590, \quad T_{\mathrm{Sa}} = 0.15 \cdot 0.02 + 0.85 \cdot 0.98 = 0.836.$$

Hence

$$\mathcal{M}_{\mathrm{PF}}(w_1; \mathrm{Cf}) = \frac{0.92 + 0.400 + 0.590 + 0.836}{4} = 0.6865.$$

For w_2:

$$T_{\mathrm{Cf}} = 0.20, \quad T_{\mathrm{Tn}} = 0.70 \cdot 0.70 + 0.30 \cdot 0.30 = 0.58,$$
$$T_{\mathrm{Ex}} = 0.40 \cdot 0.80 + 0.60 \cdot 0.20 = 0.44, \quad T_{\mathrm{Sa}} = 0.15 \cdot 0.10 + 0.85 \cdot 0.90 = 0.780.$$

Thus

$$\mathcal{M}_{\mathrm{PF}}(w_2; \mathrm{Cf}) = \frac{0.20 + 0.58 + 0.44 + 0.780}{4} = 0.5000.$$

Decision. With a booking threshold $\theta = 0.60$, the agent *books* w_1 ($0.687 \geq \theta$) but *requests clarification* for w_2 ($0.500 < \theta$). Contradictions (Exploratory/Sarcastic vs. Confirmed) drive the blend toward complements, lowering acceptance for hedged or ironic wording.

Lemma 2.22 (Range and idempotent anchor-update)**.** *For every $w \in \Sigma^*$ and $b \in Pv$, $\mathsf{Agg}(w; b) \in [0, 1]$. Moreover, if one performs a (context) update that resets $c(a, b)$ to 0 for all a (e.g. after a one-shot "contradiction neutralization" step), then applying* (4) *again leaves the scalar acceptance unchanged.*

Proof. Each summand in (4) has the form $(1 - \alpha)x + \alpha(1 - x) = (1 - 2\alpha)x + \alpha$ with $\alpha, x \in [0, 1]$, a convex combination of x and $1 - x$; hence it belongs to $[0, 1]$. Averages preserve $[0, 1]$. If $c(\cdot, b)$ is (externally) set to 0 for all values, every summand reduces to $\mu(w \mid a)$; reapplying the same reset does nothing, so the computed acceptance is idempotent under that update. □

Notation 2.23. *Consider the pair*

$$\mathcal{L}_{\mathrm{PF}} := \big(\Sigma, \mathcal{M}_{\mathrm{PF}}\big), \qquad \mathcal{M}_{\mathrm{PF}}(w; b) := \mathsf{Agg}(w; b),$$

together with the primitives (v, Pv, pdf, pCF). Then $\mathcal{L}_{\mathrm{PF}}$ is a Plithogenic Fuzzy Natural Language (PFNL) *in the sense that each $w \in \Sigma^*$ is assigned a scalar acceptance in $[0, 1]$ obtained by a contradiction-aware aggregation over the plithogenic membership vector $\boldsymbol{\mu}(w)$.*

Theorem 2.24 (PFNLP has PFNL structure and generalizes FNLP)**.**

(a) (PFNL structure*) For any PFNLP system $\mathfrak{P}$ and anchor $b \in Pv$, the map $w \mapsto \mathcal{M}_{\mathrm{PF}}(w; b)$ together with (v, Pv, pdf, pCF) constitutes a Plithogenic Fuzzy Natural Language: it assigns to each sentence a scalar acceptance in $[0, 1]$ computed from the plithogenic memberships and the contradiction function.*

(b) (Generalization of FNLP*) Let $\mathfrak{F} = (\Sigma, K, \{U_k\}, \mathcal{L}, \mathcal{H}, T, S, \mathrm{Agg}^{\mathrm{FNLP}}, \Phi, \mathcal{A}, \rho)$ be any Fuzzy Natural Language Processing system whose output on a sentence w is a scalar $\mathrm{Acc}^{\mathrm{FNLP}}(w) \in [0, 1]$ (e.g. via $\sum_k \beta_k \, \Phi_k(\llbracket w \rrbracket_k)$). Then there exists a PFNLP system $\mathfrak{P}$ and an anchor b such that, for all $w \in \Sigma^*$,*

$$\mathsf{Agg}(w; b) = \mathrm{Acc}^{\mathrm{FNLP}}(w).$$

Consequently, PFNLP generalizes *FNLP (FNLP is a special case of PFNLP).*

Proof. (a) By construction, $\boldsymbol{\mu}(w) = (\mu(w \mid a))_{a \in Pv}$ is the plithogenic membership vector produced by $\mathsf{Compose}$ from the token-level memberships and hedges under the connectives T, S. The scalar acceptance $\mathcal{M}_{\mathrm{PF}}(w; b)$ is precisely (4), a contradiction-aware aggregator over Pv; the lemma above guarantees $\mathcal{M}_{\mathrm{PF}}(w; b) \in [0, 1]$. This is exactly the PFNL pattern: a fuzzy acceptance derived from plithogenic memberships with an explicit pCF on attribute-values.

(b) Given $\mathfrak{F}$, build $\mathfrak{P}$ as follows. Set the attribute v with a *singleton* value-set $Pv = \{b\}$. Define the lexicon $pdf(w, b)$ to be the FNLP sentence meaning collapsed to a scalar:

$$pdf(w, b) \ := \ \mathrm{Acc}^{\mathrm{FNLP}}(w) \ \in [0, 1],$$

for each $w \in \Sigma^*$ (operationally, reuse the same parser, hedges, T, S and semantic builder from $\mathfrak{F}$ to produce this scalar). Define pCF by $c(b, b) = 0$ (the only admissible assignment). Then (4) reduces to

$$\mathsf{Agg}(w; b) \ = \ (1 - c(b, b))\, pdf(w, b) + c(b, b)\,(1 - pdf(w, b)) \ = \ 1 \cdot \mathrm{Acc}^{\mathrm{FNLP}}(w) + 0 \cdot (1 - \mathrm{Acc}^{\mathrm{FNLP}}(w))$$
$$= \ \mathrm{Acc}^{\mathrm{FNLP}}(w).$$

Thus every FNLP system is realized by a PFNLP with a singleton Pv and zero contradiction, showing that PFNLP subsumes FNLP. □

3 Conclusion

In this paper, we investigated *Plithogenic Fuzzy Natural Language*, a concept that extends Fuzzy Natural Language by incorporating the notion of contradiction. We expect that future research will explore further extensions using frameworks such as *HyperStructure* [32–35], *SuperHyperStructure* [36, 37], HyperFuzzy Set [38–40], *HyperLanguage*, and *SuperHyperLanguage* [29, 41], as well as the development of algorithms to analyze and implement these advanced systems.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Code Availability

No code or software was developed for this study.

Clinical Trial

This study did not involve any clinical trials.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Supplementary Information

No supplementary materials accompany this paper.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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