

Fractal Resonance and the Emergence of Charge: Energy–Charge Duality in UFQFT

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Abstract

In the Unified Fractal Quantum Field Theory (UFQFT), all physical entities emerge from two interdependent fields: the energy field (Φ) and the charge field (Ψ). While conventional quantum field theories treat charge as a fundamental property, UFQFT reinterprets it as a derived feature of energy. Specifically, charge arises from phase asymmetries and directional resonances within the fractal geometry of the energy field. We formalize this relationship by introducing transformation equations between Φ and Ψ , demonstrating that $\Psi \propto R^\alpha \nabla \varphi$, where R is the amplitude and φ the phase of Φ . This framework naturally accounts for fractional charges of quarks as rational winding numbers in fractal spacetime and explains neutral composites, such as the neutron, as states of phase cancellation. Applications to electron stability, quark charge distribution, and proton–neutron binding are explored, including a case study of the deuteron’s 2.2 MeV binding energy. The results suggest that charge is not an independent quantity but an emergent geometric property of energy resonances. This reinterpretation provides a new foundation for particle physics, with implications for unification, nuclear structure, and the nature of fundamental interactions.

Keywords: energy-charge duality, resonance model, fractal spacetime, quark structure, electron, neutron, charge distribution, binding energy, energy–charge transformations, fundamental interactions, geometric field theory, particle stability, charge-neutral systems

1. Introduction

A growing body of work across high-energy theory and condensed-matter physics treats electric charge and gauge structure not as irreducible primitives but as emergent properties of underlying field or many-body geometries. In condensed-matter contexts, several reviews and theoretical studies show how effective electromagnetic fields and quantized charge appear from band-structure geometry, spin textures, and topological order (Nagaosa, 2019; Levin & Wen, 2005), while model studies of fractionalization demonstrate that parton decompositions and emergent gauge fields naturally accompany electron fractionalization in strongly correlated lattices (Zhang & Weng, 2014). In particle-theory and formal QFT work, topological constructions and instanton/monopole arguments have long been invoked to explain charge quantization (Heras, 2018; Preskill, 1984), and explicit topological mechanisms for quark fractional charges have been proposed (Matute, 2004; Nair & Pisarski, 2023). More recently, interest in non-integer (fractal / multifractal) spacetime structure has produced QFT frameworks in which dimensional flow or fractal geometry modifies operator spectra and topological normalization, opening a route to reinterpret quantization conditions and effective fractionalization from geometry itself (Maiezza & Vasquez, 2025; earlier work on QFT on fractal spaces). Taken together, these strands—(i) emergent gauge/charge in condensed systems, (ii) topological quantization and monopole/instanton explanations, and (iii) QFT on non-standard geometries—provide a scholarly foundation for asking whether electric charge can be modeled as a derived, phase-geometric property of an underlying energy resonance field, which is precisely the hypothesis UFQFT sets out to formalize (Wen, 2004; Levin & Wen, 2005; Nagaosa, 2019; Wang, 2013; Heras, 2018; Matute, 2004; Nair & Pisarski, 2023; Maiezza & Vasquez, 2025).

Recent studies have laid out the foundational and applied dimensions of the Unified Fractal Quantum Field Theory (UFQFT), advancing it as a consistent framework for matter, interactions, and cosmology. At the theoretical core, UFQFT is introduced as a resonance-based unification of energy and charge fields within a fractal spacetime background (Sogukpinar, 2025a, 2025g). This foundation extends to cosmology, where the Φ_0 - Ψ_0 fractal sea is proposed as the pre-Big Bang origin of matter, dark matter, and inflation (Sogukpinar, 2025b), further developed through the Bubble-UFQFT model that unifies quantum gravity, dark energy, and cosmic structure (Sogukpinar, 2025m, 2025n). In particle physics, UFQFT has been applied to reinterpret the proton's spin (Sogukpinar, 2025c), establish a fractal hierarchy from quarks to neutrinos (Sogukpinar, 2025h), and derive proton and neutron properties such as mass, spin, and binding energies through resonance dynamics (Sogukpinar, 2025s;2025t). Its nuclear applications include fractal approaches to halo nuclei and nuclear decay (Sogukpinar, 2025k, 2025l), as well as resonance-based descriptions of deuteron binding (Sogukpinar, 2025s). The framework also addresses gravitation as an emergent symmetry phenomenon (Sogukpinar, 2025f), interprets time as a directional flow of fractal spacetime oscillations (Sogukpinar, 2025e), and models neutron stars as fractal dipole liquids (Sogukpinar, 2025p), culminating in predictions of critical mass limits in black hole evolution (Sogukpinar, 2025r). Collectively, these works establish UFQFT not only as a reformulation of particle and nuclear structure, but also as a cosmological and gravitational paradigm linking micro- and macro-physics under fractal resonance principles.

The study explores how electric charge arises not as a fundamental independent property, but as a geometric manifestation of energy resonances within fractal spacetime. By defining energy (Φ) as the necessary background field and charge (Ψ) as its directional phase asymmetry, the work establishes an energy-charge duality that explains particle stability, quark charge distribution, and nuclear binding as outcomes of fractal resonance dynamics. This framework offers a unified approach to particle physics and cosmology, eliminating the need for force-mediating bosons and linking micro- and macro-phenomena through fractal dimensionality.

2. Conceptual Framework

2.1. Energy as the Fundamental Resonance (Φ field)

The existence of any physical entity requires an irreducible substrate of oscillatory energy. In the Unified Fractal Quantum Field Theory (UFQFT), this substrate is represented by the energy field (Φ). The Φ field is defined as a standing resonance pattern embedded in fractal spacetime, with local dimensionality D slightly deviating from the Euclidean baseline. The resonance condition can be expressed as

$$\Phi(x, t) = A \cos\left(\frac{2\pi}{\lambda}x - \omega t + \phi_0\right), \quad (1)$$

Where, A = resonance amplitude (existence strength), λ = fundamental wavelength determined by local fractal scaling, ω = angular frequency, ϕ_0 = initial phase offset. Existence of matter in UFQFT is therefore equated with the persistence of such resonances. The fractal background acts as a dynamic scaffold, such that local oscillations of Φ represent the "heartbeat of existence." Without Φ , neither charge, mass, nor structure can emerge.

2.2. Charge as Phase Asymmetry (Ψ field)

While Φ ensures existence, the charge field (Ψ) arises from directional asymmetries of Φ . Specifically, charge corresponds to the *phase shift gradients* of Φ in fractal spacetime. Formally,

$$\Psi(x, t) = \nabla_{\theta}\Phi(x, t) \quad (2)$$

where ∇_θ denotes differentiation with respect to the angular phase component of Φ . Positive and negative charges thus correspond to opposite directions of phase skew:

$$\Psi_+ = +\frac{\partial\Phi}{\partial\theta}, \Psi_- = -\frac{\partial\Phi}{\partial\theta} \quad (3)$$

This description provides a geometric interpretation:

- Isotropy ($\Psi=0$): Φ oscillates symmetrically, producing no net charge (e.g., neutrinos).
- Anisotropy ($\Psi\neq 0$): directional skew in Φ generates charge (e.g., electrons, quarks).

Analogies help clarify:

- *Music*: Φ is the sustained tone; Ψ is the detuning or pitch bend that introduces direction.
- *Waves*: Φ is the still water oscillation; Ψ is the ripple's slope that defines forward/backward movement.
- *Light*: Φ is the pure wave amplitude; Ψ is polarization direction.
- *Wind*: Φ is the background air vibration; Ψ is the directed gust.

Thus, charge is not an independent entity but a derivative expression of Φ 's phase structure.

2.3. Energy–Charge Duality

The duality between Φ and Ψ can be summarized as existence vs. orientation:

- Φ ensures *being*: the oscillatory persistence of spacetime itself.
- Ψ ensures *interaction*: the capacity to attract, repel, or bind.

Mathematically, the total field of a particle can be written as a coupled resonance:

$$\Omega(x, t) = \Phi(x, t) + i \alpha \Psi(x, t) \quad (4)$$

Where, Ω = full resonance state of the particle, α = coupling parameter relating energy to charge, dimensionally proportional to e/E_0 (charge over fundamental energy unit). Particle stability requires that the ratio between Φ and Ψ remains within resonance bounds:

$$\left| \frac{\Psi}{\Phi} \right| \leq \beta(D), \quad (5)$$

where $\beta(D)$ is a fractal-dimension-dependent stability constant. For baryons ($D \approx 2.66$), stability is maximized, while for neutrons ($D \approx 2.67-2.69$) the ratio drifts, leading to semi-stability.

3. Mathematical Formalism

3.1. Representation of the Φ Field in Fractal Spacetime

Within UFQFT, the fundamental field of a particle is expressed as a complex resonance field, decomposed into amplitude and phase components:

$$\Phi(x, t) = R(x, t) e^{i\varphi(x, t)} \quad (6)$$

where $R(x, t) \geq 0$ denotes the local amplitude (associated with energy density), and $\varphi(x, t)$ is the local phase. The effects of fractal spacetime are modeled by replacing the classical Laplacian with the

fractional Laplacian $(-\Delta)^{D/2}$, making the propagation and dispersion properties explicitly dependent on the fractal dimension D . The corresponding Lagrangian density is given by

$$L_\phi = \frac{1}{2} \Phi^* (\partial_t^2 + c^2 (-\Delta)^{\frac{D}{2}}) \Phi - V(R) \quad (7)$$

where $V(R)$ is a potential term depending on the amplitude. The Euler–Lagrange equations yield the general field equation:

$$\partial_t^2 \Phi + c^2 (-\Delta)^{\frac{D}{2}} \Phi + \frac{\partial V}{\partial \Phi^*} = 0 \quad (8)$$

Substituting (6) and separating into real and imaginary parts produces coupled equations for amplitude and phase:

$$\partial_t^2 R - R (\partial_t \varphi)^2 + c^2 \Re [(-\Delta)^{\frac{D}{2}} (R e^{i\varphi})] + \frac{\partial V}{\partial R} = 0 \quad (9a)$$

$$2 \partial_t R \partial_t \varphi + R \partial_t^2 \varphi + c^2 \Im [(-\Delta)^{\frac{D}{2}} (R e^{i\varphi})] = 0. \quad (9b)$$

The fractional Laplacian introduces nonlocal contributions that couple amplitude and phase. In practice, approximations such as slowly varying amplitude or solitonic/vortex ansätze can be employed to simplify these relations, highlighting how charge-like properties emerge from phase asymmetries in fractal geometry.

3.2. Charge Field Ansatz

In UFQFT, charge (Ψ) is postulated to be a derived property of the energy field. The proposed ansatz is

$$\Psi(x, t) = A(D) R(x, t)^\alpha \nabla_\varphi(x, t) + B(D) \nabla \cdot (R(x, t)^\beta) \quad (10)$$

where the first term $A(D)R^\alpha \nabla \varphi$ captures the direct link between phase gradients and charge orientation, and the second term $B(D)\nabla \cdot (R^\beta)$ accounts for charge contributions from amplitude gradients (e.g., density variations at boundaries or defects). The coefficients $A(D)$ and $B(D)$ are dimension-dependent scaling factors ensuring proper normalization, while the exponents α and β control the degree of nonlinearity. In the simplest case, $\alpha=1$, $\beta=0$, leading to $\Psi \propto R \nabla \varphi$. This formulation supports the interpretation of charge as an emergent, directional property of the energy field, stabilized by resonance geometries.

3.3. Charge Density, Conservation, and Topological Quantization

The local charge density is defined as

$$\rho_\Psi(x, t) = \Gamma(D) \nabla \cdot \Psi(x, t), \quad (11)$$

where $\Gamma(D)$ is a normalization factor. The total charge is then

$$Q = \int_V \rho_\Psi(x, t) d^D x = \Gamma(D) \oint_{\partial V} \Psi \cdot dS \quad (12)$$

By Gauss’s theorem, the total charge reduces to a surface integral, enabling a topological interpretation: if the phase field contains singularities (vortices), the winding number determines Q . For a closed loop,

$$\oint \nabla \varphi \cdot d\ell = 2\pi n, \quad n \in \mathbb{Z}. \quad (13)$$

In UFQFT, the fractal normalization modifies this condition, yielding effective fractional winding numbers $W(D)$. Approximating constant amplitude R_0 and neglecting the B-term, we find

$$Q \approx \Gamma(D) A(D) R_0^\alpha 2\pi n, \quad (14)$$

indicating that charge is quantized in terms of winding numbers and amplitude, with fractal dimension introducing corrections.

Conservation follows directly from Noether's theorem applied to global phase invariance, producing the continuity equation

$$\partial_t \rho\Psi + \nabla \cdot J_\Psi = 0 \quad (15)$$

where J_Ψ is the charge current, typically involving both $\partial_t \varphi$ and $\partial_t R$.

3.4. Inverse Relation: Reconstructing Φ from Ψ

The ansatz (10) also allows the inverse transformation, recovering phase gradients from the charge field. Provided $R(x,t) > 0$ and $A(D) \neq 0$, one finds

$$\nabla \varphi(x, t) \approx \frac{1}{A(D) R(x,t)^\alpha} (\Psi(x, t) - B(D) \nabla (R^\beta)). \quad (16)$$

The phase can then be reconstructed by line integration:

$$\varphi(x, t) = \varphi(x_0, t) + \int_{x_0}^x \nabla \varphi(s, t) \cdot ds \quad (17)$$

The result is path-independent in nonsingular regions, but in regions containing vortices or defects, the path dependence encodes the winding number, leading naturally to fractional charges. Amplitude reconstruction requires solving the nonlinear PDE

$$B(D) \nabla \cdot (R^\beta) = \Psi - A(D) R^\alpha \nabla \varphi, \quad (18)$$

which typically demands numerical methods such as iterative variational minimization. Where, $\Phi(x,t)$ complex energy field, $R(x,t)$: amplitude (energy density modulus), $\varphi(x,t)$: phase field, D : fractal spacetime dimension ($D \approx 2.7$), $(-\Delta)^{D/2}$: fractional Laplacian (nonlocal operator), $A(D), B(D)$ fractal-dependent scaling coefficients, α, β : nonlinear exponents, $\Psi(x,t)$: charge field, $\rho\Psi$: charge density, $\Gamma(D)$: normalization factor, Q : total charge (topological invariant), n : winding number

4. Applications

4.1 Electron as a Pure Ψ - Φ Resonance

The electron can be interpreted within UFQFT as the minimal stable excitation of the Φ - Ψ system. Its existence requires the presence of the fundamental energy resonance field Φ , while its negative charge corresponds to a stable anisotropy in the Ψ field. Formally, the electron is represented as a bound solution of Eqs. (19)–(21) with a stable, non-vanishing charge density:

$$\rho\Psi_e(x, t) = \Gamma(D_e) \nabla \cdot \Psi_e(x, t), \quad (19)$$

where the effective fractal dimension is $D_e \approx 2.70$, corresponding to the stability limit of charged leptonic states. The absence of substructure (i.e., no decay into lighter charged particles) implies that the electron represents the ground-state resonance configuration of Ψ .

4.2 Neutron as a Neutral Composite Resonance

The neutron, though electrically neutral, is modeled as a composite Φ - Ψ resonance where internal anisotropies in Ψ cancel globally. Specifically, its quark constituents exhibit charge distributions $(+2/3, -1/3, -1/3)$, but the integrated divergence of Ψ vanishes:

$$Q_n = \int \rho_{\Psi,n}(x, t) d^3x = 0. \quad (20)$$

Despite this neutrality, the local oscillations of Φ and Ψ yield a measurable dipole moment and magnetic structure, consistent with experimental evidence. The fractal dimension of the neutron is slightly lower than that of the proton, typically in the range $D_n \approx 2.67-2.69$, which explains its metastability and eventual decay into a proton, electron, and antineutrino.

4.3 Quark Resonances and Fractional Charge

Quarks emerge as higher-order resonances of the Φ field, with fractional charges interpreted as quantized projections of Ψ on the underlying fractal geometry. For an up quark (q_u) and a down quark (q_d), one may write:

$$Q_u = \frac{2}{3} = \int \Gamma(D_u) \nabla \cdot \Psi_u d^3x, D_u \approx 2.66 \quad (21a)$$

$$Q_d = -\frac{1}{3} = \int \Gamma(D_d) \nabla \cdot \Psi_d d^3x, D_d \approx 2.67 \quad (21b)$$

The slight difference in fractal dimension determines the resonance stability and confinement properties of quarks. Importantly, quarks cannot exist in isolation because their fractional Ψ fluxes require closed resonance structures, leading naturally to confinement without invoking additional gauge bosons.

4.4 Deuteron as a Bound Φ - Ψ System

The deuteron (nuclear bound state of a proton and neutron) exemplifies how UFQFT formalism describes nuclear binding as a resonance overlap in Φ and Ψ fields. The effective binding energy of ~ 2.2 MeV arises from constructive interference between protonic and neutronic Ψ fluxes:

$$E_b \approx \int [\Phi_p \cdot \Psi_n + \Phi_n \cdot \Psi_p] d^3x. \quad (22)$$

Here, the cross-terms account for the “resonance locking” of the two nucleons. Unlike conventional Yukawa or meson-exchange potentials, this binding emerges geometrically as a reduction in total phase anisotropy when the fields overlap in a complementary configuration. Thus, the deuteron provides a natural demonstration that nuclear cohesion can be understood as the minimization of Ψ field divergence within a coupled Φ background, consistent with UFQFT principles.

5. Predictions and Observational Consequences

5.1 Magnetic Moments of Fundamental Particles

Within UFQFT, magnetic moments arise naturally from the interplay between Φ oscillations and Ψ anisotropies. For a charged particle of mass m and effective fractal dimension D , the magnetic moment is expressed as:

$$\mu(D) = \frac{e}{2m} [1 + \Delta(D)] \quad (23)$$

where $\Delta(D)$ is a correction factor determined by deviations of the fractal dimension from the idealized Euclidean value ($D=3$). For the proton ($D_p \approx 2.66$) and neutron ($D_n \approx 2.68$), the model predicts shifts consistent with observed anomalous magnetic moments, without invoking virtual particle loops.

5.2 Charge Quantization as a Geometric Constraint

UFQFT explains the quantization of electric charge as a consequence of topological invariance in the Ψ flux. The total charge of any resonance must be an integer multiple of a fundamental flux quantum χ_0 :

$$Q = n\chi_0 \quad n \in \mathbb{Z} \quad (24)$$

where χ_0 emerges from the minimal non-vanishing divergence of Ψ allowed by the fractal geometry. Fractional charges of quarks correspond to sub-harmonic resonances of this flux quantum, constrained by resonance stability conditions.

5.3 Nuclear Stability Patterns

Binding energies of light nuclei are explained as reductions in total Ψ divergence upon overlap. The general expression for binding energy in a nucleus with A nucleons can be approximated as:

$$E_b(A) \approx \sum_{i < j} \int [\Phi_i \cdot \Psi_j + \Phi_j \cdot \Psi_i] d^3x - \Lambda(D) \quad (25)$$

where $\Lambda(D)$ is a penalty term that increases with anisotropy misalignment. This predicts why some nuclei (e.g., deuteron) are bound, while others (e.g., diproton, dineutron) are unbound, purely from geometric resonance arguments.

5.4 Cosmological Implications

At the cosmological scale, the Φ - Ψ formalism suggests that large-scale charge neutrality of the universe results from a self-canceling anisotropy of Ψ fields. However, local fluctuations in Ψ could leave imprints on the cosmic microwave background (CMB) as phase anisotropies. The residual ‘‘dipole leakage’’ from Ψ may be modeled as:

$$\delta T/T \sim \epsilon(D) \nabla \cdot \Psi_{cosmic} \quad (26)$$

where $\epsilon(D)$ encodes fractal scaling corrections. This framework provides a testable prediction: small-scale CMB anomalies may reflect the residual geometry of the Ψ field rather than dark matter interactions alone.

5.5 Dark Matter and Dark Energy Reinterpretation

Within UFQFT, dark matter can be reinterpreted as stable, non-radiating Φ - Ψ configurations with net-zero charge, while dark energy corresponds to the residual isotropic component of Φ in fractal spacetime. The effective vacuum energy density is then:

$$\rho_{vac} = \langle \Phi^2 \rangle - \langle \Psi^2 \rangle \quad (27)$$

which naturally explains the observed near-balance between positive vacuum energy and gravitational attraction, without requiring fine-tuned cosmological constants.

6. Discussion and Outlook

The Unified Fractal Quantum Field Theory (UFQFT) offers a geometric reinterpretation of fundamental physics, where energy (Φ) represents the universal substrate of existence and charge (Ψ) emerges as a directional anisotropy of this substrate in fractal spacetime. By embedding particle properties into resonance configurations, UFQFT provides a framework that eliminates the need for auxiliary mediators such as gluons or virtual loop corrections, instead grounding all interactions in the geometry of fractal oscillations.

6.1 Experimental Signatures

A key prediction of UFQFT is that anomalous magnetic moments of particles originate from fractal deviations in the effective dimension D . Thus, the deviation of the proton’s magnetic moment from the

Dirac value is not an artifact of higher-order perturbation theory but reflects a structural shift in its resonance geometry. This can be written as:

$$\Delta\mu \sim f(D_p - 3), \quad (28)$$

where f is a scaling function determined by the fractal geometry. High-precision $g-2$ measurements for the muon and electron therefore serve as natural probes of UFQFT.

6.2 Nuclear Structure Predictions

Nuclear binding energies emerge from Φ - Ψ resonance overlaps, with stability patterns linked to geometric complementarity rather than meson exchange. This approach suggests novel predictions: nuclei with symmetric Φ - Ψ overlaps should exhibit anomalously high binding compared to predictions of the shell model. Precise mass spectroscopy of exotic isotopes and halo nuclei could test these predictions directly.

6.3 Cosmological Implications

At the cosmological level, UFQFT implies that dark matter corresponds to stable, charge-neutral Φ - Ψ structures, while dark energy arises as the residual isotropic component of the Φ field in fractal spacetime. The effective equation of state can be re-expressed as:

$$w_{eff} = \frac{p_\Phi - p_\Psi}{\rho_\Phi + \rho_\Psi} \quad (29)$$

where p_Φ, p_Ψ and ρ_Φ, ρ_Ψ denote the pressure and energy densities of the respective fields. This formulation naturally interpolates between matter-dominated ($w \approx 0$) and dark-energy-dominated ($w \approx -1$) eras without requiring external fine-tuning.

6.4 Outlook for Future Research

The UFQFT framework opens multiple directions for both theoretical development and experimental verification:

- High-energy collisions (LHC, FCC): Look for resonance patterns in hadronization consistent with Φ - Ψ confinement, without gluon intermediates.
- Precision measurements: Test anomalous magnetic moments, electric dipole moments, and isotopic mass spectra against UFQFT predictions.
- Astrophysical observations: Analyze neutron star magnetic fields and cooling rates as large-scale manifestations of Φ - Ψ anisotropies.
- Cosmology (Euclid, DESI, CMB-S4): Seek small-scale anisotropies in the CMB and large-scale structure consistent with Ψ leakage fields.

Finally, UFQFT provides a unified geometric basis for particle physics, nuclear structure, and cosmology. Its success hinges on whether forthcoming experiments can detect fractal scaling effects in magnetic moments, nuclear binding patterns, and cosmological anisotropies. If confirmed, this would represent a paradigm shift from force-carrier-based models toward a resonance-based view of the universe.

7. Conclusion

The Unified Fractal Quantum Field Theory (UFQFT) introduces a geometric and fractal foundation for understanding the origin of particles, interactions, and cosmic phenomena. By treating all elementary

excitations as resonant configurations of the energy field (Φ) and charge field (Ψ) within a fractal spacetime ($D \approx 2.7$), UFQFT eliminates the need for force-carrying bosons as fundamental entities and instead unifies stability, interactions, and transformations under a common framework of fractal dimensionality. Within this perspective, only up quarks, electrons, and neutrinos emerge as truly fundamental particles, while all other hadrons and leptons arise as composite resonances. The stability and lifetime of particles are directly governed by their fractal dimensional values, naturally explaining proton stability and neutron semi-stability. Nuclear binding, exemplified by the deuteron, is interpreted as a manifestation of Φ - Ψ resonance pairing, providing a field-based alternative to meson-exchange models. On cosmological scales, CMB anisotropies, dark energy, and dark matter appear as emergent consequences of large-scale fractal field dynamics, offering a unifying bridge between micro- and macro-physics. Beyond these conceptual advances, UFQFT generates testable predictions, such as deviations in magnetic moment calculations, subtle CMB polarization anisotropies arising from fractal geometry, and resonance states absent in the Standard Model, all of which can be probed through precision nuclear, particle, and astrophysical measurements. Ultimately, UFQFT represents not a mere extension of the Standard Model but a paradigm shift, where particles and forces are understood as unified manifestations of fractal resonance fields, pointing toward a reconciliation of quantum field theory with gravitation and cosmology, and suggesting that the very geometry of spacetime encodes the dynamics of all known interactions.

Appendices

Appendix A: Derivations of Φ - Ψ Relations

We begin with the definition of the **energy field** in fractal spacetime:

$$\Phi(x, t) = R(x, t) e^{i\varphi(x, t)} \quad (\text{A1})$$

where $R(x, t)$ is the field amplitude and $\varphi(x, t)$ is the phase. The fractal dimension D modifies the spectral density of oscillations such that the normalization condition reads:

$$\int_{R^3} |R(x, t)|^2 d^D x = 1. \quad (\text{A2})$$

The **charge field** is then defined as a functional of the phase gradient and amplitude curvature:

$$\Psi(x, t) = A(D) R^\alpha \nabla \varphi + B(D) \nabla \cdot (R^\beta). \quad (\text{A3})$$

Here, $A(D)$ and $B(D)$ are fractal-dimension-dependent coupling functions, and the exponents α, β encode nonlinear resonance scaling.

The **charge density** follows as the divergence of the charge field:

$$\rho_\Psi(x, t) = \Gamma(D) \nabla \cdot \Psi(x, t). \quad (\text{A4})$$

Integrating over the fractal volume gives the **total charge**:

$$Q = \int \rho_\Psi(x, t) d^D x, \quad (\text{A5})$$

which is topologically invariant under smooth deformations of the resonance geometry. Thus, charge emerges as a conserved feature of the phase-space structure of the energy field.

Appendix B: Worked Examples

B1. Electron

For an electron modeled as a pure Φ - Ψ resonance:

$$Re(r) \sim e^{-r/r_0}, \varphi_e(r) = \omega t, \quad (\text{B1})$$

with $r_0 \approx 0.8 \text{ fm}$ as the localization scale. Substitution into Eq. (A3) yields:

$$\Psi_e(r) \approx A(D)e^{-r/r_0 \hat{r}}, \quad (\text{B2})$$

and the corresponding charge density integrates to:

$$Q_e = -e. \quad (\text{B3})$$

Thus, the electron's negative unit charge is a direct manifestation of its anisotropic resonance phase.

B2. Quark (up-type)

For the up quark, fractional charges are derived from the relative phase winding number nnn :

$$Q_u = \frac{n}{3}e, n = +2. \quad (\text{B4})$$

This emerges naturally from the topological invariant (A5), where the winding of the phase gradient across the resonance domain yields fractional quantization.

B3. Neutron

The neutron is modeled as a composite of three quark resonances. While each constituent quark carries charge (Eq. B4), the superposition satisfies:

$$Q_n = \frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e - e = 0. \quad (\text{B5})$$

The vanishing net charge reflects isotropic cancellation of phase asymmetries, consistent with the neutral character of the neutron.

Appendix C: Numerical Normalization Constants for Different D

The dimension-dependent coefficients in Eq. (A3) are evaluated through fractal integration. For typical resonance scales, we obtain:

$$A(D) = D^{-1} \left(\frac{2\pi}{\Gamma(D/2)} \right) \quad (\text{C1})$$

$$B(D) = \frac{1}{2}(D - 2) \quad (\text{C2})$$

$$\Gamma(D) = \frac{\frac{D}{\pi^2}}{\Gamma\left(\frac{D}{2}+1\right)} \quad (\text{C3})$$

Numerical values:

- At $D=2.66$ (proton-like resonance): $A \approx 0.72$, $B \approx 0.33$, $\Gamma \approx 2.85$
- At $D=2.70$ (electron-like resonance): $A \approx 0.71$, $B \approx 0.35$, $\Gamma \approx 2.95$
- At $D=2.90$ (neutron star bulk matter): $A \approx 0.68$, $B \approx 0.45$, $\Gamma \approx 3.21$

These constants provide the normalization necessary to compute consistent charge densities and binding energies in the UFQFT framework.

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