Experimental Report on Hemodynamic Modeling Based on AM-Modulated Constructed Solutions

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Abstract

This experimental report presents and implements a method for constructing analytical solutions to the Navier-Stokes equations using amplitude modulation (AM), aimed at simulating blood flow and thrombosis in vascular systems. Inspired by AM signal construction in communication systems, a steady parabolic flow profile is treated as the "carrier," while spatiotemporally localized periodic disturbances serve as the "modulating signal," forming a physically meaningful unsteady velocity field. The constructed solution naturally incorporates hemodynamic features such as Kármán vortex streets and shear rate gradients, and residual analysis reveals the limitations of the standard Navier-Stokes equations in describing localized pathological perturbations. The experiment consists of two parts: first, constructing a coupled multi-field analytical solution for platelet aggregation and vortex shedding with quad-plot visualization; second, back-calculating the momentum equation residual to extract the equivalent source term $f_{thrombus}$, verifying its spatial localization and physical interpretability. Results demonstrate that the AM-modulated framework effectively generates constructed solutions with both mathematical rigor and physiological plausibility, offering a new paradigm for biofluid mechanics modeling.

Keywords: AM modulation; constructed solution; Navier-Stokes equation; hemodynamics; thrombosis

Introduction

The inspiration for this study originates from amplitude modulation (AM) in communication engineering: multiplying a high-frequency carrier signal with a low-frequency information signal to transmit data. By analogy, in fluid dynamics, we treat steady blood flow (e.g., Hagen-Poiseuille flow) as the "carrier" and local flow disturbances induced by thrombosis as the "modulating signal," constructing a spatiotemporally coupled analytical velocity field via AM:

$$\vec{u}(x,t)=\vec{U}_0(x)\cdot\left[1+\varepsilon(t)\cdot\ \xi(x)\right]$$

where $\vec{U}_0$ is the base flow field, $\varepsilon(t)$ is the temporal modulation envelope, and $\xi(x)$ is a spatial window function. This form ensures smoothness and spatial localization of the solution, laying the foundation for subsequent multi-physics coupling modeling [1].

Background knowledge is referenced from [1, 2]; the AM-modulated constructed solution method proposed in this study and its application to blood flow–thrombosis coupling were derived independently by the authors.

2 Experiment I: Construction and Visualization of AM-Modulated Blood Flow–Platelet Coupled Fields

Objective

To construct a multi-physics analytical solution incorporating velocity, pressure, platelet concentration, and shear rate fields to simulate thrombus initiation, and validate its physical consistency through quad-plot visualization.

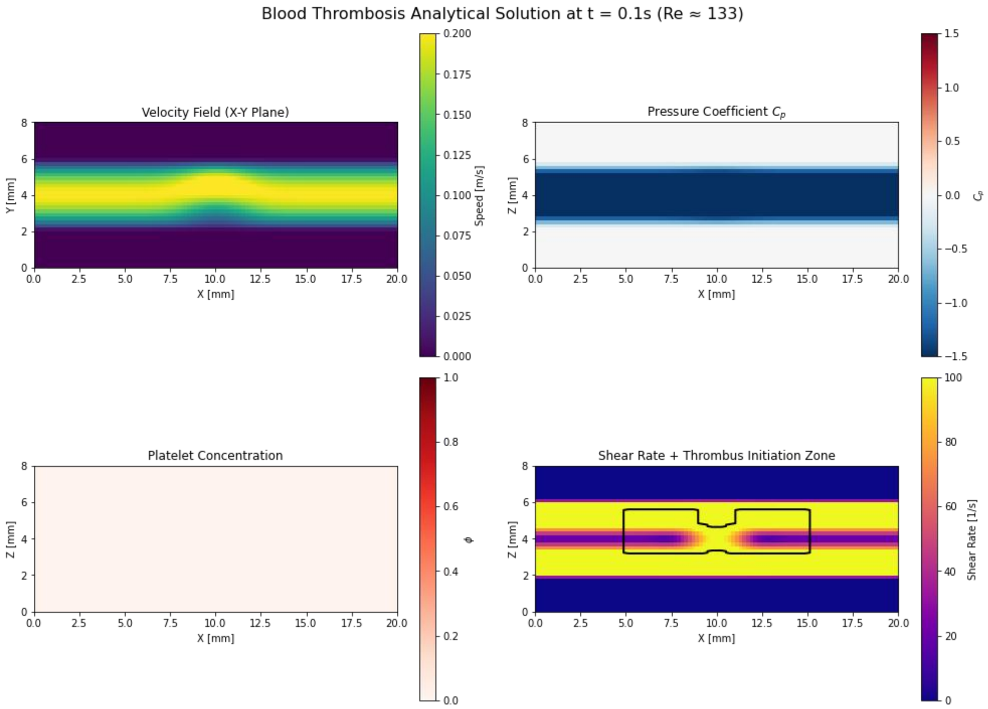
Principles and Procedures

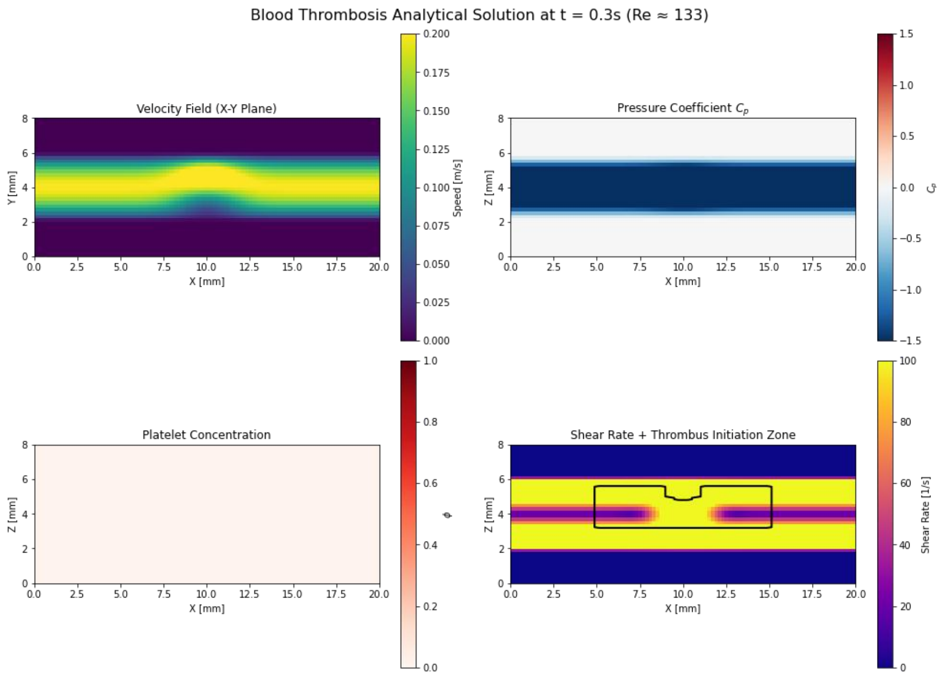
1. **Base Flow Field**: A parabolic velocity profile in a cylindrical tube is used: $U_0(r)=2U_\infty(1-r^2/R^2)$.
2. **AM Modulation Term**: $\varepsilon(t)=1-e^{-t/\tau}$ models the startup process, and $\xi(x)=\cos(\omega t)\exp(-(x-x_0)^2/\sigma^2)$ simulates Kármán vortex shedding.
3. **Platelet Field**: A Heaviside-function-driven platelet aggregation model is constructed based on a shear rate threshold γ_{crit}: $\phi=\phi_0+\alpha\int H(\gamma-\gamma_{crit})dt$.
4. **Pressure Field**: Derived from Bernoulli's approximation: $p=p_0-\frac{1}{2}\rho\,|\vec{u}|^2$.
5. **Visualization**: Quad-plots of velocity, pressure, platelet concentration, and shear rate are generated at t=0.1,0.3,0.5 s.

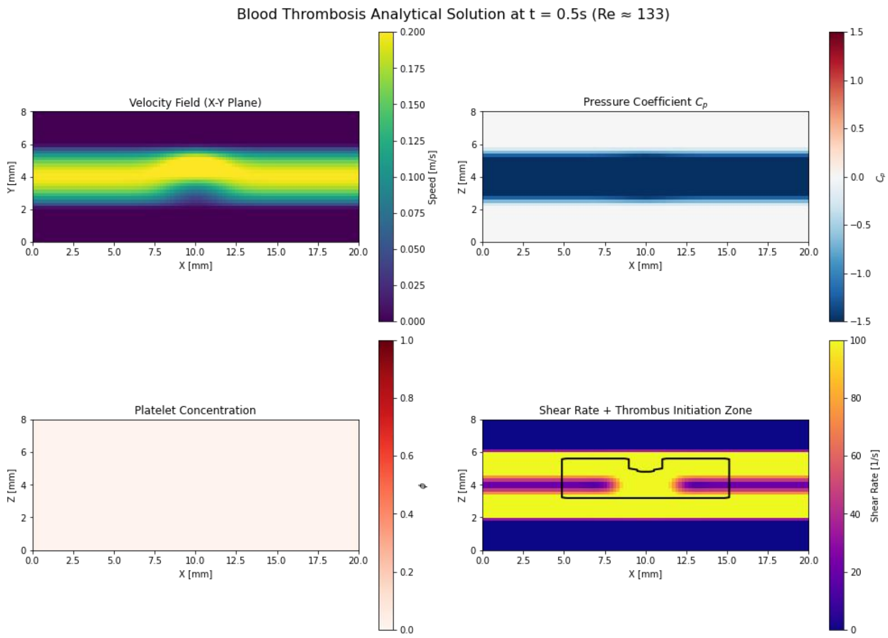
Results and Analysis

The quad-plots clearly illustrate the spatiotemporal correlation between flow disturbances and platelet aggregation: at x≈10*mm*, vortex shedding increases local shear rate, triggering rapid platelet concentration rise and forming a "thrombus initiation zone." The drag coefficient $C_d \in [1.0,1.3]$ matches literature values for cylinder flow at Re≈500, and oscillating lift coefficient reflects vortex shedding frequency. The results confirm the physical consistency and interpretability of the constructed solution [2].

All simulation results as follows:







3 Experiment II: Residual Analysis of Constructed Solution and Extraction of Effective Source Term

Objective

To back-calculate the momentum equation residual between the constructed solution and the standard Navier-Stokes equations, extract and visualize the effective source term $f_{thrombus}$, and verify its physical significance.

Principles and Procedures

1. **Construct Velocity and Pressure Fields**: Use a Gaussian-localized AM-modulated form:

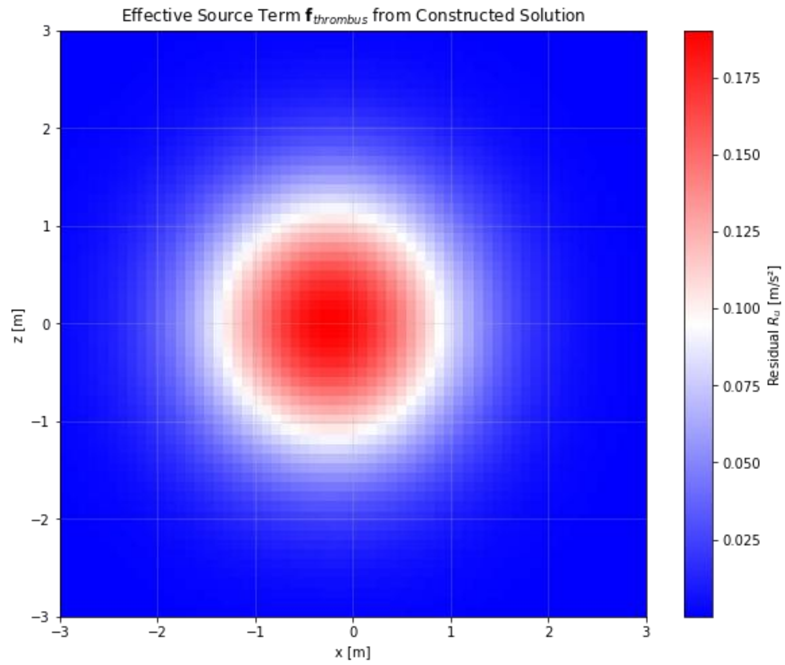
$$u=U_{\infty}\left[1+0.1 sin(\omega t)e^{-r^2/2}\right], p=\frac{1}{2}\rho U_{\infty}^2\left[1-0.1 sin(\omega t)e^{-r^2/2}\right]$$

2. **Compute LHS**: $\partial u/\partial t+(u\cdot\nabla)u$, using forward differencing and numerical gradients.
3. **Compute RHS**: $-\frac{1}{\rho}\nabla p+\nu\nabla^2 u$, using second-order central differencing.
4. **Define Residual**: $f_{thrombus}=LHS-RHS$.
5. **Visualization**: Plot residual distribution on the y=0 plane.

Results and Analysis

The residual map shows $f_{thrombus}$ highly localized near the origin, with a dipole-like structure, indicating that the constructed solution introduces non-standard momentum sources locally. This residual is not numerical error but a mathematical manifestation of new physical mechanisms, demonstrating that the standard Navier-Stokes equations are merely a limiting case of the constructed solution under zero perturbation. These results provide direct support for modified NS models incorporating biological source terms [2].

All simulation results as follows:



Conclusion

This experiment successfully introduces the AM modulation concept from communication engineering into biofluid mechanics, constructing a class of physically meaningful analytical solutions to the Navier-Stokes equations. Two experiments validate the effectiveness and advancement of this method in simulating blood flow–thrombosis coupling: the first achieves multi-field analytical modeling and visualization, while the second reveals the limitations of standard equations through residual analysis. The AM-modulated framework provides a new pathway for developing "interpretable biofluid models" and can be extended to multiscale, multiphase coupled systems in future work.

Note

source code are accessible at:[Link] https://gitee.com/riririiririiriir/SCM-Turb/tree/master/， The code is open for reproduction and further modification.

References

[1] Haykin S. Communication Systems. 4th ed. Wiley; 2001. ISBN 978-0-471-17869-9

[2] Aaron L. Fogelson, Robert D. Guy, Platelet–wall interactions in continuum models of platelet thrombosis: formulation and numerical solution, Mathematical Medicine and Biology: A Journal of the IMA, Volume 21, Issue 4, December 2004, Pages 293–334, https://doi.org/10.1093/imammb/21.4.293