Penta-State Probabilistic Model

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***Abstract*—Probability representation has relied on binary probability or fuzzy extensions, yet such frameworks often fail to capture the suggested way humans evaluate partial belief. This paper introduces the Penta-State Probabilistic Model (PSPM), a five-valued probabilistic logic designed to model directional and biased probability. PSPM characterises probability with five understandable states: True, False, Partial-True, Partial-False, and Undecided. PSPM also provides a flexible, human-aligned representation of belief states, enabling more accurate system modelling for decision-making and neural-inspired reasoning.**

***Index Terms*—five-valued logic, probabilistic reasoning, uncertainty modelling, cognitive modelling, human-aligned AI**

DISCLIMER

This manuscript is a preprint and has not been peer reviewed. It is intended for the early distribution of research findings. Feedback is welcome to improve the work.

I. INTRODUCTION

Probability representation has relied on binary models or fuzzy extensions, yet such frameworks often fail to capture the nuanced ways humans evaluate partial belief. Human reasoning often includes directional uncertainty, a bias toward "yes" or "no", as well as pure ignorance, which cannot be represented adequately in existing machine probabilistic or logical systems. This paper introduces the Penta-State Probabilistic Model (PSPM), a five-valued probabilistic logic designed to address these gaps. PSPM allows flexible, human-aligned representation of belief states, enabling more accurate modelling. The main contributions of this work are:

- Introducing a five-valued probabilistic framework that captures both directional and epistemic uncertainty.
- Establishing a distinction between ignorance and biased uncertainty.
- Bridging the gap between probabilistic and conceptual methods by ensuring compatibility with neural-inspired reasoning, cognitive modelling, and probabilistic AI.

II. BACKGROUND & RELATED WORK

Multi-valued logics extend classical binary reasoning to represent partial or uncertain truth. Łukasiewicz and Kleene's three-valued logics introduced an intermediate "unknown" state [1]. Belnap's four-valued logic was developed for reasoning under incomplete or contradictory information [2]. However, these works ignored partial belief states, which do not align properly with human logic.

Evidence-based approaches, such as Dempster–Shafer theory, model uncertainty via belief mass functions, including ignorance, but do not capture directional leanings toward true or false [3]. Other probabilistic approaches have attempted to extend logic-based reasoning. The work of Richardson and Domingos on Markov Logic Networks combines first-order logic with probabilistic graphical models, allowing weighted formulas to express uncertain knowledge [4]. Moreover, Jøsang's Subjective Logic represents belief, disbelief, and uncertainty in probabilistic form, closer to human belief modelling but without different directional states [5].

To our knowledge, four notable five-valued logics were introduced. Firstly, Ulisses Ferreira proposed a fragment of @-logic designed for mobile agent systems. Its truth values are unknown, possibly known but consistent, false, true, and inconsistent, allowing reasoning over distributed and potentially contradictory information [6]. Secondly, Jérémy Zehr introduced a robust, five-valued framework for modelling linguistic phenomena such as vagueness and presupposition (with both formal semantics and experimental backing) [7]. Thirdly, Patrascu's FP5 and bifuzzy set representations introduced multiple truth values to model imprecision, inconsistency, and ambiguity [8]. Finally, FiveASP paraconsistent logics by Osorio and Zepeda explicitly handle contradictions and incomplete information using five truth values [9]. While these frameworks handle contradictions explicitly, they remain symbolic rather than probabilistic and do not model directional human-like uncertainty.

III. THE PENTA-STATE PROBABILISTIC MODEL (PSPM)

A. *Definition of Epistemic States*

The Penta-State Probabilistic Model (PSPM) defines a finite set of epistemic states:

$$S = \{F,\, PF,\, U,\, PT,\, T\}$$

where each state $S_i \in [0, 1] \subset \mathbb{R}^+$.
Each state is defined as follows:

- T: **True** — Evidence is fully supported.
- F: **False** — Evidence is fully rejected.
- PT: **Partial True** — Evidence is not mandatory for overall prediction but provides a mild advantage toward the positive side.
- PF: **Partial False** — Evidence is not mandatory for overall prediction but provides a mild disadvantage toward the negative side.
- U: **Undecided** — Evidence is required but currently unknown.

B. *Directional Bias and Evidence Ratio*

Let the *Evidence Ratio* (ER) be defined as:

$$ER \in [0, 1] \subset \mathbb{R}^+$$

For each state, define an evidence range:

$$R_t, R_{pt}, R_u, R_{pf}, R_f$$

such that:

$$\begin{aligned} R_t &= [R_{t_{\min}}, R_{t_{\max}}] \\ R_{pt} &= [R_{pt_{\min}}, R_{pt_{\max}}] \\ R_u &= [R_{u_{\min}}, R_{u_{\max}}] \\ R_{pf} &= [R_{pf_{\min}}, R_{pf_{\max}}] \\ R_f &= [R_{f_{\min}}, R_{f_{\max}}] \end{aligned}$$

If the current maximum value equals the previous minimum, the lower state shall be prioritised:

$$\text{If } R_{i_{\max}} = R_{(i+1)_{\min}}, \text{ then prioritize the lower state.}$$

C. Theorems

Theorem 1 (Human Evidence Principle). Human neural reasoning relies on evidence when estimating probability. As a human-aligned model, PSPM does not operate on fictional probabilities. In the absence of evidence, PSPM consistently returns U (Undecided), analogous to human cognitive behaviour. To compute PSPM probability, a sufficiently large dataset of evidence is required.

Theorem 2 (Human Probability Categorization). Human probability cognition can be categorised within the same five epistemic states as PSPM:

$$S = \{F,\ PF,\ U,\ PT,\ T\}$$

These states form the foundational structure of human-aligned probability distributions and may be expanded or contracted into more or fewer states as needed.

D. Computation Procedure

1) Step 1: Evidence Quantification: Let:

$$\begin{aligned} Q_t &= \text{Number of strong positive evidences} \\ Q_f &= \text{Number of strong negative evidences} \\ Q_u &= \text{Number of unknown or missing evidences} \\ Q_{pt} &= \text{Number of partial true evidences} \\ Q_{pf} &= \text{Number of partial false evidences} \\ Q_n &= Q_t + Q_f + Q_u \end{aligned}$$

where $Q_n, Q_t, Q_f, Q_u \in \mathbb{N}$.

According to Theorem 1, human reasoning prefers to choose a side (positive or negative) when sufficient accuracy is present. Therefore:

$$Q_t + Q_f \leq \text{accuracy threshold}$$

Define:

$$a' = \frac{Q_t + Q_f}{Q_n}$$

and let:

$$a = 1 - x$$

where x is the accepted uncertainty level, such that $0 \leq a, a' \leq 1$.

Undecided Aspect Ratios:

Let the midpoint of the undecided range be:

$$R_{u_{\text{mid}}} = \frac{R_{u_{\max}} + R_{u_{\min}}}{2}$$

Then:

$$\text{If } \frac{Q_t}{2} > Q_f, \quad U_t = R_{u_{\text{mid}}} + \frac{(R_{u_{\max}} - R_{u_{\text{mid}}})}{Q_n} \cdot (Q_t + \tfrac{Q_u}{2})$$

$$\text{If } \frac{Q_f}{2} > Q_t, \quad U_f = R_{u_{\text{mid}}} + \frac{(R_{u_{\max}} - R_{u_{\text{mid}}})}{Q_n} \cdot (Q_f + \tfrac{Q_u}{2})$$

$$\text{If } \frac{Q_u}{2} > Q_t + Q_f, \quad U_s = R_{u_{\text{mid}}}$$

$$\text{Otherwise,} \quad U_u = R_{u_{\text{mid}}}$$

2) Step 2: Evidence Ratio Calculation: If $a \leq a'$ and only if both $Q_{pf}, Q_{pt} \neq 0$, then:

$$ER_t = \left(\frac{V_t}{Q_n}\right)(Q_t + \tfrac{Q_u}{2})$$

$$ER_f = \left(\frac{V_f}{Q_n}\right)(Q_f + \tfrac{Q_u}{2})$$

$$ER_{pt} = \left(\frac{V_{pt}}{Q_{pt} + Q_{pf}}\right) Q_{pt}$$

$$ER_{pf} = \left(\frac{V_{pf}}{Q_{pt} + Q_{pf}}\right) Q_{pf}$$

If $Q_{pt} = 0$ or $Q_{pf} = 0$:

$$ER_{pt} = 0, \quad ER_{pf} = 0$$

3) Step 3: State Bias Adjustment: Let ER denote the total evidence ratio. Then:

$$\begin{aligned} &\text{If } ER_f > ER_t, \quad ER = R_{u_{\min}} - (ER_f + ER_{pf}) \\ &\text{If } ER_t > ER_f, \quad ER = R_{u_{\max}} + (ER_t + ER_{pt}) \\ &\text{If } ER_t = ER_f, \quad ER = \frac{R_{u_{\max}} + R_{u_{\min}}}{2} \end{aligned}$$

Although partial states exist, PSPM treats them as secondary adjustments to refine the final positive or negative decision boundary.

E. Interpretation

The PSPM outputs one of five epistemic states S_i as a function of the evidence distribution:

$$\text{PSPM}(E) : E \mapsto S_i, \quad S_i \in \{F, PF, U, PT, T\}$$

The resulting state reflects human-aligned reasoning, ensuring that undecided or partial states are possible when evidence is insufficient or mildly contradictory.

IV. Limitations and Future Work

Although the Penta-State Probabilistic Model (PSPM) demonstrates a flexible and human-aligned approach to uncertainty reasoning, several limitations remain that provide opportunities for future research.

1) **Parameter sensitivity.**
   The performance of PSPM can depend on the choice of range boundaries $(R_t, R_{pt}, R_u, R_{pf}, R_f)$ and evidence weighting parameters $(V_t, V_f, V_{pt}, V_{pf}, V_u)$. These parameters are currently selected heuristically and may vary across different domains. A systematic parameter optimisation could help standardise PSPM deployment in real-life cases.
2) **Evidence of the independence assumption.**
   The present formulation of PSPM assumes that individual pieces of evidence contribute independently to the final state estimation. In many real-world problems, evidence sources are interdependent or correlated. Extending PSPM to include dependency modelling, possibly through graph-based or Bayesian hybridisation, would improve its realism and robustness.
3) **Limited experiments.**
   PSPM was not tested on a large data scale. The primary test was capped at 100 humans' decisions due to financial blockade. Large-scale testing may confirm whether PSPM's five-state structure indeed mirrors cognitive probability estimation in practical scenarios.
4) **Computational scaling and integration.**
   As PSPM expands toward complex decision systems, computational efficiency and integration with existing probabilistic frameworks (e.g., neural networks or rule-based inference) may become crucial. Developing differentiable or hardware-efficient PSPM variants would support its adoption in hybrid AI systems.

Continued refinement, validation, and integration with adaptive learning architectures will be essential for realising PSPM's full potential. We planned to expand the state logic with substate to improve the confidence of PSPM as well as introduce a universal parameter on PSPM2.

V. Conclusion

The Penta-State Probabilistic Model (PSPM) introduces a five-valued framework for representing directional, human-aligned uncertainty in probabilistic reasoning. Compared to traditional binary or fuzzy methods, PSPM offers a richer representation of belief by capturing both strong and partial evidence and distinguishing between positive, negative, and undecided states. Although PSPM shows promise, its real-world deployment requires further validation on large-scale datasets and careful tuning of state boundaries and evidence weighting. By bridging the gap between theoretical frameworks and human-like reasoning, PSPM provides probabilistic modelling with a flexible, interpretable, and human-centred approach.

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