Monty-Hall (parameterized strategist-host) Theorem : Correcting a Historical Error in Statistical Methodology

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ABSTRACT

The Monty-Hall (parameterized strategist-host) Theorem along with a constructive proof is presented, by solving the corresponding Monty-Hall Problem, wherein the host plays a parameterized strategy on the guest. It establishes the limits on the range of values for the probability of winning the prize. Eight extreme strategies (corresponding to the set of extreme values for the three perturbation parameters) have been well characterized. It is shown that there does not exist any strategy wherein a *switched-choice* will *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) lead to an enhancement in the chances of winning the prize. The classical Monty-Hall Problem is a special case with zero-value for each of the three perturbation parameters.

This paper is an attempt to correct the errors (of long-standing historical significance) in the application of *statistical methodology* in solving the classical Monty-Hall Problem - one of them being the erroneous use of conditional probabilities for updating the knowledge to facilitate the decision-making by the guest, based on the information about a losing-choice, which itself is dependent on the initial-choice of the guest. Similar scenarios in data science, machine learning & artificial intelligence can have serious far-reaching consequences.

Keywords: Bayes-Price Rule; Bayes Theorem; Discrete Event (Sample) Space; Parameterized Strategy; Perturbation Parameters; Monty-Hall (parameterized strategist-host) Theorem;

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1. Introduction

The *classical* "Monty-Hall Problem" [1] also referred to as the "Three-Door Problem" is based on a game show "Let's Make a Deal" [2] wherein the host reveals a losing choice to the guest, who had earlier made an initial choice, and in turn offers the guest an enticing option to switch from the initial choice to a second available choice with an aim to enhance the chances of winning the prize. The most prevalent & widely accepted position, as reported in literature, among the leading subject matter experts, mathematicians, statisticians, logicians, and rational intellectuals, is that an appropriate detailed study & analysis of the scenario using the well accepted standard approach of Probability & Statistics, would lead to a recommendation to the guest to switch to the second available choice based on the knowledge obtained from the host revealing a losing choice.

We present the statement of the Monty-Hall (parameterized strategist-host) Theorem, corresponding to the most general scenario of the Monty-Hall Problem; wherein the host may play a parameterized strategy on the guest, that may at the outset appear to result in a disadvantageous situation for the guest. We provide a constructive proof by solving the corresponding generic Monty-Hall Problem. It establishes the limits on the range of values for the probability of winning the prize, with or without a switched-choice. Eight extreme strategies have been identified and characterized (corresponding to the set of extreme values for the three perturbation parameters) while establishing that there cannot exist any strategy that would *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) be disadvantageous for the guest. This is a refutation of the position held by the leading subject matter experts, that a switched-choice *always* provides an *enhancement on the probability of winning the prize* in the classical case.

2. Notational Details

Let us develop the notational details by first considering the so-called *classical* Monty-Hall Problem as reported widely in the literature - with a prize hidden behind one of the three doors; a guest making an initial choice of a door to claim the prize; the host who knows the location of the prize as well as the initial choice made by the guest, now *reveals a distinctly different and yet a losing-choice*, by opening a third door. Then the host also offers the guest, an option to switch from the initial-choice to the now available second-choice, anticipating a possible enhancement in the chances of winning the prize, based on the knowledge obtained about a losing-choice.

Let us represent the events/actions associated with the three doors:

(1) xr∈{1,2,3}: reward/prize x is hidden behind door r;

(2) yp∈{1,2,3}: player/guest y choses door p as the initial-choice;

(3) zq∈{1,2,3}: quizmaster/host z opens door q to reveal a losing choice.

Let the symbol 'ai' denote the event/action [E{(a=i)}] for any 'agent' a∈{x,y,z} and 'door' i∈{r,p,q}={1,2,3}.

It is essential to note here that xr and yp are mutually independent of each other as well as independent of zq; whereas zq itself is dependent on both xr and yp, since {(zq ≠ yp) & (zq ≠ xr)} as per the rules of the game. Also, note that the focus must be on the decision-making process and the action to be taken by the guest. Therefore, the *problem formulation* (*modelling*) must necessarily be from the view-point of the guest/player.

3. Assumptions

From the point of view of the player/guest, it is *assumed* that the reward/prize is hidden randomly (maybe because of lack of any specific knowledge) behind one of the three doors. That is, each of the events [xr∈{1,2,3}] is considered random and equiprobable among the available three alternatives.

Also, the initial-choice of the door [yp∈{1,2,3}] chosen by the player/guest being a random (blind) choice (again because of lack of any specific knowledge) is *assumed* to be equiprobable among the available three alternatives.

The host knows the door behind which the prize is hidden and also the door that is the initial choice of the guest. Therefore, the event/action of the host z opening door q, zq∈{1,2,3} to show a losing choice, is dependent on both yp and xr, as per the rules of the game show, that is, {(zq ≠ yp) & (zq ≠ xr)}. This dependency of zq on yp and xr does indeed limit the available options for the host. It turns out that when yp ≠ xr the host doesn't have any option except to turn to the one and only one remaining door {zq ≠ (yp ≠ xr)}; whereas when yp = xr the host has the option of choosing between the two doors, that is, {zq ≠ (yp = xr)}. Because the host has this option, at least in a restricted sense, of choosing which of the two doors to open, it introduces an uncertainty for the guest/player to predict/expect/anticipate the host's decision/action in this regard, irrespective of whether the host adopts any strategy or otherwise. The only knowledge that the guest/player gets from the host's action of opening a door to show a losing-choice is exactly that the specific door is indeed a losing-choice which doesn't have the prize hidden behind it.

4. Input Data Set

In the classical Monty-Hall Problem, it is *assumed* that whenever {zq ≠ (yp = xr)} the host's choice between the two available options is indeed random and *equiprobable*. In our supermodel representing the strategist-host, we allow the host to play a generic parameterized strategy defined by three perturbations parameters; (-1/2) ≤ {α, β, γ} ≤ (1/2); as shown in the corresponding input data set.

The input data set for the generic Monty-Hall (parametrized strategist-host) Problem, presented in Table-1, lists the 12 mutually-exclusive together-exhaustive possible alternatives for the combined-triple-event space along with the relevant apriori probabilities.

Sl.No.	[xr]	[yp]	[xr&yp]	[zq]	[xr&yp&zq]	P[xr]	P[yp]	P[zq|(xr&yp)]	P[xr&yp&zq]
01	1	1	11	2	112	1/3	1/3	(1/2)+α	((1/2)+α)/9
02	1	1	11	3	113	1/3	1/3	(1/2)-α	((1/2)-α)/9
03	1	2	12	3	123	1/3	1/3	1	1/9
04	1	3	13	2	132	1/3	1/3	1	1/9
05	2	1	21	3	213	1/3	1/3	1	1/9
06	2	2	22	3	223	1/3	1/3	(1/2)+β	((1/2)+β)/9
07	2	2	22	1	221	1/3	1/3	(1/2)-β	((1/2)-β)/9
08	2	3	23	1	231	1/3	1/3	1	1/9
09	3	1	31	2	312	1/3	1/3	1	1/9
10	3	2	32	1	321	1/3	1/3	1	1/9
11	3	3	33	1	331	1/3	1/3	(1/2)+γ	((1/2)+γ)/9
12	3	3	33	2	332	1/3	1/3	(1/2)-γ	((1/2)-γ)/9
Twelve Mutually-Exclusive Together-Exhaustive Alternative-Possibilities									
[xr]: prize x behind door r; [yp]: guest y choses door p; [zq]: host z reveals door q									
Perturbation parameters (-1/2)≤{α,β,γ}≤(1/2) are non-zero for strategist-host									

Table-1: Twelve *combined-triplet-event* possibilities in three-dimensional discrete event(sample)space along with the apriori probabilities.

Table-2 gives a concise schematic 2-dimensional representation of the same input data set corresponding to the 3-dimensional event (sample) space, in the form of a 3x3 matrix (the diagonal cells being split into two parts to accommodate the two options of the host in revealing a losing choice) with xr along the rows, yp along the columns and {xrypzq}:={zq(xr,yp)} as the entries in the matrix.

{xrypzq} MHP zq(xr,yp)		player/guest y chooses door p		
		yp=1	yp=2	yp=3
reward/prize x hidden behind door r	xr=1	((1/2)-α)/9 {113} {112} ((1/2)+α)/9	{123} 1/9	{132} 1/9
	xr=2	{213} 1/9	((1/2)-β)/9 {221} {223} ((1/2)+β)/9	{231} 1/9
	xr=3	{312} 1/9	{321} 1/9	((1/2)-γ)/9 {332} {331} ((1/2)+γ)/9

Table-2: Schematic representation of the input data set

5. Monty-Hall (parameterized strategist-host) Theorem

The strategy adopted by the host, in the supermodel representing the corresponding Monty-Hall (parametrized strategist-host) Problem, is characterized by the three perturbation parameters, $(-1/2) \leq \{\alpha, \beta, \gamma\} \leq (1/2)$; as presented in the corresponding input data set in Table-1. The default classical case corresponds to the situation with zero-value for each of these three perturbation parameters, with equiprobable alternatives; whereas the non-zero values of the perturbation parameters indicate the extent of deviation from the default classical case.

Theorem Statement

Given that the initial choice of the guest is, say door-1 (event [y1]); and that the host opens the door, say door-3 (event [z3]) to reveal a losing choice, that is different from the door behind which the prize is hidden, and also different from the initial choice of the guest; then the probability of the guest winning the prize is given by the *aposteriori* (conditional to [z3]) *probability* of the prize being hidden behind the door-1 (event [x1]); that is, P[x1 | z3]. This value may be computed by the application of the Bayes-Price Rule (Bayes Theorem) for a three-dimensional discrete event(sample)space. Similarly, the values for P[xr | zq] are computed for each of the three pairs of valid combinations of xr and zq; for $xr \in \{1,2,3\}$ and for $zq \in \{1,2,3\}$ with $xr \neq zq$.

Claim MHT1:

In the case of the classical Monty-Hall Problem with zero-values for the perturbation parameters; P[xr | zq] is exactly 1/2. Therefore, the option of the switched-choice doesn't yield any enhancement in the chances of winning the prize.

Claim MHT2:

The lower & upper bounds on P[xr | zq] are given by (1/3) ≤ P[xr | zq] ≤ (2/3); as listed in Table-3 for each of the 24 distinctly different combinations of the limiting values of the perturbation parameters.

Claim MHT3:

Table-4 lists the eight extreme strategies, each identified by its characteristic signature, along with the resultant probability values for the prize being hidden behind each of the remaining two doors other than the specific door opened by the host to reveal a losing-choice. We claim that there does not exist any strategy that the host can play on the guest/player, which would result in a situation wherein a *switched-choice* will *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) lead to a universal enhancement/diminishment in the chances of winning the prize.

Proof of the Monty-Hall (parameterized strategist-host) Theorem

The proof is simply by solving the problem, following the below enumerated steps. For each required value, a general expression is given first; followed by the specific value.

(a) Input Data Set

P[x1]; P[x2]; P[x3]; P[y1]; P[y2]; P[y3];

P[z3 | x1y1]; P[z3 | x1y2]; P[z3 | x2y1]; P[z3 | x2y2];

P[z2 | x1y1]; P[z2 | x1y3]; P[z2 | x3y1]; P[z2 | x3y3];

P[z1 | x2y2]; P[z1 | x2y3]; P[z1 | x3y2]; P[z1 | x3y3];

(b) Joint Probabilities for Independent Events [xr & yp]

P[x1y1] = P[x1]*P[y1]; P[x1y2] = P[x1]*P[y2]; P[x1y3] = P[x1]*P[y3];

P[x2y1] = P[x2]*P[y1]; P[x2y2] = P[x2]*P[y2]; P[x2y3] = P[x2]*P[y3];

P[x3y1] = P[x3]*P[y1]; P[x3y2] = P[x3]*P[y2]; P[x3y3] = P[x3]*P[y3];

(c) Validity Check for *Non-Zero Apriori* Probabilities

Check and confirm the *validity of input data values* for application of Bayes-Price Rule (Bayes Theorem). The presence of *zero-value* for any of the *apriori probabilities* leading to the intended conditional used to derive the required *aposteriori* (*conditional*) *probabilities*, can result in *spurious results*. Appropriate alternative approach may be needed in such cases. The avoidance of *extreme priors* and/or *point-mass-bias* is a *regularity principle*; refer to Cromwell's Rule [13]. The nine *joint probabilities* {P[x1y1], P[x1y2], P[x1y3], P[x2y1], P[x2y2], P[x2y3], P[x3y1], P[x3y2], P[x3y3]} listed above, leading to the required conditionality of the host opening a door must necessarily have non-zero values for the valid application of the Bayes Theorem.

(d) Apriori Probability for [z3] – as per the Rules Of The Game

P[z3] = P[z3|x1y1]*P[x1y1] + P[z3|x2y1]*P[x2y1] + P[z3|x1y2]*P[x1y2] + P[z3|x2y2]*P[x2y2];
= P[x1y1z3] + P[x1y2z3] + P[x2y1z3] + P[x2y2z3];
= ((1/2)-α)*(1/9) + 1*(1/9) + 1*(1/9) + ((1/2)+β)*(1/9);
= (3+β-α)/9;

(e) Apriori (conditional w.r.t. x1; marginal w.r.t. yp) Probability for z3

P[z3 | x1] = (P[z3 | x1y1] * P[x1y1] + P[z3 | x1y2] * P[x1y2]) / (P[x1]);
= (P[z3x1y1] + P[z3x1y2]) / (P[x1]);
= ((1/2)-α) * (1/9) + 1 *(1/9)) / (1/3);
= ((1/2)-(α/3));

(f) Aposteriori (conditional w.r.t. z3; marginal w.r.t. yp) Probability for x1

P[x1 | z3] = (P[z3 | x1] * P[x1]) / (P[z3]);
= ((P[z3x1y1] + P[z3x1y2]) / (P[x1y1z3] + P[x1y2z3] + P[x2y1z3] + P[x2y2z3]);
= (((1/2)-(α/3)) * (1/3)) / ((3+β-α)/9);
= (1/2)*(3-2α)/(3+β-α);

Similarly we can compute the values for each of the six valid combinations of xr and zq; r∈{1,2,3} and q∈{1,2,3} with xr ≠ zq; and we get –

P[x1 | z3] = (1/2) * (3-2α)/(3+β-α); P[x2 | z3] = (1/2) * (3+2β)/(3+β-α);
P[x2 | z1] = (1/2) * (3-2β)/(3+γ-β); P[x3 | z1] = (1/2) * (3+2γ)/(3+γ-β);
P[x3 | z2] = (1/2) * (3-2γ)/(3+α-γ); P[x1 | z2] = (1/2) * (3+2α)/(3+α-γ);

Note that in the case of mutually independent together exhaustive discrete joint event probabilities, it is easier & safe to work directly with the appropriate joint probability values and the appropriate marginal probability values to compute the required aposteriori (conditional) probability values.

END OF COMPUTATION

Claim MHT1:

When each of the three perturbation parameters takes zero-value, the situation corresponds to the default classical case of the Monty-Hall Problem. From the computations shown in the theorem above, it results in each of the above six aposteriori (conditional) probability values take on the value of exactly 1/2. Therefore, it gets established that irrespective of whichever be the door opened by the host (thus revealing a losing choice) each of the remaining two doors have equal probability of having the prize hidden behind it – implying that the so called "switched-choice" is neither better nor worse in terms of the chances of winning, than staying with the "initial-choice" of the guest. This is a clear refutation of the claim by the leading subject matter experts.

Claim MHT2:

Table-3 lists the above six values of aposteriori (conditional) probabilities and their range-limiting bounds, corresponding to the various possible combinations of limiting values of the three perturbation parameters.

Note that the first three columns in Table-3 contain the relevant partial signature (refer to the description presented later here below) of the related extreme strategy.

From the entries in Table-3 it is clear that the lower & upper bounds on P[xr | zq] are given by (1/3) ≤ P[xr | zq] ≤ (2/3); which can also be derived independently from the corresponding algebraic expressions for each of these six values.

x1y1zq	x2y2zq	x3y3zq	α=	β=	γ=	P[xr|zq]=		
112	223		α=1/2	β=1/2		P[x1|z3]=	(1/2)*(3-2α)/(3+β-α)	=1/3
112	221		α=1/2	β=-1/2		P[x1|z3]=	(1/2)*(3-2α)/(3+β-α)	=1/2
113	223		α=-1/2	β=1/2		P[x1|z3]=	(1/2)*(3-2α)/(3+β-α)	=1/2
113	221		α=-1/2	β=-1/2		P[x1|z3]=	(1/2)*(3-2α)/(3+β-α)	=2/3
112	223		α=1/2	β=1/2		P[x2|z3]=	(1/2)*(3+2β)/(3+β-α)	=2/3
112	221		α=1/2	β=-1/2		P[x2|z3]=	(1/2)*(3+2β)/(3+β-α)	=1/2
113	223		α=-1/2	β=1/2		P[x2|z3]=	(1/2)*(3+2β)/(3+β-α)	=1/2
113	221		α=-1/2	β=-1/2		P[x2|z3]=	(1/2)*(3+2β)/(3+β-α)	=1/3
	223	331		β=1/2	γ=1/2	P[x2|z1]=	(1/2)*(3-2β)/(3+γ-β)	=1/3
	223	332		β=1/2	γ=-1/2	P[x2|z1]=	(1/2)*(3-2β)/(3+γ-β)	=1/2
	221	331		β=-1/2	γ=1/2	P[x2|z1]=	(1/2)*(3-2β)/(3+γ-β)	=1/2
	221	332		β=-1/2	γ=-1/2	P[x2|z1]=	(1/2)*(3-2β)/(3+γ-β)	=2/3
	223	331		β=1/2	γ=1/2	P[x3|z1]=	(1/2)*(3+2γ)/(3+γ-β)	=2/3
	223	332		β=1/2	γ=-1/2	P[x3|z1]=	(1/2)*(3+2γ)/(3+γ-β)	=1/2
	221	331		β=-1/2	γ=1/2	P[x3|z1]=	(1/2)*(3+2γ)/(3+γ-β)	=1/2
	221	332		β=-1/2	γ=-1/2	P[x3|z1]=	(1/2)*(3+2γ)/(3+γ-β)	=1/3
112		331	α=1/2		γ=1/2	P[x3|z2]=	(1/2)*(3-2γ)/(3+α-γ)	=1/3
113		331	α=-1/2		γ=1/2	P[x3|z2]=	(1/2)*(3-2γ)/(3+α-γ)	=1/2
112		332	α=1/2		γ=-1/2	P[x3|z2]=	(1/2)*(3-2γ)/(3+α-γ)	=1/2
113		332	α=-1/2		γ=-1/2	P[x3|z2]=	(1/2)*(3-2γ)/(3+α-γ)	=2/3
112		331	α=1/2		γ=1/2	P[x1|z2]=	(1/2)*(3+2α)/(3+α-γ)	=2/3
113		331	α=-1/2		γ=1/2	P[x1|z2]=	(1/2)*(3+2α)/(3+α-γ)	=1/2
112		332	α=1/2		γ=-1/2	P[x1|z2]=	(1/2)*(3+2α)/(3+α-γ)	=1/2
113		332	α=-1/2		γ=-1/2	P[x1|z2]=	(1/2)*(3+2α)/(3+α-γ)	=1/3

Table-3. Twenty-Four Limiting value combinations of the perturbation parameters

Claim MHT3:

There are *eight distinctly different possible extreme strategies* that can be adopted by a strategist-host in the Monty-Hall Problem; corresponding to the three situations that provide an option for the host to open one of the two available alternative doors to reveal a losing-choice to the guest. That is, whenever the initial choice of the guest matches with the door behind which the prize is hidden, the host can open one specific chosen door (as per the chosen strategy) from among the other two doors, each of which is a losing choice. Therefore, we can identify each of these eight distinct strategies by a *uniquely characteristic signature label* {x1y1zu, x2y2zv, x3y3zw} where u∈{2,3}; v∈{3,1}; w∈{1,2}; or simply by an equivalent label {11u22v33w}; as listed in Table-4.

With this notation, we can collect & collate the results of the computation presented in Table-3 corresponding to each of these eight extreme strategies. Each of these eight extreme strategies arise when the three perturbation parameters take on the limiting values within the corresponding range of valid values. Table-4 summarizes the results of the computations for each of these eight strategies, giving the probability of the prize being hidden behind one of the two doors corresponding to the case wherein the host reveals a losing-choice. The symmetry in the results as shown in Table-4 is indeed very intriguing.

Note that Table-4 presents three pairs of values for the comparison of aposteriori probabilities corresponding to each of the eight strategies, thus having a total of 24 pairs of values for comparison. For six of the eight strategies, there are two pairs (1/2, 1/2) and one pair (2/3, 1/3). The two pairs (1/2, 1/2) indicate the two scenarios wherein a switched-choice doesn't affect the chances of winning the prize; *whereas* the one pair (2/3, 1/3) indicates a scenario wherein a switched-choice affects the chances of winning the prize - an enhancement from 1/3 to 2/3 or a diminishment from 2/3 to 1/3 based on the initial-choice of the guest. Note also that the strategies S4 & S5, have all the three pairs with values (2/3, 1/3) and hence they correspond to the two special extreme strategies wherein a switched-choice always leads to either an enhancement or a diminishment in the chances of winning depending on the initial-choice of the guest relative to the placement of the prize.

Corresponding to each scenario of an enhancement there is a complementary scenario of diminishment, and these are distributed symmetrically among the eight distinctly different extreme strategies as can be observed from the Table entries.

Sl.No.	STRATEGY LABEL	P[x1|z3]	P[x2|z3]	P[x2|z1]	P[x3|z1]	P[x1|z2]	P[x3|z2]
S1	{113223331}	1/2	1/2	1/3	2/3	1/2	1/2
S2	{113223332}	1/2	1/2	1/2	1/2	1/3	2/3
S3	{113221331}	2/3	1/3	1/2	1/2	1/2	1/2
S4	{113221332}	2/3	1/3	2/3	1/3	1/3	2/3
S5	{112223331}	1/3	2/3	1/3	2/3	2/3	1/3
S6	{112223332}	1/3	2/3	1/2	1/2	1/2	1/2
S7	{112221331}	1/2	1/2	1/2	1/2	2/3	1/3
S8	{112221332}	1/2	1/2	2/3	1/3	1/2	1/2

Table-4: Eight Extreme Strategies -
each with three pairs of aposteriori probabilities for comparison

For example, in strategy S1 since P[x2 | z1] is 1/3 and P[x3 | z1] is 2/3 it is clear that if the initial-choice is door-2 [y2] then a switched-choice [y3] yields an enhancement in the chances of winning the prize, whereas if the initial-choice is door-3 [y3] then a switched-choice [y2] yields a diminishment in the chances of winning the prize.

Therefore, it is established that there does not exist any strategy that leads to a scenario wherein a switched-choice *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) yields a clear advantage (enhancement) or a clear disadvantage (diminishment) in the chances of winning the prize.

END OF PROOF

5. Earlier Erroneous Result for the Classical Case

The Monty-Hall (parameterized strategist-host) Theorem reaffirms the common-sense based rational and intellectual reasoning, confirmed by the results obtained through the computations shown in the proof, which itself is based on the most fundamental elementary concepts of probability theory. Note that the Monty-Hall Problem is not a problem with possibly multiple correct solutions. Therefore, the above theorem indirectly points out the erroneous result that has been the widely accepted position held by the leading subject matter experts who claim that a "switched-choice" has a clear advantage - with the chances of winning the prize being 2/3 as against only 1/3 for staying with the "initial-choice", in the default classical case.

There seems to be various approaches adopted by the leading subject matter experts, to derive the very same erroneous result. Almost all of them are centered around the use (rather the erroneous use) of the four apriori joint probabilities: (1) P[z3x1y1]; (2) P[z3x2y1]; (3) P[z3x1y2]; (4) P[z3x2y2]; leading to the intended conditional [z3] that is supposed to be used appropriately to derive the required *aposteriori* (*conditional*) *probabilities*: P[x1 | z3] to be compared with P[x2 | z3] in the decision-making problem faced by the guest.

Some consider only the two apriori terms (1) & (2) while leaving out the other two terms (3) & (4) mentioned above; an *error of omission*; as-if fixing [z3y1] as the conditionality rather than [z3]; and derive the aposteriori probabilities: P[x1 | z3y1] to be compared with P[x2 | z3y1] - only to recommend a switched choice from [y1] to [y2]. This is indeed a serious *Logical Fallacy* of lifting/violating the very condition [_|y1] used in that computation. This is exactly similar to the physical analogy of reaching out to the *proverbial mirage-water*, wherein that perception (of mirage-water) itself vanishes, since the very conditions that caused such a perception are violated (no more valid) by the very action of moving towards (that mirage-water) it.

Some others seem to go wrong in their application of the Bayes-Price Rule (Bayes Theorem); an *error of commission*; in a situation with zero-value associated with apriori probabilities P[y2] & P[y3]; as-if fixing [y1] as a pre-condition; an issue of concern that has been clearly mentioned in the above proof while *insisting on a* *check for the validity* (non-zero a-priori leading to the conditional) *of input data* before further processing to derive aposteriori (conditional to [z3]) probabilities.

One of the most striking errors is the claim that the chances of winning by staying with the initial choice is given by (P[x1y1z2]+P[x1y1z3]) whereas the chances of winning by a switched choice is given by (P[x1y2z3]+P[x1y3z2]); as-if fixing [x1] as a pre-condition while not taking advantage of the additional knowledge gained from the host opening the door [z3] revealing a losing choice.

Similarly, another equally intriguing approach adopted by some others is to compare (P[x1y1z3]+P[x2y2z3]) with (P[x1y2z3]+P[x2y1z3]) while correctly considering [z3] as the aposteriori condition although not updating the required probabilities for evaluation & comparison of the two possible alternatives [y1] & [y2] available for the guest.

We are amazed as to how these approaches can be justified by either any rational intellectual reasoning or any study based on the fundamental concepts of Probability & Statistics. This is indeed an atypical case of *erroneous mathematical formulation* of the problem giving rise to an *erroneous model*, and/or even possibly some erroneous problem-solving methodology leading to *erroneous results*, further confirmed (!?!) by *erroneous computer simulation* etc. involving the leading subject matter experts who are expected to warn us from such misleading possibilities.

6. A Challenge to the Leading Subject Matter Experts

Let us rephrase the well-known default case of the classical Monty-Hall Problem, now adorned with a *jewel-on-the-crown* as presented below:

(1.1) The prize is hidden behind one of the three doors;

(1.2) I the guest make an initial-choice, say door-1, to claim my prize;

(1.3) Now, Monty the host opens a different door, say door-3, revealing a losing-choice;

(2.1) I am given an option to withdraw/cancel the earlier choice of door-1 and opt for a switched-choice, that is, door-2;

(2.2) I appreciate the knowledge of a losing-choice and also Monty's offer of the option to switch;

(3.1) I grab Monty's offer, withdraw/cancel my earlier choice of door-1;

(3.2) Then I re-evaluate the two choices available for me now, namely stay with my initial-choice, that is, door-1; or go for the switched-choice, that is, door-2;

(3.3) I find that the chances of winning are exactly the same between the two available choices;

(4.1) Now that you, a subject matter expert, enter the Hall, I seek your recommendation. What is your recommendation?

(4.2) To Switch or Not To Switch : That Is The Question!

Note that your answer must necessarily be independent of my initial-choice; although Monty's choice of opening a door to reveal a losing-choice was dependent on my initial-choice which he had to avoid as per the rules of the game.

Hope your expert advice is *NEITHER* an exemplification of a well-known proverb "*the grass is always greener on the other side*" *NOR* any enticement to reach out for the proverbial mirage-water.

7. Conclusion

The parameterized supermodel represents the generic Monty-Hall Problem, wherein the host may play a parameterized strategy in choosing one of the two doors to open and reveal a losing-choice to the guest – this happens when the initial-choice of the guest matches with the door behind which the prize is hidden. We use three perturbation parameters; $(-1/2) \leq \{\alpha, \beta, \gamma\} \leq (1/2)$; to characterize this scenario. The Monty-Hall (parameterized strategist-host) Theorem is the most general result associated with such a problem. Eight extreme strategies have been identified and characterized. It is shown that there does not exist any strategy that the host may play on the guest, that will be always (irrespective of the placement of the prize and irrespective of the initial-choice of the guest) disadvantageous/advantageous to the guest.

The classical Monty-Hall Problem is the default situation arising from the general parameterized model, when each of the perturbation parameters takes zero-value; wherein it is established that the probability of winning the prize is indeed unaffected by a switched-choice; refuting the claim by the leading subject matter experts.

The most prevalent and widely accepted position held by the leading subject matter experts seems to have arisen from either some *erroneous problem formulation* giving rise to an *erroneous mathematical model* and/or *erroneous problem-solving approach*, possibly also riddled with some *Logical Fallacy*, leading to an *erroneous* *result*, that seems to have been justified by some *erroneous computer simulation* studies, etc.

This paper is an attempt to correct these errors (of long-standing historical significance) in the application of *statistical methodology* in solving the classical Monty-Hall Problem. Specifically, one of them is the erroneous use of the Bayes-Price Rule (Bayes Theorem) in computing the appropriate conditional probabilities. Similar scenarios in data science, machine learning & artificial intelligence, etc., especially in life-critical application areas can indeed have serious far-reaching consequences.

The clearly partitioned three-dimensional discrete event (sample) space with the twelve *mutually-exclusive together-exhaustive* possible alternatives, along with the corresponding apriori probabilities, as represented in a Table, is indeed a fail-safe framework to study, analyze and solve the problem; with no possibility of missing any relevant component terms or including any irrelevant component terms, while going through the required calculations in order to derive the desired results.

Also, note that a real manipulative host would rather simply keep quiet whenever the initial-choice of the guest is a losing-choice (two-third of the time); but would reveal a losing-choice along with an offer to opt for a switched-choice *only* whenever the initial-choice of the guest is indeed a winning-choice (one-third of the time); thus resulting in the overall chances of winning the prize brought down from on-third to zero if a gullible guest falls prey to such seemingly enticing traps.

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9. Acknowledgement

Let us acknowledge that, to '*right a wrong*' requires fighting a tough battle of 'refusal to accept the mistake' followed by '*resistance to unlearn the wrong*' even before or alongside the natural '*grudging reluctance to learn the right*'; indeed a challenge, although an intriguing and yet intellectually fulfilling exploratory journey. I am extremely delighted that this paper is being considered for publication in one of the most prestigious journals with a significant outreach that would provide a forum for the much needed open and unbiased discussions on this seemingly simple and yet a notorious problem, in order to facilitate that endeavor to 'right a wrong'.

10. Declaration Regarding Affiliation and Funding

I, Dr(Prof) Keshava Prasad Halemane, hereby declare that I am a Professor retired as on 2017JAN31 from National Institute of Technology Karnataka Surathkal India, and I am not affiliated to any institution or organization or corporation or any other agency or whatever. This research work has been conducted entirely by me on my own as an Independent Researcher, and that I have not received any funding from any source other than my own savings, and I do not have any obligations or encumbrances of any kind, neither financial nor legal nor of any other kind, regarding the contents of the manuscript - of which I am the original author and creator.

11. Author Rights Retention Notice

12. Dedication

To my **ಅಜ್ಜ**(ajja) Karinja Halemane Keshava Bhat & **ಅಜ್ಜಿ**(ajji) Thirumaleshwari, **ಅಪ್ಪ**(appa) Shama Bhat & **ಅಮ್ಮ**(amma) Thirumaleshwari, for their *teachings through love*, that *quality matters more than quantity*; to my wife Vijayalakshmi for her *ever consistent love & support*; to my daughter Sriwidya.Bharati and my twin sons Sriwidya.Ramana & Sriwidya.Prawina for their *love & affection.*