Monty-Hall (classical-host) Theorem and Monty-Hall (strategist-host) Theorem

Dr.(Prof.) Keshava Prasad Halemane,
Professor - retired from
Department of Mathematical And Computational Sciences,
National Institute of Technology Karnataka Surathkal,
Srinivasnagar, Mangaluru - 575025, India.
https://orcid.org/0000-0003-3483-3521
Independent Researcher (no funding)
SASHESHA, 8-129/12 Sowjanya Road, Naigara Hills,
Bikarnakatte, Kulshekar Post,
Mangaluru-575005. Karnataka State, India
https://www.linkedin.com/in/keshavaprasadahalemane/
k.prasad.h@gmail.com

ABSTRACT

The Monty-Hall (classical-host) Theorem is presented along with a constructive proof by solving the *classical* Monty-Hall Problem. It establishes the fact that the probability of winning the prize is indeed unaffected by a *switched-choice*; unlike the most prevalent and widely accepted position held by the leading subject matter experts. The Monty-Hall (strategist-host) Theorem provides a supermodel that subsumes the Monty-Hall (classical-host) Theorem. Eight extreme strategies have been identified and characterized. It is established that there does not exist any strategy, that a strategist-host may play on the guest, which would result in a situation wherein a *switched-choice* will *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) lead to an enhancement/diminishment in the chances of winning the prize for the guest.

The clearly partitioned *triple-event* space, with the twelve *mutually-exclusive together-exhaustive* possible alternatives, along with the corresponding (joint) probabilities, as represented in a Table, is a fail-safe framework to study, analyze & solve the problem; with no possibility of missing any relevant component terms or including any irrelevant component terms, while going through the required calculations in order to derive the desired results.

Keywords: Monty-Hall (classical-host) Theorem;
Monty-Hall (strategist-host) Theorem;
Monty-Hall Theorem; Bayes-Price Rule; Bayes Theorem

AMS MSC Mathematics Subject Classification: 60A99; 60C99; 62A99; 62C99.

1. INTRODUCTION

The *classical* "Monty-Hall Problem", also referred to as the "Three-Door Problem" is based on a game show "Let's Make a Deal" wherein the host reveals a losing choice to the guest, who had earlier made an initial choice, and in turn offers the guest an enticing option to switch from the initial choice to a second available choice with an aim to enhance the chances of winning the prize. The most prevalent & widely accepted position, as reported in literature, among the leading subject matter experts, mathematicians, statisticians, logicians, and rational intellectuals, is that an appropriate detailed study & analysis of the scenario using the well accepted standard approach of Probability & Statistics, would lead to a recommendation to the guest to switch to the second available choice based on the knowledge obtained from the host revealing a losing choice.

We present the statement of the Monty-Hall (Classical-Host) Theorem, and provide a constructive proof by solving the *classical* Monty-Hall Problem. It establishes that the probability of the guest winning the prize is indeed 1/2 irrespective of whether the guest stays with the initial choice or goes for a switched choice after gathering the information from the host who reveals a losing choice.

We also present the Monty-Hall (Strategist-Host) Theorem as a supermodel that subsumes the earlier Monty-Hall (Classical-Host) Theorem. It considers the situation wherein the host may adopt some strategy in playing with the guest, that may at the outset appear to result in disadvantageous situations for the guest. Eight extreme strategies have been identified & characterized in Table-2 while establishing that there cannot exist any strategy that would always be disadvantageous for the guest.

2. PROBLEM DESCRIPTION - INPUT DATA SET

Let us consider the so-called *classical* Monty-Hall Problem as reported widely in the literature - with a prize hidden behind one of the three doors; a guest making an initial choice of a door to claim the prize; the host who knows the location of the prize as well as the initial choice made by the guest, now *reveals a distinctly different and yet a losing choice*, by opening a third door. Then the host also offers the guest, an option to switch from the initial choice to the now available second choice, anticipating an enhanced chance of winning the prize, based on the knowledge obtained about a losing choice.

Let us represent the events/actions associated with the three doors:
(1) xr ∊ {1,2,3} be the door r behind which the reward/prize x is hidden;
(2) yp ∊ {1,2,3} be the initial choice of the door p chosen by the player/guest y;
(3) zq ∊ {1,2,3} be the door q opened by the host z to reveal a losing choice.

Let the symbol ‘ai’ denote the event/action [E{(a=i)}] for any ‘agent’ a ∊ {x,y,z} and ‘door’ i∊{r,p,q}={1,2,3}.

It is essential to note here that xr and yp are mutually independent of each other as well as independent of zq; whereas zq itself is dependent on both xr and yp, as per the rules of the game. Also, note that the focus must be on the decision-making process & the action to be taken by the guest. So, the *problem formulation* (*modelling*) must necessarily be from the view-point of the guest.

3. ASSUMPTIONS

It is assumed that the reward/prize is hidden randomly behind one of the three doors, each of the events [xr∊{1,2,3}] being considered equiprobable. Also, the initial choice of the door [yp∊{1,2,3}] chosen by the player/guest is a random (blind) choice.

The host knows the door behind which the prize is hidden and also the door that is the initial choice of the guest. Therefore, the event/action of the host z opening door q, zq∊{1,2,3} to show a losing choice, is dependent on both yp and xr, as per the rules of the game show. That is, (zq ≠ yp) & (zq ≠ xr). This dependency of zq on yp and xr does indeed limit the available options. It turns out that when yp ≠ xr the host doesn't have any option except to turn to the one and only one remaining door zq ≠ (yp ≠ xr); whereas when yp = xr the host has the option of choosing between the two doors, that is, zq ≠ (yp = xr). Because the host has this option, at least in a restricted sense, of choosing which of the two doors to open, it introduces an uncertainty for the guest to predict/expect/anticipate the host's decision/action in this regard. Here, it is *assumed* that whenever zq ≠ (yp = xr) the host's choice between the two available options is indeed *equiprobable*.

The Input Data Set presented in Table-1 lists the 12 mutually-exclusive together-exhaustive possible alternatives for the combined-triple-event space along with the relevant apriori probabilities.

Sl.No.	[xr]	[yp]	[xr&yp]	[zq]	[xr&yp&zq]	P[xr]	P[yp]	P[zq|(xr & yp)]	P[xr&yp&zq]
01	1	1	11	2	112	1/3	1/3	1/2	1/18
02	1	1	11	3	113	1/3	1/3	1/2	1/18
03	1	2	12	3	123	1/3	1/3	1	1/9
04	1	3	13	2	132	1/3	1/3	1	1/9
05	2	1	21	3	213	1/3	1/3	1	1/9
06	2	2	22	1	221	1/3	1/3	1/2	1/18
07	2	2	22	3	223	1/3	1/3	1/2	1/18
08	2	3	23	1	231	1/3	1/3	1	1/9
09	3	1	31	2	312	1/3	1/3	1	1/9
10	3	2	32	1	321	1/3	1/3	1	1/9
11	3	3	33	1	331	1/3	1/3	1/2	1/18
12	3	3	33	2	332	1/3	1/3	1/2	1/18

Twelve Mutually-Exclusive Together-Exhaustive Alternative-Possibilities

[xr]: prize x behind door r; [yp]: guest y choses door p; [zq]: host z reveals door q

Table-1: Twelve *combined-triplet-event* possibilities along with its *joint-probabilities*.

4. MONTY-HALL (CLASSICAL-HOST) THEOREM

Given that the initial choice of the guest is, say door-1 (event [y1]); and that the host opens the door, say door-3 (event [z3]) to reveal a losing choice, that is different from the door behind which the prize is hidden, and also different from the initial choice of the guest; then the probability of the guest winning the prize is given by the *aposteriori* (conditional to [z3]) *probability* of the prize being hidden behind the door-1 (event [x1]); that is, P[x1 | z3]. In the case of the *classical* Monty-Hall Problem, this value may be computed by the application of the *Bayes-Price Rule* (*Bayes Theorem*) for the case of *three parameter event(sample)space*; and it is equal to 0.50; therefore, the option of the switched-choice doesn't yield any enhancement in the chances of winning the prize.

PROOF

The proof is simply by solving the problem, following the below enumerated steps. For each required value, a general expression is given first; followed by the *classical* case.

(a) INPUT DATA SET

$$\begin{aligned}
&P[x1]; \quad P[x2]; \quad P[x3]; \quad P[y1]; \quad P[y2]; \quad P[y3];\\
&P[z3 \mid x1y1]; \quad P[z3 \mid x1y2]; \quad P[z3 \mid x2y1]; \quad P[z3 \mid x2y2];\\
&P[z2 \mid x1y1]; \quad P[z2 \mid x1y3]; \quad P[z2 \mid x3y1]; \quad P[z2 \mid x3y3];\\
&P[z1 \mid x2y2]; \quad P[z1 \mid x2y3]; \quad P[z1 \mid x3y2]; \quad P[z1 \mid x3y3];
\end{aligned}$$

(b) JOINT PROBABILITIES FOR INDEPENDENT EVENTS [xr & yp]

$$\begin{aligned}
P[x1y1] &= P[x1]*P[y1]; & P[x1y2] &= P[x1]*P[y2]; & P[x1y3] &= P[x1]*P[y3];\\
P[x2y1] &= P[x2]*P[y1]; & P[x2y2] &= P[x2]*P[y2]; & P[x2y3] &= P[x2]*P[y3];\\
P[x3y1] &= P[x3]*P[y1]; & P[x3y2] &= P[x3]*P[y2]; & P[x3y3] &= P[x3]*P[y3];
\end{aligned}$$

(c) VALIDITY CHECK FOR NON-ZERO APRIORI PROBABILITIES

Check and confirm the *validity of input data values* for application of Bayes-Price Rule (Bayes Theorem). The presence of *zero-value* for any of the *apriori probabilities* leading to the intended conditional used to derive the required *aposteriori* (*conditional*) *probabilities*, can result in *spurious results*. Appropriate alternative approach may be needed in such cases. For the *classical* Monty-Hall Problem, the *joint probabilities* listed above leading to the required conditionality of the host opening a door (say z3) will be used in the below calculations.

(d) APRIORI PROBABILITY FOR [z3] – PER RULES OF THE GAME

$$\begin{aligned}
P[z3] &= P[z3|x1y1]*P[x1y1] + P[z3|x2y1]*P[x2y1] + P[z3|x1y2]*P[x1y2] + P[z3|x2y2]*P[x2y2];\\
&= P[x1y1z3] + P[x1y2z3] + P[x2y1z3] + P[x2y2z3];\\
&= 1/18 + 1/9 + 1/9 + 1/18;\\
&= 1/3;
\end{aligned}$$

(e) APRIORI (conditional w.r.t. x1; marginal w.r.t. yp) PROBABILITY for z3

$$\begin{aligned}
P[z3 \mid x1] &= (P[z3 \mid x1y1] * P[x1y1] + P[z3 \mid x1y2] * P[x1y2]) / (P[x1]);\\
&= (P[z3x1y1] + P[z3x1y2]) / (P[x1]);\\
&= (1/18 + 1/9) / (1/3);\\
&= 1/2;
\end{aligned}$$

(f) APOSTERIORI (conditional w.r.t. z3; marginal w.r.t. yp) PROBABILITY for x1

P[x1 | z3] = (P[z3 | x1] * P[x1]) / (P[z3]);
= (1/2 * 1/3) / (1/3);
= 1/2;

END OF PROOF

5. DISCUSSION

It is to be noted here that the theorem and the proof uses some specific labels for the doors, just for convenience; namely, door-3 [z3] for the door opened by the host to reveal a losing choice, and door-1 [y1] for the guest's initial choice, thus leading to a decision-making problem for the guest that requires the computation of the value P[x1 | z3] and its complementary value P[x2 | z3]. However, the result is neither restricted by nor dependent on these specific labels. This is evident from the symmetry in the data entries in Table-1, which shows that xr and yp are interchangeable for any given zq, and that the entries are identical for the three subsets corresponding to each of the values for zq ∊ {1,2,3}. Or, if one wishes, one can re-write the theorem in three parts, one corresponding to each of the specific cases with the host opening a door zq ∊ {1,2,3}. Also, one can always consider a scenario wherein the three doors are exactly identical from the viewpoint of the guest, and that the initial choice of the guest is then labelled as door-1, and that the door that is opened by the host is then labelled as door-3, thus leaving the remaining door to be labelled as door-2.

Therefore, it gets established that *irrespective of whichever be the door opened by the host, thus revealing a losing choice, each of the remaining two doors have equal probability of having the prize hidden behind it* – implying that the so called "switched-choice" is neither better nor worse in terms of the chances of winning, than staying with the initial-choice of the guest.

6. EARLIER ERRONEOUS RESULT

The Monty-Hall (classical-host) Theorem reaffirms common-sense based rational & intellectual reasoning, confirmed by the results obtained through the computations shown in the proof, which itself is based on the most fundamental elementary concepts of probability theory. Note that the Monty-Hall Problem is not a problem with possibly multiple correct solutions. Therefore, the above theorem indirectly points out the erroneous result that has been the widely accepted position held by the leading subject matter experts who claim that a switched choice has a clear advantage, with the chances of winning the prize being 2/3 as against only 1/3 for staying with the initial choice.

There seems to be various approaches adopted by the leading subject matter experts, to derive the very same erroneous result. Almost all of them are centered around the use (rather the erroneous use) of the four apriori joint probabilities: (1) P[z3x1y1]; (2) P[z3x2y1]; (3) P[z3x1y2]; (4) P[z3x2y2]; leading to the intended conditional [z3] that is supposed to be used appropriately to derive the required *aposteriori* (*conditional*) *probabilities*: P[x1 | z3] to be compared with P[x2 | z3] in the decision-making problem faced by the guest.

Some consider only the two apriori terms (1) & (2) while leaving out (*error of omission*) the other two terms (3) & (4) mentioned above; an *error of omission*; as-if fixing [z3y1] as the conditionality rather than [z3]; and derive the aposteriori probabilities: P[x1 | z3y1] to be compared with P[x2 | z3y1] - only to recommend a switched choice from [y1] to [y2]. This is indeed a serious *Logical Fallacy* of lifting/violating the very condition [|y1] used in that computation. This is exactly similar to the physical analogy of chasing the proverbial mirage-waters, wherein that perception itself vanishes, since the very conditions that caused such a perception are violated (no more valid) by the very action of moving towards it.

Some others seem to go wrong in their application of the Bayes-Price Rule (Bayes Theorem) - *error of commission* - in a situation with zero value associated with apriori probabilities P[y2] & P[y3]; as-if fixing [y1] as a pre-condition – an issue of concern that has been clearly mentioned in the above proof while *insisting on a check for the validity of input data* before further processing to derive aposteriori (conditional to [z3]) probabilities.

One of the most striking errors is the claim that the chances of winning by staying with the initial choice is given by (P[x1y1z2]+P[x1y1z3]) whereas the chances of winning by a switched choice is given by (P[x1y2z3]+P[x1y3z2]); as-if fixing [x1] as a pre-condition while not taking advantage of the additional knowledge gained from the host opening the door [z3] revealing a losing choice!

Similarly, another equally intriguing approach adopted by some others is to compare (P[x1y1z3]+P[x2y2z3]) with (P[x1y2z3]+P[x2y1z3]) while correctly considering [z3] as the aposteriori condition although not updating the required probabilities for evaluation & comparison of the two possible alternatives [y1] & [y2] available for the guest!

We are amazed as to how these approaches can be justified by either any rational intellectual reasoning or any study based on the fundamental concepts of Probability & Statistics. This is indeed an atypical case of *erroneous mathematical formulation* of the problem giving rise to an *erroneous model*, and/or even possibly some erroneous problem-solving methodology leading to *erroneous results*, further confirmed (!?!) by *erroneous computer simulation* etc. involving the leading subject matter experts who are expected to warn us from such misleading possibilities.

7. A CHALLENGE TO THE LEADING SUBJECT MATTER EXPERTS

Let us rephrase the classical Monty-Hall Problem,
now adorned with a *jewel-on-the-crown* as below:

(1.1) The prize is hidden behind one of the three doors.
(1.2) I the guest make an initial-choice, say door-1, to claim my prize.
(1.3) Then Monty the host opens a different door, say door-3, revealing a losing-choice.
(2.1) I am given an option to withdraw/cancel the earlier choice of door-1 and switch to door-2.
(2.2) I appreciate the knowledge of a losing-choice and also Monty's offer of the option to switch.
(3.1) I grab Monty's offer, withdraw/cancel my earlier choice of door-1.
(3.2) Then I re-evaluate the two choices available for me now, namely door-1 or door-2.
(3.3) I find that the chances of winning are exactly the same between the two available choices;
(4.1) Now that you, a subject matter expert enter the Hall, I seek your recommendation. What is your recommendation?
(4.2) TO SWITCH OR NOT TO SWITCH : THAT IS THE QUESTION!

Note that your answer must necessarily be independent of my initial-choice; although Monty's choice of opening a door to reveal a losing choice was dependent on my initial choice which he had to avoid as per the rules of the game.

Hope your expert advice is *NEITHER* an exemplification of a well-known proverb "*the grass is always greener on the other side*" *NOR* any enticement to chase the proverbial mirage-waters wherein that perception (of mirage) itself vanishes, since the very conditions that caused such a perception (of mirage) are violated by the very action of moving towards (the mirage) it.

8. COOL-HEADED BRAVE-HEARTS PLAY WITH STRATEGIST HOST

This is somewhat *far from the so-called classical version* of the Monty-Hall Problem. Here, we allow the host to exercise whatever '*strategic game-playing*' that one wishes to play with the guest. The situation can be captured by the terms P[z3 | x1 & y1] and P[z3 | x2 & y2] that are fully under the control of the host. An extreme situation is when the host adopts a certain strategy that pulls down one of them to zero and pushes the other one to its maximum value of the restricted-probability, namely 1/9. Then it turns out that the values of the two aposteriori(conditional)marginal-probabilities P[x1 | z3] and P[x2 | z3] can't be the same anymore; in the *extreme case*, one will be 1/3 and the other will be 2/3; which then may lead to the two possibilities: One extreme case with a *specific strategy* wherein a *switched choice has a clear disadvantage*; and a second extreme case with a *specific strategy* wherein a *switched choice has a clear advantage*. It was left (refer: [12]) as an exercise to the cool-headed brave-hearts to figure out the two specific strategies that would lead to such extreme situations. To close this issue once for all, let us present the Monty-Hall Theorem for the case of strategist-host.

9. MONTY-HALL (STRATEGIST-HOST) THEOREM

There does not exist any strategy, that can be adopted by a strategist-host, in the Monty-Hall (strategist-host) Problem, that would result in a situation wherein a *switched-choice* will *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) lead to an enhancement/diminishment in the chances of winning the prize for the guest.

PROOF

Note that the Monty-Hall (strategist-host) Theorem subsumes the Monty-Hall (classical-host) Theorem for the classical case wherein the host randomly chooses between the two available doors to reveal a losing choice; the random choice itself being a possible strategy. However, we will focus on the extreme strategy situations, here.

There are *eight distinctly different possible extreme strategies* that can be adopted by a strategist-host in the Monty-Hall Problem; corresponding to the three situations that provide an option for the host to open one of the two available alternative doors to reveal a losing choice to the guest. That is, whenever the initial choice of the guest matches with the door behind which the prize is hidden, the host can open one specific chosen door from among the other two doors, each of which is a losing choice. Therefore, we can identify each of these eight distinct strategies by a *uniquely characteristic signature label* {x1y1zu, x2y2zv, x3y3zw} where u∈{2,3}; v∈{3,1}; w∈{1,2}; or simply by an equivalent label {11u22v33w}.

Referring back to Table-1, strategy S1 presented in Table-2 corresponds to the scenario that includes each of the three combined-triple-events [x1y1z3] and [x2y2z3] and [x3y3z1] with the joint probability of 1/9 for each of them, whereas the associated three combined-triple-events [x1y1z2] and [x2y2z1] and [x3y3z2] are eliminated from consideration; while the remaining six combined-triple-events [x1y2z3] and [x2y1z3] and [x1y3z2] and [x3y1z2] and [x2y3z1] and [x3y2z1] remain as such with the joint probability of 1/9 for each of them. It is exactly the same as deriving an Input Data Set IDS^1 for the strategy S1 by appropriately eliminating the unwanted three rows from Table-1 and updating the joint probabilities for each of their corresponding complementary combined-triple-events therein.

Similarly, we can derive the Input Data Sets IDS^k with k∈{1,2,3, . . . 8} for each of the eight strategies mentioned in Table-2, using which the *required computations* can be carried out similar to what is presented in Section-4 for the proof of the Monty-Hall (classical-host) Theorem. The proof for the Monty-Hall (strategist-host) Theorem is exactly similar to that for the Monty-Hall (classical-host) Theorem, wherein each of the eight strategies S^k with k∈{1,2,3, . . . 8} is associated with the specific modified Input Data Set IDS^k instead of using the Input Data entries presented in Table-1.

Table-2 summarizes the results of the computations for each of these eight strategies, giving the probability of the prize being hidden behind one of the two doors corresponding to the case wherein the host reveals a losing choice. Note that the detailed computations associated with each of these eight extreme strategies require exactly the very same steps that were gone through in the entire proof for the Monty-Hall (classical-host) Theorem earlier; once for each strategy, using the appropriate Input Data Set for each strategy.

The symmetry in the results as shown in Table-2 above is indeed very intriguing.

Note that Table-2 presents three pairs of values for the comparison of aposteriori probabilities corresponding to each of the eight strategies, thus having a total of 24 pairs of values for comparison. For six of the eight strategies, there are two pairs (1/2, 1/2) and one pair (2/3, 1/3). The two pairs (1/2, 1/2) indicate the two scenarios wherein a switched-choice doesn't affect the chances of winning the prize; *whereas* the one pair (2/3, 1/3) indicates a scenario wherein a switched-choice affects the chances of winning the prize - an enhancement from 1/3 to 2/3 or a diminishment from 2/3 to 1/3 based on the initial-choice of the guest. Note also that the strategies S4 & S5, have all the three pairs with values (2/3, 1/3) and hence both of them address the question posed in Section-8 above.

Sl.No.	STRATEGY LABEL	P[x1|z3]	P[x2|z3]	P[x2|z1]	P[x3|z1]	P[x1|z2]	P[x3|z2]
S1	{113223331}	1/2	1/2	1/3	2/3	1/2	1/2
S2	{113223332}	1/2	1/2	1/2	1/2	1/3	2/3
S3	{113221331}	2/3	1/3	1/2	1/2	1/2	1/2
S4	{113221332}	2/3	1/3	2/3	1/3	1/3	2/3
S5	{112223331}	1/3	2/3	1/3	2/3	2/3	1/3
S6	{112223332}	1/3	2/3	1/2	1/2	1/2	1/2
S7	{112221331}	1/2	1/2	1/2	1/2	2/3	1/3
S8	{112221332}	1/2	1/2	2/3	1/3	1/2	1/2

Table-2: Eight Extreme Strategies -
each with three pairs of aposteriori probabilities for comparison

Corresponding to each scenario of an enhancement there is a complementary scenario of diminishment, and these are distributed symmetrically among the eight distinctly different extreme strategies as can be observed from the Table entries.

For example, in strategy S1 since P[x2 | z1] is 1/3 and P[x3 | z1] is 2/3 it is clear that if the initial-choice is door-2 [y2] then a switched-choice [y3] yields an enhancement in the chances of winning the prize, whereas if the initial-choice is door-3 [y3] then a switched-choice [y2] yields a diminishment in the chances of winning the prize.

Therefore, it is established that there is no strategy which presents any scenario wherein a switched-choice *always* (*irrespective of the placement of the prize* and *irrespective of the initial-choice of the guest*) yields a clear advantage or a clear disadvantage (enhancement or diminishment) in the chances of winning the prize.

END OF PROOF

10. CONCLUSION

The Monty-Hall (classical-host) Theorem establishes the correct approach in formulating and solving the *classical* Monty-Hall Problem. It establishes the fact that the probability of winning the prize is indeed unaffected by a switched-choice.

The most prevalent and widely accepted position held by the leading subject matter experts seems to have arisen from either some *erroneous problem formulation* giving rise to an *erroneous mathematical model* and/or *erroneous problem-solving approach*, possibly also riddled with some *Logical Fallacy*, leading to an *erroneous result*, that seems to have been justified by some *erroneous computer simulation* studies, etc.

The clearly partitioned *triple-event* space, with the twelve *mutually-exclusive together-exhaustive* possible alternatives, as represented in Table-1, is a fail-safe framework to study, analyze & solve the problem; no possibility of missing any relevant component terms and/or including any irrelevant component terms while going through the required calculations in order to derive whatever desired results.

11. RECOMMENDED READING

[1]. Wikipedia Page – https://en.wikipedia.org/wiki/Monty_Hall_problem

[2]. Jason Rosenhouse; "The Monty Hall Problem:
The Remarkable Story of Math's Most Contentious Brain Teaser";
Oxford University Press, ISBN 978-0-19-536789-8, 2009.

[3]. Jason Rosenhouse; "Games-for-Your-Mind_History-&-Future-of-Logic-Puzzles";
Princeton University Press, 2020.

[4]. Anthony B. Morton; "Prize insights in probability, and one goat of a recycled error";
Arxiv:1011.3400v2 2010.

[5]. Matthew A. Carlton;
"Pedigrees, Prizes, and Prisoners: The Misuse of Conditional Probability";
Journal of Statistics Education Volume 13, Number 2 (2005);
ww2.amstat.org/publications/jse/v13n2/carlton.html

[6]. Richard D. Gill; "The Monty Hall Problem is not a Probability Puzzle :
It's a challenge in mathematical modelling";
arXiv:1002.0651v4 2023.

[7]. Torsten Enßlin and Margret Westerkamp;
"The rationality of irrationality in the Monty Hall problem";
arXiv:1804.04948v4 2018.

[8]. Jeffrey S. Rosenthal; "Monty Hall, Monty Fall, Monty Crawl";
probability.ca/jeff/writing_montyfall

[9]. Christopher A. Pynes; "IF MONTY HALL FALLS OR CRAWLS";
EuJAP Vol.9, No.2, pp 33-47; 2013.

[10]. Andrew Vazsonyi, Feature Editor; "Which Door Has the Cadillac?";
The Real-Life Adventures of a Decision Scientist – featured column
www.decisionsciences.org/DecisionLine/Vol30/30_1/vazs30_1.pdf

[11]. Richard Isaac (1995); "The Pleasure of Probability"
Springer-Verlag Undergraduate Texts in Mathematics.

[12]. Halemane, K. P. (2025);
"Refutation of the Logical Fallacy Committed by the Subject Matter Experts on the Monty-Hall Problem";
https://engrxiv.org/preprint/view/5102

[13]. Halemane, K. P. (2014);
"Unbelievable O(L$^{1.5}$) worst case computational complexity
achieved by *spdspds* algorithm for linear programming problem";
arxiv:1405.6902 2025.

12. DECLARATION REGARDING AFFILIATION AND FUNDING

I, Dr(Prof) Keshava Prasad Halemane, hereby declare that I am a Professor retired as on 2017JAN31 from National Institute of Technology Karnataka Surathkal India, and I am not affiliated to any institution or organization or corporation or any other agency or whatever. This research work has been conducted entirely by me on my own as an Independent Researcher, and that I have not received any funding from any source other than my own savings, and I do not have any obligations or encumbrances of any kind, neither financial nor legal nor of any other kind, regarding the contents of the manuscript - of which I am the original author and creator.

13. AUTHOR RIGHTS RETENTION NOTICE

The author as the original creator of this work asserts exclusive ownership and retains full-&-complete Intellectual Property Rights (IPR) and CopyRight for this (as well as all its previous versions, including all the versions archived in various repositories) in perpetuity, for the maximum duration permitted by law. For the purpose of publication, the author hereby applies a CC BY 4.0 Public Copyright License to this manuscript, and any/every Author Accepted Manuscript (AAM) version arising from this manuscript, and grants only a non-exclusive publishing rights to the publisher. The author's IPR including all commercial and derivative rights are retained in full as the exclusive intellectual property of the author and shall pass to his designated legal heirs (the three children mentioned herein below) in perpetuity, for the maximum duration permitted by law.

14. ACKNOWLEDGEMENT

Let us acknowledge that, looking back, it seems as if Marilyn cast an enchantingly deep spell over the Frequentists who in turn pushed the Probabilists to mistake Bayes-Price, paying the price through errors of commission and/or errors of omission, riddled with some Logical Fallacy as well. We need to come out of that long drawn intellectual hibernation of over six decades, and wake up to reality.

I must necessarily confess here that the *core idea behind this analysis is so stunningly & elusively simple*, that one may simply be taken aback in a profound wonder-struck jaw-drop-silence, maybe with an after-thought: "*oh my goodness, how could it be that it never flashed on me any time earlier*"! as was also the case in an earlier research work reported in [13] by this author.

On this most auspicious vidyaa(vijaya)daSami day [2025OCT02] I was blessed with the vision to formulate the *Monty-Hall Theorem* and its *proof* - as a *concise* & *precise* approach - the preferred style of presentation for the target audience consisting of Mathematicians, Statisticians, Logicians, eminent scientists etc.

15. DEDICATION

To my **ಅಜ್ಜ**(ajja) Karinja Halemane Keshava Bhat & **ಅಜ್ಜಿ**(ajji) Thirumaleshwari, **ಅಪ್ಪ**(appa) Shama Bhat & **ಅಮ್ಮ**(amma) Thirumaleshwari, for their *teachings through love*, that *quality matters more than quantity*; to my wife Vijayalakshmi for her *ever consistent love & support*; to my daughter Sriwidya.Bharati and my twin sons Sriwidya.Ramana & Sriwidya.Prawina for their *love & affection*.

To all the *cool-headed brave-hearts*, eagerly awaited but probably yet to be visible among the world professionals, especially the *Subject matter experts*, who would be attracted to and certainly capable of effectively understanding without any prejudice and appreciating the deeper insights enshrined in this short research report, who may opt for innate rational-&-intellectual common-sense and simple creativity over any sophisticated and/or complex theory in problem-solving to resolve any seemingly paradoxical scenarios possibly arising therefrom.

Further, let us be aware that, to '*right a wrong*' requires fighting a tough battle of 'refusal to accept the mistake' followed by '*resistance to unlearn the wrong*' even before or alongside the natural '*grudging reluctance to learn the right*'. I am sure you as a *cool-headed brave-heart*, have come forward to take up that challenge, although an intriguing and yet intellectually fulfilling exploratory journey - all my best wishes to you.

ॐतत्सत्