Entropy $\neq$ Disorder

James Oliver

Abstract

Boltzmann's formula, $S = k_B \ln(\Omega)$, is not a measure of "disorder," but a tool for counting accessible configurations. This is seen when arranging cards: one card has one configuration ($\Omega = 1$, $S = 0$), while a full deck has an astronomical number ($\Omega = 52!$, $S \approx 156\, k_B$). The formula quantifies the size of this possibility space. Common examples reinforce this: the entropy of a "messy room" or the anagrams LISTEN/SILENT depends entirely on the constraints of the description, not on subjective labels. By focusing on what Boltzmann's formula actually calculates, this paper clarifies a persistent pedagogical confusion, showing that entropy measures the size of the explorable space defined by physical constraints.

Contribution: This paper synthesizes established results from statistical mechanics to make Boltzmann's definition of entropy explicit. The contribution is not new physics but a clarification of persistent pedagogical confusion, building on critiques by Styer (2019) and Lambert (2002). We argue that entropy measures the size of the accessible possibility space, a framework that would be falsified if systems with identical macroscopic constraints could have different entropies or if the formula $S = k_B \ln(\Omega)$ failed to match measured data.

The Deck Of Cards

Take a single card from a deck: the 2 of clubs. How many ways can you arrange it? Only one. The entropy is:

$$S = k_B \ln(1) = 0 \tag{1}$$

There is no possibility space. It's a single point. Nothing to explore, nowhere to go. Entropy is zero. Now add a second card: the 9 of diamonds. Suddenly there are two arrangements: you could have $[2\clubsuit, 9\diamondsuit]$ or $[9\diamondsuit, 2\clubsuit]$. The possibility space opens. The entropy becomes:

$$S = k_B \ln(2) \approx 0.69\, k_B \tag{2}$$

Add a third card: the King of hearts, and the space explodes into six configurations. Every new card doesn't just add possibilities, but rather multiplies them. With three cards, entropy jumps to:

$$S = k_B \ln(6) \approx 1.79\, k_B \tag{3}$$

Add a fourth card, then a fifth, then ten, then twenty. Each addition expands the space. By the time you hold a full deck of 52 cards in your hands, the number of possible arrangements is astronomical:

$$\Omega = 52! \approx 8 \times 10^{67} \tag{4}$$

To put that in perspective: if you shuffled a deck every second since the Big Bang began 14 billion years ago, you'd have performed about 10^{17} shuffles, roughly 10^{50} times fewer than the number of possible arrangements. Every time you shuffle a deck, you are almost certainly creating an arrangement that has never existed before in the history of the universe, and will likely never exist again. The entropy of a shuffled deck:

$$S = k_B \ln(52!) \approx 156\, k_B \tag{5}$$

When you shuffle, you're moving from one point in this possibility space to another. And this exploration is happening right now, across millions of decks worldwide. Somewhere in Las Vegas, someone at a casino blackjack table shuffles. In London, a bridge club deals the next hand. In Tokyo, cards move through a living room. Each shuffle visits a point in the 52!-point space that has almost certainly never existed before in the history of the universe, and will likely never exist again. Every second, new points in that space are visited for the first time and the last. The possibility space dictated by Boltzmann's formula, a living pulse of exploration across the planet.

Is Entropy Disorder?

Open a physics textbook and you'll likely read that entropy measures disorder. So let's look carefully at what that means.

Take your shuffled deck and arrange the cards in what most people call "perfect order":

$$[A\spadesuit, 2\spadesuit, 3\spadesuit, ..., K\spadesuit, A\heartsuit, 2\heartsuit, ..., K\heartsuit, A\diamondsuit, ..., K\diamondsuit, A\clubsuit, ..., K\clubsuit]$$

Spades, then hearts, then diamonds, then clubs. Ascending within each suit. Aces low. Now take the same 52 cards and arrange them randomly:

$$[7\clubsuit, Q\heartsuit, 2\diamondsuit, 9\spadesuit, K\clubsuit, 3\heartsuit, ...]$$

Most people would say the first is "ordered" and the second is "disordered." But look at what's actually there. Both are specific arrangements of 52 cards. Both are single points in the 52!-point space. Both have exactly the same probability of occurring from a random shuffle: $1/52! \approx 10^{-68}$. Both are equally specific microstates. Physics treats them identically.

The first seems "ordered" because we chose that arrangement as our standard. But watch what happens when someone else looks at it. Your friend walks up: "I organize my decks differently. Clubs first, then diamonds, then hearts, then spades. And aces are high, not low." His "ordered" deck:

$$[2\clubsuit, 3\clubsuit, ..., K\clubsuit, A\clubsuit, 2\diamondsuit, 3\diamondsuit, ..., A\diamondsuit, 2\heartsuit, ..., A\heartsuit, 2\spadesuit, ..., A\spadesuit]$$

Your "order" looks wrong to him. His looks wrong to you. Another person organizes by color, putting all red cards together and all black cards together. Each person has a different "order." Each sees everyone else's arrangement as "disordered" relative to their standard.

Who's right? Nobody, at least not in physics. The standard sequence $[A\spadesuit, 2\spadesuit, 3\spadesuit, ...]$ may be conventional or practical, but it has no special status in the formula. Every arrangement is just one specific configuration out of 52!. "Order" only exists relative to a chosen reference. Change the reference, change what counts as "ordered."

But entropy is supposed to be a physical quantity, something objective and calculable that appears in fundamental laws. So how can it depend on arbitrary human preferences about card order? Every specific arrangement, whether $[A\spadesuit, 2\spadesuit, 3\spadesuit ...]$ or $[7\clubsuit, Q\heartsuit, 2\diamondsuit ...]$, is a **single microstate**. As a microstate, each has entropy:

$$S = k_B \ln(1) = 0 \tag{6}$$

They're points, not spaces. Individually, they have zero entropy. But when you say "any shuffled arrangement," you're describing a **macrostate**, a huge set containing 52! different microstates. *That* macrostate has entropy:

$$S = k_B \ln(52!) \approx 156\, k_B \tag{7}$$

The entropy isn't in any particular arrangement. It's in how many arrangements satisfy your description.

Is Entropy Missing Information?

You may also read that entropy measures missing information or uncertainty. Let's explore that.

You open a fresh deck of cards. They come in standard order: $[A\spadesuit, 2\spadesuit, 3\spadesuit, ..., K\clubsuit]$. You know exactly where every card is. Complete knowledge, zero uncertainty.

Now you shuffle the deck thoroughly, realizing that this shuffle almost certainly creates an arrangement that has never existed before in cosmic history. You now have no idea where any card is. Zero knowledge, complete uncertainty. So, given that your knowledge has gone from complete to none, the entropy must have changed dramatically, right?

But does the entropy reflect our knowledge? Let's see what the formula calculates, which depends only on the constraints we set. If we describe the deck by the macrostate "52 cards in any arrangement," the entropy is the same whether we know the order or not:

Known order (new deck): $\Omega = 52!$, $S \approx 156\, k_B$

Unknown order (shuffled): $\Omega = 52!$, $S \approx 156\, k_B$

Our knowledge changed completely, but under this consistent description, the entropy remained identical. If instead we describe the deck by the macrostate "the single, specific arrangement it is in right now," the entropy is zero in both cases: $\Omega = 1$ and $S = 0$. Once again, the entropy is identical, regardless of our knowledge. What's actually varying here is not your knowledge, but which constraint you choose.

"Order" and "disorder" require arbitrary reference points
"Information" and "knowledge" depend on what observers know
Possibility space exists independent of both

The constraint is a choice about your description. The entropy given that constraint is what the formula calculates.

What Entropy Is

Boltzmann gave us [1]:

$$S = k_B \ln(\Omega) \tag{8}$$

where:

- S is entropy (joules per kelvin)
- k_B is Boltzmann's constant (1.38×10^{-23} J/K)
- Ω is the number of microstates accessible to the system

This is counting. Entropy measures how many configurations are possible.

Two Concepts: Microstates and Macrostates

To understand what Ω counts, we need to be precise about two things.

A **microstate** is a complete specification of the system, every detail, every degree of freedom. For our deck, a microstate specifies which card is in which position: [7♣ at position 1, $Q\heartsuit$ at position 2, $2\diamondsuit$ at position 3, ...] is one microstate.

A **macrostate** is a coarse-grained description using only certain variables, the constraints you impose. For the deck, we can define different macrostates by choosing different constraints.

Macrostate A: "52 cards in standard sequence $[A♠, 2♠, 3♠, ...]$"

$$\Omega_A = 1, \quad S_A = k_B \ln(1) = 0 \tag{9}$$

Only one microstate satisfies this constraint. Notice something striking: this 52-card deck has the same entropy as a single card. Both have $S = 0$ for the same reason, there's only one accessible arrangement. The deck has 52 cards, but the constraint eliminates all degrees of freedom. There's nowhere to go, nothing to explore.

This reveals something fundamental: having multiple components is necessary for entropy, but it doesn't guarantee it. The 52 cards create the *possibility* of 52! different arrangements, but constraints determine which possibilities are *accessible*. With one card, you have no possibility of rearrangement. With 52 cards under a tight constraint, you have the potential for rearrangement but the constraint blocks access to it. Entropy measures freedom to explore, not the number of components.

Let's see what happens when we loosen the constraint.

Macrostate B: "52 cards with suits grouped, any order within suits"

$$\Omega_B = (13!)^4 \approx 1.5 \times 10^{39}, \quad S_B = k_B \ln((13!)^4) \approx 90.6\, k_B \tag{10}$$

Now many microstates satisfy this constraint, any arrangement that keeps suits together. By loosening the constraint from "exact sequence" to "suits grouped," we've opened up a vast region of possibility space. The entropy jumps from zero to $90.6\, k_B$, more than half the maximum entropy despite still maintaining suit order.

Macrostate C: "52 cards in any arrangement"

$$\Omega_C = 52! \approx 8 \times 10^{67}, \quad S_C = k_B \ln(52!) \approx 156\, k_B \tag{11}$$

All possible microstates satisfy this constraint. Loosen the constraint further and the accessible region explodes. The same physical deck now has entropy $156\, k_B$.

Same physical deck. Different constraints. Different Ω. Different entropy. The deck hasn't changed, but how much of its possibility space we allow it to access has changed dramatically.

To calculate Ω, you must first specify which constraints you're imposing, which variables you track, which details you ignore. This constraint-based view, formalized by Jaynes [2], makes explicit that entropy calculations depend on the chosen macroscopic description.

Tight constraints (tracking more details) give small Ω and low S. Loose constraints (tracking fewer details) give large Ω and high S. Entropy depends on your choice of constraints. Given a choice of constraints, the calculation is completely objective. But different observers can choose different constraints, and therefore calculate different entropy for the same physical system.

Physical Systems: The Dynamic Picture

This becomes vivid when we look at systems evolving over time. Consider water at atmospheric pressure in three phases, all with the same number of H_2O molecules.

Ice (solid): Molecules are locked in a crystalline lattice. Each molecule vibrates around a fixed position but can't roam freely. Each molecule has minimal degrees of freedom, restricted to small vibrations around one lattice site. The accessible region of phase space: tiny. The system explores minimal space.

Liquid water: Molecules are loosely bound. They can slide past each other, rearranging continuously within the container. Each molecule has moderate degrees of freedom, able to move throughout the container but remaining somewhat correlated with neighbors. The accessible region: moderate. The system explores moderate space.

Water vapor (gas): Molecules move freely throughout the container, bouncing off walls and each other. Each molecule has enormous degrees of freedom, able to occupy any position in the container with any momentum direction and magnitude. The accessible region: vast. The system explores huge space.

Same H_2O molecules. Same number of molecules. Different phases. Different entropy. What changed? The degrees of freedom per molecule.

This is the same principle we saw with cards. Ice is like Macrostate A, the deck locked in exact sequence. The molecules are there, the potential for rearrangement exists, but constraints (low energy, lattice bonds) eliminate the degrees of freedom. Vapor is like Macrostate C, any arrangement allowed. The constraints are loose, the degrees of freedom maximal, and the accessible space enormous.

How much of phase space the system can explore determines whether we call it ice, water, or vapor. Entropy measures the size of the explorable space. A gas molecule at room temperature explores an enormous region, it could be anywhere in the container with any of billions of possible momentum values. An ice molecule explores a tiny region, vibrating around one lattice site with limited energy. Same molecules. Different degrees of freedom. Different entropy.

Why Adding Energy Increases Entropy

You may also read that adding energy increases entropy. That's usually true, but let's see why.

Remember our card examples. Adding cards creates the *potential* for higher entropy, but constraints determine whether that potential is realized. One card at any constraint level has $S = 0$ because there's no possibility of rearrangement. Fifty-two cards at maximum constraint (exact sequence) also has $S = 0$, the possibility exists but it's blocked. Fifty-two cards at minimum constraint has $S \approx 156\, k_B$, now the possibility is accessible.

Adding energy works the same way. Energy unlocks new regions of phase space by giving molecules enough energy to overcome barriers, but whether entropy increases depends on whether your constraints allow access to those regions. A cold gas explores only low-momentum configurations. The molecules move slowly, occupying a small region of momentum space. Heat the gas, and molecules gain enough energy to access high-momentum configurations. If constraints allow it, the accessible region expands dramatically. The system can now explore both low and high momentum states. More accessible configurations means higher Ω means higher entropy.

The mathematics is identical. Adding cards or energy creates the potential for more arrangements. Loosening constraints makes that potential accessible. Entropy only increases when both happen. Adding energy to a highly constrained system is like adding cards to a deck you're not allowed to shuffle—the possibilities exist, but they are locked away, and the entropy remains low.

When you add energy to ice under normal conditions, you unlock configurations that were previously inaccessible. The molecules gain enough energy to break free from lattice sites (melting) and eventually to break free from each other entirely (boiling). Each phase transition opens vast new regions of phase space:

- **Ice:** Confined to lattice sites (small Ω)
- **Water:** Mobile but remain near neighbors (medium Ω)
- **Vapor:** Move freely and independently (large Ω)

Energy creates potential. Constraints allow or block it. Entropy quantifies the possibilities.

The Second Law

With this picture, the Second Law of Thermodynamics becomes clear:

In an isolated system, entropy never decreases.

Translation: *Given time, a system explores all accessible regions of phase space.*

An isolated system has some fixed total energy and particle number. These define a region of phase space, all the microstates compatible with that energy and particle count. Over time, the system wanders through this region, visiting different microstates. Eventually, it visits all accessible microstates with roughly equal frequency. That's equilibrium.

Entropy is a snapshot. At any instant, it measures the size of the accessible set given your current constraints. But systems don't stay frozen at a single microstate. They explore. Gas molecules at room temperature collide billions of times per second, hopping from microstate to microstate. Over time, they visit the entire accessible region.

This is why isolated systems evolve toward maximum entropy. They're exploring the space, and there are vastly more microstates in high-entropy macrostates than low-entropy ones. The system naturally spends most of its time where most of the microstates are. Maximum entropy means you've explored it all, there's nowhere new to go.

Cards don't explore spontaneously, they need someone to shuffle them. But the principle is identical. Since playing cards were invented, every shuffle across human history has been exploring that 52!-point space.

Entropy measures how much space there is to explore. The Second Law says systems will explore it.

One More Example: LISTEN and SILENT

Consider two English words made from identical letters: **LISTEN** and **SILENT**. Both use exactly six letters: L, I, S, T, E, N.

If entropy measured disorder, we'd have to say one is disordered relative to the other depending on which word you wanted. But watch what the formula calculates under different constraints:

- **Constraint A:** "Letters spell LISTEN exactly" $\rightarrow \Omega = 1$, $S = 0$
- **Constraint B:** "Letters spell SILENT exactly" $\rightarrow \Omega = 1$, $S = 0$
- **Constraint C:** "Six letters {L, I, S, T, E, N} in any arrangement" $\rightarrow \Omega = 6! = 720$, $S \approx 6.58\, k_B$

Under Constraint C, 720 arrangements are possible. LISTEN is one of them. SILENT is another. Both are equally specific points in a 720-point possibility space. Neither word is "more ordered" than the other. Both are single microstates with identical status under the natural constraint "which letters are present."

The only reason one might seem "disordered" relative to the other is if you chose one as your target. But that choice depends on your goal, not on physics. There is no objective disorder. There's only: how many arrangements satisfy your constraints?

The same principle applies whether you're arranging cards, molecules, or letters.

What the Formula Counts

Look again at Boltzmann's formula: $S = k_B \ln(\Omega)$. It has always been counting. One card, one possibility: $S = 0$. Fifty-two cards, 10^{67} possibilities: $S = k_B \ln(52!)$.

The labels "disorder" and "missing information" are human projections. The formula doesn't define disorder and it doesn't care what an observer knows. It asks one question: How many configurations are accessible?

Energy creates potential. Constraints determine access. Entropy quantifies the result.

References

[1] Boltzmann, L. (1877). Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung. *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften in Wien*, 76, 373–435.

[2] Jaynes, E. T. (1957). Information Theory and Statistical Mechanics. *Physical Review*, 106(4), 620–630.

[3] Styer, D. F. (2019). Entropy as disorder: History of a misconception. *The Physics Teacher*, 57(7), 454–458.

[4] Lambert, F. L. (2002). Disorder—A cracked crutch for supporting entropy discussions. *Journal of Chemical Education*, 79(2), 187.