

# Relative density estimation from shallow depth CPTs in siliceous sand: an updated approach

Mathias Rolf Jensen<sup>1</sup> BE, MSc

<sup>1</sup>Corresponding author  
Geotechnical Engineer,  
Geo, Copenhagen, Denmark  
Maglebjergvej 1, 2800 Kongens Lyngby  
Phone +45 3174 0205  
Email: [mtj@geo.dk](mailto:mtj@geo.dk)

## Abstract

Cone penetration tests (CPTs) in sands are governed by two distinct mechanisms: shallow failure mode, which dominates in the upper ~0.2–1.0 m of standard field CPTs, and a deep failure mode, which develops once sufficient embedment is achieved. Industry-standard correlations between CPT cone resistance ( $q_c$ ) and relative density ( $D_r$ ) are calibrated under deep failure conditions, and their application in the shallow failure zone produces erroneous results. This paper reviews existing shallow depth interpretation methods and introduces an updated global model that integrates shallow and deep penetration mechanisms. Building on Jensen (2024), the new formulation addresses identified limitations through a comprehensive recalibration using controlled laboratory tests and a new internal database of 132 onshore and offshore CPTs, enabling improved differentiation of near-surface densities and closer agreement with measured  $q_c$  profiles. The model shows significant reduction in shallow-zone  $D_r$  bias relative to earlier approaches and is most reliable for clean, young, uncemented, uniformly graded siliceous sands under fully saturated or dry conditions. Deviations occur in sands of higher compressibility, increased fines content, or pronounced particle angularity, and within the top 2-4 cone diameters where mudline definition and minor cone disturbance become influential. Site-specific validation of the model against direct  $D_r$  measurements remains essential, even where soils fall within the calibrated range.

## Keywords

CPT; cone resistance; sands; shallow depth; relative density.

## **Introduction**

When sufficient embedment is achieved during cone penetration in sand, a deep failure mechanism develops, characterized by a localized deformation zone near the cone tip (Arshad et al., 2014). Deep penetration has been widely analyzed using the analogy between cone penetration and spherical cavity expansion (e.g., Salgado and Prezzi, 2007). In contrast, the upper ~0.2-1.0 m of standard field cone penetration tests (CPTs) is typically governed by a shallow failure mechanism, associated with upward displacement of surface sand (Biarez and Gresillon, 1972). A transition zone separates the shallow and deep mechanisms, marking a shift in the governing soil deformation pattern (Puech and Foray, 2002).

Industry-standard methods for estimating the relative density ( $D_r$ ) of sand from CPT cone resistance ( $q_c$ ) are largely empirical, based on CPTs in calibration chambers under deep failure conditions (e.g., Baldi et al., 1986; Jamiolkowski et al., 2003). Applying these methods to near-surface sands, where shallow failure governs, leads to erroneous  $D_r$  estimates. Although important for various geotechnical applications, such as low-stress scenarios, the resulting estimates are unreliable and should be disregarded.

Several researchers have advanced understanding of shallow failure penetration and the transition zone (Puech and Foray, 2002; Emerson et al., 2008; Kim et al., 2015; Lehane et al., 2022; Jensen, 2024), yet practical implementation of these insights remains limited. This paper provides a critical review of existing methods for estimating  $D_r$  at shallow depths and introduces an updated global model that extends the formulation of Jensen (2024). The model is recalibrated using data from published controlled laboratory tests and a newly compiled internal database of 132 onshore and offshore CPTs, providing a more robust basis for assessing  $D_r$  in the shallow penetration regime.

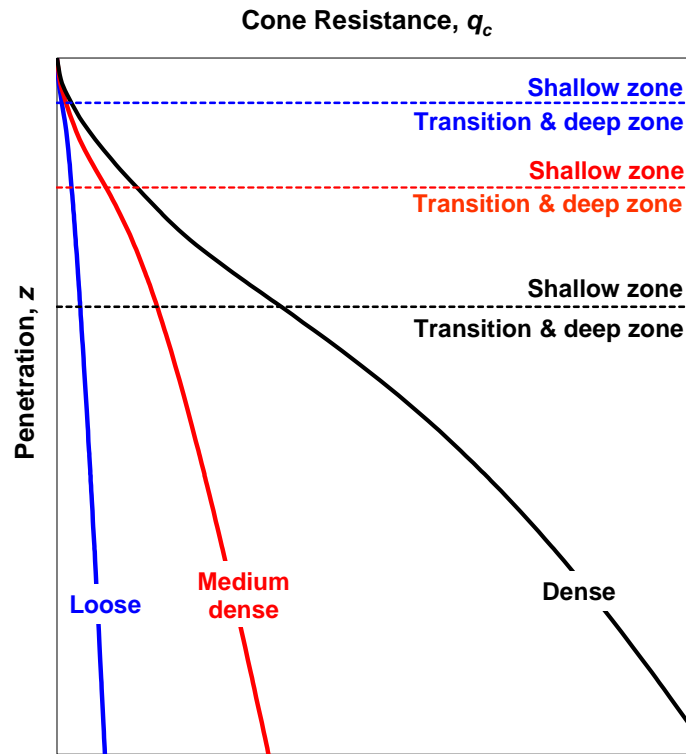


Fig. 1. Idealized shallow depth cone resistance profiles in uniform loose, medium dense, and dense sand.

## 2. Background

### 2.1 Shallow failure penetration

As illustrated in Fig. 1, cone penetration in uniform sand deposits generally exhibits a transition in the measured cone resistance profile,  $q_c(z)$ , with depth ( $z$ ), from a shallow failure mode to a deep failure mode. In the very near-surface zone, shallow failure dominates, producing an upward-concave  $q_c(z)$  profile characterized by an increasing gradient ( $dq_c/dz$ ). The mechanics of shallow failure penetration are governed primarily by the mobilized friction angle and soil dilatancy (Durgunoglu and Mitchell, 1973). This penetration mode has been the subject of several studies (e.g., Puech and Foray, 2002; Balachowski, 2007; Kim et al., 2015).

Experimental observations by Kim et al. (2015), consistent with the general findings of Lehane et al. (2022), indicate that the transition from shallow to deep failure occurs at a depth of penetration of approximately 3-6 cone diameters in loose, clean silica sand, and about 15–20 cone diameters in dense silica sand. Based on their tests, Kim et al. (2015) proposed the following expression with a form that describes the upward-concave  $q_c(z)$  profile representative of shallow failure penetration ( $q_{c.shallow}$ ) in clean silica sand:

$$q_{c.shallow} = (0.96 \cdot D_r - 0.25) \cdot \left(\frac{\sigma'_v}{p_a}\right)^{0.5} \cdot \left(\frac{z}{d_c}\right) \cdot p_a \quad (1)$$

where  $p_a$  is a reference stress (= 100 kPa),  $\sigma'_v$  is vertical effective stress, and  $d_c$  is the cone diameter. Following Kim et al. (2015), both  $q_{c.shallow}$  and  $\sigma'_v$  are expressed in MPa in Eq. (1). Although this convention may appear dimensionally inconsistent, it is retained to ensure comparability with their formulation. Equation (1), along with other approaches, is discussed further later in this paper.

## 2.2 Deep failure penetration

At greater depth, the penetration response transitions to a deep failure mode with decreasing  $dq_c/dz$ , as shown in Fig. 1. Deep failure penetration is primarily governed by sand compressibility characteristics (Konrad, 1998; Yu and Mitchell, 1998). Widely used CPT- $D_r$  correlations for deep failure are based on Schmertmann's (1976) formulation:

$$q_{c.deep} = C_{0.deep} p_a \left(\frac{\sigma'}{p_a}\right)^{C_{1.deep}} \exp(C_{2.deep} D_{r.deep}) \quad (2)$$

$$D_{r.deep} = \frac{1}{C_{2.deep}} \ln \left( \frac{q_{c.deep}/p_a}{C_{0.deep} (\sigma'/p_a)^{C_{1.deep}}} \right) \quad (3)$$

where  $C_{0.deep}$ ,  $C_{1.deep}$ , and  $C_{2.deep}$  are empirical fitting parameters, and  $\sigma'$  is the normalizing effective stress.

In normally consolidated (NC) sands, calibration is often performed using the vertical effective stress  $\sigma'_v$  (e.g., Schmertmann, 1978; Baldi et al., 1986). However, Houlsby and Hitchman (1988) demonstrated that the horizontal effective stress ( $\sigma'_h$ ) has a stronger influence on  $q_c$ . Consequently, many methods adopt the mean effective stress,  $p' = (\sigma'_v + 2\sigma'_h) / 3$ , improving generality, particularly in overconsolidated (OC) sands where the coefficient of earth pressure at rest ( $K_o$ ), on which  $\sigma'_h$  depends, is significantly higher (e.g., Jamiolkowski et al., 2003; Krogh et al., 2022).

In the upper 3-5 m of OC sands, where  $K_o$  and the overconsolidation ratio (OCR) vary significantly with depth, stress normalization using  $p'$  becomes particularly important to avoid overestimation of  $D_r$ , as may occur when normalizing solely with  $\sigma'_v$ . Depth-dependent  $K_o(z)$  can be evaluated using the approach of Krogh et al. (2022) (procedure given in the flowchart in Fig. 7 presented later in the paper), where preconsolidation stress is estimated following Agaiby and Mayne (2019). Importantly, Emerson et al. (2008) and Jensen (2024) demonstrated that better interpretation of shallow CPT data requires integrating the  $K_o(z)$ -based stress normalization with shallow failure considerations, as discussed in the following.

### **3. Review of CPT-based interpretation methods for shallow $D_r$ estimation**

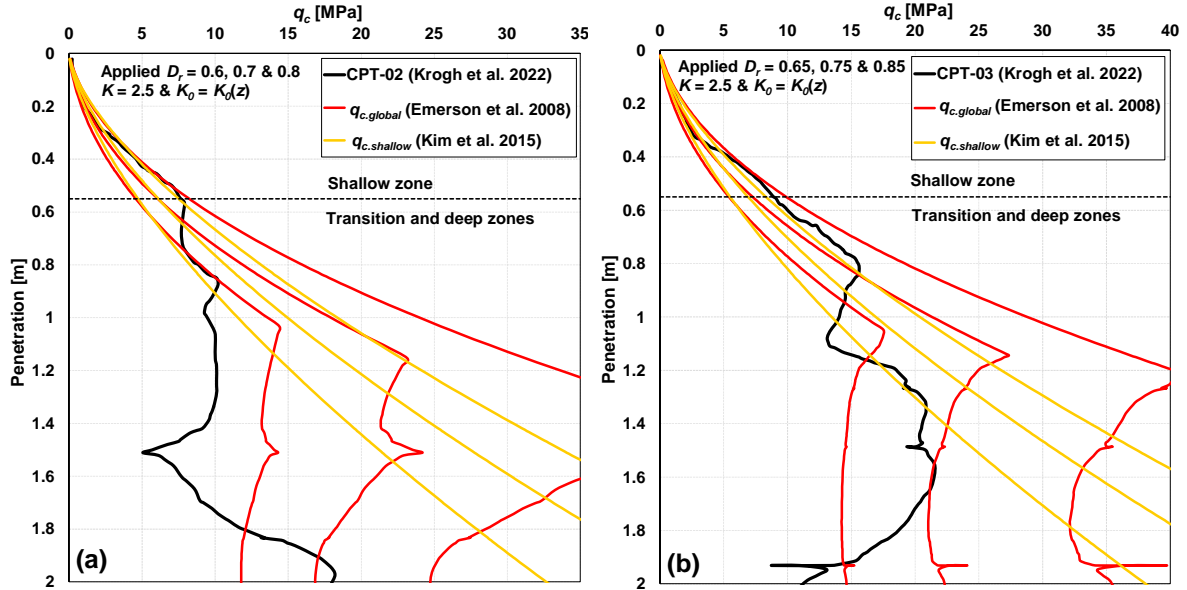
#### **3.1 Limit equilibrium method (Puech and Foray, 2002; Emerson et al., 2008)**

Puech and Foray (2002) developed a limit equilibrium model for shallow failure penetration in siliceous sand based on bearing capacity theory, incorporating friction angle, cone diameter, and lateral friction:

$$q_{c,shallow} = \gamma' \cdot z \cdot N_q \left( 1 + \frac{K \cdot \sin \varphi'}{L} \cdot z \right) \quad (4)$$

**Table 1. Calibration parameters for  $q_{c,deep}$  (Eq. (2)) and  $q_{c,shallow}$  (Eq. (6)) for various references.**

Reference	Deep ( $\sigma' = p'$ )			Shallow ( $\sigma' = \sigma'_v$ )		
	$C_{0,deep}$	$C_{1,deep}$	$C_{2,deep}$	$C_{0,shallow}$	$C_{1,shallow}$	$C_{2,shallow}$
Jamiolkowski et al. (2003)	24.94	0.46	2.96	-	-	-
Lehane et al. (2022)	-	-	-	19	0.7	3.2
Jensen (2024)	25.6	0.7	3.2	19	0.7	3.2
This study	25.6	0.7	3.2	19	0.7	3.8



**Fig. 2. Performance of Emerson et al. (2008) for shallow depth interpretation in siliceous sand compared with Kim et al. (2015).**

where  $\gamma'$  is effective unit weight of sand,  $\phi'$  the peak friction angle,  $L$  is the dimension of a soil cylinder considered around the cone ( $L = d_c \cdot \exp(\tan(\phi' \cdot (\pi/2))) \cdot \tan(\pi/4 + \phi'/2)$ ),  $N_q = 1.058 \cdot \exp(6.168 \cdot \tan \phi')$ , and  $K$  is a lateral stress coefficient, different from  $K_0$  and ranging from approximately 1 to 3 (Emerson et al., 2008). Equation (4) is to be applied with curve fitting while relating  $D_r$  via the following correlation with  $\phi'$ :

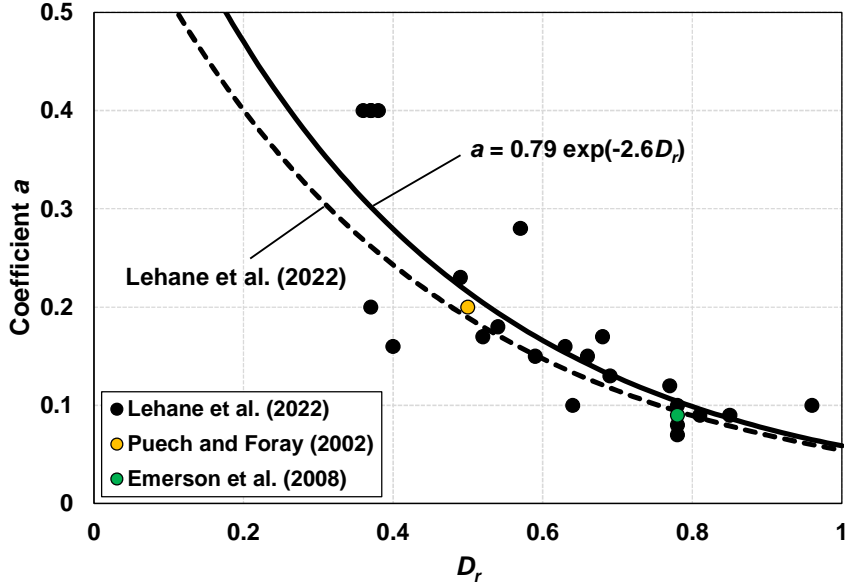
$$\phi' = 20.4^\circ \cdot D_r + 25^\circ \quad (5)$$

Distinguishing between shallow and deep failure penetration is critical, since deep failure penetration in loose sand can produce  $q_c$ -profiles that resemble those from shallow penetration in dense sand, leading to misinterpretation of relative density. To address this

issue, Emerson et al. (2008) proposed a global  $q_c$  model ( $q_{c.global}$ ) that via a transition function combines Puech and Foray's (2002) model for  $q_{c.shallow}$  with the widely applied calibration of deep failure penetration ( $q_{c.deep}$ ; Eq. (3)) by Jamiolkowski et al. (2003); see Table 1. In their global model, the transition between shallow and deep failure penetration is defined by a critical depth ( $z_{cr}$ ), primarily dependent on  $D_r$ ; lower  $D_r$  causes shallower onset of deep failure.

Figures 2a and 2b show Emerson et al.'s (2008) global model applied to two CPTs at a German test site with homogeneous, clean, uniformly graded, fine to medium OC siliceous sand, using a standard 10 cm<sup>2</sup> cone ( $d_c = 3.57$  cm) (data from Krogh et al., 2022). Nuclear densometer (ND) tests performed at the site indicate  $D_r = 60\% \pm 10\%$  (CPT-02; Fig. 2a) and  $D_r = 80\% \pm 10\%$  (CPT-03; Fig. 2b). A weakness in Emerson et al.'s model lies in the parameter  $K$  (Eq. (4)), which is somewhat arbitrary. Although  $K$  strongly influences the predicted  $q_{c.shallow}$  profile in the model, no robust guidance exists for selecting an appropriate value in practice. For comparison within the shallow failure zone, Emerson et al.'s (2008) global model is evaluated against the experimentally derived expression for  $q_{c.shallow}$  proposed by Kim et al. (2015) (Eq. (1)). The two approaches show good agreement in the shallow zone when Emerson et al.'s model is applied with  $K = 2.5$  (Figs. 2a and 2b). In these comparisons, representative relative density ranges of  $D_r = 0.6-0.8$  (Fig. 2a) and  $0.65-0.85$  (Fig. 2b) were used, corresponding to the ND test results.

Although the curves now show reasonable agreement with the direct ND measurements of  $D_r$  in the shallow zone, Emerson et al.'s model does not accurately capture the transition from shallow to deep failure, predicting the shift at a considerably greater penetration depth than indicated by the CPT data (Fig. 2). Furthermore, the abrupt shift from increasing  $dq_c/dz$



**Fig. 3. Variation of coefficient  $a$  with  $D_r$ , showing data from Lehane et al. (2022), values inferred from data of by Puech and Foray (2002) and Emerson et al. (2008), and the updated expression proposed in this study (Eq. (10)).**

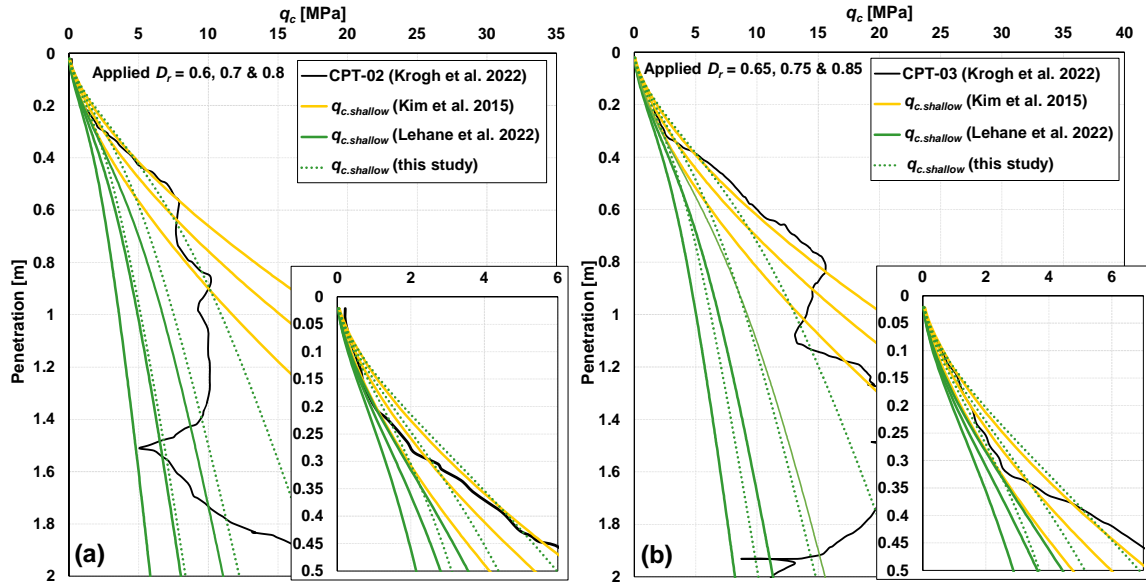
(shallow zone) to decreasing  $dq_c/dz$  (deep zone) does not fit the general observation of a gradually changing gradient (Fig. 1). If applying a lower value of  $K$ , the transition to the deep failure penetration could have been captured better, but that would have then compromised the accuracy significantly in the shallow failure zone. Similar discrepancies were noted by the author comparing with the extensive internal CPT database, presented later in this paper.

### 3.2 Centrifuge-based model (Lehane et al., 2022)

Centrifuge CPTs, due to limited penetration, are well suited for studying shallow failure penetration (e.g., Bolton et al., 1999; Balachowski, 2007). Lehane et al. (2022) used a large centrifuge database to describe the shallow-to-deep transition with a hyperbolic tangent function:

$$q_{c,shallow} = \tanh\left(\frac{az}{d_c}\right) C_{0,shallow} p_a \left(\frac{\sigma'_v}{p_a}\right)^{C_{1,shallow}} \exp(C_{2,shallow} D_{r,shallow}) \quad (6)$$

where  $C_{0,shallow}$ ,  $C_{1,shallow}$ , and  $C_{2,shallow}$  are empirical parameters calibrated by Lehane et al. (2022) for clean, uniformly graded, fine to medium, freshly deposited NC silica sands using



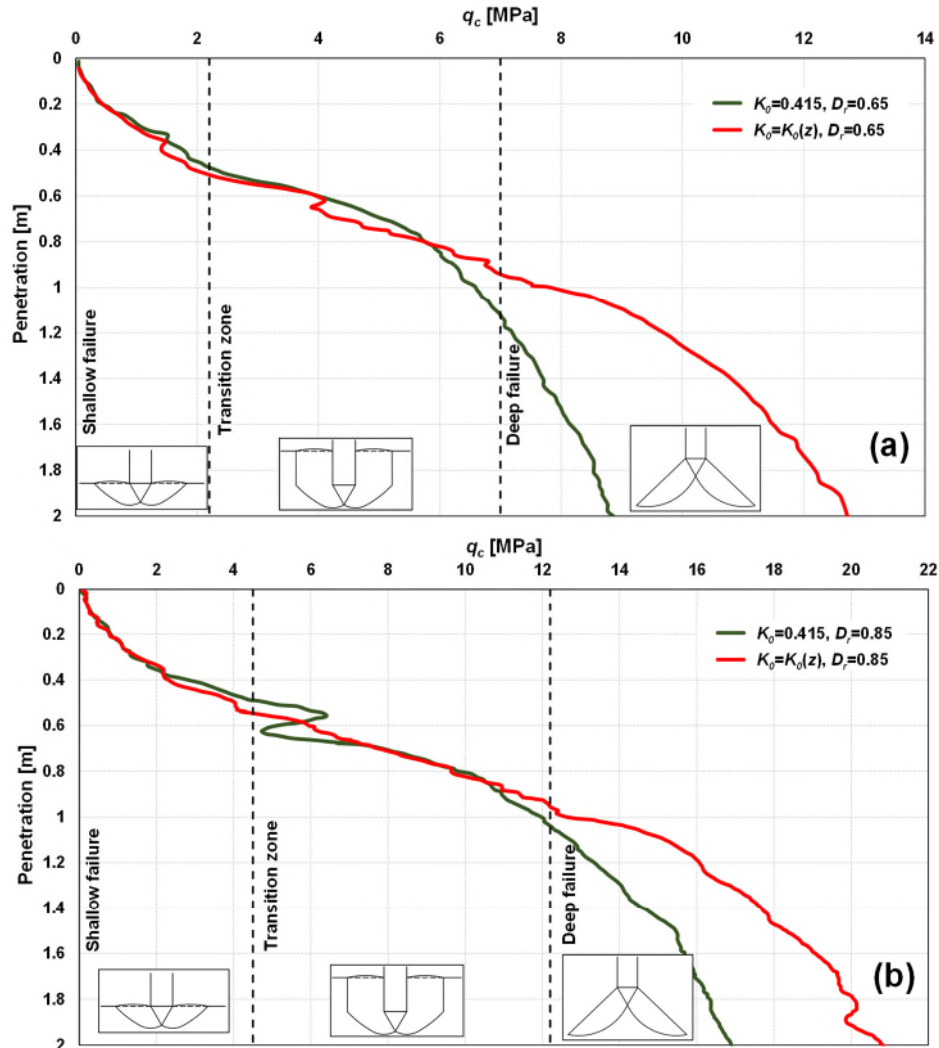
**Fig. 4. Performance of Lehan et al. (2022) and a modification in this study for shallow depth interpretation in siliceous sand compared with Kim et al. (2015).**

$\sigma' = \sigma'_v$  (Table 1), and  $a$  is an empirical curve fitting parameter, given by Lehan et al. (2022)

as:

$$a = 0.66\exp(-2.5D_{r,shallow}) \quad (7)$$

Equation (7) (Lehan et al., 2022) is shown in Fig. 3 together with the  $a$  values derived from their centrifuge database, and supplemented with values inferred by the author from data presented by Puech and Foray (2002) and Emerson et al. (2008) from CPTs performed with a standard 10 cm<sup>2</sup> cone. Although designated “shallow”, Eq. (6) may also apply for deep failure penetration in freshly deposited NC sands, since it essentially transitions into a deep failure expression (Eq. (2)) when the tangent hyperbolic function  $q_c$  approaches unity with depth, as controlled by the  $a$  parameter. In this study, however, it is primarily used to model the shallow zone and the transition.



**Fig. 5. Mechanisms observed with  $q_c$  profiles from numerical simulations of Krogh et al. (2022) for (a)  $D_r = 0.65$  with  $K_0 = 0.415$  and  $K_0 = K_0(z)$ , and (b)  $D_r = 0.8$  with  $K_0 = 0.415$  and  $K_0 = K_0(z)$ .**

When applied using  $D_r = 0.6-0.8$  (Fig. 4a) and  $D_r = 0.65-0.85$  (Fig. 4b), predicted  $q_c$  values fall well below the field data, with the discrepancy increasing beyond  $\sim 0.3-0.4$  m, as expected due to overconsolidation effects. Jensen (2024) noted that Eqs. (6) and (7) effectively captures the shape of field  $q_c$  profiles in low-medium compressible siliceous sands. However, overconsolidation effects influences deep  $q_c$  significantly and must be accounted for.

For shallow failure penetration, Jensen (2024) argued that overconsolidation has little, if any, effect on shallow failure penetration resistance. This conclusion was supported by two main

pieces of evidence. Firstly, numerical simulations by Krogh et al. (2022) (Fig. 5) demonstrated comparable  $q_{c,shallow}$  for relative densities of  $D_r = 0.65$  (Fig. 5a) and  $D_r = 0.85$  (Fig. 5b), under both constant  $K_0 = 0.414$  (representative of NC sand) and depth-dependent  $K_0(z)$  (representative of OC sand). Secondly, similar trends can be inferred from the initial penetration curves reported in experimental centrifuge CPT studies by El-Sekelly et al. (2015), Roy (2020), and Richards et al. (2021). These experiments, conducted at varying g-levels, effectively simulated a range of OCR values, including NC conditions.

Building on these observations, Jensen (2024) proposed a global  $q_c$  model that combines Eqs. (2) and (6), with the transition governed by OCR or  $K_0$ . An assumption in this approach is therefore that very shallow  $q_c$  values are equivalent for both normally consolidated and overconsolidated sands, with OCR effects becoming more pronounced with increasing embedment approaching deep failure penetration.

### 3.3 Global model (Jensen, 2024)

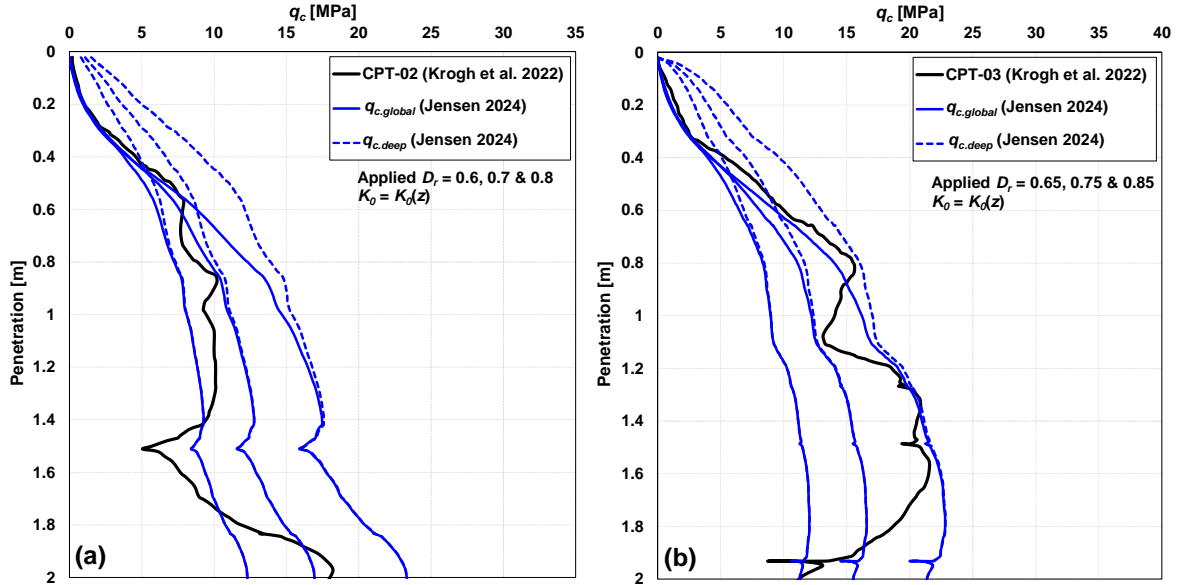
To capture both failure regimes and the transition, Jensen (2024) proposed:

$$q_{c,global} = \begin{cases} q_{c,shallow} & \text{if } \tanh(az/d_c) \leq n_1 \\ q_{c,shallow} + f_{transition}(q_{c,deep} - q_{c,shallow}) & \text{if } \tanh(az/d_c) > n_1 \end{cases} \quad (8)$$

where

$$f_{transition} = \left( \frac{\tanh\left(\frac{az}{d_c}\right) - n_1}{1 - n_1} \right)^{n_2} \quad (9)$$

with  $n_1 = 0.5$  and  $n_2 = 2$  suggested as initial values, subject to further calibration. Jensen (2024) also proposed that the calibration of Eq. (6) could be extended to deep failure (Eq. (2)) by making small adjustments to  $C_{0,shallow}$ , thereby enabling a shift from  $\sigma_v'$  to  $p'$  in stress normalization (Table 1).



**Fig. 6. Performance of Jensen (2024) for shallow depth interpretation in siliceous sand.**

The global model, defined by Eqs. (8) and (9) and incorporating Jensen’s (2024) preliminary calibration, is presented in Figs. 6a and 6b. The predicted  $q_{c,global}$  profiles closely match the shape of the measured CPT data and align well with the ND measurements. However, a current key limitation is evident in the upper  $\sim 0.4$  m: the profiles converge in the initial penetration for the applied  $D_r$  range, preventing objective differentiation of  $D_r$ . Although the model successfully captures the overall profile shape, this shortcoming highlights the need for refinement, which is addressed in the following section.

#### 4. Updated global model

To support refinement of the global model, an internal database of 132 shallow CPTs from onshore and offshore industry projects was compiled and analysed in conjunction with previously published controlled laboratory tests. The internal database comprises onshore tests conducted in Denmark and Germany, and offshore tests performed in the North Sea and Irish Sea; details are summarized in Table 2. The onshore CPTs were mainly carried out in

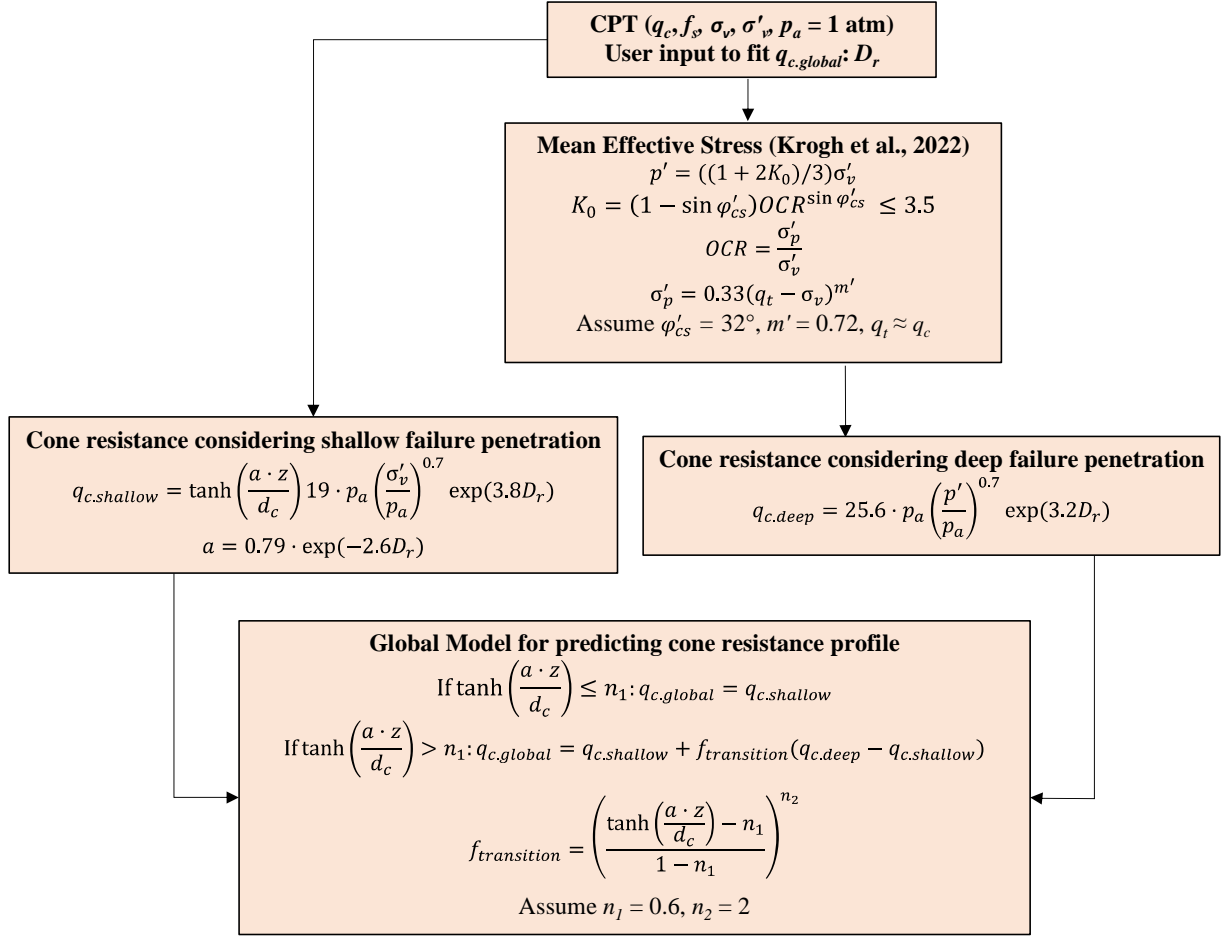
**Table 2. Details of the internal CPT database**

Site	No. of tests	Cone size	Geography	Offshore/ onshore	General sand description
A	3	10 cm <sup>2</sup>	Denmark	Onshore	Sand fill, siliceous, clean, medium, uniform
B(1)	24	10 cm <sup>2</sup>	Denmark	Onshore	Sand fill, siliceous, clean, medium to coarse, uniform to well graded
B(2)	36	15 cm <sup>2</sup>	Denmark	Onshore	Sand fill, siliceous, clean, medium to coarse, uniform to well graded
C	6	10 cm <sup>2</sup>	Germany	Onshore	Siliceous, clean, fine to medium, uniform
D	3	10 cm <sup>2</sup>	North Sea	Offshore	Post-glacial, siliceous, clean, fine to coarse, uniform
E	1	10 cm <sup>2</sup>	North Sea	Offshore	Post-glacial, siliceous, clean, fine, uniform
F	12	10 cm <sup>2</sup>	North Sea	Offshore	Post-glacial, siliceous, clean, fine to medium, uniform
G	10	10 cm <sup>2</sup>	North Sea	Offshore	Post-glacial, siliceous, clean, fine to medium, uniform
H	37	10 cm <sup>2</sup>	Irish Sea	Offshore	Siliceous (no detailed sample descriptions)

recently placed, clean siliceous sand fill with varying degrees of compaction, whereas the offshore CPTs were conducted in young, surficial, clean siliceous deposits. As noted in the descriptions for each site (Table 2), the tested sands generally consist of fine to coarse, uniformly graded sand, as confirmed by sampling. Of the total tests, 96 CPTs were performed with a 10 cm<sup>2</sup> cone and 36 CPTs using a 15 cm<sup>2</sup> cone, all from the same site (Site B(2)).

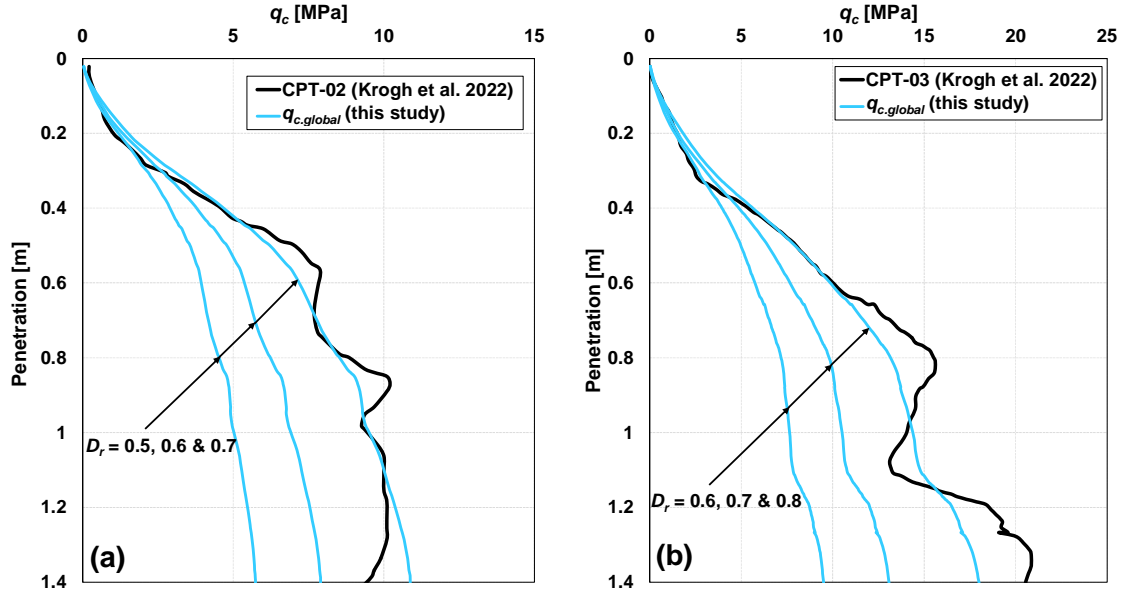
A qualitative review of the internal CPT database indicated that the transition to deep failure penetration generally occurs at a slightly shallower depth, particularly for loose to medium dense sands, than predicted by the current model governed by the coefficient  $a$  defined in Eq. (7). To account for this, a minor modification to Eq. (7) is proposed, resulting in an improved fit to the internal database and a closer overall correspondence with the data shown in Fig. 3:

$$a = 0.79 \exp(-2.6D_{r,shallow}) \quad (10)$$



**Fig. 7. Flow chart summarizing the updated global model for interpretation of shallow depth CPTs in clean siliceous sand.**

The review of the internal database also indicates that Lehane et al. (2022) profiles (Eq. (6)), which forms the basis for the global model by Jensen (2024) in the very shallow zone, consistently underestimate  $q_{c.shallow}$  in the near-surface range for both loose and dense sands. This trend is further supported by the distinct discrepancy observed in Fig. 4, where comparisons between Lehane et al. (2022) (Eq. (6)) profiles and  $q_{c.shallow}$  profiles from Kim et al. (2015) reveal systematic large deviations in the shallow failure zone. As was demonstrated in Fig. 2, the profiles by Kim et al. (2015) show good agreement in the shallow failure zone with both the theoretically derived model of Puech and Foray (2002) and the measured data by Krogh et al. (2022), further confirming the observed underestimation of  $q_{c.shallow}$  with Lehane et al. (2022) profiles in natural deposits.



**Fig. 8. Performance of updated global model ( $q_{c.global}$ ) for CPT-02 and CPT-03 (measured data from Krogh et al. 2022).**

To improve agreement, an increase in  $C_{2.shallow}$  from 3.2 to 3.8 is proposed (Table 1), and a slight adjustment to  $f_{transition}$  (Eq. (9)) is similarly made; specifically, increasing  $n_1$  to 0.6, while retaining  $n_2 = 2$ . As shown in Fig. 4 (profiles named ‘this study’), these adjustments enable Eq. (6) to follow the upward concave portion of the Kim et al. (2015) profiles closely in the shallow failure penetration zone. The complete updated approach is summarized in the flow chart in Fig. 7.

As shown in Fig. 8, curve fitting with the updated global model indicates  $D_r \approx 0.5-0.7$  for CPT-02 (Fig. 8a) and  $D_r \approx 0.6-0.8$  for CPT-03 (Fig. 8b), both in excellent agreement with the ND measurements ( $D_r = 60\% \pm 10\%$  and  $D_r = 80\% \pm 10\%$ , respectively). The updated model achieves two key improvements: (1) it prevents convergence of the  $q_{c.global}$  profiles in the surficial sand, allowing objective estimation of  $D_r$ ; and (2) it yields strong agreement with the internal database, as demonstrated with numerous representative examples from the

**Table 3. Summary of interpreted  $D_r$  within the upper 1 m of CPT data from the internal database, based on the updated global model. Descriptions correspond to interpretations from top to bottom of the 1 m profile, with numbering indicating the count of CPTs in each category.**

Interpretation <sup>1</sup>	Site								
	A	B(1)	B(2)	C	D	E	F	G	H
Loose	-	-	-	-	1	-	2	1	3
Loose to medium dense	-	1	-	-	-	-	-	1	4
Loose to dense	-	-	-	-	-	-	-	2	-
Medium dense	-	7	1	-	1	-	1	2	12
Medium dense to dense	-	4	-	3	-	-	1	2	4
Medium dense to very dense	-	-	-	-	-	-	4	-	2
Dense to medium dense	-	-	-	-	-	-	-	-	8
Dense	1	12	1	1	1	-	1	1	4
Dense to very dense	-	-	-	2	-	1	3	1	-
Very dense to medium dense	1	-	-	-	-	-	-	-	-
Very dense to dense	1	-	2	-	-	-	-	-	-

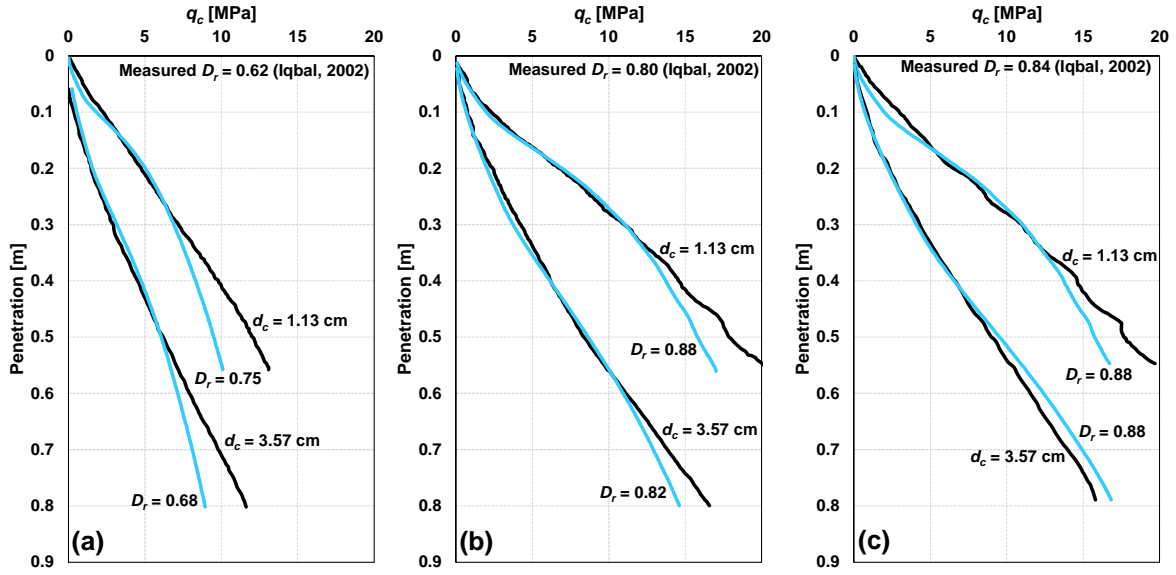
<sup>1</sup>Very loose:  $D_r = 0-0.15$ , Loose:  $D_r = 0.15-0.35$ , Medium dense:  $D_r = 0.35-0.65$ , Dense:  $0.65-0.85$ , Very dense:  $D_r = 0.85-1$

database later in this paper. Furthermore, the following section discusses cone size effects, in which the updated global model similarly show great performance with controlled tests by Iqbal (2002) and Larsen and Ibsen (2006).

The variability observed in the internal database is reflected in Table 3, which summarizes relative density interpretations for the upper 1 m of CPT data, using the updated global model. The descriptive categories from ‘loose’ to ‘very dense’ that are linked to the  $D_r$  values (see Table 3; descriptions correspond to interpretations from top to bottom of the upper 1 m profile) indicate substantial variation in the database.

## 5. Cone size effects

As embedded in Eq. (1) (Kim et al., 2015) and Eq. (4) (Puech and Foray, 2002), the cone diameter ( $d_c$ ) plays an important role in the  $q_{c,shallow}$  profiles. Specifically, an increase in  $d_c$  leads to a reduction in cone resistance at shallow failure penetration. By analysing CPT data obtained with varying cone diameters in uniform sand samples, Lehane et al. (2022) observed

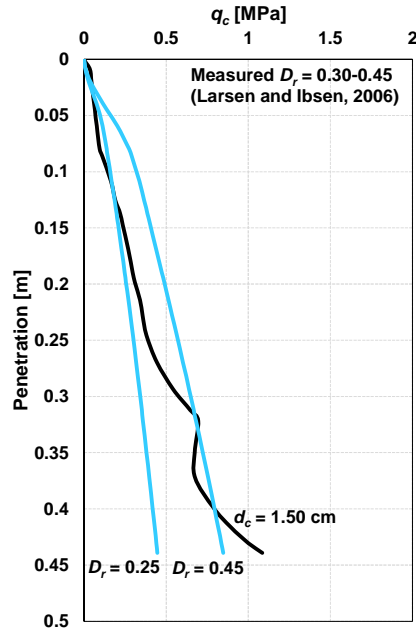


**Fig. 9. Evaluation of the updated global model ( $q_{c,global}$ ) against CPT data from Iqbal (2002) for two cone diameters ( $d_c = 1.13$  cm and  $3.57$  cm) in uniformly graded silica sand, illustrating cone size effects.**

that data fall into a consistent trend when presented in a normalized chart, where the stress normalised cone resistance is plotted against the normalised penetration depth ( $z/d_c$ ). This observation formed the basis for their  $q_{c,shallow}$  profile expression (Eq. (6)), upon which the updated global model is an extension.

Essentially, Eq. (6) expresses a strong influence of cone diameter during shallow failure penetration, which diminishes with large penetration well into the deep failure penetration zone where profiles of varying  $d_c$  will eventually converge. This is consistent with experimental observations (e.g., Balachowski, 2007).

The updated global model is evaluated in Fig. 9 using CPTs conducted under controlled laboratory conditions by Iqbal (2002). The tests were performed with two significantly different cone diameters,  $d_c = 1.13$  cm and  $d_c = 3.57$  cm (standard field size cone), in fine to medium, uniformly graded silica sand. Three separate sand bodies with  $D_r$  ranging from 0.62 to 0.84 were tested, with densities confirmed by direct measurements following uniform



**Fig. 10. Comparison between measured and predicted cone resistance profiles from Larsen and Ibsen (2006) in loose, uniformly graded silica sand ( $d_c = 1.50$  cm), showing performance of the updated global model ( $q_{c,global}$ ) assuming  $K_0 = 0.5$ .**

compaction using a vibration screen unit. As shown in Fig. 9, the updated global model exhibits excellent agreement with the measurements, remaining within about 5-10% (in absolute terms) of the measured values for both  $d_c = 1.13$  cm and  $d_c = 3.57$  cm in all three sand bodies.

Because shallow failure penetration is more pronounced in denser sand, the cone size effect is correspondingly greater. In contrast, shallow failure effects are minimal in loose sand. In Fig. 10, the global model is evaluated against a CPT with  $d_c = 1.50$  cm performed in a loose, fine to medium, uniformly graded silica sand by Larsen and Ibsen (2006). Direct measurements indicated  $D_r$  ranging about 0.3 to 0.45. The sand specimen was prepared by water pluviation followed by minimal external vibration. Accordingly, a representative value of  $K_0 = 0.5$ , consistent with NC conditions, was assumed in the global model rather than using the recommended  $K_0(z)$ -procedure. Under these controlled conditions with minimal compaction disturbance, overconsolidation effects are expected to be negligible. As shown

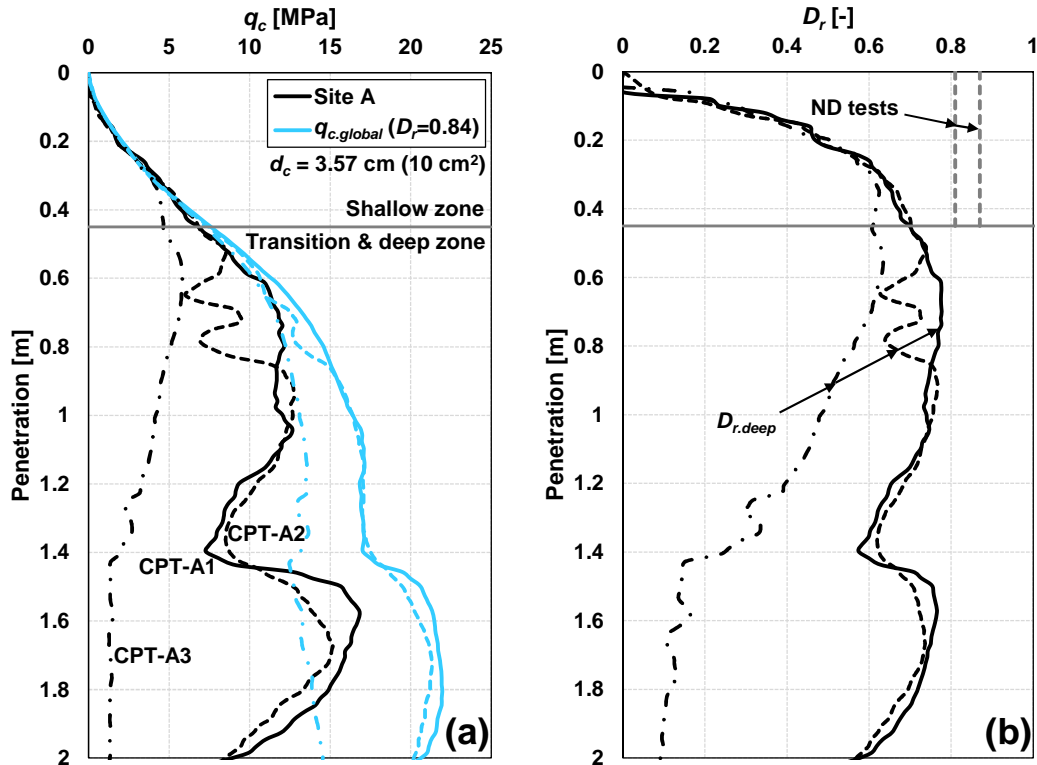


Fig. 11. Performance of updated global model ( $q_{c,global}$ ) for Site A

in Fig. 10, the global model again performs well, with curve fitting ranging about  $D_r = 0.25$ - $0.45$ .

## 6. Case histories from the internal database

### 6.1 Onshore ground improvement

#### 6.1.1 Site A

At a Danish onshore site, three CPTs (CPT-A1 to A3) using a standard 10 cm<sup>2</sup> cone and two nuclear densometer tests were conducted within a 15 m radius to assess compaction of a sand fill, see Fig. 11. Limiting void ratios of  $e_{min} = 0.45$  and  $e_{max} = 0.79$  were measured for the sand fill, typical for clean, subrounded to subangular silica sand (Youd, 1973). The ND tests are considered representative within the shallow failure zone, approximately the upper 0.45 m (Fig. 11a).

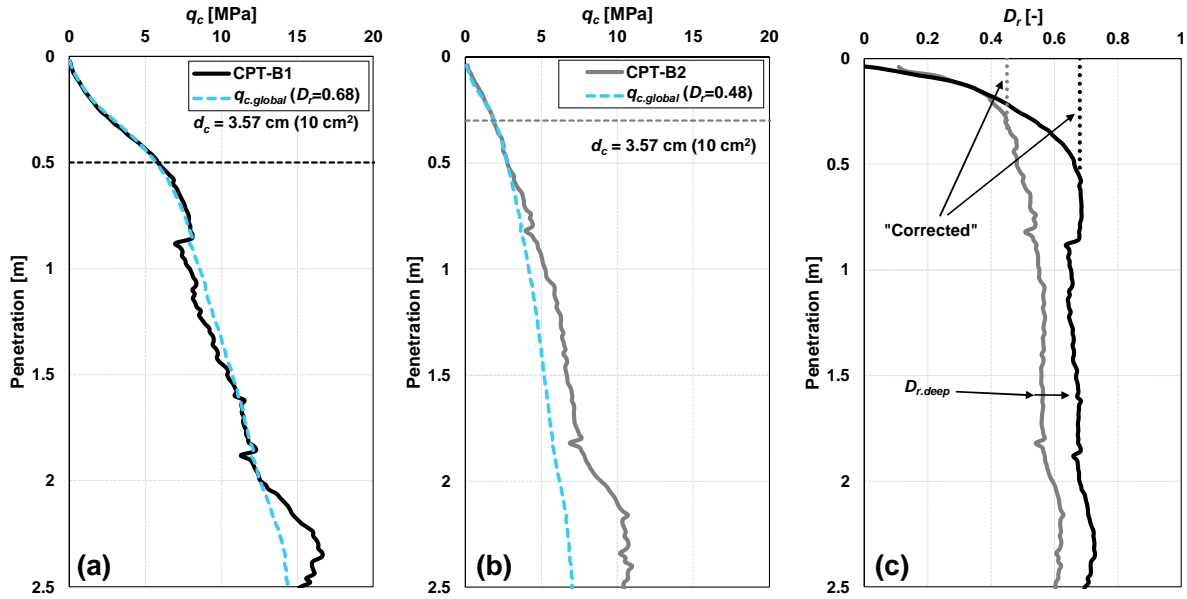
In this zone,  $D_r$  values with Eq. (3) ( $D_{r,deep}$ ) are unreliable; therefore, ND data are used as the primary reference (Fig. 11b). The ND measurements yielded  $D_r = 0.81$  and  $0.87$ , indicating dense to very dense sand ( $D_r \approx 0.85$ ) in the near-surface layer. Below this,  $D_r$  decreases to  $\sim 0.6$  at a depth of 2 m, except in CPT-A3, where the sand fill ends at about 1.4 m.

Applying  $D_r = 0.85$  in the updated global model results in  $q_{c,global}$  profiles closely matching all three CPTs in the shallow zone (Fig. 11a). At greater depths, the measured CPT data diverge from  $q_{c,global}$ , consistent with the generally decreasing  $D_{r,deep}$  profile (Fig. 11b), indicating that the compaction activities produced a denser sand deposit near the surface, which is not unusual of surface vibration compaction.

### 6.1.2 Site B

At a second Danish site (Site B), CPTs were performed using 10 cm<sup>2</sup> and 15 cm<sup>2</sup> cones (Table 2) to evaluate compaction of a medium to coarse sand fill. Particle size distribution (PSD) tests showed a uniformity coefficient ( $UC$ ) ranging from 3 to 6, indicating minor local variations in gradation. These characteristics differ slightly from those of the uniformly graded sands ( $UC \approx 2$ ) used in the development of the reference model.

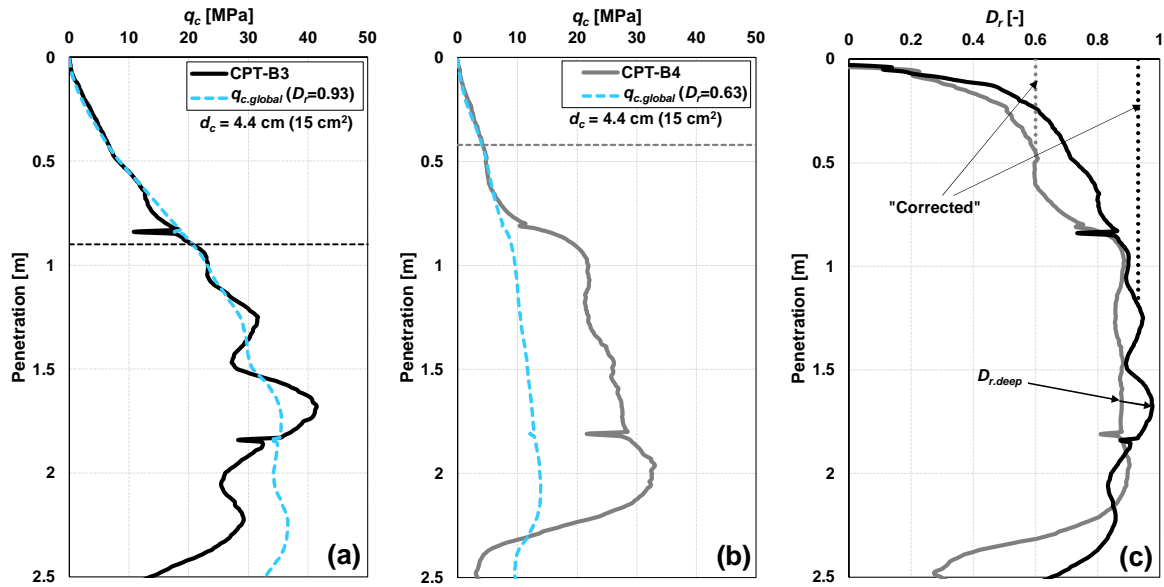
Two example CPTs at Site B, conducted with a 10 cm<sup>2</sup> cone, are presented in Fig. 12 (CPT-B1, Fig. 12a; CPT-B2, Fig. 12b). As illustrated in Fig. 11c, the  $D_{r,deep}$  profiles indicate that at CPT-B1, the relative density remains nearly constant with depth ( $D_r \approx 0.65$ – $0.70$ ), except in the shallow zone. In contrast, CPT-B2 shows a slight gradual increase in relative



**Fig. 12.** Example application of updated global model ( $q_{c,global}$ ) for Site B with 10 cm<sup>2</sup> cone.

density with depth, from  $D_r \approx 0.5$  to  $D_r \approx 0.60$  at greater depths. Application of the updated global model (Figs. 12a and 12b) yields excellent agreement in the shallow zone, with estimated  $D_r$  values of 0.68 and 0.48, respectively. These results are consistent with the trends indicated by the  $D_{r,deep}$  profiles in Fig. 12c, where indicative “corrected” values are provided for the shallow zone.

At Site B, 36 CPTs were carried out using a 15 cm<sup>2</sup> cone. These are the only tests in the internal database not performed with a 10 cm<sup>2</sup> cone and thus provide a valuable case for examining the applicability of the global model, although further validation is required to gain more confidence with this cone size. Two representative CPTs are shown in Fig. 13 (CPT-B3, Fig. 13a; CPT-B4, Fig. 13b). The  $D_{r,deep}$  profiles (Fig. 13c) indicate that CPT-B3 is dominated by very dense sand ( $D_r \approx 0.90$ – $0.95$ ), except in the shallow zone. In contrast, CPT-B4 reveals two distinct layers: a medium-dense near-surface sand ( $D_r \approx 0.60$ ) overlying a very dense layer ( $D_r \approx 0.85$ – $0.90$ ). Application of the updated global model (Figs. 13a and 13b) shows close agreement in the shallow zone, with estimated  $D_r$  values of 0.93 and 0.63,



**Fig. 13.** Example application of updated global model ( $q_{c,global}$ ) for Site B with 15 cm<sup>2</sup> cone.

respectively, consistent with the  $D_{r,deep}$  trends. The indicative “corrected” values shown in Fig 13c illustrate that the shallow zone increases with increasing relative density.

## 6.2 Offshore CPTs from the North Sea

Due to hydrostatic pore pressures, stress conditions during offshore CPTs differ from those in onshore tests, with the effective stresses typically being much lower. Figure 14 presents four offshore CPTs from the North Sea (Sites D and E, Table 2), where the upper layer consists of clean, uniformly graded siliceous sands (confirmed by sampling), illustrating a range of conditions. For clarity, the x-axis is plotted with a consistent scale across all charts in Fig. 14.

In Fig. 14a, the updated global model is applied to a CPT in loose sand within the top 1 m. Using  $D_r = 0.26$ , the  $q_{c,global}$  profile reproduces the observed shape well and indicates only limited shallow failure (~0.1-0.2 m). By contrast, methods that assume deeper shallow failure (e.g., Emerson et al., 2008) would significantly overestimate  $D_r$  in this case.

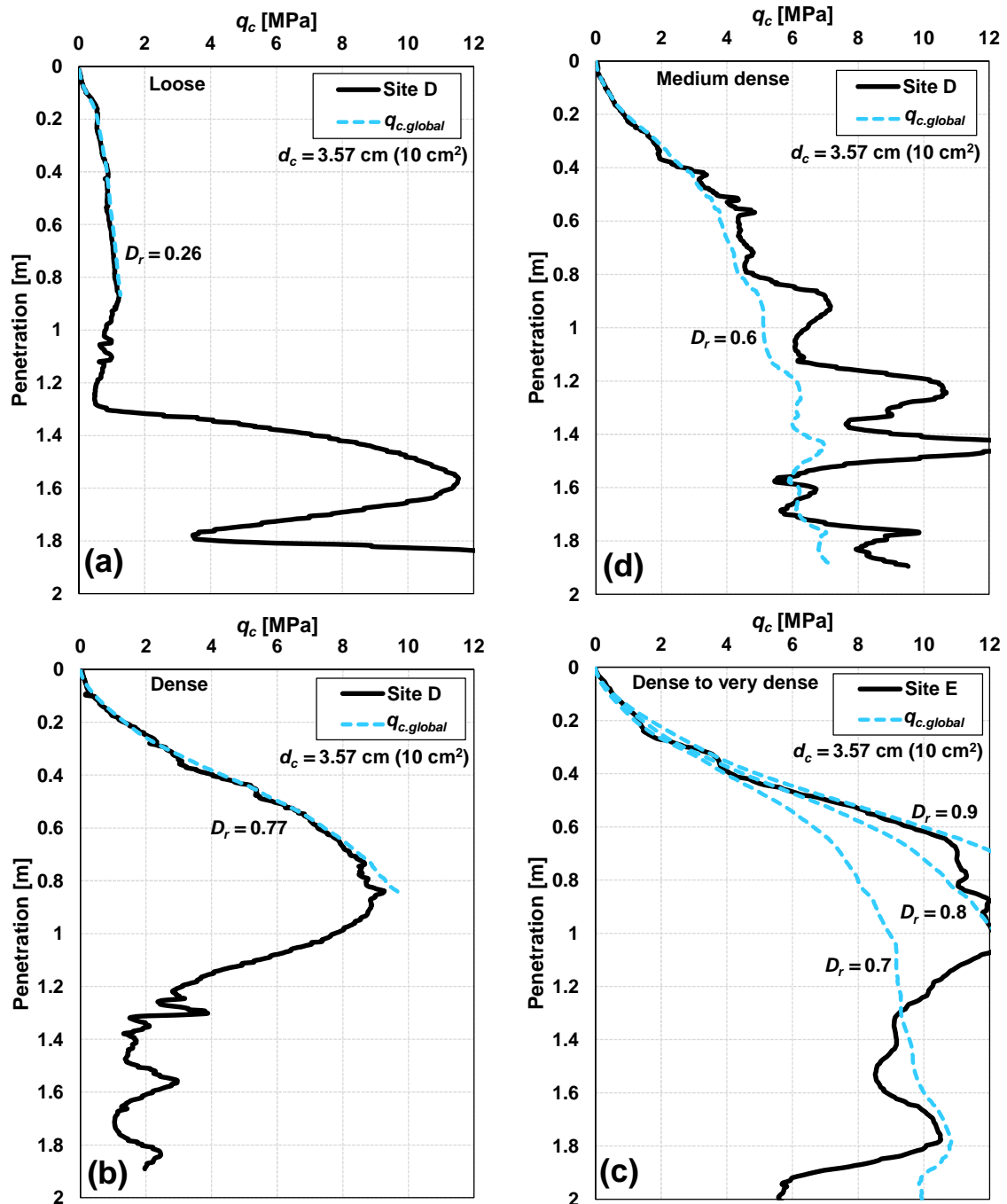
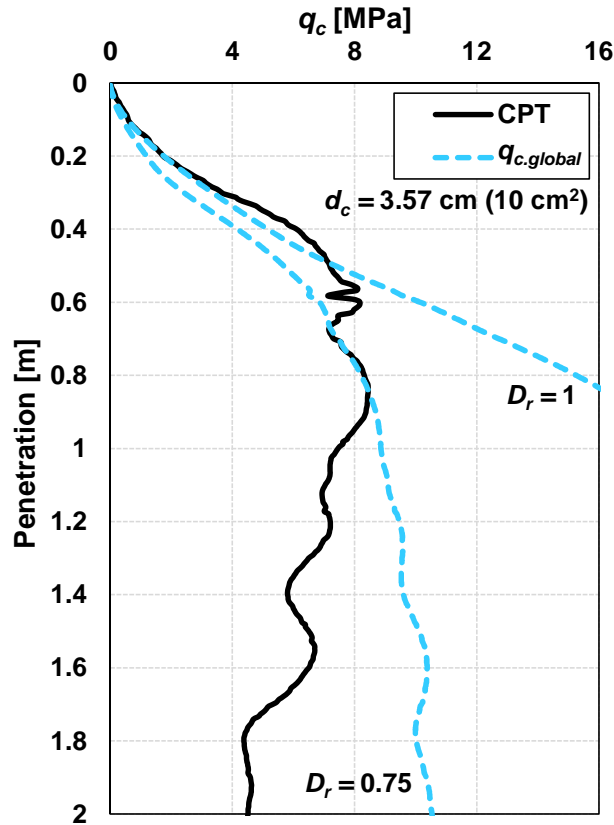


Fig. 14. Example application of updated global model ( $q_{c,global}$ ) for offshore CPTs from the North Sea.

Figure 14b shows a CPT in medium dense sand, where  $q_{c,global}$  reflects  $D_r = 0.60$ . Figure 14c presents a near-uniform dense top layer (~1 m thick), with  $q_{c,global}$  indicating  $D_r \approx 0.77$  at the



**Fig. 15. Example application with poor performance of the updated global model ( $q_{c.global}$ ) for an offshore CPT from the North Sea.**

surface. In both cases,  $D_{r.deep}$  would misclassify the deposits as loose to very loose in the near-surface.

In Fig. 14d,  $q_{c.global}$  captures slight variations from dense to very dense sand within the upper  $\sim 1.5$  m, with fitted  $D_r$  values ranging from  $\sim 0.7$  to  $0.9$ . Here again,  $D_{r.deep}$  would misclassify the surficial sand as loose to very loose.

## 7. Discussion

Figure 15 presents a CPT in a sand deposit described as slightly silty to silty and possibly heavily overconsolidated due to past glacial activity. In this case, the updated global model underperforms near the surface, probably due to increased sand compressibility not

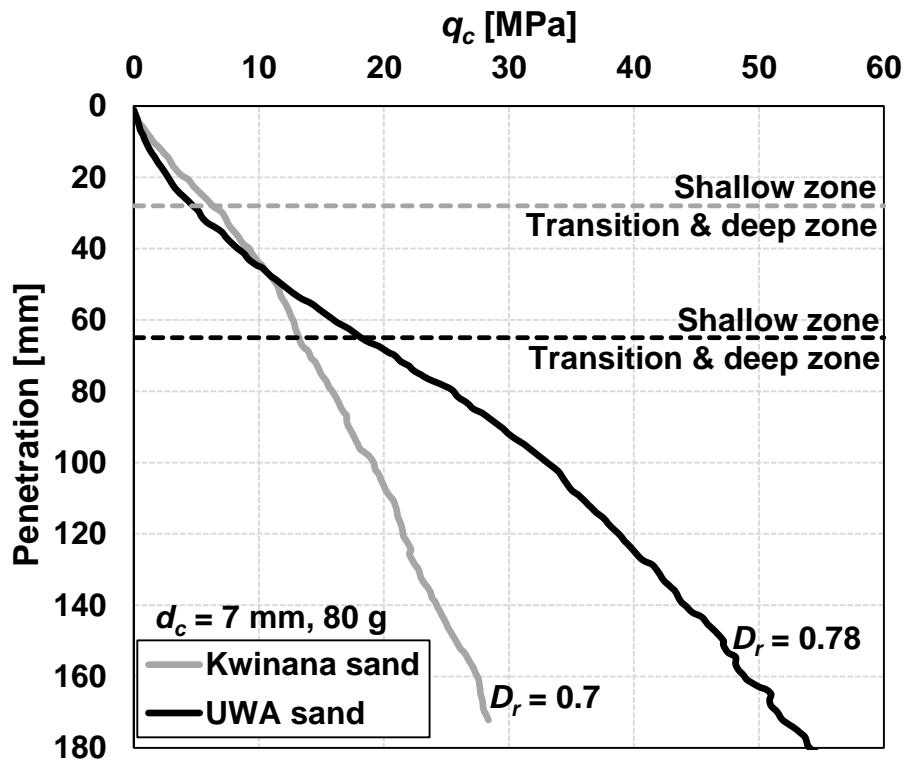


Fig. 16. Centrifuge CPTs in a high compressibility carbonate sand (Kwinana sand) and a low-medium compressible sand (UWA sand) (measured data from Liu and Lehane, 2020).

represented in the model calibration. The shallow failure zone appears smaller than predicted by the global model, with deep failure initiating at  $\sim 0.3\text{--}0.4$  m. This behavior is likely attributable to the fines content, although in other cases it may arise from angular particles, which increase compressibility while also enhancing dilatancy during shallow failure.

Figure 16 illustrates these effects: centrifuge CPTs in highly compressible, angular carbonate sand (Kwinana sand) and in low-medium compressible silica sand (UWA sand), both at similar relative densities ( $D_r = 0.70$  and  $0.78$ ), show that Kwinana sand develops higher shallow failure  $q_c$  ( $q_{c,shallow}$ ) due to greater dilatancy from particle angularity, but lower  $q_c$  at deep failure penetration due to higher compressibility.

Discrepancies such as those in Fig. 15 could also reflect ageing, bonding, or cementation - factors not captured by the model, which was developed for young, unaged, clean sands. Similarly, intense man-made compaction activities can modify fabric and yield very high near-surface  $q_c$  values that are not reproduced by the model, necessitating careful engineering judgement when applying the global model at shallow depth.

The global model is more reliable for fully saturated or dry siliceous sands, as partially saturated conditions can compromise effective stress estimation due to pore pressure effects. Near the surface, particularly within the top 2-4 cone diameters, model reliability is increasingly affected by second-order effects and measurement sensitivity. In this shallow zone, factors such as precise mudline definition and minor cone disturbances can influence the estimated  $D_r$ . Improved methods for mudline identification are therefore recommended to enhance interpretation in the near-surface region.

The model is derived from clean, unaged, uncemented, low-medium compressible siliceous sands. Application to sands with differing characteristics may introduce bias. A simple comparison of  $D_{r,deep}$  estimates performed by the author, using the calibrations of Jamiolkowski et al. (2003) and Jensen (2024) (Table 1), applied to a database of 480 calibration chamber tests on 11 siliceous sands (compiled by Jefferies and Been, 2015), yielded standard deviations of 0.15 and 0.14, respectively, between measured and predicted  $D_r$ . These results support the general applicability of Jensen's (2024) calibration for  $q_{c,deep}$  while also underscoring the uncertainties associated with  $D_r$  estimation, partly due to the inherent difficulty of defining limiting void ratios (Lunne et al., 2019).

## **8. Conclusion**

This paper has critically reviewed existing approaches for estimating relative density ( $D_r$ ) from shallow depth CPTs in sands and presented an updated global model that unifies shallow and deep failure mechanisms, calibrated against CPTs performed under controlled laboratory testing and a comprehensive internal database of 132 onshore and offshore CPTs. The main conclusions are:

### 1. Improved shallow penetration interpretation

- Conventional CPT- $D_r$  correlations, calibrated for deep failure, are not valid in the shallow failure zone, where they often misclassify siliceous sands as loose to very loose.
- The updated global model incorporates refinements to the shallow penetration expression and transition function, preventing near-surface profile convergence and enabling objective  $D_r$  estimation.

### 2. Validation

- The model has been evaluated against numerous controlled laboratory tests and an extensive internal database of 132 suitable CPTs from onshore and offshore sites, covering a broad range of conditions.
- Validation is primarily based on tests with the most widely used 10 cm<sup>2</sup> cone, which represents the current basis of calibration. While controlled laboratory testing as well as the limited number of tests with 15 cm<sup>2</sup> cones show promising agreement, further validation is required before general application across cone sizes.
- Case studies confirm strong agreement with independent direct relative density measurements.

### 3. Applicability and limitations

- The model is most reliable for clean, young, uncemented, uniformly graded siliceous sands under fully saturated or dry conditions.
- Shortcomings are illustrated for sands with higher compressibility where factors such as fines content and very angular particle shape may alter the penetration response beyond the calibrated range.
- Within the upper 2-4 cone diameters, reliability decreases due to mudline definition and cone disturbance effects, reinforcing the need for improved mudline definition method.

#### 4. Recommendations for practice

- As natural soils rarely exhibit idealized behaviour, site-specific validation of the model against direct  $D_r$  measurements remains essential, even when soils appear to fall within the calibrated range of the model.
- Engineering judgment is required to assess potential effects of ageing, bonding, cementation, or intense man-made compaction, where soil fabric effects can produce very high near-surface  $q_c$  values not reproduced by the model.
- Future work could focus on extending calibration to a wider variety of sands (including angular and highly compressible types), further validating across cone sizes, and improving mudline identification methods for shallow penetration analysis.

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## List of notations

$\sigma'$	Normalizing effective stress
$\sigma'_h$	Horizontal effective stress
$\sigma'_v$	Vertical effective stress
$a$	Empirical fitting parameter (Eqs. (7) & (10))
$d_c$	Cone diameter
$C_{0.deep}$	Empirical fitting parameter (Eqs. (2) & (3))
$C_{1.deep}$	Empirical fitting parameter (Eqs. (2) & (3))
$C_{2.deep}$	Empirical fitting parameter (Eqs. (2) & (3))
$C_{0.shallow}$	Empirical fitting parameter (Eq. (6))
$C_{1.shallow}$	Empirical fitting parameter (Eq. (6))
$C_{2.shallow}$	Empirical fitting parameter (Eq. (6))
CPT	Cone penetration test
$D_r$	Relative density
$D_{r.deep}$	Predicted relative density considering deep failure penetration
$D_{r.shallow}$	Predicted relative density considering shallow failure penetration
$f_{transition}$	Transition function (Eq. (9))
$K_o$	Coefficient of earth pressure at rest
$n_1$	Fitting parameter (Eq. (9))
$n_2$	Fitting parameter (Eq. (9))
NC	Normally consolidated
ND	Nuclear densometer
OC	Overconsolidated
OCR	Overconsolidation ratio
$p_a$	Reference stress (=100 kPa)
$p'$	Mean effective stress
PSD	Particle size distribution
$q_c$	Cone resistance
$q_{c.deep}$	Predicted cone resistance considering deep failure penetration
$q_{c.global}$	Predicted cone resistance considering both shallow and deep failure penetration
$q_{c.shallow}$	Predicted cone resistance considering shallow failure penetration
$z$	Depth of penetration
$z_{cr}$	Critical dept
$z/d_c$	Normalized penetration depth