

1 **Relative density estimation from shallow depth CPTs in siliceous sand:**
2 **an updated approach**

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13
14 **Abstract**

15 Cone penetration tests (CPTs) in sands are governed by two distinct mechanisms: a shallow failure
16 mode, which dominates in the upper ~0.2–1.0 m of standard field CPTs, and a deep failure mode,
17 which develops once sufficient embedment is achieved. Industry-standard correlations between CPT
18 cone resistance (q_c) and relative density (D_r) are calibrated under deep failure conditions, and their
19 application in the shallow zone produces erroneous results. This paper reviews existing shallow depth
20 interpretation methods and introduces an updated global model integrating shallow and deep
21 penetration mechanisms. Building on Jensen (2024), the new formulation addresses prior limitations
22 through comprehensive recalibration using controlled laboratory tests and a database of 132 onshore
23 and offshore CPTs, enabling improved differentiation of near-surface densities and closer agreement
24 with measured q_c profiles. The model shows significant reduction in shallow-zone D_r bias relative to
25 earlier approaches and is most reliable for clean, young, uncemented, uniformly graded siliceous
26 sands under fully saturated or dry conditions. Deviations occur in sands of higher compressibility,
27 increased fines content, or pronounced angularity, and within the top 2–4 cone diameters where
28 mudline definition and minor cone disturbance become influential. Site-specific validation of the
29 model against direct D_r measurements is required for reliable application, even within the calibrated
30 range.

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35
36 **Keywords**

37 CPT; cone resistance; sands; shallow depth; relative density.

38 List of notations

39	φ'	Peak friction angle
40	φ'_{cs}	Critical state friction angle
41	γ'	Effective unit weight
42	σ'	Normalizing effective stress
43	σ'_h	Horizontal effective stress
44	σ'_p	Preconsolidation stress (Fig. 7)
45	σ'_v	Vertical effective stress
46	σ_v	Vertical total stress
47	a	Empirical fitting parameter (Eqs. (7) & (10))
48	d_c	Cone diameter
49	$C_{0.deep}$	Empirical fitting parameter (Eqs. (2) & (3))
50	$C_{1.deep}$	Empirical fitting parameter (Eqs. (2) & (3))
51	$C_{2.deep}$	Empirical fitting parameter (Eqs. (2) & (3))
52	$C_{0.shallow}$	Empirical fitting parameter (Eq. (6))
53	$C_{1.shallow}$	Empirical fitting parameter (Eq. (6))
54	$C_{2.shallow}$	Empirical fitting parameter (Eq. (6))
55	CPT	Cone penetration test
56	D_r	Relative density
57	$D_{r.deep}$	Predicted relative density considering deep failure penetration
58	$D_{r.shallow}$	Predicted relative density considering shallow failure penetration
59	e_{max}	Maximum void ratio
60	e_{min}	Minimum void ratio
61	$f_{transition}$	Transition function (Eq. (9))
62	K	Lateral stress coefficient (Eq. (4))
63	K_0	Coefficient of earth pressure at rest
64	m'	Stress exponent (Fig. 7)
65	n_1	Fitting parameter (Eq. (9))
66	n_2	Fitting parameter (Eq. (9))
67	N_q	Bearing capacity factor (Eq. (4))
68	NC	Normally consolidated
69	ND	Nuclear densometer
70	OC	Overconsolidated
71	OCR	Overconsolidation ratio

72	p_a	Reference stress (=100 kPa)
73	p'	Mean effective stress
74	PSD	Particle size distribution
75	q_c	Cone resistance
76	$q_{c.deep}$	Predicted cone resistance considering deep failure penetration
77	$q_{c.global}$	Predicted cone resistance considering both shallow and deep failure penetration
78	$q_{c.shallow}$	Predicted cone resistance considering shallow failure penetration
79	UC	Uniformity coefficient
80	z	Depth of penetration
81	q_t	Corrected cone resistance (Fig. 7)
82	z_{cr}	Critical depth
83	z/d_c	Normalized penetration depth

84 **Introduction**

85 When sufficient embedment is achieved during cone penetration in sand, a deep failure
86 mechanism develops, characterized by a localized deformation zone near the cone tip
87 (Arshad et al., 2014). Deep penetration has been widely analyzed using the analogy between
88 cone penetration and spherical cavity expansion (e.g., Salgado and Prezzi, 2007). In contrast,
89 the upper ~0.2-1.0 m of standard field cone penetration tests (CPTs) is typically governed by
90 a shallow failure mechanism, associated with upward displacement of surface sand (Biarez
91 and Gresillon, 1972). A transition zone separates the shallow and deep mechanisms, marking
92 a shift in the governing soil deformation pattern (Puech and Foray, 2002).

93

94 Industry-standard methods for estimating the relative density (D_r) of sand from CPT cone
95 resistance (q_c) are largely empirical, based on CPTs in calibration chambers under deep
96 failure conditions (e.g., Baldi et al., 1986; Jamiolkowski et al., 2003). Applying these
97 methods to near-surface sands, where shallow failure governs, leads to erroneous D_r
98 estimates. Accurate estimation of relative density at shallow depths is of practical importance
99 in various geotechnical applications. Near-surface sands often control the performance of
100 shallow foundations, pipelines, cables, and earthworks. Erroneous classification of shallow
101 sand as loose, when it is in fact dense, may lead to overly conservative designs, unnecessary
102 ground improvement, or incorrect assessment of serviceability performance. Conversely,
103 overestimation of D_r may result in unconservative predictions of foundation performance.

104

105 Despite its practical importance, guidance for interpreting CPT data at shallow depths
106 remains limited. Although several researchers have advanced understanding of shallow
107 failure penetration and the transition zone (Puech and Foray, 2002; Emerson et al., 2008;
108 Kim et al., 2015; Lehane et al., 2022; Jensen, 2024), these developments have not yet been

109 widely incorporated into routine engineering. This paper provides a critical review of existing
110 methods for estimating D_r at shallow depths and introduces an updated global model that
111 extends the formulation of Jensen (2024).

112

113 While Jensen (2024) introduced a unified framework for combining shallow and deep
114 penetration mechanisms, the initial calibration of the model exhibited limitations in the very
115 near-surface zone and was based on a small dataset. The present study addresses these
116 limitations through recalibration using comparisons with alternative theoretical and empirical
117 interpretation methods, previously published controlled laboratory tests, field CPTs with
118 measured D_r , and a new database of 132 onshore and offshore CPTs. This approach enables
119 both quantitative validation and broader qualitative evaluation. Refinement of the shallow
120 penetration coefficient and transition function improves predictive performance in the
121 shallow failure zone and reduces bias in near-surface D_r estimation, thereby strengthening
122 the model's practical applicability.

123

124 **2. Background**

125 **2.1 Shallow failure penetration**

126 As illustrated in Fig. 1, cone penetration in uniform sand deposits generally exhibits a
127 transition in the measured cone resistance profile, $q_c(z)$, with depth (z), from a shallow failure
128 mode to a deep failure mode. In the very near-surface zone, shallow failure dominates,
129 producing an upward-concave $q_c(z)$ profile characterized by an increasing gradient (dq_c/dz).
130 The mechanics of shallow failure penetration are governed primarily by the mobilized
131 friction angle and soil dilatancy (Durgunoglu and Mitchell, 1973). This penetration mode has
132 been the subject of several studies (e.g., Puech and Foray, 2002; Balachowski, 2007; Kim et
133 al., 2015).

134

135 Experimental observations by Kim et al. (2015), consistent with the general findings of
136 Lehane et al. (2022), indicate that the transition from shallow to deep failure occurs at a depth
137 of penetration of approximately 3-6 cone diameters in loose, clean silica sand, and about
138 15-20 cone diameters in dense silica sand. Based on their tests, Kim et al. (2015) proposed
139 the following expression with a form that describes the upward-concave $q_c(z)$ profile
140 representative of shallow failure penetration ($q_{c.shallow}$) in clean silica sand:

$$141 \quad q_{c.shall} = (0.96 \cdot D_r - 0.25) \cdot \left(\frac{\sigma'_v}{p_a}\right)^{0.5} \cdot \left(\frac{z}{d_c}\right) \cdot p_a \quad (1)$$

142 where p_a is a reference stress (= 100 kPa), σ'_v is vertical effective stress, and d_c is the cone
143 diameter. Following Kim et al. (2015), both $q_{c.shallow}$ and σ'_v are expressed in MPa in Eq. (1).
144 Although this convention may appear dimensionally inconsistent, it is retained to ensure
145 comparability with their formulation. Equation (1), along with other approaches, is discussed
146 further later in this paper.

147

148 **2.2 Deep failure penetration**

149 At greater depth, the penetration response transitions to a deep failure mode with decreasing
150 dq_c/dz , as shown in Fig. 1. Deep failure penetration is primarily governed by sand
151 compressibility characteristics (Konrad, 1998; Yu and Mitchell, 1998). Widely used CPT- D_r
152 correlations for deep failure are based on Schmertmann's (1976) formulation:

$$153 \quad q_{c.deep} = C_{0.deep} p_a \left(\frac{\sigma'}{p_a}\right)^{C_{1.deep}} \exp(C_{2.deep} D_{r.deep}) \quad (2)$$

$$154 \quad D_{r.deep} = \frac{1}{C_{2.deep}} \ln \left(\frac{q_{c.deep}/p_a}{C_{0.deep} (\sigma'/p_a)^{C_{1.deep}}} \right) \quad (3)$$

155 where $C_{0.deep}$, $C_{1.deep}$, and $C_{2.deep}$ are empirical fitting parameters, and σ' is the normalizing
156 effective stress.

157

158 In normally consolidated (NC) sands, calibration is often performed using the vertical
159 effective stress σ'_v (e.g., Schmertmann, 1978; Baldi et al., 1986). However, Houlsby and
160 Hitchman (1988) demonstrated that the horizontal effective stress (σ'_h) has a stronger
161 influence on q_c . Consequently, many methods adopt the mean effective stress,
162 $p' = (\sigma'_v + 2\sigma'_h) / 3$, improving generality, particularly in overconsolidated (OC) sands where
163 the coefficient of earth pressure at rest (K_o), on which σ'_h depends, is significantly higher
164 (e.g., Jamiolkowski et al., 2003; Krogh et al., 2022).

165

166 In the upper 3-5 m of OC sands, where K_o and the overconsolidation ratio (OCR) vary
167 significantly with depth, stress normalization using p' becomes particularly important to
168 avoid overestimation of D_r , as may occur when normalizing solely with σ'_v . Depth-dependent
169 $K_o(z)$ can be evaluated using the approach of Krogh et al. (2022) (procedure given in the
170 flowchart in Fig. 7 presented later in the paper), where preconsolidation stress is estimated
171 following Agaiby and Mayne (2019). Importantly, Emerson et al. (2008) and Jensen (2024)
172 demonstrated that better interpretation of shallow CPT data requires integrating the
173 $K_o(z)$ -based stress normalization with shallow failure considerations, as discussed in the
174 following.

175

176 **3. Review of CPT-based interpretation methods for shallow D_r estimation**

177 **3.1 Limit equilibrium method (Puech and Foray, 2002; Emerson et al., 2008)**

178 Puech and Foray (2002) developed a limit equilibrium model for shallow failure penetration
179 in siliceous sand based on bearing capacity theory, incorporating friction angle, cone
180 diameter, and lateral friction:

$$181 \quad q_{c,shallow} = \gamma' \cdot z \cdot N_q \left(1 + \frac{K \cdot \sin \phi'}{L} \cdot z \right) \quad (4)$$

182 where γ' is effective unit weight of sand, φ' the peak friction angle, L is the dimension of a
183 soil cylinder considered around the cone ($L = d_c \cdot \exp(\tan(\varphi' \cdot (\pi/2))) \cdot \tan(\pi/4 + \varphi'/2)$),
184 $N_q = 1.058 \cdot \exp(6.168 \cdot \tan\varphi')$, and K is a lateral stress coefficient, different from K_o and
185 ranging from approximately 1 to 3 (Emerson et al., 2008). Equation (4) is to be applied with
186 curve fitting while relating D_r via the following correlation with φ' :

$$187 \quad \varphi' = 20.4^\circ \cdot D_r + 25^\circ \quad (5)$$

188

189 Distinguishing between shallow and deep failure penetration is critical, since deep failure
190 penetration in loose sand can produce q_c -profiles that resemble those from shallow
191 penetration in dense sand, leading to misinterpretation of relative density. To address this
192 issue, Emerson et al. (2008) proposed a global q_c model ($q_{c.global}$) that via a transition function
193 combines Puech and Foray's (2002) model for $q_{c.shallow}$ with the widely applied calibration of
194 deep failure penetration ($q_{c.deep}$; Eq. (3)) by Jamiolkowski et al. (2003); see Table 1. In their
195 global model, the transition between shallow and deep failure penetration is defined by a
196 critical depth (z_{cr}), primarily dependent on D_r ; lower D_r causes shallower onset of deep
197 failure.

198

199 Figures 2a and 2b show Emerson et al.'s (2008) global model applied to two CPTs at a
200 German test site with homogeneous, clean, uniformly graded, fine to medium OC siliceous
201 sand, using a standard 10 cm² cone ($d_c = 3.57$ cm) (data from Krogh et al., 2022). Nuclear
202 densometer (ND) tests performed at the site indicate $D_r = 60\% \pm 10\%$ (CPT-02; Fig. 2a) and
203 $D_r = 80\% \pm 10\%$ (CPT-03; Fig. 2b). A weakness in Emerson et al.'s model lies in the
204 parameter K (Eq. (4)), which is somewhat arbitrary. Although K strongly influences the
205 predicted $q_{c.shallow}$ profile in the model, no robust guidance exists for selecting an appropriate
206 value in practice. For comparison within the shallow failure zone, Emerson et al.'s (2008)

207 global model is evaluated against the experimentally derived expression for $q_{c.shallow}$ proposed
 208 by Kim et al. (2015) (Eq. (1)). The two approaches show good agreement in the shallow zone
 209 when Emerson et al.'s model is applied with $K = 2.5$ (Figs. 2a and 2b). In these comparisons,
 210 representative relative density ranges of $D_r = 0.6-0.8$ (Fig. 2a) and $0.65-0.85$ (Fig. 2b) were
 211 used, corresponding to the ND test results.

212

213 Although the curves now show reasonable agreement with the direct ND measurements of
 214 D_r in the shallow zone, Emerson et al.'s model does not accurately capture the transition from
 215 shallow to deep failure, predicting the shift at a considerably greater penetration depth than
 216 indicated by the CPT data (Fig. 2). Furthermore, the abrupt shift from increasing dq_c/dz
 217 (shallow zone) to decreasing dq_c/dz (deep zone) does not fit the general observation of a
 218 gradually changing gradient (Fig. 1). If applying a lower value of K , the transition to the deep
 219 failure penetration could have been captured better, but that would have then compromised
 220 the accuracy significantly in the shallow failure zone. Similar discrepancies were noted by
 221 the author comparing with the extensive internal CPT database, presented later in this paper.

222

223 **3.2 Centrifuge-based model (Lehane et al., 2022)**

224 Centrifuge CPTs, due to limited penetration, are well suited for studying shallow failure
 225 penetration (e.g., Bolton et al., 1999; Balachowski, 2007). Lehane et al. (2022) used a large
 226 centrifuge database to describe the shallow-to-deep transition with a hyperbolic tangent
 227 function:

$$228 \quad q_{c.shallow} = \tanh\left(\frac{az}{d_c}\right) C_{0.shallow} p_a \left(\frac{\sigma'_v}{p_a}\right)^{C_{1.shallow}} \exp(C_{2.shallow} D_{r.shallow}) \quad (6)$$

229 where $C_{0.shallow}$, $C_{1.shallow}$, and $C_{2.shallow}$ are empirical parameters calibrated by Lehane et al.
 230 (2022) for clean, uniformly graded, fine to medium, freshly deposited NC silica sands using

231 $\sigma' = \sigma'_v$ (Table 1), and a is an empirical curve fitting parameter, given by Lehane et al. (2022)
232 as:

$$233 \quad a = 0.66 \exp(-2.5 D_{r, \text{shallow}}) \quad (7)$$

234

235 Equation (7) (Lehane et al., 2022) is shown in Fig. 3 together with the a values derived from
236 their centrifuge database, and supplemented with values inferred by the author from data
237 presented by Puech and Foray (2002) and Emerson et al. (2008) from CPTs performed with
238 a standard 10 cm² cone. Although designated “shallow”, Eq. (6) may also apply for deep
239 failure penetration in freshly deposited NC sands, since it essentially transitions into a deep
240 failure expression (Eq. (2)) when the tangent hyperbolic function approaches unity with
241 depth, as controlled by the a parameter. In this study, however, it is primarily used to model
242 the shallow zone and the transition.

243

244 When applied using $D_r = 0.6-0.8$ (Fig. 4a) and $D_r = 0.65-0.85$ (Fig. 4b), predicted q_c values
245 fall well below the field data, with the discrepancy increasing beyond $\sim 0.3-0.4$ m, as expected
246 due to overconsolidation effects. Jensen (2024) noted that Eqs. (6) and (7) effectively
247 captures the shape of field q_c profiles in moderately compressible siliceous sands. However,
248 overconsolidation effects influences deep q_c significantly and must be accounted for.

249

250 For shallow failure penetration, Jensen (2024) argued that overconsolidation has little, if any,
251 effect on shallow failure penetration resistance. This conclusion was supported by two main
252 pieces of evidence. Firstly, numerical simulations by Krogh et al. (2022) (Fig. 5)
253 demonstrated comparable $q_{c, \text{shallow}}$ for relative densities of $D_r = 0.65$ (Fig. 5a) and $D_r = 0.85$
254 (Fig. 5b), under both constant $K_0 = 0.414$ (representative of NC sand) and depth-dependent
255 $K_0(z)$ (representative of OC sand). Secondly, similar trends can be inferred from the initial

256 penetration curves reported in experimental centrifuge CPT studies by El-Sekelly et al.
 257 (2015), Roy (2020), and Richards et al. (2021). These experiments, conducted at varying g-
 258 levels, effectively simulated a range of OCR values, including NC conditions.

259

260 Building on these observations, Jensen (2024) proposed a global q_c model that combines Eqs.
 261 (2) and (6), with the transition governed by OCR (or K_θ). An assumption in this approach is
 262 therefore that very shallow q_c values are equivalent for both normally consolidated and
 263 overconsolidated sands, with OCR effects becoming more pronounced with increasing
 264 embedment approaching deep failure penetration.

265

266 3.3 Global model (Jensen, 2024)

267 To capture both failure regimes and the transition, Jensen (2024) proposed:

$$268 \quad q_{c,global} = \begin{cases} q_{c,shall} & \text{if } \tanh(az/d_c) \leq n_1 \\ q_{c,shallow} + f_{transition}(q_{c,deep} - q_{c,shall}) & \text{if } \tanh(az/d_c) > n_1 \end{cases} \quad (8)$$

269 where

$$270 \quad f_{transition} = \left(\frac{\tanh\left(\frac{az}{d_c}\right) - n_1}{1 - n_1} \right)^{n_2} \quad (9)$$

271 with $n_1 = 0.5$ and $n_2 = 2$ suggested as initial proof of concept values, subject to further
 272 calibration. Jensen (2024) also proposed that the calibration of Eq. (6) could be extended to
 273 deep failure (Eq. (2)) by making small adjustments to $C_{0,shallow}$, thereby enabling a shift from
 274 σ_v' to p' in stress normalization (Table 1).

275

276 The global model, defined by Eqs. (8) and (9) and incorporating Jensen's (2024) preliminary
 277 calibration, is presented in Figs. 6a and 6b. The predicted $q_{c,global}$ profiles closely match the
 278 shape of the measured CPT data and align well with the ND measurements. However, a
 279 current key limitation is evident in the upper ~ 0.4 m: the profiles converge in the initial

280 penetration for the applied D_r range, preventing objective differentiation of D_r . Although the
281 model successfully captures the overall profile shape, this shortcoming highlights the need
282 for refinement, which is addressed in the following section.

283

284 **4. Updated global model**

285 To support refinement of the global model, an internal database of 132 shallow CPTs from
286 onshore and offshore industry projects was compiled and analysed in conjunction with
287 previously published controlled laboratory tests. The internal database comprises onshore
288 tests conducted in Denmark and Germany, and offshore tests performed in the North Sea and
289 Irish Sea; details are summarized in Table 2. The onshore CPTs were mainly carried out in
290 recently placed, clean siliceous sand fill with varying degrees of compaction, whereas the
291 offshore CPTs were conducted in young, surficial, clean siliceous deposits. As noted in the
292 descriptions for each site (Table 2), the tested sands generally consist of fine to coarse,
293 uniformly graded sand, as confirmed by sampling. Of the total tests, 96 CPTs were performed
294 with a 10 cm² cone and 36 CPTs using a 15 cm² cone, all from the same site (Site B(2)). Each
295 CPT was advanced to a depth constraining both shallow and deep failure penetration.

296

297 A qualitative review of the internal CPT database indicated that the transition to deep failure
298 penetration generally occurs at a slightly shallower depth, particularly for loose to medium
299 dense sands, than predicted by the current model governed by the coefficient a defined in Eq.
300 (7). That is, a controls the hyperbolic tangent function in Eq. (6), which drives the transition
301 of the q_c profile to deep failure penetration as the function approaches unity. To account for
302 this, a minor modification to Eq. (7) is proposed, resulting in an improved fit to the internal
303 database and a closer overall correspondence with the data shown in Fig. 3:

$$304 \quad a = 0.79\exp(-2.6D_{r,shallow}) \quad (10)$$

305

306 The review of the internal database also indicates that Lehane et al. (2022) profiles (Eq. (6)),
307 which forms the basis for the global model by Jensen (2024) in the very shallow zone,
308 consistently underestimate $q_{c.shallow}$ in the near-surface range for both loose and dense sands.
309 This trend is further supported by the distinct discrepancy observed in Fig. 4, where
310 comparisons between Lehane et al. (2022) (Eq. (6)) profiles and $q_{c.shallow}$ profiles from Kim
311 et al. (2015) reveal systematic large deviations in the shallow failure zone. As was
312 demonstrated in Fig. 2, the profiles by Kim et al. (2015) show good agreement in the shallow
313 failure zone with both the theoretically derived model of Puech and Foray (2002) and the
314 measured data by Krogh et al. (2022), further confirming the observed underestimation of
315 $q_{c.shallow}$ with Lehane et al. (2022) profiles in natural deposits.

316

317 To improve agreement, an increase in $C_{2.shallow}$ from 3.2 to 3.8 is proposed (Table 1), and a
318 slight adjustment to $f_{transition}$ (Eq. (9)) is similarly made; specifically, increasing n_1 to 0.6,
319 while retaining $n_2 = 2$. As shown in Fig. 4 (profiles named ‘this study’), these adjustments
320 enable Eq. (6) to follow the upward concave portion of the Kim et al. (2015) profiles closely
321 in the shallow failure penetration zone. The complete updated approach is summarized in the
322 flow chart in Fig. 7, which is easily implemented in a spreadsheet. In the proposed approach,
323 D_r is treated as a representative constant value, whereas curve fitting may be performed over
324 selected depth intervals depending on the desired level of detail.

325

326 As shown in Fig. 8, curve fitting with the updated global model indicates $D_r \approx 0.5-0.7$ for
327 CPT-02 (Fig. 8a) and $D_r \approx 0.6-0.8$ for CPT-03 (Fig. 8b), both in excellent agreement with the
328 ND measurements ($D_r = 60\% \pm 10\%$ and $D_r = 80\% \pm 10\%$, respectively). The updated model
329 achieves two key improvements: (1) it prevents convergence of the $q_{c.global}$ profiles in the

330 surficial sand, allowing objective estimation of D_r ; and (2) it yields strong agreement with
331 the internal database, as demonstrated with numerous representative examples from the
332 database later in this paper. Furthermore, the following section discusses cone size effects, in
333 which the updated global model similarly show great performance with controlled tests by
334 Iqbal (2002) and Larsen and Ibsen (2006).

335

336 The variability observed in the internal database is reflected in Table 3, which summarizes
337 relative density interpretations for the upper 1 m of CPT data, using the updated global model.
338 The descriptive categories from ‘loose’ to ‘very dense’ that are linked to the D_r values (see
339 Table 3; descriptions correspond to interpretations from top to bottom of the upper 1 m
340 profile) indicate substantial variation in the database.

341

342 **5. Cone size effects**

343 As embedded in Eq. (1) (Kim et al., 2015) and Eq. (4) (Puech and Foray, 2002), the cone
344 diameter (d_c) plays an important role in the $q_{c.shallow}$ profiles. Specifically, an increase in d_c
345 leads to a reduction in cone resistance at shallow failure penetration. By analysing CPT data
346 obtained with varying cone diameters in uniform sand samples, Lehane et al. (2022) observed
347 that data fall into a consistent trend when presented in a normalized chart, where the stress
348 normalised cone resistance is plotted against the normalised penetration depth (z/d_c). This
349 observation formed the basis for their $q_{c.shallow}$ profile expression (Eq. (6)), upon which the
350 updated global model is an extension.

351

352 Essentially, Eq. (6) expresses a strong influence of cone diameter during shallow failure
353 penetration, which diminishes with large penetration well into the deep failure penetration

354 zone where profiles of varying d_c will eventually converge. This is consistent with
355 experimental observations (e.g., Balachowski, 2007).

356

357 The updated global model is evaluated in Fig. 9 using CPTs conducted under controlled
358 laboratory conditions by Iqbal (2002). The tests were performed with two significantly
359 different cone diameters, $d_c = 1.13$ cm and $d_c = 3.57$ cm (standard field size cone), in fine to
360 medium, uniformly graded silica sand. Three separate sand bodies with D_r ranging 0.62 to
361 0.84 were tested, with densities confirmed by direct measurements following uniform
362 compaction using a vibration screen unit. As shown in Fig. 9, the updated global model
363 exhibits excellent agreement with the measurements, remaining within about 5-10% (in
364 absolute terms) of the measured values for both $d_c = 1.13$ cm and $d_c = 3.57$ cm in all three
365 sand bodies.

366

367 Because shallow failure penetration is more pronounced in denser sand, the cone size effect
368 is correspondingly greater. In contrast, shallow failure effects are minimal in loose sand. In
369 Fig. 10, the global model is evaluated against a CPT with $d_c = 1.50$ cm performed in a loose,
370 fine to medium, uniformly graded silica sand by Larsen and Ibsen (2006). Direct
371 measurements indicated D_r ranging about 0.3 to 0.45. The sand specimen was prepared by
372 water pluviation followed by minimal external vibration. Accordingly, a representative value
373 of $K_0 = 0.5$, consistent with NC conditions, was assumed in the global model rather than
374 using the recommended $K_0(z)$ -procedure. Under these controlled conditions with minimal
375 compaction disturbance, overconsolidation effects are expected to be negligible. As shown
376 in Fig. 10, the global model again performs well, with curve fitting ranging about
377 $D_r = 0.30$ - 0.45 .

378

379 6. Case histories from the internal database

380 6.1 Onshore ground improvement

381 6.1.1 Site A

382 At a Danish onshore site, three CPTs (CPT-A1 to A3) using a standard 10 cm² cone and two
383 nuclear densometer tests were conducted within a 15 m radius to assess compaction of a sand
384 fill, see Fig. 11. Limiting void ratios of $e_{min} = 0.45$ and $e_{max} = 0.79$ were measured for the
385 sand fill, typical for clean, subrounded to subangular silica sand (Youd, 1973). The ND tests
386 are considered representative within the shallow failure zone, approximately the upper
387 0.45 m (Fig. 11a).

388

389 In this zone, D_r values with Eq. (3) ($D_{r,deep}$) are unreliable; therefore, ND data are used as the
390 primary reference (Fig. 11b). The ND measurements yielded $D_r = 0.81$ and 0.87 , indicating
391 dense to very dense sand ($D_r \approx 0.84$) in the near-surface layer. Below this, D_r decreases to
392 ~ 0.6 at a depth of 2 m, except in CPT-A3, where the sand fill ends at about 1.4 m.

393

394 Applying $D_r = 0.84$ in the updated global model results in $q_{c,global}$ profiles closely matching
395 all three CPTs in the shallow zone (Fig. 11a). At greater depths, the measured CPT data
396 diverge from $q_{c,global}$, consistent with the generally decreasing $D_{r,deep}$ profile (Fig. 11b),
397 indicating that the compaction activities produced a denser sand deposit near the surface,
398 which is not unusual of surface vibration compaction.

399

400 6.1.2 Site B

401 At a second Danish site (Site B), CPTs were performed using 10 cm² and 15 cm² cones
402 (Table 2) to evaluate compaction of a medium to coarse sand fill. Particle size distribution
403 (PSD) tests showed a uniformity coefficient (UC) ranging from 3 to 6, indicating minor local

404 variations in gradation. These characteristics differ slightly from those of the uniformly
405 graded sands ($UC \approx 2$) used in the development of the reference model.

406

407 Two example CPTs at Site B, conducted with a 10 cm² cone, are presented in Fig. 12
408 (CPT-B1, Fig. 12a; CPT-B2, Fig. 12b). As illustrated in Fig. 12c, the $D_{r,deep}$ profiles indicate
409 that at CPT-B1, the relative density remains nearly constant with depth ($D_r \approx 0.65$ – 0.70),
410 except in the shallow zone. In contrast, CPT-B2 shows a slight gradual increase in relative
411 density with depth, from $D_r \approx 0.5$ to $D_r \approx 0.60$ at greater depths. Application of the updated
412 global model (Figs. 12a and 12b) yields excellent agreement in the shallow zone, with
413 estimated D_r values of 0.68 and 0.48, respectively. These results are consistent with the trends
414 indicated by the $D_{r,deep}$ profiles in Fig. 12c, where indicative “corrected” values are provided
415 for the shallow zone.

416

417 At Site B, 36 CPTs were carried out using a 15 cm² cone. These are the only tests in the
418 internal database not performed with a 10 cm² cone and thus provide a valuable case for
419 examining the applicability of the global model, although further validation is required to
420 gain more confidence with this cone size. Two representative CPTs are shown in Fig. 13
421 (CPT-B3, Fig. 13a; CPT-B4, Fig. 13b). The $D_{r,deep}$ profiles (Fig. 13c) indicate that CPT-B3
422 is dominated by very dense sand ($D_r \approx 0.90$ – 0.95), except in the shallow zone. In contrast,
423 CPT-B4 reveals two distinct layers: a medium-dense near-surface sand ($D_r \approx 0.60$) overlying
424 a very dense layer ($D_r \approx 0.88$). Application of the updated global model (Figs. 13a and 13b)
425 shows close agreement in the shallow zone, with estimated D_r values of 0.93 and 0.63,
426 respectively, consistent with the $D_{r,deep}$ trends. The indicative “corrected” values shown in
427 Fig 13c illustrate that the shallow zone increases with increasing relative density.

428

429 6.2 Offshore CPTs from the North Sea

430 Due to hydrostatic pore pressures, stress conditions during offshore CPTs differ from those
431 in onshore tests, with the effective stresses typically being much lower. Figure 14 presents
432 four offshore CPTs from the North Sea (Sites D and E, Table 2), where the upper layer
433 consists of clean, uniformly graded siliceous sands (confirmed by sampling), illustrating a
434 range of conditions. For clarity, the x-axis is plotted with a consistent scale across all charts
435 in Fig. 14.

436

437 In Fig. 14a, the updated global model is applied to a CPT in loose sand within the top 1 m.
438 Using $D_r = 0.26$, the $q_{c,global}$ profile reproduces the observed shape well and indicates only
439 limited shallow failure ($\sim 0.1-0.2$ m). By contrast, methods that assume deeper shallow failure
440 (e.g., Emerson et al., 2008) would significantly overestimate D_r in this case.

441

442 Figure 14b shows a CPT in medium dense sand, where $q_{c,global}$ reflects $D_r = 0.60$. Figure 14c
443 presents a near-uniform dense top layer (~ 1 m thick), with $q_{c,global}$ indicating $D_r \approx 0.77$ at the
444 surface. In both cases, $D_{r,deep}$ would misclassify the deposits as loose to very loose in the
445 near-surface.

446

447 In Fig. 14d, $q_{c,global}$ captures slight variations from dense to very dense sand within the
448 upper ~ 1.5 m, with fitted D_r values ranging from ~ 0.7 to 0.9 . Here again, $D_{r,deep}$ would
449 misclassify the surficial sand as loose to very loose.

450

451 7. Discussion

452 The updated model builds on Jensen (2024), which was presented with an initial proof of
453 concept calibration and emphasised the need for further calibration. Nevertheless, Jensen

454 (2024) demonstrated an improvement over existing approaches in capturing the realistic
455 shape of shallow CPT profiles in sand. The shortcomings associated with the initial
456 calibration are addressed in the present study through a recalibration based on previously
457 published controlled laboratory tests and a large internal database of 132 CPTs compiled
458 from industry projects.

459

460 As was demonstrated in Fig. 6, the initial Jensen (2024) model results in convergence of q_c
461 profiles in the very shallow zone for certain ranges of D_r , preventing objective differentiation
462 of D_r ; this limitation is resolved with the updated model. Furthermore, the initial calibration
463 tends to underestimate q_c in the shallow zone, with the discrepancy becoming increasingly
464 pronounced with increasing D_r . This behaviour is illustrated in Fig. 15 through comparison
465 of the initial calibration and the updated model using tests that were presented in Figs. 12 and
466 14. Overall, the updated model captures the shape of the q_c profiles more accurately, in
467 agreement with the full internal database, and the difference between the initial and updated
468 formulations increases with D_r .

469

470 Figure 16 presents a CPT in a sand deposit visually described as slightly silty to silty and
471 possibly heavily overconsolidated due to past glacial activity. In this case, the updated global
472 model underperforms near the surface, probably due to increased sand compressibility not
473 represented in the model calibration. The shallow failure zone appears smaller than predicted
474 by the global model, with deep failure initiating at $\sim 0.3\text{--}0.4$ m. This behavior is likely
475 attributable to the fines content, although in other cases it may arise from angular particles,
476 which increase compressibility while also enhancing dilatancy during shallow failure.

477

478 Figure 17 illustrates these effects: centrifuge CPTs in highly compressible, angular carbonate
479 sand (Kwinana sand) and in moderately compressible silica sand (UWA sand), both at similar
480 relative densities ($D_r = 0.70$ and 0.78), show that Kwinana sand develops higher shallow
481 failure q_c ($q_{c,shallow}$) due to greater dilatancy from particle angularity, but lower q_c at deep
482 failure penetration due to higher compressibility.

483

484 Discrepancies such as those in Fig. 16 could also reflect ageing, bonding, or cementation -
485 factors not captured by the model, which was developed for young, unaged, clean sands.
486 Similarly, intense man-made compaction activities can modify fabric and yield very high
487 near-surface q_c values that are not reproduced by the model, necessitating careful engineering
488 judgement when applying the global model at shallow depth.

489

490 The global model is more reliable for fully saturated or dry siliceous sands, as partially
491 saturated conditions can compromise effective stress estimation due to pore pressure effects.
492 Near the surface, particularly within the top 2-4 cone diameters, model reliability is
493 increasingly affected by second-order effects and measurement sensitivity. In this shallow
494 zone, factors such as precise mudline definition and minor cone disturbances can influence
495 the estimated D_r . Improved methods for mudline identification are therefore recommended
496 to enhance interpretation in the near-surface region.

497

498 The model is derived from clean, unaged, uncemented, moderately compressible siliceous
499 sands. Application to sands with differing characteristics may introduce bias. A simple
500 comparison of $D_{r,deep}$ estimates performed by the author, using the calibrations of
501 Jamiolkowski et al. (2003) and Jensen (2024) (Table 1), applied to a database of 480
502 calibration chamber tests on 11 siliceous sands (compiled by Jefferies and Been, 2015),

503 yielded standard deviations of 0.15 and 0.14, respectively, between measured and predicted
504 D_r . These results support the general applicability of Jensen's (2024) calibration for $q_{c,deep}$
505 while also underscoring the uncertainties associated with D_r estimation, partly due to the
506 inherent difficulty of defining limiting void ratios (Lunne et al., 2019).

507

508 **8. Conclusion**

509 This paper has critically reviewed existing approaches for estimating relative density (D_r)
510 from shallow depth CPTs in sands and presented an updated global model that unifies shallow
511 and deep failure mechanisms, calibrated against CPTs performed under controlled laboratory
512 testing and a comprehensive internal database of 132 onshore and offshore CPTs. The main
513 conclusions are:

514 1. Advancement beyond Jensen (2024)

- 515 ○ The updated calibration refines the shallow penetration formulation and
516 transition function proposed in Jensen (2024), enabling more objective
517 differentiation of D_r in the shallow zone.
- 518 ○ The revised model reduces shallow-zone bias and provides improved
519 agreement with measured q_c profiles across a substantially expanded dataset.

520 2. Validation

- 521 ○ The model has been evaluated against numerous controlled laboratory tests,
522 by comparison with alternative theoretical and empirical approaches, and an
523 extensive internal database of 132 suitable CPTs from onshore and offshore
524 sites, covering a broad range of conditions.
- 525 ○ Validation is primarily based on tests with the most widely used 10 cm² cone,
526 which represents the current basis of calibration. While controlled laboratory
527 testing as well as the limited number of tests with 15 cm² cones show

528 promising agreement, further validation is required before general application
529 across cone sizes.

530 ○ Case studies confirm strong agreement with independent direct relative
531 density measurements.

532 3. Applicability and limitations

533 ○ The model is most reliable for clean, young, uncemented, uniformly graded
534 siliceous sands under fully saturated or dry conditions.

535 ○ Shortcomings are illustrated for sands with higher compressibility where
536 factors such as fines content and very angular particle shape may alter the
537 penetration response beyond the calibrated range.

538 ○ Within the upper 2-4 cone diameters, reliability decreases due to mudline
539 definition and cone disturbance effects, reinforcing the need for improved
540 mudline definition method.

541 4. Recommendations for practice

542 ○ As natural soils rarely exhibit idealized behaviour, site-specific validation of
543 the model against direct D_r measurements is required, even when soils appear
544 to fall within the calibrated range of the model.

545 ○ Engineering judgement is required to assess potential effects of ageing,
546 bonding, cementation, or intense man-made compaction, where soil fabric
547 effects can produce very high near-surface q_c values not reproduced by the
548 model.

549 ○ Future work could focus on extending calibration to a wider variety of sands
550 (including angular and highly compressible types), further validating across
551 cone sizes, and improving mudline identification methods for shallow
552 penetration analysis.

553

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562

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Table 1. Calibration parameters for $q_{c.deep}$ (Eq. (2)) and $q_{c.shallow}$ (Eq. (6)) for various references.

Reference	Deep ($\sigma' = p'$)			Shallow ($\sigma' = \sigma'_v$)		
	$C_{0.deep}$	$C_{1.deep}$	$C_{2.deep}$	$C_{0.shallow}$	$C_{1.shallow}$	$C_{2.shallow}$
Jamiolkowski et al. (2003)	24.94	0.46	2.96	-	-	-
Lehane et al. (2022)	-	-	-	19	0.7	3.2
Jensen (2024)	25.6	0.7	3.2	19	0.7	3.2
This study	25.6	0.7	3.2	19	0.7	3.8

Table 2. Details of the internal CPT database

Site	No. of tests	Cone size	Geography	Offshore/ onshore	General sand description
A	3	10 cm ²	Denmark	Onshore	Sand fill, siliceous, clean, medium, uniform
B-1	24	10 cm ²	Denmark	Onshore	Sand fill, siliceous, clean, medium to coarse, uniform to well graded
B-2	36	15 cm ²	Denmark	Onshore	Sand fill, siliceous, clean, medium to coarse, uniform to well graded
C	6	10 cm ²	Germany	Onshore	Siliceous, clean, fine to medium, uniform
D	3	10 cm ²	North Sea	Offshore	Post-glacial, siliceous, clean, fine to coarse, uniform
E	1	10 cm ²	North Sea	Offshore	Post-glacial, siliceous, clean, fine, uniform
F	12	10 cm ²	North Sea	Offshore	Post-glacial, siliceous, clean, fine to medium, uniform
G	10	10 cm ²	North Sea	Offshore	Post-glacial, siliceous, clean, fine to medium, uniform
H	37	10 cm ²	Irish Sea	Offshore	Siliceous (no detailed sample descriptions)

Table 3. Summary of interpreted D_r within the upper 1 m of CPT data from the internal database, based on the updated global model. Descriptions correspond to interpretations from top to bottom of the 1 m profile, with numbering indicating the count of CPTs in each category.

Interpretation ¹	Site								
	A	B-1	B-2	C	D	E	F	G	H
Loose	-	-	-	-	1	-	2	1	3
Loose to medium dense	-	1	-	-	-	-	-	1	4
Loose to dense	-	-	-	-	-	-	-	2	-
Medium dense	-	7	1	-	1	-	1	2	12
Medium dense to dense	-	4	-	3	-	-	1	2	4
Medium dense to very dense	-	-	-	-	-	-	4	-	2
Dense to medium dense	-	-	-	-	-	-	-	-	8
Dense	1	12	1	1	1	-	1	1	4
Dense to very dense	-	-	-	2	-	1	3	1	-
Very dense to medium dense	1	-	-	-	-	-	-	-	-
Very dense to dense	1	-	2	-	-	-	-	-	-

¹Very loose: $D_r = 0-0.15$, Loose: $D_r = 0.15-0.35$, Medium dense: $D_r = 0.35-0.65$, Dense: $0.65-0.85$, Very dense: $D_r = 0.85-1$

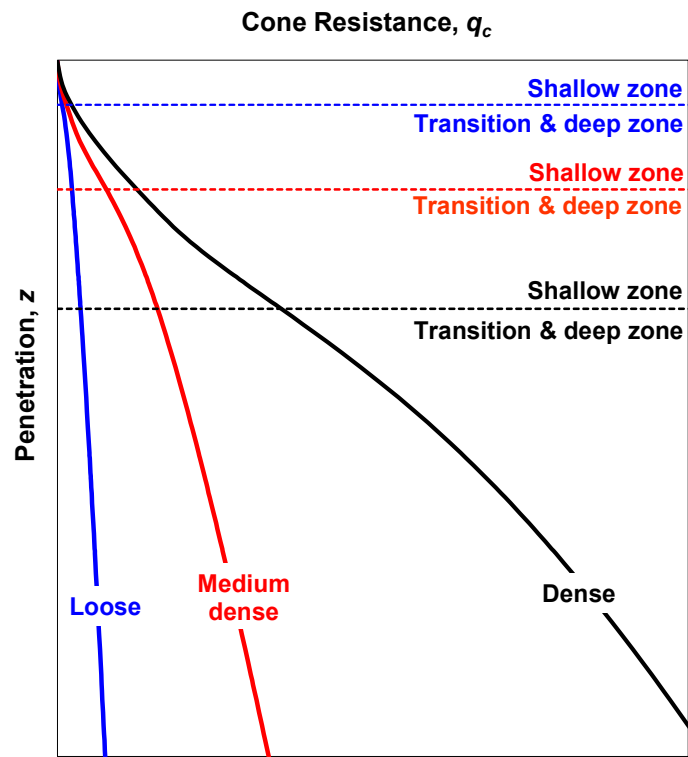


Fig. 1. Idealized shallow depth cone resistance profiles in uniform loose, medium dense, and dense sand.

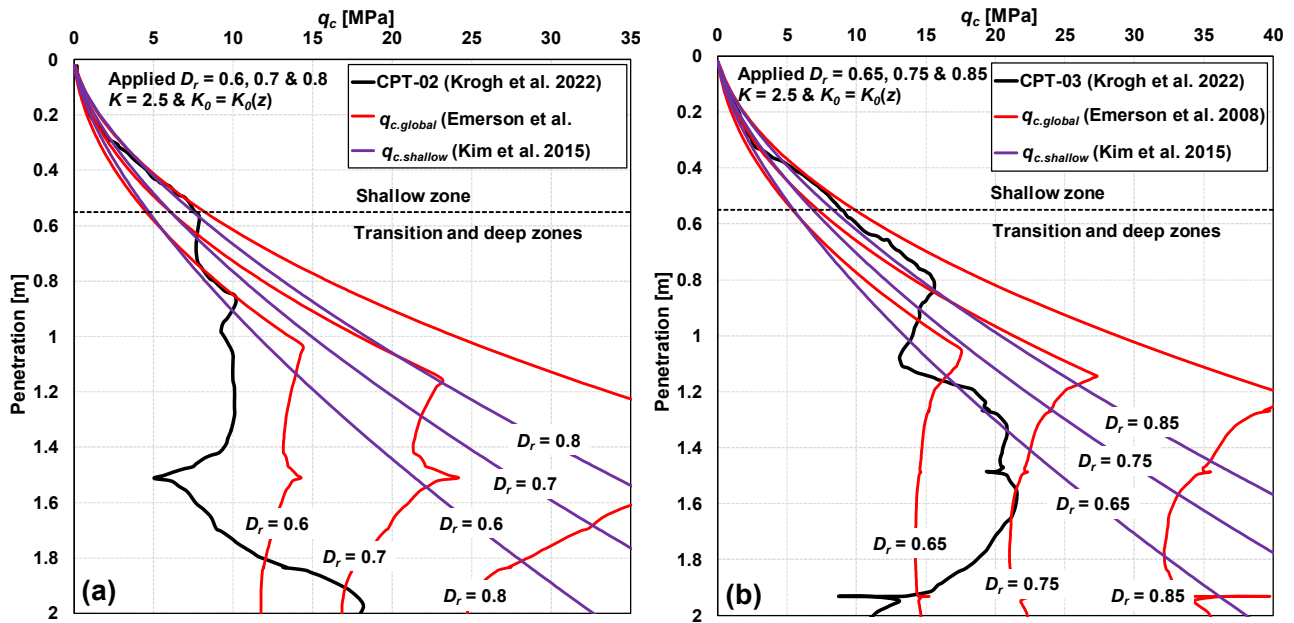


Fig. 2. Performance of Emerson et al. (2008) for shallow depth interpretation in siliceous sand compared with Kim et al. (2015). (a) CPT-02 (Krogh et al. 2022): measured q_c profile compared with $q_{c.global}$ (Emerson et al. 2008) and $q_{c.shallow}$ (Kim et al., 2015) for $D_r = 0.6-0.8$ ($K = 2.5$, $K_0 = K_0(z)$). (b) CPT-03 (Krogh et al. 2022): measured q_c profile compared with $q_{c.global}$ (Emerson et al. 2008) and $q_{c.shallow}$ (Kim et al. 2015) for $D_r = 0.65-0.85$ ($K = 2.5$, $K_0 = K_0(z)$).

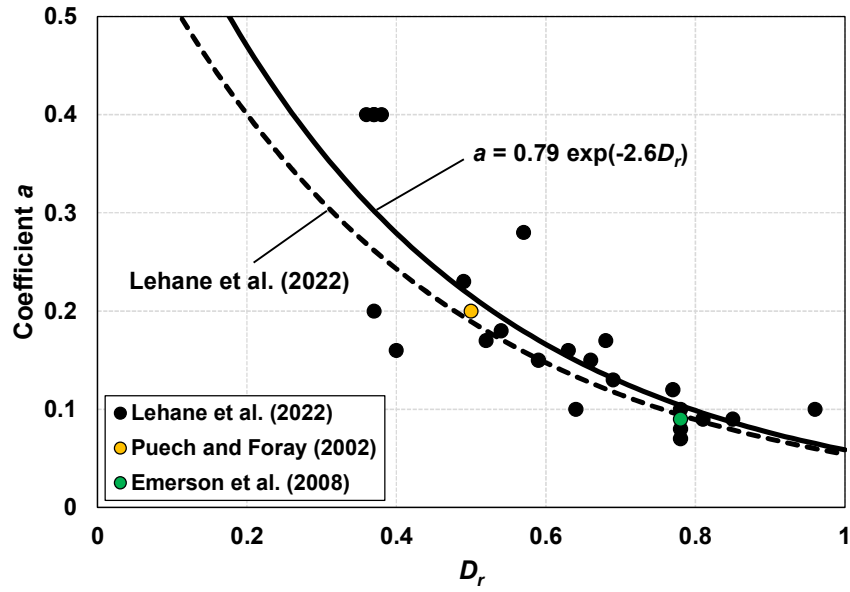


Fig. 3. Variation of coefficient a with D_r , showing data from Lehane et al. (2022), values inferred from data of by Puech and Foray (2002) and Emerson et al. (2008), and the updated expression proposed in this study (Eq. (10)).

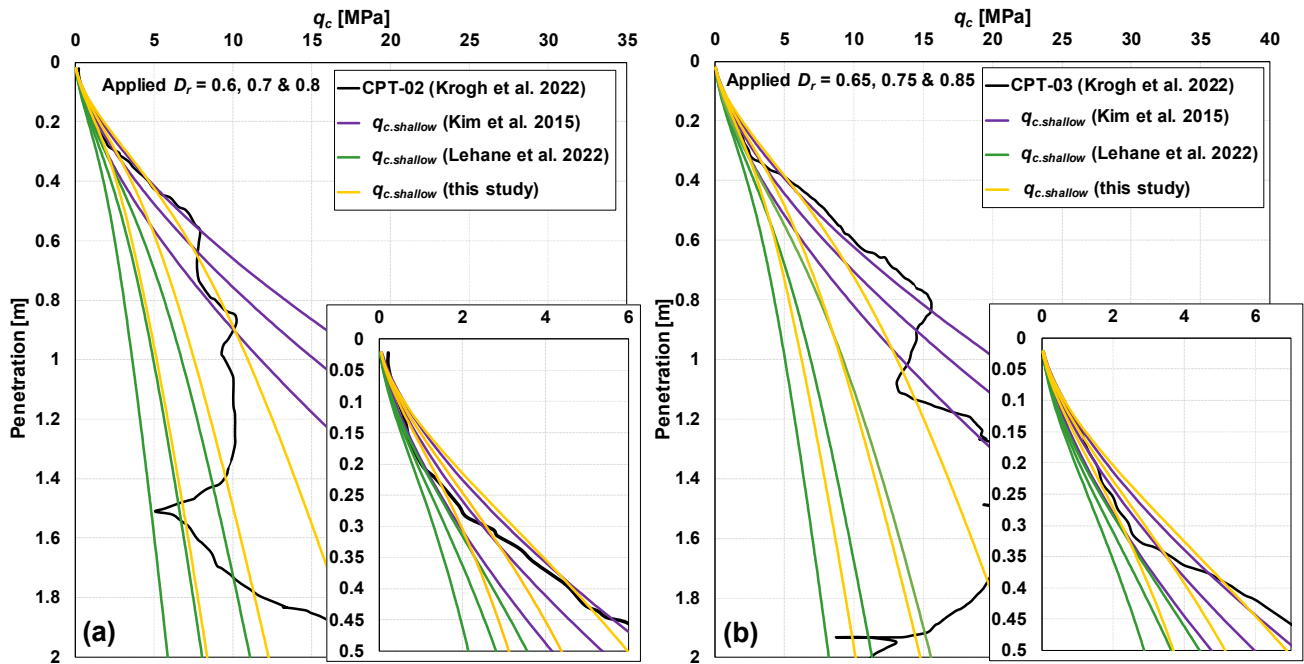


Fig. 4. Comparison of shallow penetration formulations in siliceous sand for CPT-02 and CPT-03 (Krogh et al. 2022). (a) CPT-02: measured q_c compared with $q_{c,shallow}$ from Kim et al. (2015), Lehane et al. (2022), and the modified formulation proposed in this study for $D_r = 0.6-0.8$. (b) CPT-03: measured q_c compared with $q_{c,shallow}$ from Kim et al. (2015), Lehane et al. (2022), and the modified formulation proposed in this study for $D_r = 0.65-0.85$.

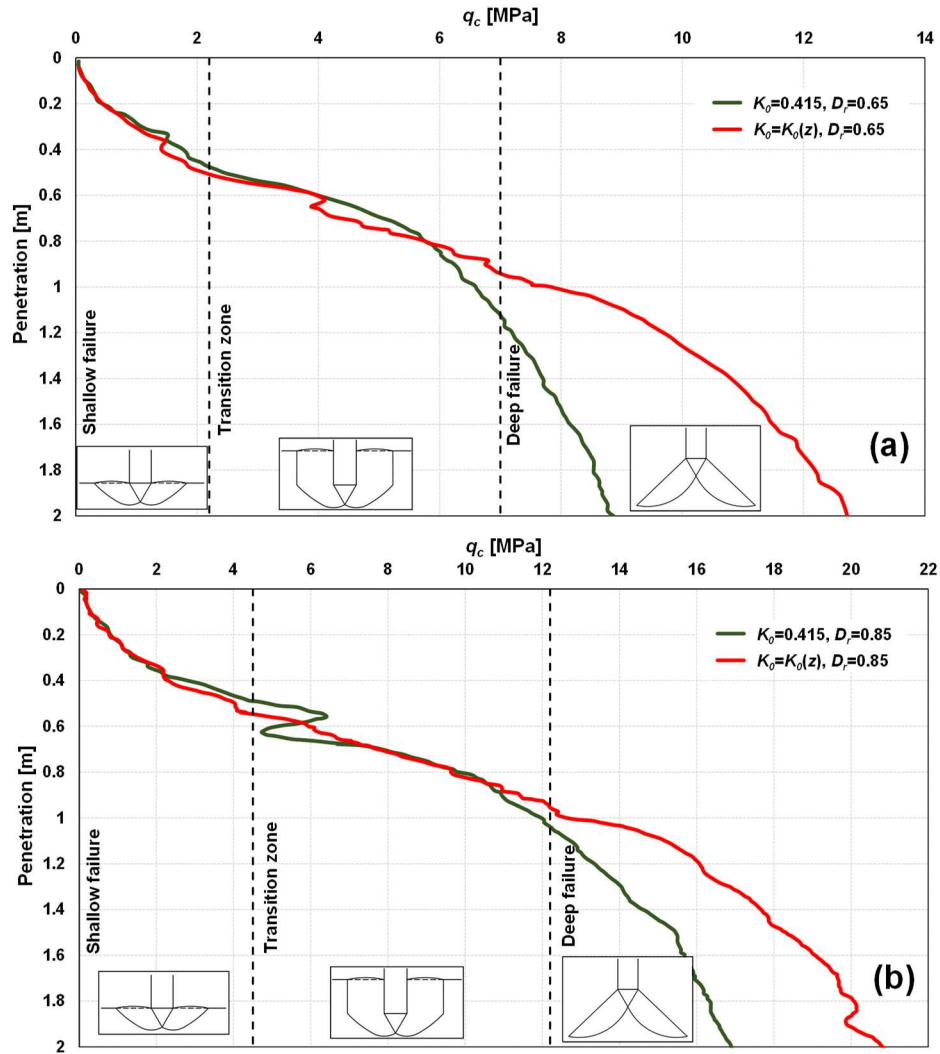


Fig. 5. Numerical simulations by Krogh et al. (2022) illustrating shallow and deep penetration mechanisms. (a) $D_r = 0.65$: comparison of q_c profiles for constant $K_0 = 0.415$ and depth-dependent $K_0(z)$. (b) $D_r = 0.85$: comparison of q_c profiles for constant $K_0 = 0.415$ and depth-dependent $K_0(z)$.

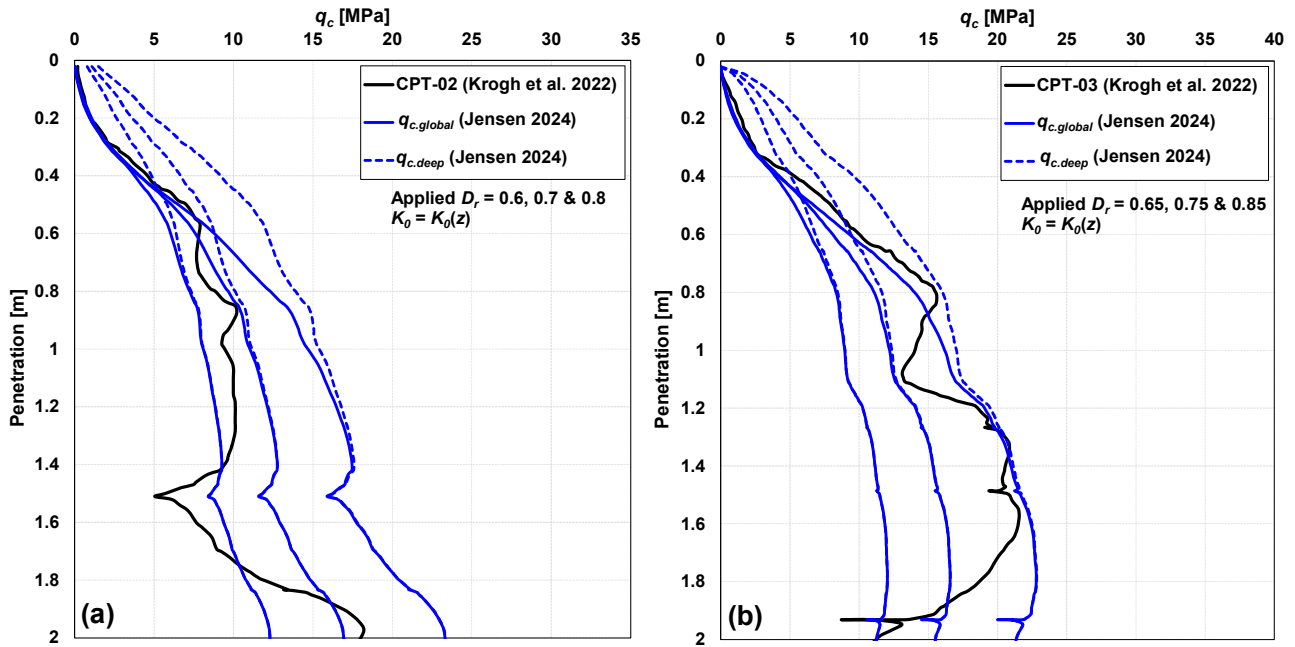


Fig. 6. Performance of Jensen (2024) global model for shallow depth interpretation in siliceous sand. (a) CPT-02 (Krogh et al. 2022): measured q_c compared with $q_{c,global}$ and $q_{c,deep}$ for $D_r = 0.6-0.8$ using $K_0 = K_0(z)$. (b) CPT-03 (Krogh et al. 2022): measured q_c compared with $q_{c,global}$ and $q_{c,deep}$ for $D_r = 0.65-0.85$ using $K_0 = K_0(z)$.

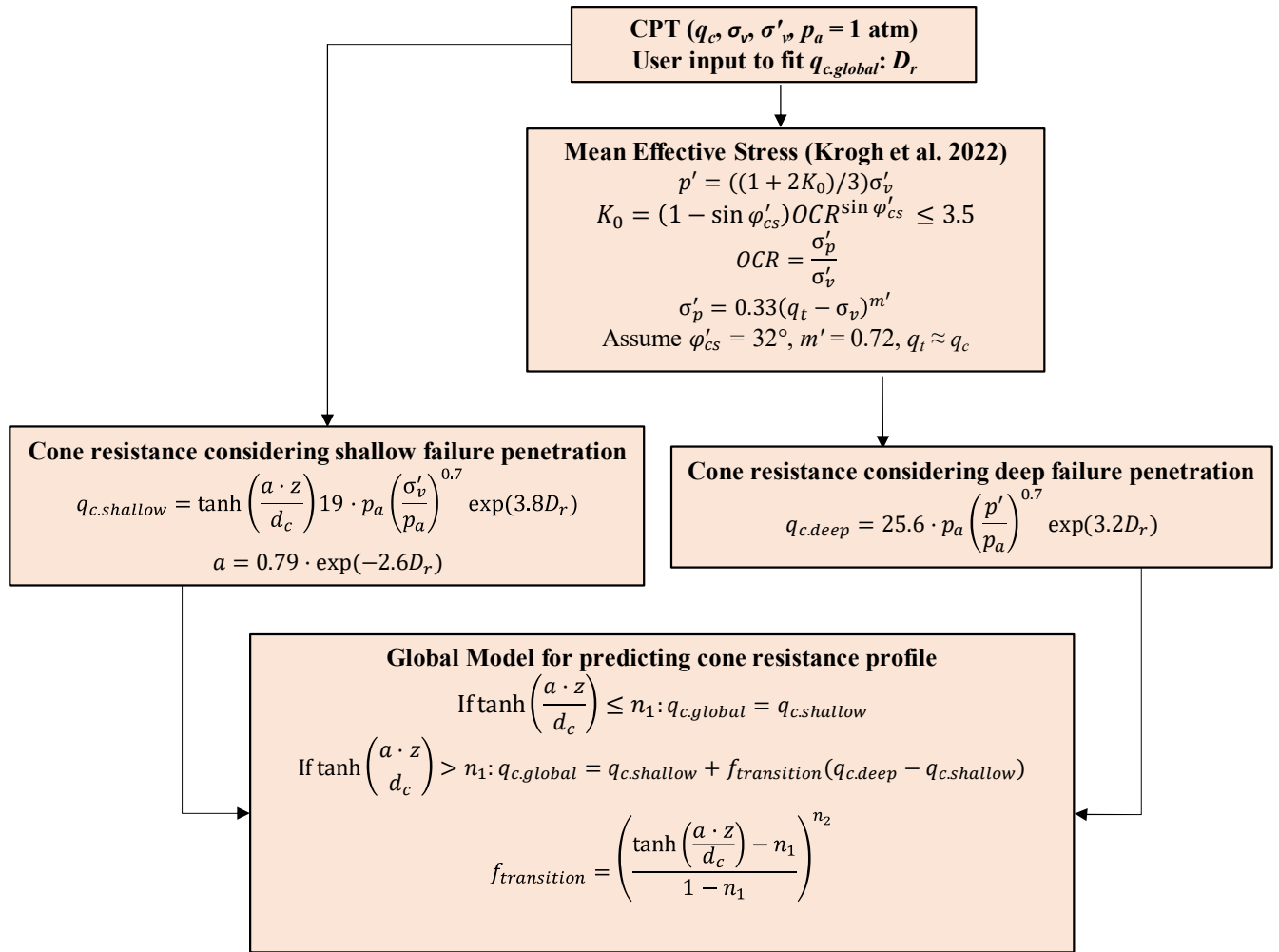


Fig. 7. Flow chart summarizing the updated global model for interpretation of shallow depth CPTs in siliceous sand. The CPT input parameters include cone resistance (q_c), vertical total stress (σ_v), vertical effective stress (σ'_v), and reference stress ($p_a = 100 \text{ kPa}$). The corrected cone resistance is denoted q_t . The preconsolidation stress is σ'_p , the overconsolidation ratio is OCR, and m' is an empirical stress exponent. The transition parameters are suggested as $n_1 = 0.6$ and $n_2 = 2$.

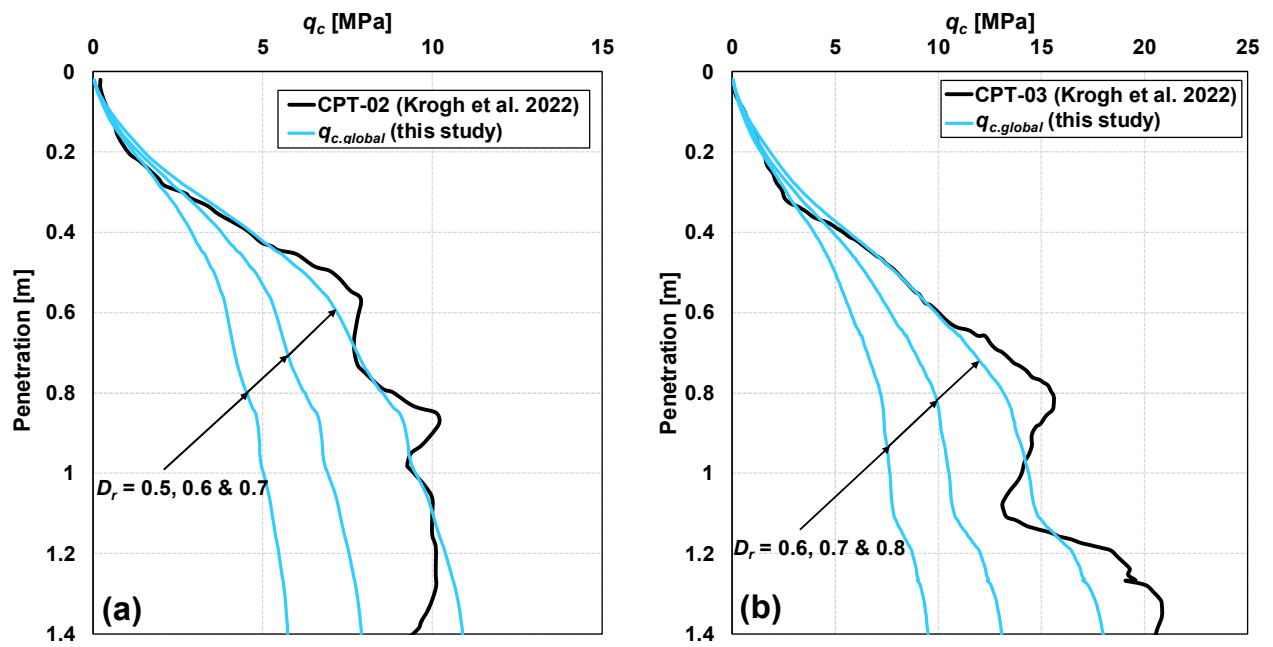


Fig. 8. Performance of updated global model ($q_{c.global}$) for CPTs from Krogh et al. (2022). (a) CPT-02: measured q_c compared with $q_{c.global}$ for $D_r = 0.5$ – 0.7 . (b) CPT-03: measured q_c compared with $q_{c.global}$ for $D_r = 0.6$ – 0.8 .

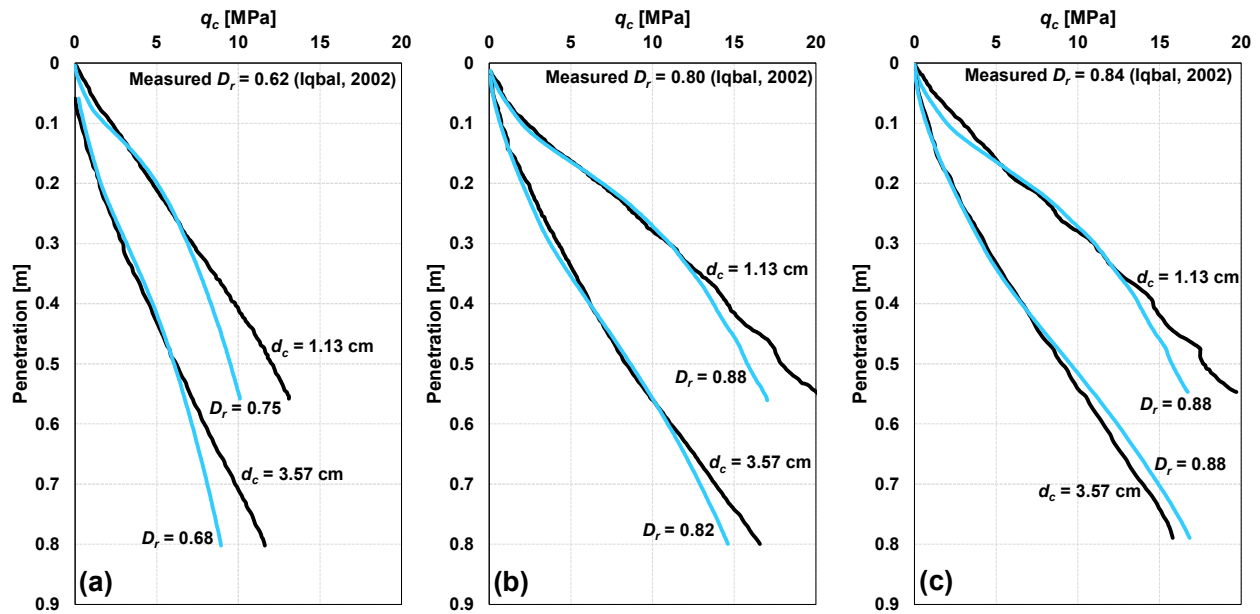


Fig. 9. Evaluation of the updated global model ($q_{c.global}$) against laboratory CPT data from Iqbal (2002), illustrating cone size effects in uniformly graded silica sand. (a) Sand body 1: measured q_c for $d_c = 1.13$ cm and $d_c = 3.57$ cm compared with $q_{c.global}$ for corresponding D_r values. (b) Sand body 2: measured q_c for $d_c = 1.13$ cm and $d_c = 3.57$ cm compared with $q_{c.global}$. (c) Sand body 3: measured q_c for $d_c = 1.13$ cm and $d_c = 3.57$ cm compared with $q_{c.global}$.

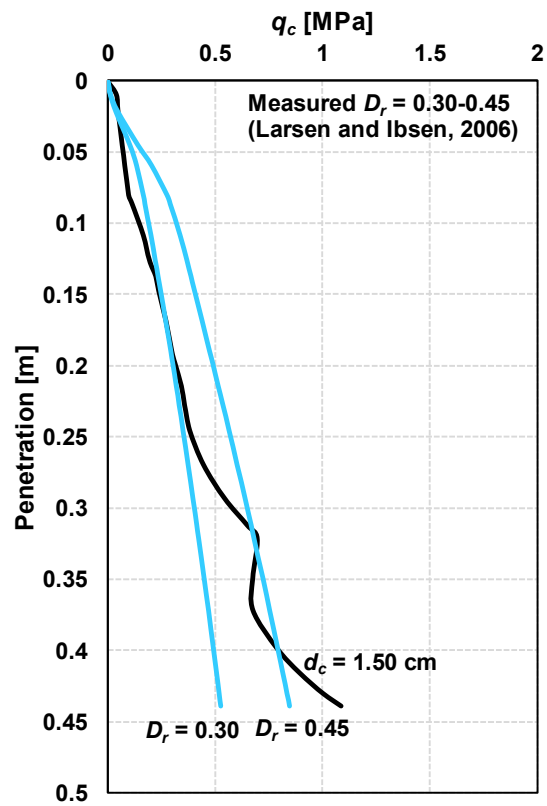


Fig. 10. Comparison between measured and predicted cone resistance profiles from Larsen and Ibsen (2006) in loose, uniformly graded silica sand ($d_c = 1.50$ cm), showing performance of the updated global model ($q_{c,global}$) assuming $K_0 = 0.5$.

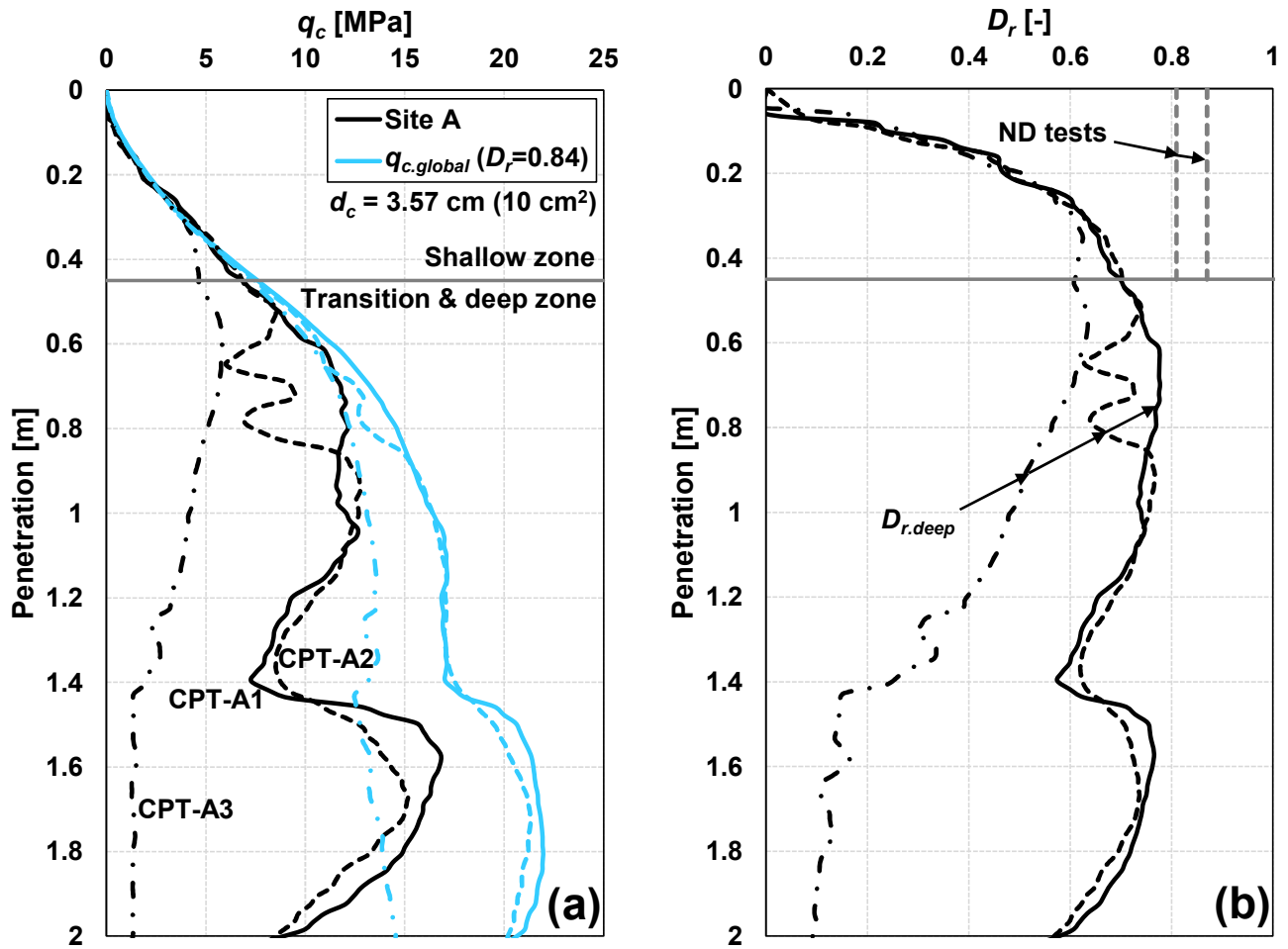


Fig. 11. CPT interpretation at Site A (onshore sand fill). (a) Measured q_c profiles (CPT-A1 to A3) compared with $q_{c.global}$ for $D_r \approx 0.84$ in the shallow zone. (b) Relative density profiles: comparison between $D_{r,deep}$ and nuclear densometer (ND) measurements.

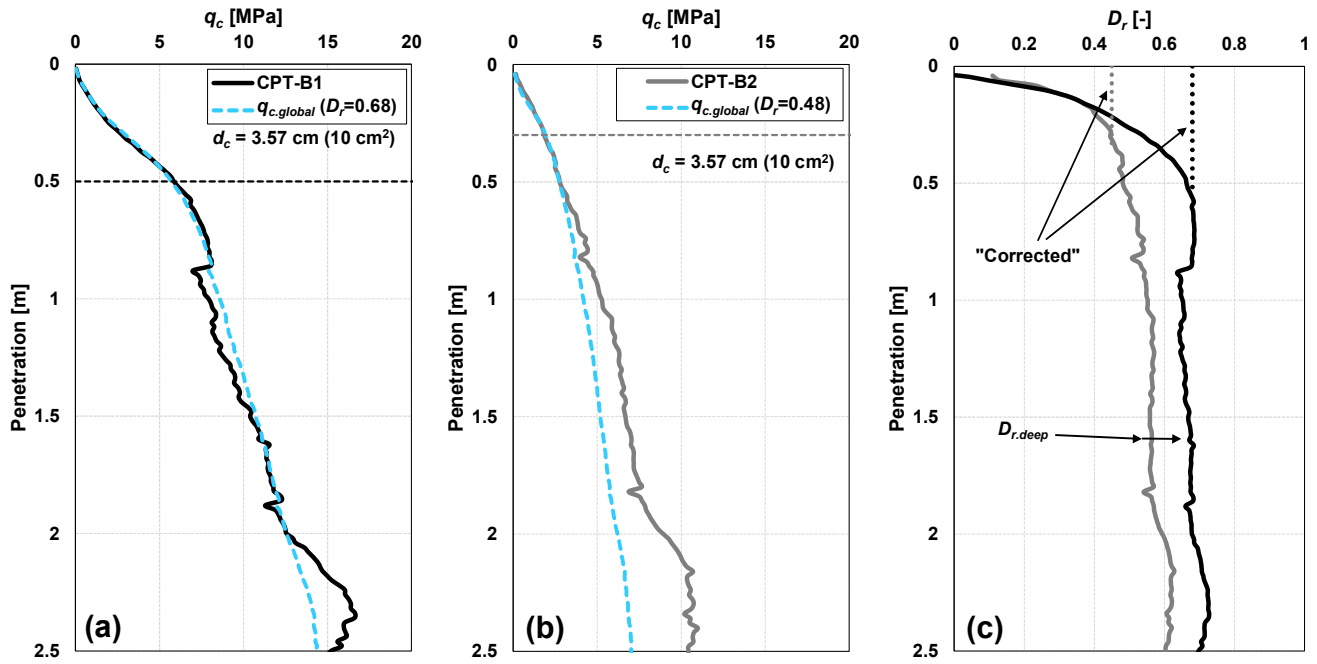


Fig. 12. CPT interpretation at Site B using 10 cm² cone. (a) CPT-B1: measured q_c compared with $q_{c,global}$; shallow $D_r \approx 0.68$. (b) CPT-B2: measured q_c compared with $q_{c,global}$; shallow $D_r \approx 0.48$. (c) Corresponding $D_{r,deep}$ profiles and indicative corrected shallow D_r values.

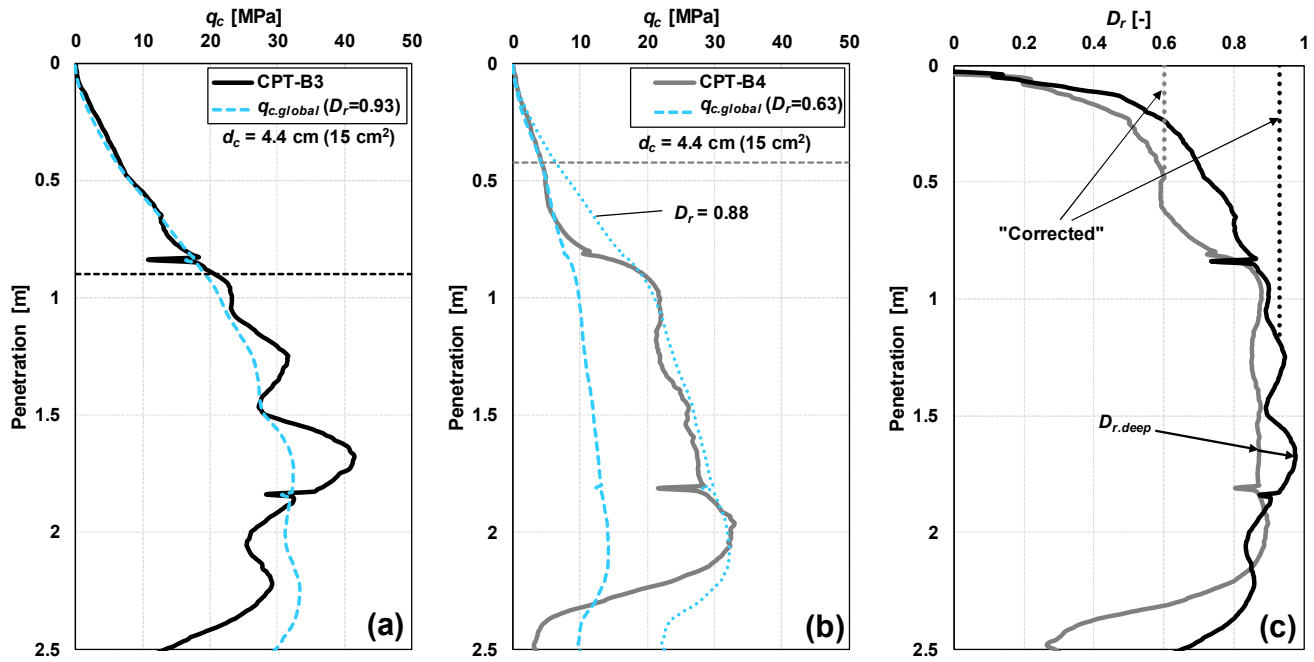


Fig. 13. CPT interpretation at Site B using 15 cm² cone. (a) CPT-B3: measured q_c compared with $q_{c,global}$; shallow $D_r \approx 0.93$. (b) CPT-B4: measured q_c compared with $q_{c,global}$; shallow $D_r \approx 0.63$. (c) Corresponding $D_{r,deep}$ profiles illustrating density variation with depth.

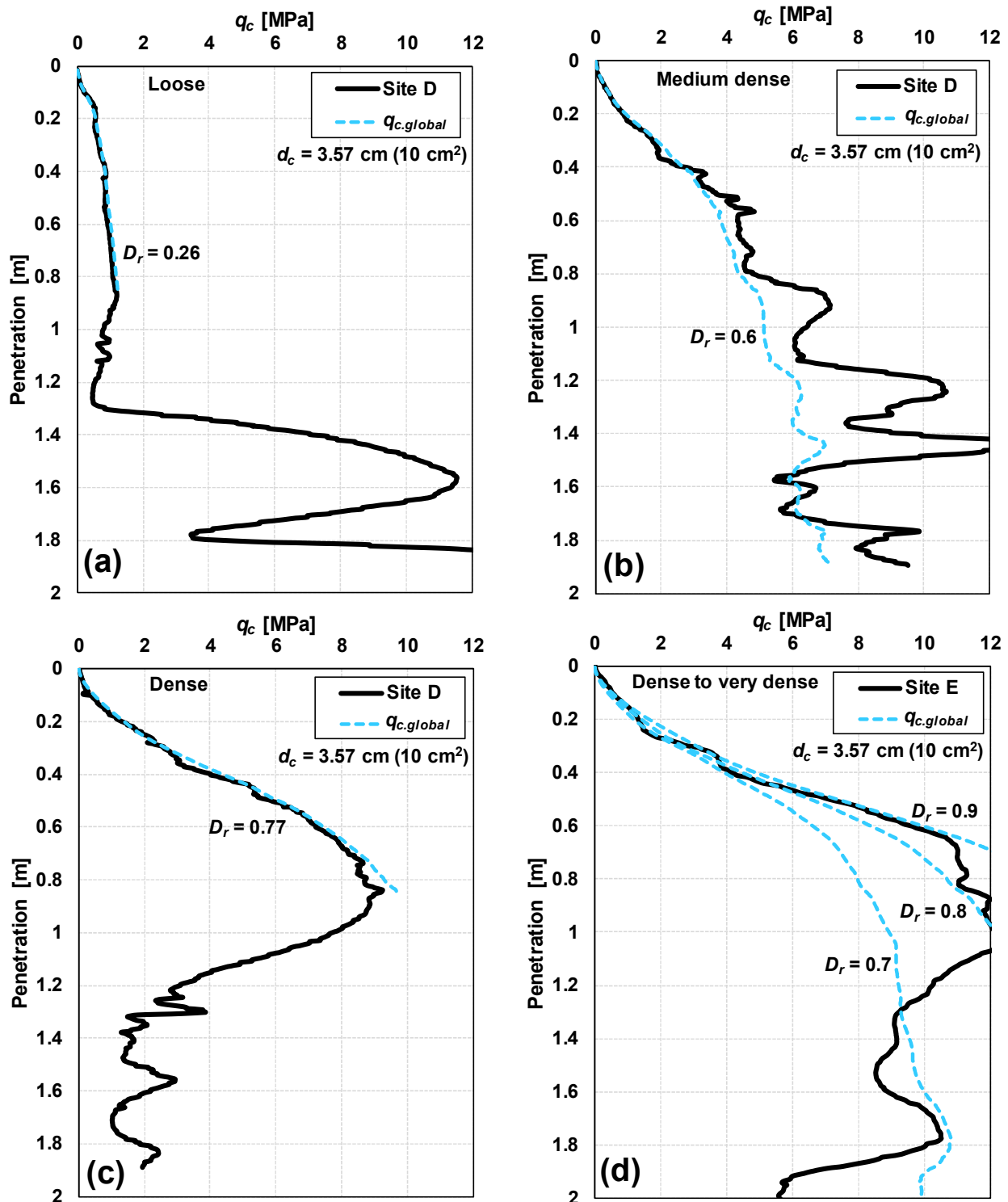


Fig. 14. Offshore CPTs from the North Sea illustrating shallow depth interpretation using the updated global model. (a) Loose sand ($D_r \approx 0.26$). (b) Medium dense sand ($D_r \approx 0.60$). (c) Dense near-surface sand layer ($D_r \approx 0.77$). (d) Dense to very dense sand with slight vertical variability ($D_r \approx 0.7-0.9$).

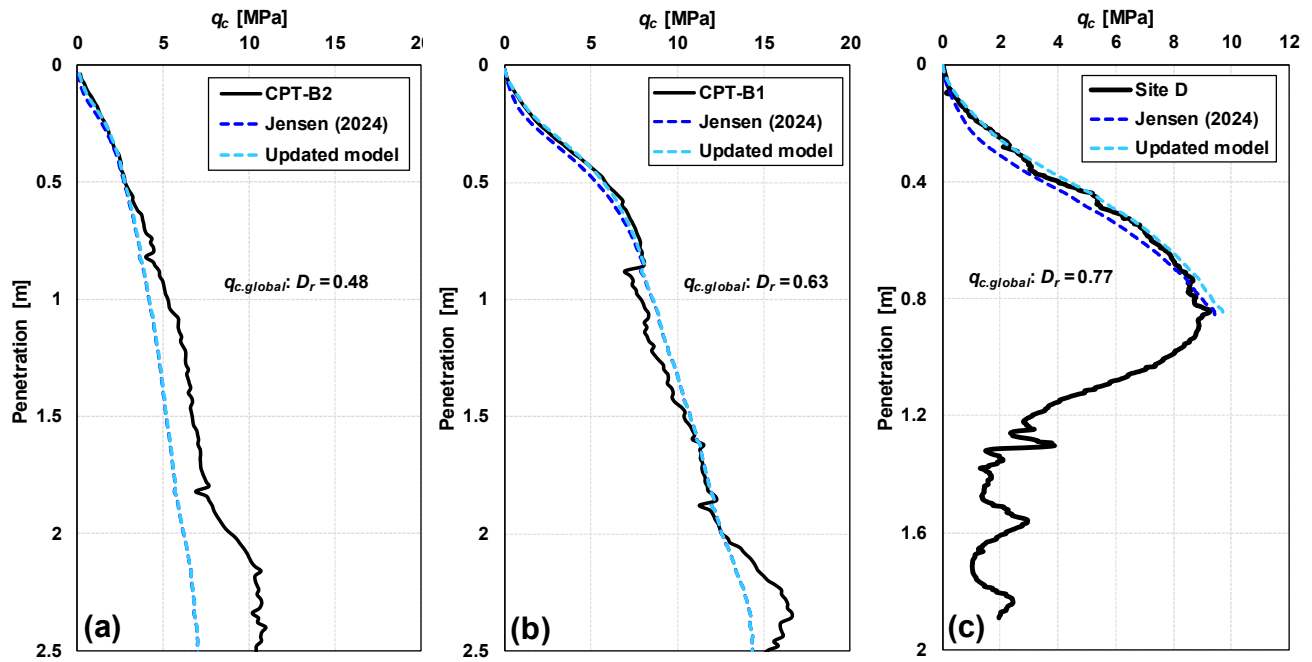


Fig. 15. Comparison between Jensen (2024) global model and the updated model. (a) CPT-B2; $q_{c,global}: D_r = 0.48$. (b) CPT-B1; $q_{c,global}: D_r = 0.63$. (c) Site D; $q_{c,global}: D_r = 0.77$.

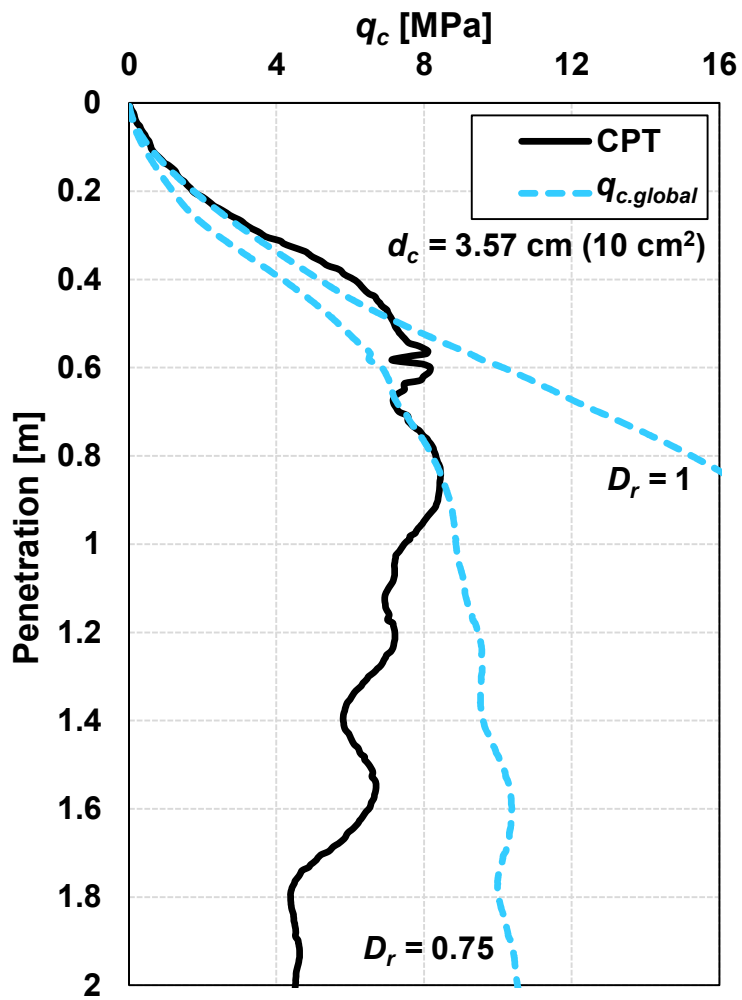


Fig. 16. Example application with poor performance of updated global model ($q_{c,global}$) for an offshore CPT from the North Sea.

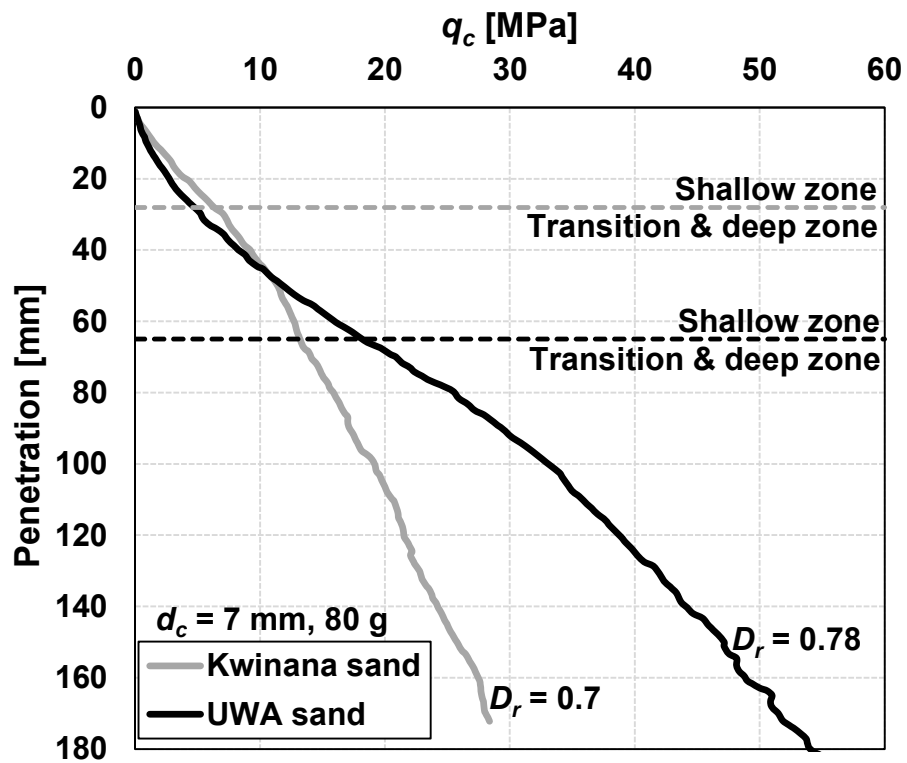


Fig. 17. Centrifuge CPTs in a high compressibility carbonate sand (Kwinana sand) and a moderately compressible sand (UWA sand) (measured data from Liu and Lehane, 2020).