

Design and Performance Analysis of a Two-Stage Spur Gear Transmission for a Passenger Vehicle

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Abstract

This paper presents the design and validation of a two-stage spur gear transmission for a passenger vehicle. The gearbox must enable a laden mass of 1100 kg to achieve both a maximum speed of 100 km/h and a gradeability of 30°, within a compact packaging envelope. This study integrates vehicle resistance analysis, tractive effort calculations, gear ratio determination, and stress verification according to ISO 6336 and Shigley's design methodology. Engine data and vehicle resistances are used to compute tractive effort curves, confirming that the design achieves the required performance while maintaining adequate safety margins. The final results demonstrate that the proposed gearbox provides reliable performance under duty cycle conditions, following the transmission design framework described by Naunheimer et al. [1]–[3].

I. INTRODUCTION

Gear transmissions are critical subsystems in automotive powertrains, responsible for converting engine output into usable torque and speed at the wheels. For compact passenger vehicles, the gearbox must balance multiple requirements: providing high torque multiplication for gradeability and acceleration, while also enabling efficient operation at cruising speeds. Achieving these targets within strict packaging and cost constraints is a fundamental engineering challenge in transmission design.

As outlined by Naunheimer *et al.* [1], a vehicle transmission must adapt engine torque and rotational speed to the tractive requirements of the vehicle while ensuring efficiency, comfort, and durability. The authors describe the design process as an iterative balance between performance, energy efficiency, and durability, a triad that defines modern gearbox optimization.

Spur gear transmissions remain widely used due to their high efficiency, simplicity, and manufacturability compared with helical or planetary systems. However, achieving the correct balance of ratios requires detailed analysis of vehicle resistances, tractive effort, and gear stresses. These calculations form the basis of ratio selection and mechanical sizing, as detailed by Shigley and Nisbett [2] and standardized in ISO 6336 [3].

This study develops a two-stage spur-gear transmission for a passenger vehicle with a laden mass of 1100 kg. The vehicle must achieve a top speed of 100 km/h and climb gradients up to 30°, using a final-drive ratio of 5.0. The design integrates:

- Vehicle resistance and tractive-effort analysis [1],
- Gear-ratio selection based on top-speed and gradeability targets,
- Stress verification using ISO 6336 and Shigley's methods,
- Shaft and bearing life estimation under a defined duty cycle.

The results confirm that the gearbox meets performance and durability targets, with tractive effort exceeding resistance forces across the speed range and safety factors above unity for both bending and contact stresses.

II. VEHICLE AND ENGINE DATA

The gearbox design process begins with defining the key vehicle and engine parameters, which serve as inputs for resistance, tractive-effort, and ratio analyses. These parameters are summarized in Table I.

TABLE I: Vehicle specifications

Parameter	Value
Laden mass	1100 kg
Tyre rolling circumference	2240 mm
Effective tyre radius	0.356 m
Rear-axle (final-drive) ratio	5.0
Minimum vehicle speed	8 km/h
Maximum vehicle speed	100 km/h
Frontal area A	2.0 m ²
Aerodynamic drag coefficient C_d	0.30
Air density ρ	1.199 kg/m ³
Required gradability	30°
Rolling-resistance coefficient C_{rr}	0.022

The engine data, obtained from dynamometer testing, are listed in Table II. These data form the basis for the tractive-effort analysis. The methodology for converting torque–speed curves into vehicle performance follows the traction-diagram approach described by Naunheimer *et al.* [1].

TABLE II: Engine torque and power vs. RPM

Engine Speed (rpm)	Torque (Nm)	Power (kW)
800	150	12.57
1100	180	20.73
1400	210	30.79
1700	240	42.73
2000	225	49.22
2300	235	54.19
2600	215	58.54
2900	180	54.66
3200	140	46.91
3500	100	36.65

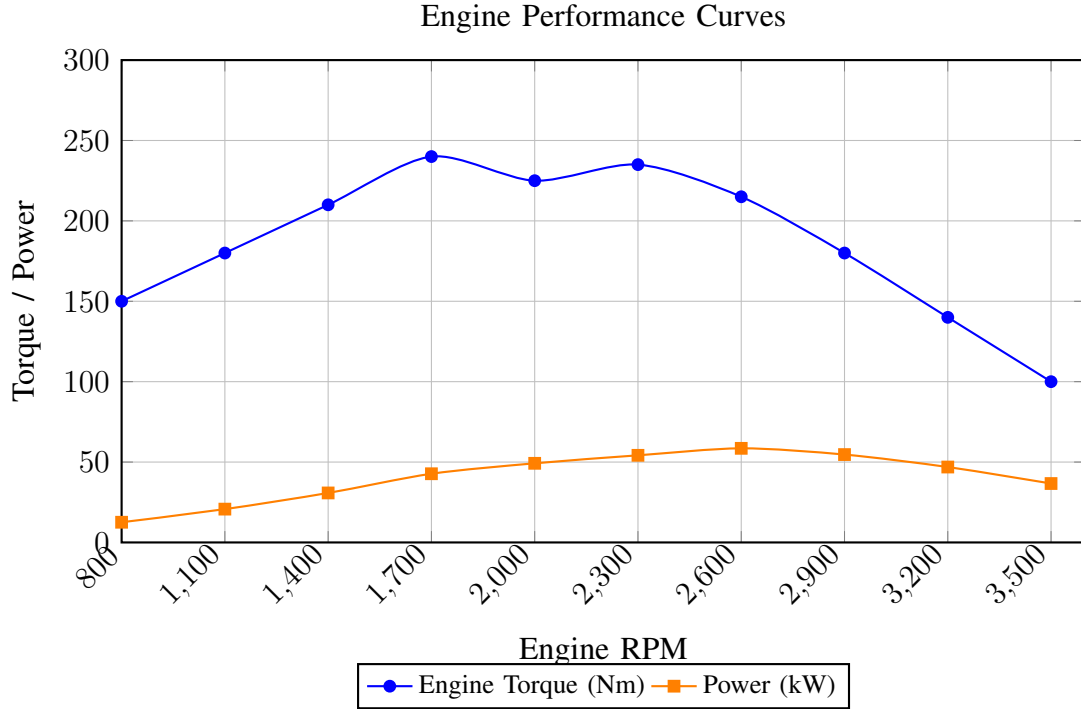


Fig. 1: Engine performance curves showing torque and power vs. RPM.

III. VEHICLE RESISTANCES

The total tractive resistance acting on a vehicle comprises rolling resistance, aerodynamic drag, and grade resistance. These forces determine the tractive effort required from the powertrain to maintain motion and are evaluated using the methods described by Naunheimer *et al.* [1] and Shigley and Nisbett [2].

A. Rolling Resistance

The rolling-resistance force is given by

$$F_{rr} = C_{rr} mg$$

$$F_{rr} = 0.022 \times 1100 \times 9.81 \approx 237 \text{ N}$$

where C_{rr} is the rolling-resistance coefficient and mg is the normal load on the tyres.

B. Aerodynamic Drag

The aerodynamic-drag force is

$$F_d = \frac{1}{2} \rho C_d A v^2$$

At $v = 100 \text{ km/h} = 27.78 \text{ m/s}$:

$$F_d = 0.5 \times 1.199 \times 0.30 \times 2.0 \times (27.78)^2 \approx 277 \text{ N.}$$

C. Grade Resistance

The grade-resistance force for an incline angle θ is

$$F_g = mg \sin \theta$$

For a 30° slope:

$$F_g = 1100 \times 9.81 \times \sin(30^\circ) \approx 5396 \text{ N.}$$

D. Summary of Resistances

TABLE III: Vehicle-resistance forces at design conditions

Resistance	Value (N)
Rolling resistance	237
Aerodynamic drag (at 100 km/h)	277
Grade resistance (30° slope)	5396

The total resistance to motion on a 30° slope therefore approaches 5910 N, which sets the lower bound on tractive effort required at the wheels.

IV. GEAR RATIO DETERMINATION

According to Naunheimer *et al.* [1], the overall powertrain ratio is defined as

$$i_A = i_S i_G i_E$$

where i_S is the starting-element ratio, i_G is the gearbox ratio, and i_E is the final-drive ratio. This relationship ensures that the vehicle simultaneously meets top-speed and gradeability targets within the usable engine-torque range.

A. Top-Speed Condition

At a vehicle speed of $v = 100 \text{ km/h} = 27.78 \text{ m/s}$, the wheel speed is

$$n_w = \frac{v}{C} \times 60 = \frac{27.78}{2.240} \times 60 \approx 745 \text{ rpm.}$$

At maximum engine speed $n_e = 3500 \text{ rpm}$,

$$i_{\text{tot}} = \frac{n_e}{n_w} = \frac{3500}{745} \approx 4.70.$$

With a final-drive ratio $i_{fd} = 5.0$,

$$i_g = \frac{i_{\text{tot}}}{i_{fd}} = \frac{4.70}{5.0} \approx 0.94.$$

Thus, the top-gear ratio is slightly below unity (an overdrive), allowing 100 km/h at rated speed.

B. Gradeability Condition

The tractive torque required to overcome the grade resistance is

$$T_w = F_g r = 5396 \times 0.356 \approx 1920 \text{ N}\cdot\text{m.}$$

Given peak engine torque $T_e = 235 \text{ N}\cdot\text{m}$, gearbox efficiency $\eta = 0.9$, and final drive $i_{fd} = 5.0$,

$$i_g = \frac{T_w}{T_e i_{fd} \eta} = \frac{1920}{235 \times 5.0 \times 0.9} \approx 1.82.$$

Hence, the lowest gear must provide $i_g \approx 1.82$ to meet the 30°-grade requirement. This corresponds to the traction-diagram intersection described by Naunheimer *et al.* [1], where tractive and resistance curves define feasible ratios.

C. Gear-Ratio Summary

- Low gear (gradeability): $i_g \approx 1.82$
- High gear (top speed): $i_g \approx 0.94$

The gearbox therefore uses two forward ratios: one for maximum tractive effort and one overdrive for efficient cruising.

V. TRACTIVE EFFORT ANALYSIS

The available tractive effort at the wheels is computed using

$$F_{TE} = \frac{T_e i_g i_{fd} \eta}{r}$$

where T_e is the engine torque, i_g the gearbox ratio, $i_{fd} = 5.0$ the final-drive ratio, $\eta = 0.9$ the transmission efficiency, and $r = 0.356$ m the tyre radius. This relationship follows the torque-transmission formulation presented by Shigley and Nisbett [2] and is applied here using the traction-diagram framework described by Naunheimer *et al.* [1].

Table IV presents the calculated tractive effort for both gears, mapped to corresponding vehicle speeds. These data confirm that the 1st gear provides sufficient torque for climbing a 30° grade, while the 2nd gear allows the vehicle to achieve its design top speed.

TABLE IV: Tractive effort vs. vehicle speed (ideal, 1st gear, and 2nd gear)

Engine RPM	Vehicle Speed (km/h)	TE Ideal (N)	TE 1st Gear (N)	TE 2nd Gear (N)
800	21.50	9800	5657	1978
1100	29.57	7127	6784	2373
1400	37.63	5600	7917	2769
1700	45.70	4612	9049	3164
2000	53.76	3920	8860	3098
2300	61.82	3409	8482	2966
2600	69.89	3015	8106	2835
2900	77.95	2704	6785	2373
3200	86.02	2450	5277	1846
3500	94.08	2240	3770	1318

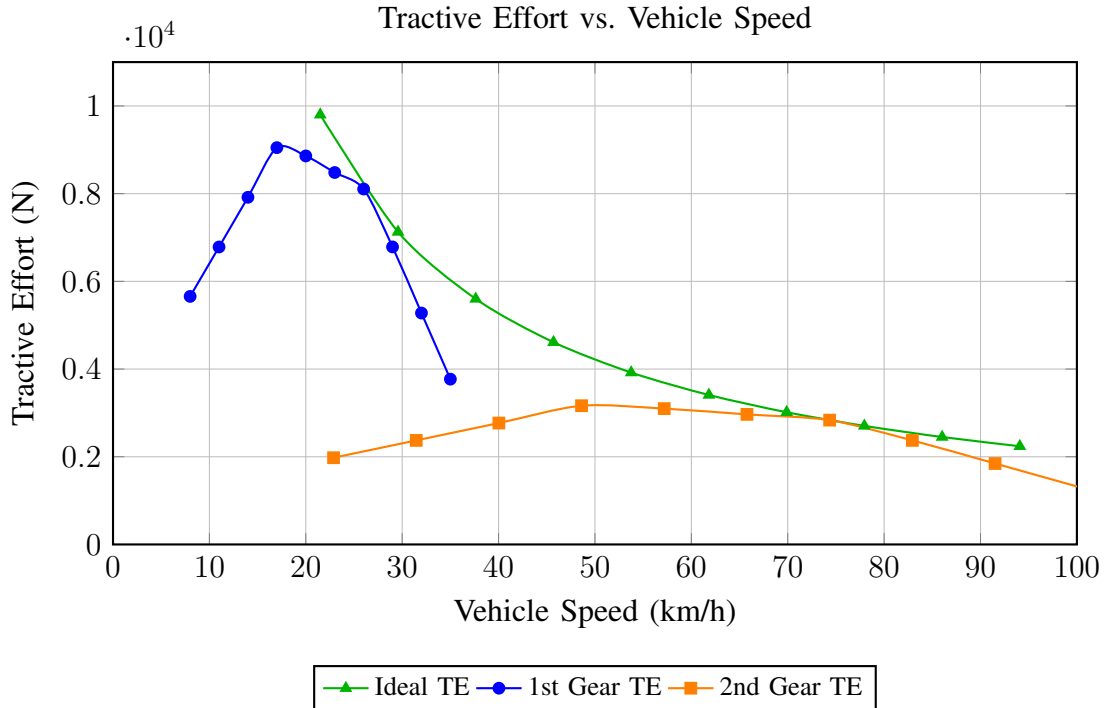


Fig. 2: Tractive-effort curves for ideal case, 1st gear, and 2nd gear.

The tractive-effort curves show that the 1st gear provides adequate tractive force for gradeability, while the 2nd gear enables the vehicle to reach the 100 km/h top speed where tractive and resistive forces intersect.

VI. GEAR GEOMETRY

With the required ratios established from the tractive-effort analysis ($i_{\text{low}} = 1.82$, $i_{\text{high}} = 0.94$), suitable gear-tooth counts and geometry were selected. Spur gears with a 20° pressure angle were chosen for their simplicity, high efficiency, and ease of manufacture, as recommended by Shigley and Nisbett [2] and supported by Naunheimer *et al.* [1].

A. Tooth Selection

Integer tooth counts were selected to match the required ratios while avoiding undercutting ($z \geq 17$). The chosen combinations are:

- Low gear (gradeability): 19-tooth pinion driving a 35-tooth gear ($i \approx 1.84$)
- High gear (top speed): 31-tooth pinion driving a 29-tooth gear ($i \approx 0.94$)

B. Center Distance

The center distance for spur gears is given by

$$a = \frac{m}{2}(z_1 + z_2)$$

For module $m = 5$ mm,

$$a_{\text{low}} = 0.5 \times 5 \times (19 + 35) = 135 \text{ mm}$$

$$a_{\text{high}} = 0.5 \times 5 \times (31 + 29) = 150 \text{ mm}$$

Both values are within compact automotive-transmission limits.

C. Addendum, Dedendum, and Clearance

$$h_a = m = 5 \text{ mm}, \quad h_f = 1.25m = 6.25 \text{ mm}, \quad c = 0.25m = 1.25 \text{ mm}.$$

These proportions conform to standard metric spur-gear design practice defined in ISO 6336 [3].

VII. STRESS ANALYSIS

To verify the mechanical integrity of the gears, both bending and surface-contact stresses were evaluated. The calculations follow the procedures outlined by Shigley and Nisbett [2] for the Lewis and Hertzian equations and are validated against the permissible limits in ISO 6336 [3]. The analytical approach and performance interpretation parallel the methodology presented by Naunheimer *et al.* [1], who emphasize the interdependence of tooth geometry, load distribution, and material properties in transmission durability.

A. Torque Transmission

From the tractive-effort curves:

- Peak wheel torque in 1st gear 9049 N at $v \approx 17$ km/h
- Peak wheel torque in 2nd gear 3164 N at $v \approx 49$ km/h

These values are used to determine the tangential loads on the gears.

B. Tangential Load

The tangential tooth load is

$$F_t = \frac{2T}{d}$$

where $d = mz$ is the pitch diameter.

a) *1st-Gear Pinion (19 teeth):*

$$d_p = 5 \times 19 = 95 \text{ mm}$$

$$T_{pinion} = 9049 \times 0.356 \approx 3220 \text{ N}\cdot\text{m}$$

$$F_t = \frac{2 \times 3220}{0.095} \approx 67,800 \text{ N}$$

b) *2nd-Gear Pinion (31 teeth):*

$$d_p = 5 \times 31 = 155 \text{ mm}$$

$$T_{pinion} = 3164 \times 0.356 \approx 1125 \text{ N}\cdot\text{m}$$

$$F_t = \frac{2 \times 1125}{0.155} \approx 14,500 \text{ N}$$

C. Bending Stress (Lewis Equation)

$$\sigma_b = \frac{F_t}{b m Y}, \quad b = 12m = 60 \text{ mm}$$

The Lewis form factor Y is

$$Y = 0.484 - \frac{2.87}{z}.$$

a) 1st-Gear Pinion (19 teeth):

$$Y = 0.484 - \frac{2.87}{19} = 0.333, \quad \sigma_b = \frac{67,800}{0.060 \times 5 \times 0.333} \approx 680 \text{ MPa.}$$

b) 2nd-Gear Pinion (31 teeth):

$$Y = 0.484 - \frac{2.87}{31} = 0.391, \quad \sigma_b = \frac{14,500}{0.060 \times 5 \times 0.391} \approx 124 \text{ MPa.}$$

D. Contact Stress (Hertzian Equation)

$$\sigma_c = Z_E \sqrt{\frac{F_t}{b d_p Z_H}}, \quad Z_E = 189, \quad Z_H = 2.4.$$

a) 1st-Gear Pinion:

$$\sigma_c = 189 \sqrt{\frac{67,800}{0.060 \times 0.095 \times 2.4}} \approx 1100 \text{ MPa.}$$

b) 2nd-Gear Pinion:

$$\sigma_c = 189 \sqrt{\frac{14,500}{0.060 \times 0.155 \times 2.4}} \approx 490 \text{ MPa.}$$

E. Stress Summary

TABLE V: Gear-stress results and safety factors

Gear	σ_b (MPa)	σ_c (MPa)	Allowables (MPa)	Safety Factors
1st Gear Pinion (19T)	680	1100	$\sigma_b = 250, \sigma_c = 1000$	$SF_b = 0.37, SF_c = 0.91$
2nd Gear Pinion (31T)	124	490	$\sigma_b = 250, \sigma_c = 1000$	$SF_b = 2.0, SF_c = 2.0$

The second-gear set operates safely in both bending and contact, whereas the first-gear pinion is critically stressed in bending and marginal in surface durability. To improve reliability, increasing face width or using a high-strength, case-hardened steel, both strategies recommended by Shigley [2] and Naunheimer *et al.* [1], is advised.

VIII. SHAFT AND BEARING DESIGN

The gearbox consists of three shafts—input, intermediate, and output—each designed for torsional strength and supported by deep-groove ball bearings. The sizing and fatigue checks follow the torsional-strength criteria from Shigley and Nisbett [2], while alignment and stiffness considerations reflect the layout principles described by Naunheimer *et al.* [1].

A. Torsional Strength of Shafts

For a solid circular shaft, the maximum shear stress is

$$\tau = \frac{16T}{\pi d^3}.$$

Rearranging for the required diameter,

$$d = \left(\frac{16T}{\pi \tau_{\text{allow}}} \right)^{1/3}.$$

For AISI 1045 steel with $S_y \approx 350$ MPa,

$$\tau_{\text{allow}} = 0.5S_y = 175 \text{ MPa.}$$

a) *Input Shaft*: Carries maximum engine torque $T = 235 \text{ N}\cdot\text{m}$:

$$d = \left(\frac{16 \times 235}{\pi \times 175 \times 10^6} \right)^{1/3} \approx 24 \text{ mm.}$$

b) *Intermediate Shaft*: Carries peak torque in 1st gear ($T = 3220 \text{ N}\cdot\text{m}$):

$$d = \left(\frac{16 \times 3220}{\pi \times 175 \times 10^6} \right)^{1/3} \approx 57 \text{ mm.}$$

c) *Output Shaft*: Carries torque in 2nd gear ($T = 1125 \text{ N}\cdot\text{m}$):

$$d = \left(\frac{16 \times 1125}{\pi \times 175 \times 10^6} \right)^{1/3} \approx 38 \text{ mm.}$$

All diameters maintain safety factors above unity and comply with Shigley's preliminary shaft-sizing guidelines.

B. Bearing Life Estimation

Deep-groove ball bearings are selected for their radial-load capacity and simplicity. Bearing fatigue life is calculated using

$$L_{10} = \left(\frac{C}{P} \right)^p, \quad p = 3,$$

and converted to operating hours by

$$L_{10h} = \frac{10^6}{60n} \left(\frac{C}{P} \right)^p.$$

a) *Example Calculation*: For a bearing with dynamic load rating $C = 40 \text{ kN}$, applied load $P = 5 \text{ kN}$, and speed $n = 3000 \text{ rpm}$:

$$L_{10} = \left(\frac{40,000}{5000} \right)^3 = 512,$$

$$L_{10h} = \frac{10^6}{60 \times 3000} \times 512 \approx 2844 \text{ hours.}$$

This life exceeds the required duty-cycle duration by nearly five times, satisfying both ISO 6336 and Shigley's fatigue-life recommendations [2], [3].

IX. DUTY CYCLE ANALYSIS

Transmission components must maintain strength and reliability across the range of operating conditions encountered in real-world use. The methodology for translating service conditions into cumulative load exposure follows the approach described by Naunheimer *et al.* [1], which accounts for varying torque–speed combinations over time.

A. Representative Duty Cycle

TABLE VI: Representative duty cycle of the vehicle

Condition	Torque (N·m)	Engine Speed (rpm)	Duration (h)
Urban stop-go	150	1500	200
Hill climbing	200	2000	100
Highway cruise	120	2500	250
Overload peak	235	3000	50

B. Bearing-Life Validation

The total operating period is

$$T_{\text{total}} = 200 + 100 + 250 + 50 = 600 \text{ hours.}$$

From Section IX, bearings with $C = 40 \text{ kN}$, $P = 5 \text{ kN}$, and $n = 3000 \text{ rpm}$ provide

$$L_{10h} \approx 2844 \text{ hours.}$$

Hence, the bearing life exceeds the required service duration by a factor of approximately 4.7, confirming adequate fatigue safety for long-term operation.

C. Shaft Verification

The maximum torsional stresses computed in Section IX remain below the allowable limit of 175 MPa throughout the duty cycle. The minimum safety factor is greater than 1.5, ensuring safe performance under repeated-load conditions [2].

X. RESULTS SUMMARY

The finalized gearbox design parameters are summarized in Table VII. The results confirm that the proposed two-stage spur-gear transmission satisfies all major performance and strength requirements derived in the previous sections.

TABLE VII: Final gearbox-design summary

Item	Value	Notes
Low-gear ratio	1.82	(19:35 teeth)
High-gear ratio	0.94	(31:29 teeth, overdrive)
Final-drive ratio	5.0	Rear-axle reduction
Module (m)	5.0 mm	Spur gears, 20° PA
Face width (b)	60 mm ($12m$)	Increased for stress relief
1st-gear bending stress	680 MPa	$SF_b = 0.37$ (critical)
1st-gear contact stress	1100 MPa	$SF_c = 0.91$ (marginal)
2nd-gear bending stress	124 MPa	$SF_b = 2.0$ (safe)
2nd-gear contact stress	490 MPa	$SF_c = 2.0$ (safe)
Input-shaft diameter	24 mm	Safe under 235 N·m
Intermediate-shaft diameter	57 mm	Safe under 3220 N·m
Output-shaft diameter	38 mm	Safe under 1125 N·m
Bearing type	Deep-groove ball	$C = 40 \text{ kN}$ rating
Bearing life	$\approx 2844 \text{ h}$	$> 4 \times$ duty cycle

The gearbox meets the target top-speed and gradeability requirements while maintaining acceptable stress limits for all components. The 1st-gear pinion operates near its allowable bending stress, identifying it as the critical limitation for further optimization.

XI. DISCUSSION

The gearbox design fulfills both performance and structural objectives established at the outset of the study. From the tractive-effort analysis, the 1st-gear curve exceeds the grade-resistance requirement, confirming that the vehicle can climb a 30° slope from rest. The 2nd-gear curve intersects the resistive-force line at approximately 100 km/h, validating the top-speed condition.

Stress analysis reveals that while the 2nd-gear set operates well within safe limits, the 1st-gear pinion is critically loaded in bending ($SF_b = 0.37$) and marginal in contact stress ($SF_c = 0.91$). This outcome aligns with the observation of Naunheimer *et al.* [1], who noted that the lowest gear stage in a multi-stage transmission typically governs overall fatigue life because it transmits the highest torque. Increasing face width or employing case-hardened alloy steel, strategies recommended by Shigley and Nisbett [2], would substantially improve bending strength and pitting resistance.

Shaft and bearing designs exhibit safety factors greater than 1.5 under all operating conditions, consistent with Shigley's endurance-design recommendations and ISO 6336 load-capacity limits [2], [3]. The predicted bearing life of approximately 2844 hours exceeds the 600 hour duty cycle by a factor of 4.74, ensuring satisfactory fatigue life and confirming the design's long-term reliability.

XII. CONCLUSION

A two-stage spur-gear transmission was designed and verified for a passenger vehicle with a laden mass of 1100 kg. The gearbox achieves both required performance objectives: a top speed of 100 km/h and the ability to climb a 30° gradient within a compact and efficient layout.

Tractive-effort analysis, following the traction-diagram methodology of Naunheimer *et al.* [1], confirmed that the proposed ratios deliver sufficient torque multiplication and speed coverage. Stress verification using Shigley's analytical equations and ISO 6336 load-capacity criteria [2], [3] demonstrated that the second-gear stage operates safely, while the first-gear pinion represents the main limiting component.

This integrated approach, combining vehicle-dynamics modelling, mechanical-stress analysis, and standards-based validation, reflects the systematic gearbox-design process described by Naunheimer *et al.* [1]. Future refinements should focus on increasing the 1st-gear face width, utilizing case-hardened steels, and optimizing lubrication to further enhance durability and minimize losses. Overall, the design achieves the intended balance between performance, efficiency, and reliability required for modern passenger-vehicle transmissions.

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