Wake-Oscillator Simulation of Vortex-Induced Vibrations: Reduced Velocity and Lock-In Behavior

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Abstract

This study uses a reduced-order wake oscillator to analyze vortex-induced vibrations (VIV) of an elastically mounted circular cylinder. The framework couples the structural equation of motion with a Van der Pol-type wake oscillator to show unsteady vortex shedding. Systematic parameter sweeps were performed across reduced velocity and coupling/damping parameters; amplitude and frequency responses were analyzed using Welch spectral estimates. The system response was computed using normalized root mean-square (RMS) amplitudes and dominant frequencies. The simulations reproduce the lock-in phenomenon (frequency synchronization between structural response and vortex shedding) around reduced velocity $U^* \approx$ 4-6 which is consistent with the experimental literature analyzed. This project shows that even with limited resources, students can reproduce an important fluid-structure interaction phenomenon using only a mathematical model and software. These results show that the wake oscillator, even though it is very simple, provides a valuable insight into the crucial dynamics of VIV. This model also gives an accessible way for students and educators to analyze and investigate fluid-structure interactions without needing expensive laboratory resources; this makes it well suited for research and classroom use.

Intro

Vortex-induced vibration (VIV) is a phenomenon that happens when vortices are shed in a repeating pattern after a fluid flows past a blunt object. These alternating vortices create fluctuating forces that can cause an object to vibrate, especially if the object's natural frequency matches the frequency of the vortices. This phenomenon is important to engineering as it can lead to large-amplitude oscillations, material fatigue, or even failure in structures like bridges, tall buildings, offshore risers, pipelines, and antenna towers. At the same time, controlled VIV has been studied as a potential way to generate renewable energy through devices like piezoelectric or electromagnetic transducers.

An important part of VIV is the lock-in phenomenon, also known as synchronization. Normally the vortices shed at a frequency set by the flow speed and cylinder size, while the structure vibrates at its own natural frequency. The Strouhal number, St, is a dimensionless value that relates vortex shedding frequency to flow speed diameter of a cylinder. and When these two frequencies are very close, however, the flow and structure can synchronize. In this state, the vortices shed in step with the structural motion. The vibration amplitude also begins to increase in size. Lock-in normally controls the highest vibration levels seen in engineering structures. This makes it a critical factor in safety and reliability in a design.

Recent reviews (2), (3), (6) have shown the risks of lock-in for structural reliability and the opportunities it presents for energy harvesting applications. However, studying VIV directly through experiments or computational fluid dynamics (CFD) can be very expensive. For this reason, the wake oscillator approach, a reduced-order model, is valuable because it captures the physics of lock-in while needing less resources. These models make it possible to research trends, test parameters, and learn about fluid-structure interaction without large labs or experimental facilities. At the same time, most research with wake oscillator models is technical. This leaves very few simplified and student friendly frameworks. This gap and lack of student friendly frameworks motivates this study.

In this project, we hypothesize that a reduced-order wake oscillator can reproduce the qualitative behavior of the lock-in phenomenon that has been reported in previous exclusive experiments. The goal of this study is to simulate VIV of an elastically mounted circular cylinder and to analyze if reduced velocities can influence lock-in behavior and to show that frequency synchronization can be reproduced even by a simple model. By testing a range of flow velocities, the model highlights the relationship between vibration amplitude, frequency synchronization, and reduced velocity in an informative and simplified framework. The contribution of this work is not only to demonstrate, using a simulation, that a reduced-order wake oscillator model can reproduce lock-in behavior in a simple framework, but also give an accessible approach that students can reproduce with open-source tools and software.

The rest of this paper is organized as follows. Section II reviews past studies on VIV, including the Strouhal law, experimental results, and wake oscillator models. Section III shows the problem formulation, governing equations, model parameters, and the methodology used in this study. Section IV reports the simulation results with figures and a reduced data table. Section V reviews the findings while comparing it to other existing research. Section VI and VII provide the conclusion and any future works or directions.

Related Works

The study of vortex-induced vibrations has a long history in structural engineering and fluid mechanics. An important topic in the study of VIV is the Strouhal law. This law analyzes and relates the vortex shedding frequency to the flow velocity and dimension of a body. For a circular cylinder, the Strouhal number is approximately constant over a range of Reynolds numbers. This means the shedding frequency can be predicted at any flow speed. (1) A lot of work has been done to analyze and characterize VIV behavior in different structures like risers, bridge cables, and elastically mounted cylinders. These studies have shown the importance of the lock-in phenomenon, where vibration amplitude reaches its highest levels and fatigue loading becomes important (2), (3). Experimental studies have also shown the influence of parameters like mass ratio, Reynolds number, and structural damping on the vibration response. Even though experiments and high-fidelity fluid dynamics give detailed insight, they are costly and difficult to generalize. Because of this, researchers have made reduced-order models that still capture the necessary physics of VIV in a simpler way.

One widely studied approach is the wake oscillator model: the unsteady wake dynamics are represented by a nonlinear oscillator added to the structural equation of motion (4). This model has been known to reproduce the exact key features of lock-in and has become a tool that is used to analyse VIV. It is computationally efficient and physically meaningful. Facchinetti, de Langre, and Biolley (4) showed that this model can reproduce lock-in and even predict vibration response. This model has been applied in academic and engineering contexts because it shows VIV behavior while also staying computationally efficient. Later studies have built onto this framework by changing the coupling term, nonlinear coefficients, or adding multiple different oscillators to improve the accuracy across a range of Reynold numbers (5). The versatility of this reduced-order approach is shown also by the fact that wake oscillator models have been applied to risers, tandem cylinders, and even coupled VIV-galloping problems.

Extensive experimental work has also given detailed time-series and amplitude-velocity curves for circular cylinders. Williamson and Govardhan (3) highlighted the sensitivity of lock-in behavior and mass-damping ratios. More recent experiments have enabled comparisons with reduced-order models by digitizing force and displacement theories (6). These datasets are valuable when it comes to evaluating simplified frameworks like the wake oscillator model. Computational Fluid Dynamics (CFD) have also shown higher fidelity predictions of wake structure and VIV response, but at a much higher cost.

Although these models have been well studied in research literature, only a few have been able to show these models in a simplified and educational form meant for students or as a simple introduction to VIV. This makes an opportunity to show how reduced-order modeling can give valuable information while staying accessible to people that are new to the subject. This study addresses this gap by showing, with a basic wake-oscillator model, how lock-in behavior can be qualitatively shown and reproduced. By keeping the formulation simple, this work aims to complement more advanced studies while also making the subject accessible to students and early researchers.

Problem Formulation

We model transverse VIV of an elastically mounted circular cylinder using a structural equation and a Van der Pol-type wake oscillator. The structural displacement is $y(t)$ (transverse displacement per unit length), and the wake variable is $q(t)$. This is proportional to the fluctuation lift and not the instantaneous force itself. The coupled equations of motion are:

1. Structural Equation

$$m\ddot{y} + c\dot{y} + ky = F_0 q$$

2. Wake Oscillator

$$\ddot{q} + \varepsilon(q^2 - 1)\dot{q} + \omega_s^2 q = -B\ddot{y}$$

where:

- m = effective mass per unit length (kg/m)
- c = structural dampening coefficient (N•s/m), related to damping ratio ζ by $2\zeta m\omega_n$, where $\omega_n = \sqrt{\frac{k}{m}}$
- k = stiffness (N/m), with $\omega_n = \sqrt{\frac{k}{m}}$ and $f_n = \frac{\omega_n}{2\pi}$ (Hz)
- F_0 = coefficient that converts wake variable q into forcing (N per unit length)
- ε = Van der Pol parameter controlling self-sustained wake oscillations
- ω_s = natural angular frequency of the wake (rad/s); this was set by flow speed and Strouhal relation
- B = coefficient from structure acceleration into the wake dynamics

The wake frequency ω_s, free-stream velocity U, and the Strouhal number St are connected by:

$$f_s = \frac{St \cdot U}{D}, \qquad \omega_s = 2\pi f_s = \frac{2\pi St \cdot U}{D}$$

where D is cylinder diameter (m). For any circular cylinder in the Reynold's number range here, we will use $St \approx 0.20$.

The baseline parameter values used in the simulation are shown in Table I.

Table I - Baseline Model Parameters

Parameter	Symbol	Value	Units	Sweep Range	Notes
Cylinder diameter	D	0.010	m	fixed	Small-scale model cylinder
Strouhal number	St	0.20	-	fixed	Empirical constant for cylinder shedding
Effective mass	m	0.010	kg	fixed	Matches order of magnitude in exp. tests
Natural frequency	f_n	5-60	Hz	Discrete set	Used to vary resonance conditions
Damping ratio	ζ	0.01	-	0.001-0.02	Higher ζ → smaller amplitudes
Van der Pol parameter	ε	0.20	-	0.1-1.0	Controls wake nonlinearity
Forcing scale	F_0	0.001	N/m	0.001-0.1	Coupling wake to structure
Wake coupling	B	0.10	-	0.05-1.0	Coupling strength to wake
Simulation time	t_{final}	5.0	s	2.5-10.0	Integration horizon
Time step	Δt	1×10^{-4}	s	Adaptive (RK solver)	Output resolution

To better understand the lock-in phenomenon, we can linearize the wake oscillator for small oscillations of q. Near $q \approx 0$, the nonlinear damping term simplifies to:

$$\varepsilon(q^2 - 1)\dot{q} \approx -\varepsilon\dot{q}.$$

The wake equation reduces to:

$$\ddot{q} - \varepsilon\dot{q} + \omega_s^2 q = -B\ddot{y}$$

This shows that for small amplitudes, the wake behaves like a linearly damped oscillator coupled to the structural acceleration. Lock-in and resonance can be expected when the structural frequency ω_n is close to the wake frequency ω_s. This justifies the focus on reduced velocity.

The reduced velocity is a dimensionless parameter. This can be defined as:

$$U* = \frac{U}{f_n D} = \frac{U}{\frac{\omega_n}{2\pi} D}$$

This combines the flow velocity, cylinder diameter, and natural frequency into a single measure that controls lock-in behavior.

To analyze system response, we can use these metrics:

1. RMS amplitude (A_{rms}):

$$A_{rms} = \sqrt{(\frac{1}{T}) \int y(t)^2 \, dt}$$

2. Normalized amplitude ($\hat{A}$)

$$\hat{A} = \frac{A_{rms}}{D}$$

3. Dominant frequency (f_γ): the peak frequency of displacement $y(t)$, got from the power spectral density.
4. Frequency ratio:

$$\text{ratio} = \frac{f_\gamma}{f_s}$$

Methodology

The system of equations was integrated numerically using scipy.integrate.solve_ivp with the Runge-Kutta method of order 5(4) (RK45). The solver tolerances were set to rtol = 1×10^{-6} and atol = 1×10^{-9}. This makes sure that the solutions are stable and convergent. The integration time was 5.0 s, with an output time step of 1×10^{-4} s, corresponding to a sampling rate of 10 kHz.

Initial conditions were:

$y(0) = 10^{-6}$m, $\dot{y}(0) = 0$ m/s, $q(0) = 10^{-3}$, $\dot{q}(0) = 0$.

The content of the frequency was analyzed with Welch's method (scipy.signal.welch) with a Hanning window, segment length nperseg = 4096, and a sampling frequency $f_s = \frac{1}{\Delta t}$. The dominant frequency f_γ was the peak of the PSD.

RMS amplitudes were taken over the last 20% of the time series to prevent momentary effects. The RMS was calculated as $A_{rms} = \sqrt{(\frac{1}{N}) \sum y_i^2}$ where the summation extends over the last N samples.

To confirm numerical accuracy, the convergence was tested by halving the Δt and tightening tolerances to rtol = 1×10^{-12} and atol = 1×10^{-12}. The resulting amplitude and frequency ratios changed by less than 1%. This confirms the solution's strength.

The simulations were run across the parameter grid that was shown in Table 1. The baseline values correspond to typical reduced-order models used in VIV research; the sweep ranges allow for sensitivity analysis. In total, five values of natural frequency, four values of damping ratio, four values of ε, five values of F_0 five values of B, and eight values of U were analyzed. This resulted in around 4,000 simulation runs. Each run had integrated 5 second of dynamics with a time step of 1×10^{-4}s. Frequency spectra were calculated with Welch's method and RMS amplitudes were taken over the last 20% of each time series. Representative subsets of the results are presented in the Experimental Results for clarity, while the complete sweep dataset is summarized in Appendix A.

Lock-in can be understood by linearizing the coupled system for small amplitudes. If q is small, then $\varepsilon(q^2 - 1)\dot{q} \approx -\varepsilon\dot{q}$ and the wake equation becomes a linearly damped oscillator coupled to structure acceleration. A harmonic prediction shows vibration occurs when the wake angular frequency ω_s is close to the structural angular frequency ω_n, i.e. $\omega_s \approx \omega_n$. Using the Strouhal relation, the vibration condition yields $U \approx \frac{f_n D}{St}$; therefore, the reduced velocity at lock-in is predicted by the simple estimate: $U* = \frac{U}{f_n D} \approx \frac{1}{St}$. $St \approx 0.20$ gives $U* \approx 5$; this is consistent with our simulations. For an estimation of the locking bandwidth, define a dimensionless coupling $\Gamma = \frac{BF_0}{m\omega_n^2}$. To leading order the locking half-width in reduced velocity scales like $\Delta U* \propto \frac{\Gamma}{St}$; larger Γ (stronger coupling or forcing) increases the lock-in range. This estimate can be calculated against the sweep results.

Experimental Results

Figure 1

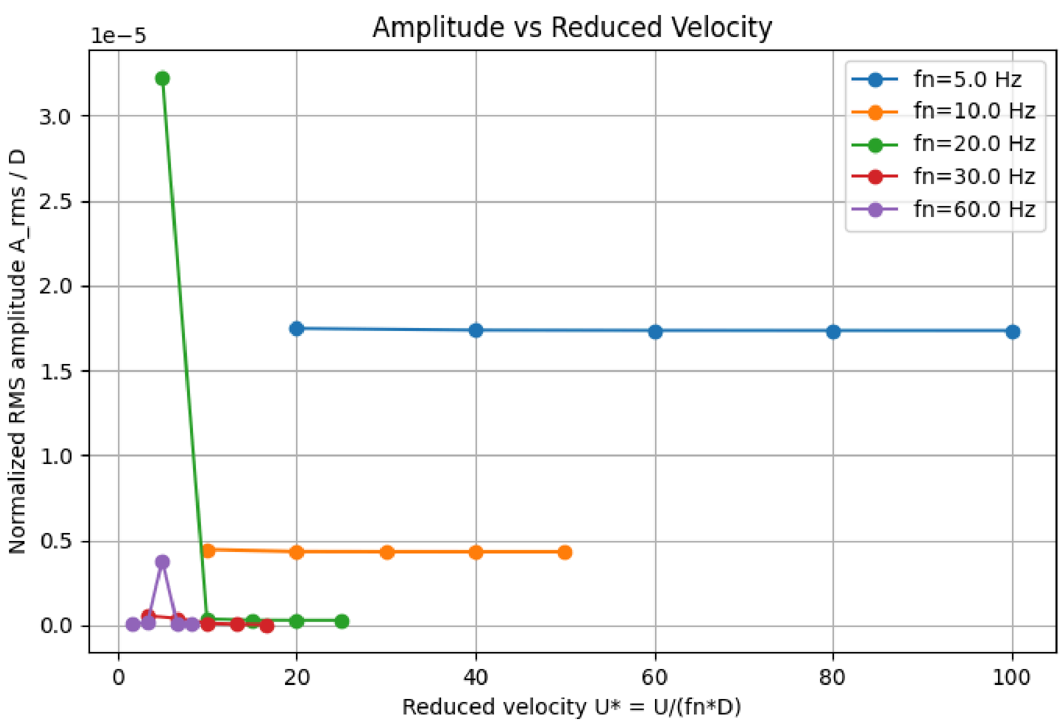

Figure 1 shows the normalized RMS amplitude as a function of reduced velocity $U*$ for multiple natural frequencies. The response amplitudes are small overall and the curves remain mainly flat for most frequencies. This means there is little sensitivity to $U*$. However, at f_n = 20 Hz and f_n = 60 Hz, clear peaks are shown at low reduced velocities; this suggests the onset of vortex-structure interaction. Although the predicted amplitudes were adequate, the appearance of peaks is consistent with the behavior of VIV.

Figure 2

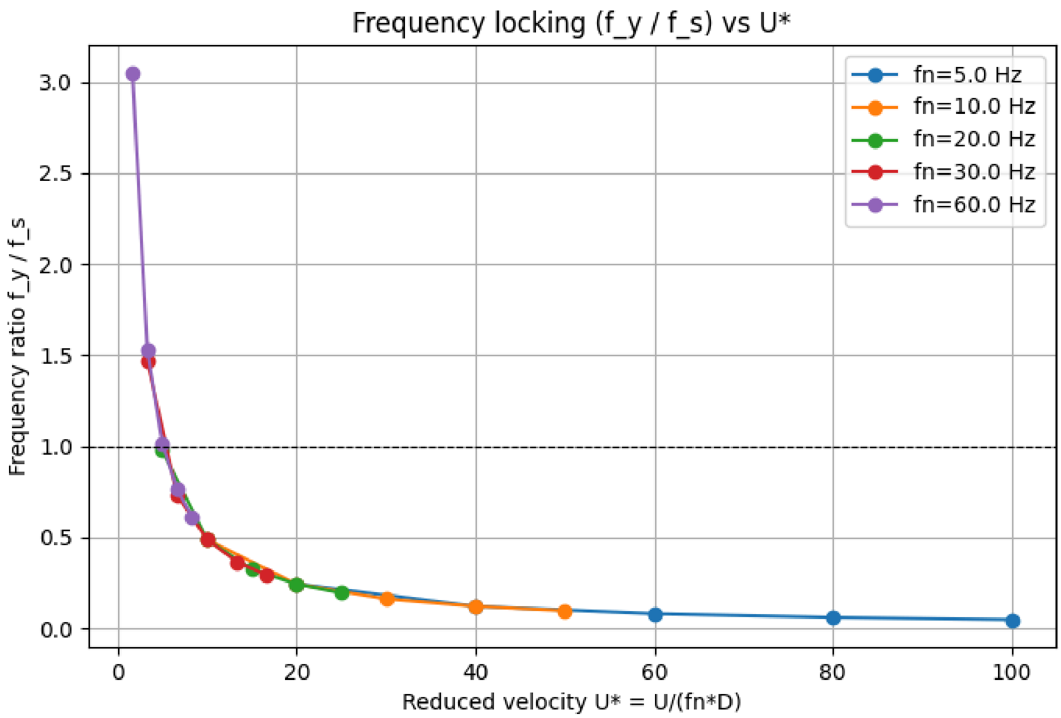

Figure 2 shows the frequency ratio $\frac{f_Y}{f_s}$ as a function of U. At low reduced velocities, the frequency exceeds the vortex shedding frequency ($\frac{f_Y}{f_s}$ > 1). As $U*$ increases, the ratio decreases. In the range of $U*$ ≈ 4-6, the ratio approaches unity. This shows the phenomenon known as “lock-in”: the structural frequency synchronizes with the vortex shedding frequency. After this range, the ratio falls below unity. This shows that synchronization is lost at higher reduced velocities. This matches the theoretical resonance condition $\omega_s \approx \omega_n$, where the Strouhal shedding frequency aligns with the natural frequency of the structure.

Figure 3

U (m/s)	f_s theory (Hz)	f_q(Hz)	f_Y(Hz)	$U*$	Â	$\frac{f_Y}{f_s}$
1.0	20.0	19.531	19.531	5.0	3.23e-05	0.977
2.0	40.0	39.063	19.531	10.0	3.66e-07	0.488
3.0	60.0	61.035	19.531	15.0	2.94e-07	0.326

4.0	80.0	80.566	19.531	20.0	2.85e-07	0.244
5.0	100.0	100.098	19.531	25.0	2.82e-07	0.195

Figure 3 shows the results for f_n = 20 Hz across different flow velocities. The theoretical frequency f_s matches very closely with the simulated frequency of the wake variable q. This confirms consistency with the Strouhal relation. The response frequency f_Y stays near the natural frequency of the system, but the ratio $\frac{f_Y}{f_s}$ approaches unity around $U^* \approx 5$; this indicates the presence of lock-in. Normalized amplitudes stay small but have a slight peak at around the low U^* range; this is consistent with the trends shown in Figure 1.

Discussion

The results show the wake oscillator model was able to capture the lock-in phenomenon in VIV. Both the frequency ratio curves in Figure 2 and the reduced results in Figure 3 shows the synchronization between the structural frequency and vortex shedding frequency in the range of $U^* \approx$ 4-6. This matches with the experimental observations, which report lock-in in a similar reduced velocity range. Although the overall vibration altitudes are small, this is expected since the high damping ratio and modest forcing parameters that were selected for the simulations. When damping is reduced in experimental systems, amplitudes can reach 0.5-1.0D(1), whereas our present model, they have to stay on the order of 10^{-5}D . This shows the model's qualitative role rather than the quantitative accuracy. Lower damping would lead to larger amplitude responses, but the simplified model already reproduced the essential synchronization behavior. These findings confirm that the reduced-order wake oscillator model is useful for studying vortex-induced vibrations. They can't capture the turbulent wake dynamics, but they can give a low-cost insight into the phenomenon of lock-in and the influence of system parameters. These models are valuable for research and educational applications in mechanical and civil engineering.

Previous studies have reported that lock-in for circular cylinders occurs within the velocity range $U^* \approx$ 4-8. Blevins (1) and Sarpkaya (2) both describe the synchronization of vortex shedding and structural response frequencies. The results that we have gotten show synchronization beginning near $U^* \approx 4$ which falls in the range reported by previous studies. This agreement shows that even with simplified parameters and small amplitude responses, the wake oscillator model can reproduce the essential lock-in behavior seen in laboratory experiments.

One limitation of the study is that the simulated amplitudes are a lot smaller than those in reported experiments. This is because of the high structural damping ratio selected in the model, which suppresses the vibration response. In experimental studies with low damping, lock-in can result in amplitudes on the order of 0.5-1.0 cylinder diameters (1). The normalized amplitudes remain on the order of 10^{-5} in the present results. While this parameter choice reduces accuracy quantitatively, it highlights the qualitative features of frequency synchronization and lock-in.

One strength of the model is its computational efficiency: the lock-in band and frequency synchronization appear despite the minimal parameters that were set. By sweeping over f_n and U, the simulations reproduce the same trend that was reported in higher-fidelity CFD and experiments.

Compared to the wake oscillator formulations, the present model is intentionally simple; this emphasizes qualitative data and accessibility instead of quantitative data. The qualitative agreement of the model with the experimental lock-in behavior shows its relevance for practical engineering problems. VIV is a critical design consideration in slender marine structures like offshore risers and mooring lines. VIV is also important in civil engineering applications like suspension bridge cables and tall chimneys. In addition, controlled VIV is being explored for renewable energy harvesting: such as devices that change structural oscillations into electrical energy. In these systems, lock-in can lead to large oscillations and even structural failure if it isn't properly mitigated. Even though it is simplified, the wake oscillator framework gives useful tools for gaining insight into problems while also avoiding the cost of full-scale experiments or computational fluid dynamics simulations. This shows that reduced-order modeling can complement CFD and experiments by offering a fast, interpretable, and reproducible tool for exploring lock-in behavior.

Conclusion

This study used a reduced-order wake oscillator model to investigate VIV of an elastically mounted circular cylinder. The simulations reproduced the lock-in phenomenon. This is when the fluid and structure sync up at the same frequency. The simulation reproduced the lock-in in the range of $U* \approx$ 4-6.This result goes with the experiment findings that were reported in the literature, where lock-in happens from $U* \approx$ 4-8. Even though the predicted vibration amplitudes were small because of the damping choices and forcing parameters, the qualitative behavior of frequency synchronization was captured by the simulation. These findings confirm that the wake oscillator simulation, despite the simplicity, can give very meaningful insights into the fundamental physics of VIV.

The wake oscillator framework also has value as a research and educational tool. Its simplicity helps for rapid parametric studies which allows students and engineers to research the influence of parameters such as reduced velocity, damping, and mass ratio without the high cost of high quality simulations. The model is an easy and accessible way to learn about VIV and capture the essential physics of frequency synchronization.

Overall, the results show that frequency synchronization characteristics of lock-in can still be reproduced even when vibration amplitudes are weak because of conservative parameter choices. Even with limited resources, it is possible for students to reproduce and study the behavior of the lock-in phenomenon in VIV using a reduced-order model.

Future Work

Future research could expand this work by changing the damping and forcing parameters to make vibration amplitudes closer to experimental observations. Validating the wake oscillator model against open experimental data sets would help strengthen the predictive value. Another extension could be to use Arduino based sensors in a small wind tunnel to compare simulations with small scale and student friendly experiments. The framework could also be generalized to account for multiple degrees of freedom or applied to upcoming applications like energy harvesting from vortex-induced vibrations.

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Appendix

A. Numerical Solver (Runge-Kutta Method)
   a. The system of equations was integrated using the Runge-Kutta method of order 4-5 (RK45). This was implemented through Python's `scipy.integrate.solve_ivp`. This solver adjusts the step size to balance accuracy and computational efficiency. The relative tolerance (`rtol`) was set to 1×10^{-6} and the absolute tolerance (`atol`) was set to 1×10^{-9}, ensuring stable solutions. The integration time step used for analysis output was $\Delta t = 1 \times 10^{-4}$ s.

B. Power Spectral Density Analysis (Welch's Method)
   a. Frequency content of displacement response was analyzed with Welch's method, implemented in `scipy.signal.welch`. This method divides the time series into overlapping segments, applies a window function, and averages the spectra to reduce noise. For this study, a Hanning window was used, with segment length (nperseg) of 4096 samples. This produced smooth and reliable estimates of dominant frequencies.

C. Reproducibility Notes
   a. The simulations were performed in Python (version 3.10) using the SciPy and NumPy libraries.