Large-amplitude Dithering Mitigates Glitches in Digital-to-analogue Converters

Arnfinn A. Eielsen, John Leth, and Andrew J. Fleming

***Abstract*—This paper develops and experimentally evaluates a dither-based method for improved generation of arbitrary signals in digital-to-analogue converters that exhibits glitches — essentially converting the glitches from high-frequency to low-frequency disturbances. One major benefit of this behaviour appears in closed-loop control applications, as the glitch disturbance can be moved from outside control law bandwidth to inside control law bandwidth, enabling suppression by feedback. A behavioural model of glitches is presented and the effect of applying a dither signal is analysed in detail. Analytical and experimental results demonstrate that a dither signal with sufficiently large amplitude can mitigate the effect of glitches, when used in conjunction with a low-pass filter. Severe glitches appear in various digital-to-analogue converter topologies, including converter topologies that are used in high-precision motion control applications, such as adaptive optics and scanning force microscopy. Glitches introduce impulse-like disturbances which have a broadband frequency distribution. Low-pass filtering alone does not provide sufficient attenuation, and in applications with feedback control only frequency content within the control law bandwidth is attenuated. Hence, a high-frequency disturbance such as a glitch will not be suppressed. The use of dithering to suppress glitches is therefore beneficial in applications where errors in signal conversion are a primary concern, such as high-precision motion control or accurate reference signal generation.**

***Index Terms*—digital analogue conversion, glitch reduction, disturbance reduction, convolution, nonlinear distortion, dither, motion control**

I. Introduction

Physical implementations of digital-to-analogue converters (DACs) introduce various non-ideal effects, including element mismatch, thermal and semiconductor noise, slew-rate limitations, and glitches caused by non-ideal transistor switching [1]–[7]. These effects come in addition to the fundamental error sources of aliasing and quantisation that occur in a digital signal processing system, due to discretisation in both time and value [8].

When applying a DAC for high-precision motion control, DAC-errors deteriorate the achieved performance [9]. Hence, it is of interest to mitigate them. Repeated spectra and aliasing is reduced by reconstruction filtering and interpolation [6, 10], and quantisation error is eliminated using small-scale dithering [8,11]–[14]. Element mismatch can be compensated for using several methods [15], including dynamic element matching [5,16] and large-scale dithering [17], and slewing can be partially reduced by oversampling [4,6].

This paper targets glitches. A behavioural model is developed and the effect of applying a dither signal is analysed: The glitches are modelled as short pulses and it is shown that it is possible to reduce the impact of these pulses by applying dithering and low-pass filtering. Dithering and low-pass filtering effectively converts a glitch from a large amplitude disturbance with a short duration, to a smaller amplitude disturbance with a longer duration, i.e. glitches are converted from high-frequency to low-frequency disturbances. One major benefit of this behaviour is that a low-frequency disturbance can be affected, and further attenuated, using feedback control.

A glitch is typically described as the transient generated when a DAC switches between two output levels [3,18]. The glitch is commonly associated with timing errors within the DAC: the non-simultaneous action of the switches generating the output levels. Glitches are usually measured as the area of the transient. Triggering of glitches is dependent on the input to the DAC. Glitches are typically minimised in the physical circuit using a sample-and-hold amplifier [3,4,6] or muliplexer [19], or by using optimised switching schemes, such as return-to-zero (RTZ) [4,6,20,21].

However, many DACs suitable for high-precision motion control applications do not have sufficient glitch compensation [9], since applicable DACs are limited to devices that have both high accuracy and low latency. Low latency is required to avoid compromising the phase margin in closed loop applications. Furthermore, it is of interest to improve the performance in existing systems where replacing the DACs is not an option.

Systems with high-precision motion control include adaptive optics, interferometry and scanning probe microscopy. Such systems are used for semiconductor production and inspection, nano-fabrication, precision machining, metrology, and lithography [22]–[25]. Improved signal generation capability in these systems directly translate to improved accuracy and capabilities. In addition, an accurate glitch model enables the use of observer-based control, such as model predictive control (MPC), to further improve performance. MPC have been shown to work well for signal generation using switching [26].

Glitches cause a form of inter-symbol interference (ISI). Variations of dynamic element matching (DEM) techniques have been shown to be effective in reducing ISI [16,27]–

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[29]; essentially by limiting the switching-rate between output elements. DEM techniques are limited to DAC topologies with redundancy in the representation of output values, and techniques for limiting the switching-rate are only applicable to very specific DAC topologies that utilise unary weighting (thermometer code) [16,27]–[29]. Furthermore, in segmented topologies [30], using e.g. binary weighing, DEM has demonstrated significant deterioration of performance [15]. Hence, the applicability of these methods is limited.

There are several behavioural models for glitches that occur in DACs [4,18,31]–[34]. While these models can provide excellent results in terms of predicting distortion spectra, power loss and simulating the effect of glitches in general circuits, they are not amenable to mathematical analysis of the effect of adding dithering and averaging, as discussed in this paper.

Dithering is an old technique in control theory [35], and has been used to improve stability in non-linear feedback control systems [36]–[38]. More recently, dithering has been applied to non-linearities in digital-analogue converters: Mitigation of the harmonic distortion caused by uniform quantisation [8,11]–[14], and mitigation of the effects due to the static non-linear transfer-function caused by element mismatch [17,39]–[44].

A. Contribution

A glitch model is developed which is intended to provide a description of glitches of any shape and to predict the performance impact. Furthermore, a glitch mitigation scheme using dithering is proposed. The model is structured in a manner that makes it amenable to analysis using established theory for dither in non-linear systems. The effect of applying stochastic and periodic dither signals is analysed in detail. The method for mitigation is intended for retrofitting to existing systems and to provide improved performance when applying a DAC or any actuation device with glitches for high-precision motion control or accurate reference signal generation.

B. Notation

A definition is denoted by $\triangleq$ and $*$ denote the convolution product. The Laplace operator is denoted $\mathcal{L}$. Functions of time t are usually denoted by lower case, e.g. $g(t)$, and the Laplace transformed by upper case, e.g. $G(s) = \mathcal{L}[g](s)$. The standard notation $L^p(\mathbb{R})$ indicate the L^p-space $p = 1, 2, \ldots, \infty$ and $||g||_p$ the p-norm of $g(t)$. For a stochastic process $d(t, \omega)$ the dependency on the sample variable ω is omitted.

II. Glitch Model

There are several models for glitches that occur in DACs. The model in [31] approximate glitch behaviour by opposite ramp-signals that are offset in time, and similar models are presented in [34] using piece-wise linear approximations. In [4,32] glitches are modelled as rectangular pulses, and in [33] a function comprised of hyperbolic and trigonometric expressions is used to approximate the glitch shape. The model in [18] superimposes two arbitrary functions to obtain a good glitch approximation, which also allows Fourier-analysis of the distortion. Unfortunately the structures of these models are not amenable to analysis of the effects of dithering; hence a new model is developed.

In the presented model, glitches are modelled as short, symmetric or asymmetric rectangular pulses driving a linear-time invariant (LTI) filter. If the response of a glitch can be measured with sufficient time resolution, then this approach can describe arbitrary glitch shapes, as an LTI filter can define any causal pulse response [45]. The LTI filter can be found using e.g. system identification techniques [46], or can be determined from first-principles if the components in the signal path are known. With an appropriately chosen LTI filter, the presented model will have an identical response, or be a close approximation, to previously published models.

A. Symmetric Glitch Model

Consider the signal $u(t)$, and the same signal $u(t-\tau)$ delayed by $\tau > 0$. If a DAC has N_T transition glitches, with polarity depending on the direction of crossing, at the thresholds T_i, and $T_i = T_j$ only when $i = j$, then the effect of the square pulses generated when crossing these thresholds can be described using the symmetric glitch model

$$n_g(u(t)) \triangleq \sum_{i=1}^{N_T} (g_i * \Delta y_i)(t)\,. \tag{1}$$

Here, the square pulses are constructed using

$$\Delta y_i(t) \triangleq y_i\,(t) - y_i\,(t-\tau)\ , \tag{2}$$

where

$$y_i(t) \triangleq n_i\,(u(t)) \tag{3}$$

are the outputs of the functions $n_i(u)$. These functions (or non-linearities) are used to generate the pulses triggered at the thresholds T_i as

$$n_i(u) = \frac{1}{\tau} H(u - T_i)\,, \tag{4}$$

where $H(u)$ is the Heaviside step-function:

$$H(u) = \begin{cases} 0\,, & u \leq 0 \\ 1\,, & u > 0 \end{cases} \tag{5}$$

The construction of $\Delta y_i(t)$ is illustrated in Fig. 1.

Note that in Sec. III and onward the input signal $u(t)$ is the combination of two different signals

$$u(t) = x(t) + d(t)\,, \tag{6}$$

where $x(t)$ is the deterministic signal that should be reconstructed by the DAC. The dither signal $d(t)$ is added to smooth the glitch responses and can be either stochastic or periodic, but is otherwise unwanted in the output.

The rectangular pulse $\Delta y_i(t)$ has unit area. The shape of the glitch is determined by the linear time-invariant (LTI) filter $G_i(s) = \mathcal{L}[g_i](s)$. That is, the shape of a glitch can be described by the response of the filter $G_i(s)$ due to the pulse

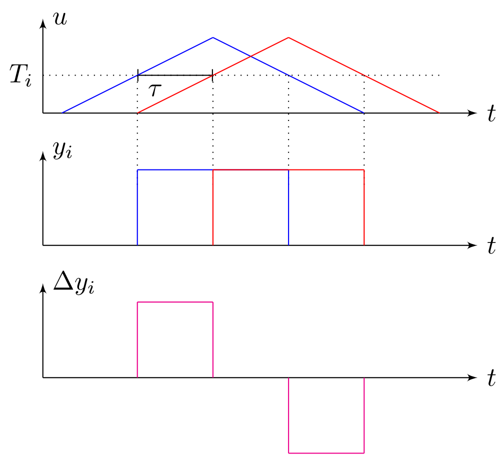


Fig. 1: Symmetric glitch model response to a triangle-wave.

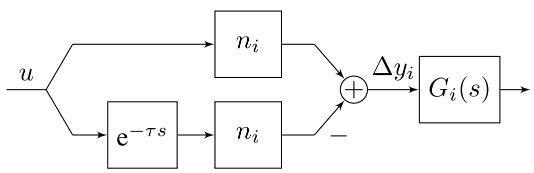


Fig. 2: Glitch sub-model for a symmetric response.

$\Delta y_i(t)$. As the time-delay $\tau \to 0$, the response converges to the impulse response. The net area A_i of a glitch,

$$A_i \triangleq \lim_{t\to\infty} \int_0^t \int_{-\infty}^{\infty} g_i(\xi - w)\Delta y_i(w)\mathrm{d}w\mathrm{d}\xi = \lim_{s\to 0} s\frac{1}{s}\frac{1}{\tau}\frac{1-\mathrm{e}^{-\tau s}}{s}G_i(s) = G_i(0)\,, \quad (7)$$

is determined by the DC-gain of $G_i(s)$. The area of this response is typically referred to as the glitch energy [18]. A block diagram of the model is shown in Fig. 2.

B. Asymmetric Glitch Model

If the glitches are asymmetric, i.e. the amplitude is different depending on whether the input is falling or rising, then, with the same assumptions as before, the effect of the square pulses generated when crossing the thresholds can be described using the asymmetric glitch model

$$\tilde{n}_g(u(t)) \triangleq \sum_{i=1}^{N_T} (g_i^+ * y_i^+)(t) + (g_i^- * y_i^-)(t)\,, \quad (8)$$

where a rising pulse is described by

$$y_i^+(t) \triangleq \frac{1}{2}\left(|\Delta y_i(t)| + \Delta y_i(t)\right) \geq 0\,, \quad (9)$$

and a falling pulse is described by

$$y_i^-(t) \triangleq \frac{1}{2}\left(|\Delta y_i(t)| - \Delta y_i(t)\right) \geq 0\,. \quad (10)$$

A block diagram of the model is shown in Fig. 3 and the construction of $y_i^+(t)$ is illustrated in Fig. 4. The LTI filters $G_i^+(s)$ determine the glitch shape when $u(t)$ is rising,

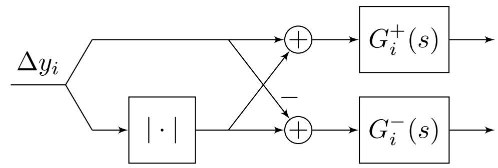


Fig. 3: Glitch sub-model for an asymmetric response.

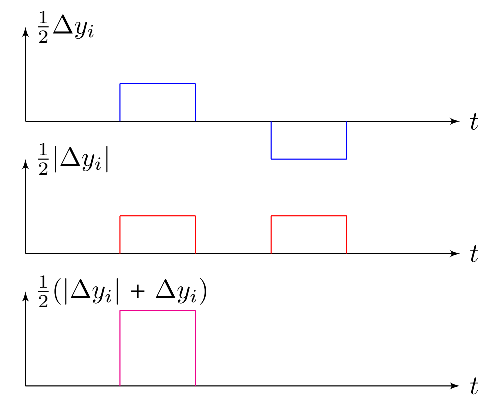


Fig. 4: Asymmetric glitch model response to a triangle-wave.

and $G_i^-(s)$ when $u(t)$ is falling. The model is equivalently expressed as

$$\tilde{n}_g(u(t)) = \frac{1}{2}\sum_{i=1}^{N_T} \left((g_i^+ - g_i^-) * \Delta y_i\right)(t) + \left((g_i^+ + g_i^-) * |\Delta y_i(t)|\right)(t)\,. \quad (11)$$

If $g_i^-(t) = -g_i^+(t)$, the symmetric model (1) is obtained.

C. Model Validation

The response of a realisation of the model is illustrated in Fig. 5, where it is compared to the measured response of one DAC channel in the experimental set-up. The DAC used is a Texas Instruments DAC8544, which is a 16-bit DAC. It has large transition glitches at code intervals of 4096, hence there are 16 transition glitches over the full range. There is a minor glitch for every least-significant bit. The filters $G_i^{\pm}(s)$ are in this case determined to be on the form

$$G_i^{\pm}(s) \triangleq \frac{\hat{A}_i^{\pm}}{\tau}W(s)\,. \quad (12)$$

The areas $\hat{A}_i^{\pm}$ were found to be $\hat{A}_i^+ = -60.6$ nVs for the major glitches with rising input, $\hat{A}_i^- = 51.9$ nV for the major glitches with falling input, and $\hat{A}_i^{\pm} = \pm 2.40$ nVs for the minor glitches.

The areas $\hat{A}_i^{\pm}$ were estimated using the trapezoidal method, as illustrated in Fig. 6, where the model response of a major glitch when the input is rising is compared to the measured response. Furthermore, $\tau = 1/625$ μs, which is the sampling time of the analogue-to-digital converter (ADC) used; an Analog Devices AD7674 with a sample rate of 625 kS/s. The filter $W(s)$ is the anti-aliasing filter,

$$W(s) \triangleq \left(\frac{2\pi f_c}{s + 2\pi f_c}\right)^2\,, \quad (13)$$

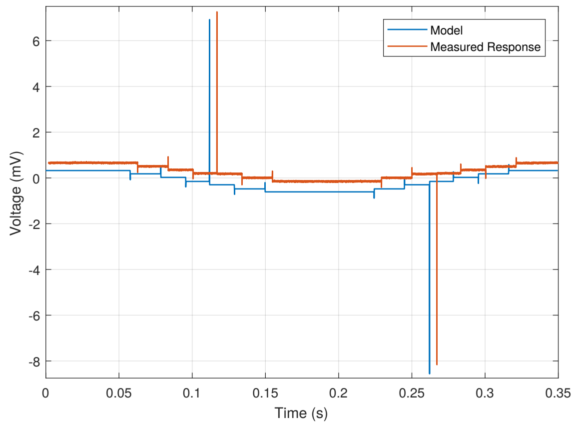


Fig. 5: Measured and model response (11) for a TI DAC8544, using a sinusoidal input signal with an amplitude of 0.01% (6.5535) of full range (65535 → 20 V). Plots offset for clarity.

with cut-off frequency $f_c = 62.5$ kHz.

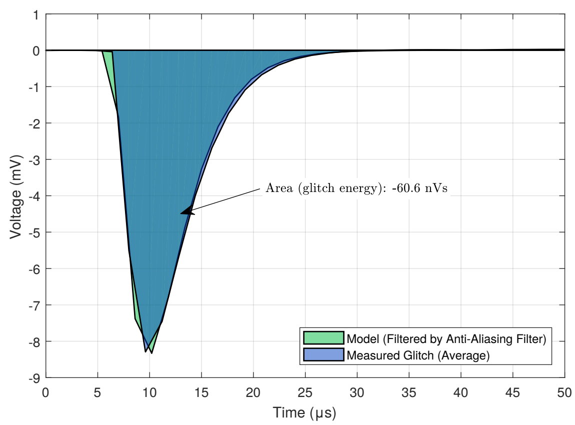


Fig. 6: A measured glitch and the corresponding model response. In this case the model glitch duration τ is set to the sampling time of the ADC. The measurements were averaged over several periods of the input signal.

III. Types of Dither

There are in general two types of dither signals: stochastic and periodic. Both achieve similar effects, and both deliver identical results in terms of the expected value. As we do not aim for complete generality, we assume in the upcoming Sections the existence of various densities. However, the result can without much difficulty be formulated in terms of the corresponding distribution functions. To avoid unnecessary notation we consider DACs with only one transition glitch in this Section: the subscript i is omitted.

A. Averaging Due to Stochastic Dither

The first case to be considered is where the dither signal in (6) is of a stochastic nature. By a stochastic dither signal $d(t)$ we understand an identically distributed stochastic process having the strictly white noise property, i.e. $d(t_i)$ and $d(t_j)$ are independent for each pair t_i and t_j. We follow [47] on the subject of white noise — for a rigorous mathematical exposition see [48]. In this Section we let $n(u)$ denote a function of bounded variation (on $\mathbb{R}$). It will be thought of as a threshold, e.g. as (4), and referred to as a non-linearity.

Consider the strictly white noise input signal

$$u(t) \triangleq x(t) + d(t)\,, \tag{14}$$

defined as the sum of a deterministic signal $x(t)$ and a stochastic dither signal $d(t)$. Let $f_d(z)$ be the probability density function corresponding to $d(t)$, and $f_x(z,t) = \delta(z - x(t))$ the probability density function corresponding to $x(t)$. Here, $\delta(\cdot)$ denotes the Dirac delta function. The signal $u(t)$ will then have probability density function

$$f_u(z,t) = \int_{\mathbb{R}} f_x(z-w,t) f_d(w)\,\mathrm{d}w = f_d(z - x(t))\,. \tag{15}$$

Consider the strictly white noise output

$$y(t) = n(u(t))\,. \tag{16}$$

The application of the dither signal makes it possible to define an averaged[1] non-linearity

$$N(z) \triangleq \int_{\mathbb{R}} n(z+w) f_d(w)\,\mathrm{d}w\,, \tag{17}$$

related to $n(u)$ and the stochastic dither signal $d(t)$. Hence

$$\|N\|_\infty \le \|f_d\|_1 \|n\|_\infty = \|n\|_\infty \tag{18}$$

since $u \in L^\infty(\mathbb{R})$ by the bounded variation property of $n(u)$. The average non-linearity thus reduces the effect caused by the non-linearity. Moreover, if $L_D \triangleq \sup_{z\in\mathbb{R}} |f_p(z)| < \infty$ then by [49, Lemma A.1]

$$L_N \le L_D TV(n) \tag{19}$$

with L_N a Lipschitz constant of $N(z)$ and $TV(n)$ the total variation of $n(u)$.

Now, by using (17) the expected value of the output is

$$\begin{aligned} \mathrm{E}[y(t)] &= \int_{\mathbb{R}} n(u) f_d(u - x(t))\,\mathrm{d}u & \text{(20a)}\\ &= \int_{\mathbb{R}} n(x(t)+w) f_d(w)\,\mathrm{d}w & \text{(20b)}\\ &= N(x(t))\,, & \text{(20c)} \end{aligned}$$

leading to the output auto-covariance

$$\begin{aligned} C_y(t,s) &\triangleq \mathrm{E}[y(t)y(s)] - \mathrm{E}[y(t)]\mathrm{E}[y(s)] & \text{(21a)}\\ &= \begin{cases} 0 & t \ne s \\ \mathrm{E}[y^2(t)] - N^2(x(t)) & t = s \end{cases} & \text{(21b)}\\ &\triangleq q(t)\chi_{\{t\}}(s) & \text{(21c)} \end{aligned}$$

where $\chi_{\{t\}}(s)$ denotes the indicator function, and the first case in (21b) follows by the strictly white noise property of $y(t)$ and the second case follows by (20c).

Consider the zero-mean strictly white noise error signal

$$\varepsilon(t) = y(t) - N(x(t))\,, \tag{22}$$

[1] sometimes also referred to as “effective” or “smoothing”

having auto-covariance $C_\varepsilon(t,s) = C_y(t,s)$. The time-varying variance of the error signal is thus

$$\begin{aligned} q(t) &= \mathrm{E}[\varepsilon^2(t)] \\ &= \int_{\mathbb{R}} n^2(u) f_d(u - x(t))\, \mathrm{d}u - N^2(x(t))\,. \end{aligned}$$

That is, for the non-linearity (4) it follows that $n^2(u(t)) = n(u(t))/\tau$, and the expression for $q(t)$ can be found as

$$q(t) = \frac{1}{\tau} N(x(t)) - N^2(x(t))\,. \tag{23}$$

A well-known result in linear system theory is that for a stationary white noise input signal w, with auto-covariance $C_w(t,s) = {\sigma_w}^2 \chi_{\{t\}}(s)$ where ${\sigma_w}^2$ is a constant, the variance of the response of the output v of an LTI-filter $G(s)$ is [50]

$${\sigma_v}^2 = \|G(s)\|_2 {\sigma_w}^2\,, \tag{24}$$

where $\|G\|_{\mathcal{H}_2}$ denotes the $\mathcal{H}_2$ norm of $G(s)$. Due to the frequency selective properties of an LTI-filter, it is possible to devise a filter that has smaller variance on the output than the input, i.e. reduces the noise. A typical signal chain contains several low-pass filters; in particular the reconstruction filter (needed to satisfy the Nyquist–Shannon theorem). This filtering can be expressed by the filters $g_i(t)$, $g_i^{\pm}(t)$ in the models (1) and (11). These filters can be expected to reduce the unwanted signal component due to the dither signal.

Now, the error signal (22) is not stationary, and (24) does not apply. However, an LTI-filter will reduce the variance of this signal: Consider the impulse response $g(t)$ corresponding to the LTI filter $G(s)$ and define the filtered error signal

$$\bar{\varepsilon}(t) \triangleq (g * \varepsilon)(t)\,,$$

as the output of the filter $g(t)$ due to the error signal $\varepsilon(t)$. The variance of the filtered error signal is then

$$\begin{aligned} C_{\bar{\varepsilon}}(t,t) &= \mathrm{E}[\bar{\varepsilon}^2(t)] && (25)\\ &= (q * g^2)(t) && (26)\\ &= \int_{\mathbb{R}} q(t-w) g^2(w)\, \mathrm{d}w\,, && (27) \end{aligned}$$

where (26) follows by [47, Theorem 9-3]. Hence if $q \in L^1(\mathbb{R})$ and $g^2 \in L^2(\mathbb{R})$ then

$$\begin{aligned} \|C_{\bar{\varepsilon}}\|_2 &= \|q * g^2\|_2 && (28a)\\ &\le \|q\|_1 \|g^2\|_2 && (28b)\\ &= \|q\|_1 \|\mathcal{L} g^2\|_{\mathcal{H}_2} && (28c) \end{aligned}$$

where (28b) follows by Young's convolution theorem [51] and (28c) follows by Parseval's theorem. By (28) the variance of the filtered error signal will be attenuated by the energy of the impulse response squared $\left\|g^2\right\|_2$. For strictly proper, unity gain low-pass filters, the energy of the impulse response is directly related to the cut-off frequency: A lower cut-off frequency yields lower energy, hence smaller variance for the filtered error signal $\bar{\varepsilon}$.

B. Averaging Due to Periodic Dither

In terms of smoothing, similar results are obtained using either stochastic or periodic dither signals. In the stochastic case, low-pass filtering on the output reduces the variance of the non-stationary error signal. For the periodic case it is the dither frequency and how well the low-pass filtering on the output resembles a windowed integrator that determines the residual error.

In this case the input signal

$$u(t) \triangleq x(t) + p(t)\,, \tag{29}$$

consists of a deterministic signal $x(t)$ and a periodic dither signal $p(t)$ with sufficiently high frequency and amplitude. For a non-linearity $n(u)$ and periodic dither $p(t)$ one can, similar to (17), define an averaged non-linearity [49,52,53]:

$$\bar{N}(z) \triangleq \int_{\mathbb{R}} n(z+w) f_p(w)\, \mathrm{d}w\,. \tag{30}$$

Here $f_p(z)$ is the amplitude density function of the periodic dither signal $p(t)$, which is the deterministic equivalent to the probability density function. If $L_P \triangleq \sup_{z\in\mathbb{R}} |f_p(z)| < \infty$ then [49, Lemma A.1]

$$L_{\bar{N}} \le L_P TV(n) \text{ and } \|\bar{N}\|_\infty \le \|n\|_\infty \tag{31}$$

with $L_{\bar{N}}$ a Lipschitz constant of $\bar{N}(z)$ and $TV(n)$ the total variation of $n(u)$. Hence, as in the case of stochastic dither, the average non-linearity induced by a periodic dither reduces the effect caused by the non-linearity.

For each $t \ge 0$ let $J_t = [t-\rho, t]$ denote the interval of length $\rho > 0$. The method of averaging via a periodic dither signal, with period ρ, relies on the equivalence (see [49] for a reference)

$$\int_{\mathbb{R}} n(z+w) f_p(w)\, \mathrm{d}w = \frac{1}{\rho} \int_{J_\rho} n(z + p(\tau))\, \mathrm{d}\tau \tag{32}$$

between the average non-linearity and a time-average over one period $J_\rho = [0,\rho]$ of the periodic dither. Note that the right-hand side of (32) holds for any interval of length ρ, in particular intervals on the form J_t. Hence averaging can be implemented by using a windowed integrator (a continuous-time moving average)

$$\frac{1}{\rho} \int_{J_t} n(z + p(\tau))\, \mathrm{d}\tau. \tag{33}$$

From a practical point of view this is advantageous, as there is no readily apparent way to implement a physical realisation of the convolution product on the left-hand side of (32), but an integrator like (33) can be realised using analogue circuitry. This realisation can take the form in (36), shown below, but as the dither frequency is finite, an error is introduced. To make this more precise recall first from [49] that; if $x(t)$ has a Lipschitz constant $L_X = L_X(J_\rho)$ on J_ρ then

$$\begin{aligned} &\left| \int_{J_\rho} n(x(\tau) + p(\tau))\, \mathrm{d}\tau - \int_{J_\rho} n(\tilde{x} + p(\tau))\, \mathrm{d}\tau \right| \\ &\qquad\qquad \le 2 L_P L_X TV(n) \rho^2 \end{aligned} \tag{34}$$

with $\tilde{x}$ satisfying

$$\min_{t\in J_\rho} x(t) \leq \tilde{x} \leq \max_{t\in J_\rho} x(t)\,. \tag{35}$$

As with (32) the error bound (34) holds for any interval of length ρ. Using (30) and (32), with J_ρ replaced by J_t, and

$$\alpha(t) \triangleq \frac{1}{\rho}\int_{J_t} \gamma(\tau)\mathrm{d}\tau \triangleq \frac{1}{\rho}\int_{J_t} n(x(\tau)+p(\tau))\mathrm{d}\tau, \tag{36}$$

the above result (34) yields

$$\begin{aligned}\left|\alpha(t)-\bar{N}(x(t))\right| &= \frac{1}{\rho}\left|\int_{J_t}\gamma(\tau)\,\mathrm{d}\tau - \int_{J_t} n(x(t)+p(\tau))\,\mathrm{d}\tau\right| \\ &\leq 2L_P L_X TV(n)\rho \end{aligned}\tag{37}$$

with $L_X = L_X(J_t)$ a Lipschitz constant of $x(t)$ on J_t. If $\bar{L}_X \triangleq \sup_{t\in[0,\infty)} L_X(J_t) < \infty$, the error bound (37) implies the error bound

$$|A(s) - G_{\bar{N}}(s)| \leq 2L_P\bar{L}_X TV(n)\rho/\sigma\,,\ \sigma > 0 \tag{38}$$

between the Laplace transformed $A(s)$ and $G_{\bar{N}}(s)$ of $\alpha(t)$ and $\bar{N}(x(t))$, respectively, with $s = \sigma + i\omega$. The error bound (38) can further be elaborated on to obtain a conclusion in analogue with (28). For this, let $r(u) \triangleq H(u+\rho) - H(u)$ denote a rectangle function expressing the interval J_0 of width ρ, with $H(u)$ the Heaviside step function (5). The signal $\alpha(t)$ can then be expressed as

$$\alpha(t) = \frac{1}{\rho}\int_0^\infty r(t-\tau)\gamma(\tau)\mathrm{d}\tau = \frac{1}{\rho}(r*\gamma)(t).$$

The Laplace transformed $A(s)$ is thus given by

$$A(s) = \frac{1}{\rho}R(s)\Gamma(s) = \frac{1-\mathrm{e}^{-\rho s}}{\rho s}\Gamma(s) \tag{39}$$

with $R(s)$ and $\Gamma(s)$ the Laplace transformed of $r(t)$ and $\gamma(t)$ respectively.

It is possible to realise a windowed integrator (36), which frequency response is expressed by $R(j\omega)/\rho$, but due to practical limitations of the resulting analogue circuitry, the actual performance will not be satisfactory. However, as noted in Sec. III-A, a typical signal chain contains several low-pass filters. The frequency response $|R(j\omega)/\rho|$ is the sinc-function, which has a low-pass characteristic. If the filters in the circuit resemble the sinc-function, i.e. $\||G(j\omega)| - |R(j\omega)/\rho|\|_2$ is small, then the error bound (38) is a good approximation and

$$A'(s) \triangleq G(s)\Gamma(s) \tag{40}$$

therefore represents an approximation of $G_{\bar{N}}(s)$ which is implementable. It can be concluded that to minimise the error when using a periodic dither, the dither frequency should be as high as practically possible, as it makes the period ρ and hence the bound (38) small, and the frequency response of the low-pass filter on the output should resemble the corresponding sinc-function.

IV. Properties

In this Section we study the effect of a specific stochastic and periodic dither signal applied to a DAC system with transition glitches.

A. Dither With Uniform Distribution

Assume that the stochastic dither signal $d(t) \in [-A, A]$ is uniformly distributed white noise and the periodic dither signal $p(t)$ is a triangle wave with amplitude A. Then both the probability density function $f_d(z)$ and the amplitude density function $f_p(z)$ will equal a rectangle function $f(z)$ as

$$f(z) = \frac{1}{2A}\Big(H(z/A+1) - H(z/A-1)\Big) \tag{41a}$$

$$= \begin{cases} 0\,, & |z| \geq A \\ \frac{1}{2A}\,, & |z| < A \end{cases}\,, \tag{41b}$$

giving Lipschitz constants $L_D = L_P = 1/2A$. Using (19) and (31) for the non-linearity in (4), with total variation $TV(n_i) = 1/\tau$, the average non-linearity $N_i(z) = \bar{N}_i(z)$ is bounded in growth by the Lipschitz constant $L_{N_i} = L_{\bar{N}_i} = 1/2A\tau$.

Since $f(z)$ is even, (17) and (30) are equivalent to

$$N_i(x(t)) = \int_{\mathbb{R}} n_i(\xi)f(x(t)-\xi)d\xi\,. \tag{42}$$

Hence, for a non-linearity (4) with $T_i = 0$, the average non-linearity is

$$N_i(x(t)) = \frac{1}{2A\tau}\int_{\mathbb{R}} H(\xi)\Big(H(x(t)-\xi+A) \tag{43a}$$

$$- H(x(t)-\xi-A)\Big)d\xi \tag{43b}$$

$$= \frac{1}{2A\tau}\Big((x(t)+A)H(x(t)+A) \tag{43c}$$

$$- (x(t)-A)H(x(t)-A)\Big) \tag{43d}$$

$$= \begin{cases} 0\,, & x(t) \leq -A \\ \frac{1}{2A\tau}(x(t)+A)\,, & |x(t)| \leq A \\ \frac{1}{\tau}\,, & x(t) \geq A \end{cases}\,, \tag{43e}$$

and therefore $N_i(x(t)) \in [0, 1/\tau]$. Fig. 7 illustrates the effect: The non-linearity $n_i(u(t))$ is shown, as well as $(g_i * n_i(x+d))(t)$ representing the response of a low-pass filter $g_i(t)$ applied to the output $y_i(t) = n_i(x(t)+d(t))$ when using a uniform white-noise dither $d(t)$. Moreover, $N_i(x(t))$, given by (43), is also shown which according to (20c) correspond to the expected value $E[y_i(t)]$. The effect of noise-modulation (23) is noticeable, as the variance is larger around zero, than towards the dither amplitude limits at $-A$ and A. From Fig. 7 it seems like the expected value $E[y(t)]$ of the output is close to the filtered mean $E[(g_i*y_i)(t)] = (g_i*E[y_i])(t)$ — this is indeed the case as seen in Fig. 8.

1) Symmetric Glitches: In the case of symmetric glitches it is straightforward to construct an average model of (1), or equivalently (11) with $g_i^-(t) = -g_i^+(t)$. Consider first the mean value

$$\begin{aligned} E[(g_i*y_i)(t)] &= E\left[\int_{\mathbb{R}} g_i(w)y_i(t-w)\,\mathrm{d}w\right] \\ &= \int_{\mathbb{R}} g_i(w)E[y_i(t-w)]\,\mathrm{d}w \\ &= (g_i * E[y_i])(t) \\ &= (g_i * N_i(x))(t)\,. \end{aligned}$$

Applying this to each term in the model (1) yields

$$E[(g_i*\Delta y_i)(t)] = (g_i*\Delta N_i)(t)$$

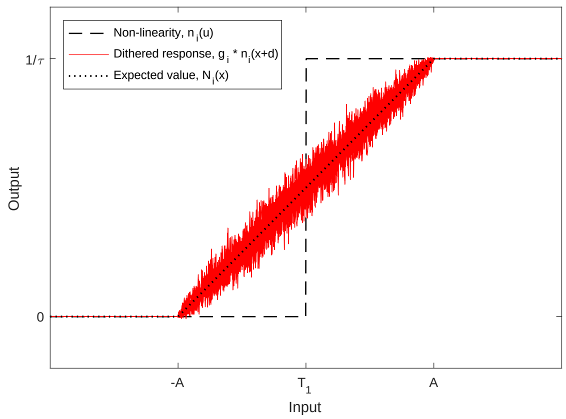


Fig. 7: Average non-linearity: Expected value (43) compared to simulated response when using uniform white-noise dither.

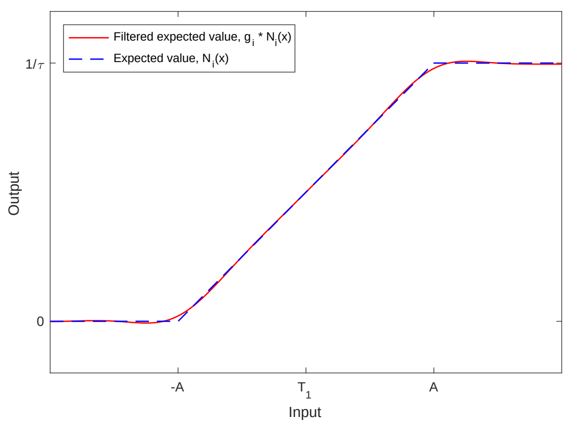


Fig. 8: Comparison between the expected value $E[y_i(t)] = N_i(x(t))$ and the filtered expected value $(g_i * N_i(x))(t)$.

with

$$\Delta N_i(t) \triangleq E[y_i(t) - y_i(t-\tau)] \tag{44a}$$
$$= E[y_i(t)] - E[y_i(t-\tau)] \tag{44b}$$
$$= N_i(x(t)) - N_i(x(t-\tau)) \tag{44c}$$

The averaged model corresponding to (1) is therefore

$$E\left[n_g\left(u(t)\right)\right] = \sum_{i=1}^{N_T} (g_i * \Delta N_i)(t). \tag{45}$$

For practical implementation it is important to note that (for a suitable choice of dither amplitude A) the average symmetric glitch model (45) can be approximated by a linear combination of the averaged non-linearities — see (46). To see this assume that for a given time-delay τ, the dither amplitude A is chosen such that $E[y_i(t)] = N_i(x(t))$ can be considered approximately constant. Typically, τ is small and A therefore needs to be large, according to (43). It then follows that

$$\begin{aligned} E[(g_i * y_i)(t)] &= \int_{\mathbb{R}} g_i(w) E[y_i(t-w)]\,\mathrm{d}w \\ &\approx \int_{\mathbb{R}} g_i(w)\,\mathrm{d}w N_i(x(t)) \\ &= A_i N_i(x(t))\,, \end{aligned}$$

with $A_i = G_i(0)$ as in (7). The mean of each term in the symmetric glitch model (1) can then be approximated as

$$E[(g_i * \Delta y_i)(t)] \approx A_i \Delta N_i(t)$$

where $\Delta N_i(x(t))$ typically is close to zero as the dither amplitude A is chosen very large. The average symmetric glitch model (45) is therefore approximately given by

$$E\left[n_g\left(u(t)\right)\right] \approx \sum_{i=1}^{N_T} A_i\, \Delta N_i(t)\,. \tag{46}$$

2) Asymmetric Glitches: In the case of asymmetric glitch model (11) it is not possible to find an average model on the same form as in (45) but it is possible to find a bound for the response. For the first term in (11)

$$\left(\left(g_i^+(t) - g_i^-\right) * \Delta y_i\right)(t) \triangleq (\tilde{g}_i(t) * \Delta y_i)(t)$$

we proceed as in the symmetric case to obtain

$$E[(\tilde{g}_i * \Delta y_i)](t) = (\tilde{g}_i * \Delta N_i)(t)$$

with the approximation (for large A)

$$E[(\tilde{g}_i * \Delta y_i)](t) \approx \tilde{A}_i \Delta N_i(t)$$

where $\tilde{A}_i = A_i^+ - A_i^-$ and $A_i^\pm = \tilde{G}_i^\pm(0)$. For the second term in (11)

$$(g_i^+(t) + g_i^-(t)) * |\Delta y_i(t)| \triangleq \bar{g}_i(t) * |\Delta y_i(t)|$$

we derive two bounds. For the first bound the triangle inequality is applied as

$$|\Delta y_i(t)| = |y_i(t) - y_i(t-\tau)| \leq |y_i(t)| + |y_i(t-\tau)| \tag{47}$$

to obtain

$$\begin{aligned} E[(\bar{g}_i * |\Delta y_i|)](t) &\leq (\bar{g}_i * E[|y_i|])(t) + (\bar{g}_i * E[|y_i|])(t-\tau) \\ &\triangleq B_i^+(t). \end{aligned} \tag{48}$$

Note that since the function $n(z)$, given by (4), is positive

$$E[|y_i(t)|] = \int_{\mathbb{R}} |n_i(z)| f(z - x(t)) dz \tag{49}$$
$$= \int_{\mathbb{R}} n_i(z) f(x(t) - z) dz \tag{50}$$
$$= N_i(x(t))\,, \tag{51}$$

with $f(z)$ given by (41). Hence the bound (48) can equivalently be expressed as

$$B_i^+(t) = (\bar{g}_i * N_i(x))(t) + (\bar{g}_i * N_i(x))(t-\tau) \tag{52}$$

and for large dither amplitude A be approximated by

$$B_i^+(t) \approx \bar{A}_i \Big(N_i(x(t)) + N_i(x(t-\tau))\Big) \triangleq \bar{B}_i^+(t) \tag{53}$$

where $\bar{A}_i = A_i^+ + A_i^-$.

For the second bound we note that $n(u) = -n(-u) + 1$, hence similar to (47) we apply the triangle inequality to obtain

$$\begin{aligned} |\Delta y_i(t)| &= |n_i(u(t)) - n_i(u(t-\tau))| \\ &= |n_i(-u(t)) - n_i(-u(t-\tau))| \\ &\leq |n_i(-u(t))| + |n_i(-u(t-\tau))| \end{aligned}$$

and therefore also a second bound

$$\begin{aligned} E[(\bar{g}_i * |\Delta y_i|)](t) \leq (\bar{g}_i * E[|n_i(-u)|])(t) \\ + (\bar{g}_i * E[|n_i(-u)|])(t-\tau) \\ \triangleq B_i^-(t) \end{aligned} \quad (54)$$

similar to (48). Moreover,

$$B_i^-(t) = (\bar{g}_i * N(-x))(t) + (\bar{g}_i * N(-x))(t-\tau) \quad (55)$$

since $E[|n_i(-u)|] = N(-x(t))$, hence for large A

$$B_i^-(t) \approx \bar{A}_i\Big(N_i(-x(t)) + N_i(-x(t-\tau))\Big) \triangleq \bar{B}_i^-(t) \quad (56)$$

Collecting the two bounds (48) and (54) in

$$B_i(t) = \min_t\{B_i^+(t), B_i^-(t)\},$$

an upper bound for the mean of the asymmetric glitch model (11) is

$$\begin{aligned} E[\tilde{n}_g(x(t))] &\leq \frac{1}{2}\sum_{i=1}^{N_T}(\tilde{g}_i * \Delta N_i)(t) + B_i(t) \quad (57) \\ &\approx \frac{1}{2}\sum_{i=1}^{N_T}\tilde{A}_i \Delta N_i(t) + \bar{B}_i(t) \quad (58) \end{aligned}$$

with

$$\bar{B}_i(t) = \min_t\{\bar{B}_i^+(t), \bar{B}_i^-(t)\}\,.$$

B. Single Glitch Responses to a Ramp Signal

For simplicity consider a single threshold T_1, which means $N_T = 1$, and suppose $T_1 = 0$. Assume that the signal $x(t)$ can be approximated locally by a ramp

$$x(t) = L_X t\,, \quad (59)$$

where $0 < L_X\tau < A$.

1) Symmetric Glitch Response: From (43e) and (44c) it follows that:

$$\Delta N_1(t) = \frac{1}{2A\tau}\begin{cases} 0\,, & L_X t \in (-\infty, -A) \\ L_X t + A\,, & L_X t \in (-A, -A + L_X\tau) \\ L_X\tau\,, & L_X t \in (-A + L_X\tau, A) \\ A + L_X\tau - t\,, & L_X t \in (A, A + L_X\tau) \\ 0\,, & L_X t \in (A + L_X\tau, \infty) \end{cases} \quad (60)$$

Using a periodic (triangle-wave) dither with density (41), Fig. 9 shows the the response of the symmetric glitch model (1) in blue (with $u(t) = x(t)$), the averaged symmetric glitch model (45) in red and the approximation (46) is indicated by the dotted line. In this example $L_X = 1000$, $\tau = 10^{-6}$ s = 0.001 ms, and $A = 1$. The output was filtered, using zero-phase filtering [54], by $g_1(t) = k_{g_1} w(t)$, with $k_{g_1} = 5 \cdot 10^{-5}$ and $w(t) = \mathcal{L}^{-1}[W](t)$, with $W(s)$ given by (13).

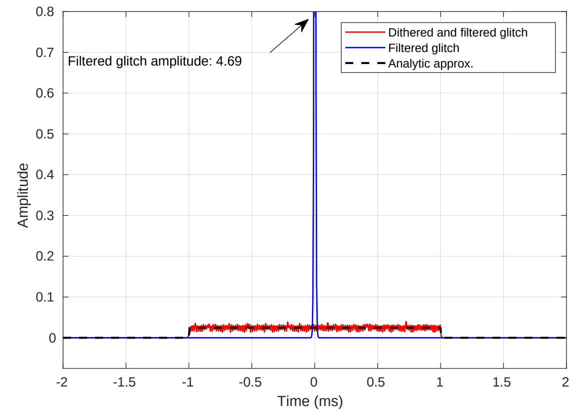


Fig. 9: Symmetric glitch response, using a periodic (triangle wave) dither. Ripple is due to finite dither frequency and imperfect averaging.

2) Asymmetric Glitch Response Bound: In this case the bound $\bar{B}_1(t)$ is:

$$\bar{B}_1(t) = \frac{1}{2A\tau}\begin{cases} 0\,, & L_X t \in (-\infty, -A) \\ L_X t + A\,, & L_X t \in (-A, -A + L_X\tau) \\ 2L_X t + 2A - L_X\tau\,, & L_X t \in (-A + L_X\tau, L_X\tau/2) \\ -2L_X t + 2A + L_X\tau\,, & L_X t \in (L_X\tau/2, A) \\ -L_X t + A + L_X\tau\,, & L_X t \in (A, A + L_X\tau) \\ 0\,, & L_X t \in (A + L_X\tau, \infty) \end{cases} \quad (61)$$

The analytic approximate bound (57), as well as a simulated response is shown in Fig. 10 for a periodic (triangle-wave) dither signal, and in Fig. 11 for a stochastic dither signal. Again, $L_X = 1000$, $\tau = 0.001$ ms, and $A = 1$. The output was filtered, using zero-phase filtering [54], by

$$\tilde{g}_1 = \frac{1}{2}(k_{g_1}^+ + k_{g_1}^-)w(t)\,, \quad (62)$$

with $k_{g_1}^+ = -5.0 \cdot 10^{-5}$, $k_{g_1}^- = 5.1 \cdot 10^{-5}$, and $w(t) = \mathcal{L}^{-1}[W](t)$, where $W(s)$ is as given by (13).

V. Experimental Set-up

A diagram of the experimental set-up used is shown in Fig. 12. The DAC is a Texas Instruments DAC8544, as described in Sec. II-C. Sixteen DAC-channels were used in order to reduce the stochastic noise-floor and thus emphasise the deterministic and systematic error introduced by the glitches.

Note that averaging of several channels is not necessary for the method to work: Let $y_k(t) = y_d(t) + e_k(t)$ denote the output of DAC-channel k, where y_d is the deterministic output generated by a DAC due to the input signal u and e_k are the independent and identically distributed stochastic output noise signals associated with the channels, having variance ${\sigma_e}^2$. The channel averaging is described by

$$y = \frac{1}{N_C}\sum_{k=1}^{N_C}(y_d(t) + e_k(t)) = y_d(t) + \frac{1}{N_C}\sum_{k=1}^{N_C} e_k(t)$$

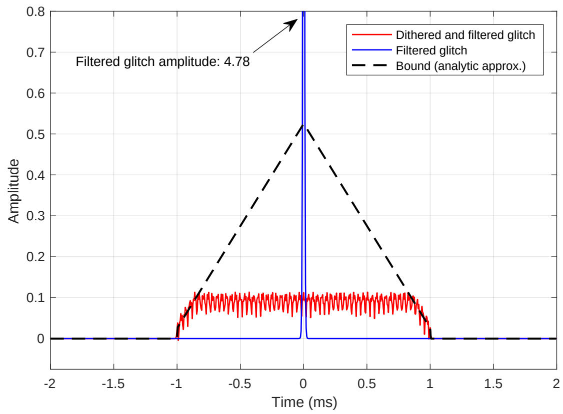


Fig. 10: Asymmetric glitch response, using a periodic (triangle-wave) dither. The maximum amplitude of the smoothed (dithered and filtered) glitch depends on parameters such as dither frequency or input signal amplitude but the expected value will not exceed the bound.

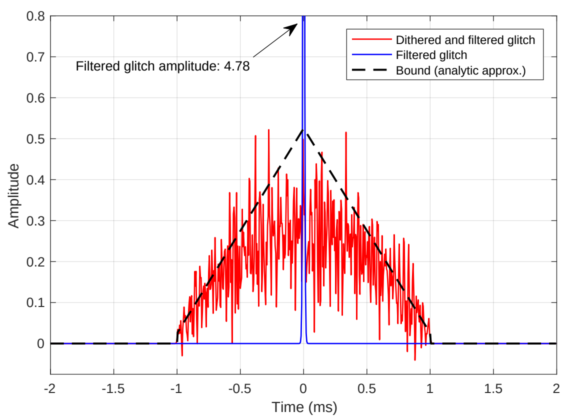


Fig. 11: Asymmetric glitch response, using a stochastic dither. The variation of the smoothed (dithered and filtered) glitch is due to imperfect filtering on the output. A lower cut-off frequency reduces the variance and brings the signal towards the bound. The bound is for the mean value.

where N_C is the number of channels. The output variance at some time instant t_0 is therefore:

$$\begin{aligned}\text{Var}(y(t_0)) &= \text{Var}(y_d(t_0)) + \text{Var}\left(\frac{1}{N_C}\sum_{k=1}^{N_C} e_k(t_0)\right) \\ &= 0 + \frac{1}{{N_C}^2}\sum_{k=1}^{N_C} {\sigma_e}^2 = \frac{1}{N_C}{\sigma_e}^2\end{aligned}$$

The variance of any deterministic and systematic signal is zero at any time instant, and will not be affected by the averaging of several channels; as the same signal is reproduced for each channel, then summed and scaled, recovering the same signal. The noise components e_k, however, are independent for each channel, and summing and scaling will cause the overall variance at any time instant to be reduced by a factor $1/N_C$.

In order to minimise the dither in the output of the summing stage, the inverse of the dither signal was applied to half of the DACs. This caused the dithers to approximately cancel each other out. When using dither signals with opposite polarity, large voltage differences can occur in the summing stage, potentially drawing higher currents than the DAC is designed for. Hence, the DAC outputs were buffered, using the Texas Instruments LME49990 operational amplifier, which has sufficient bandwidth and linearity. The printed circuit board is pictured in Fig. 13.

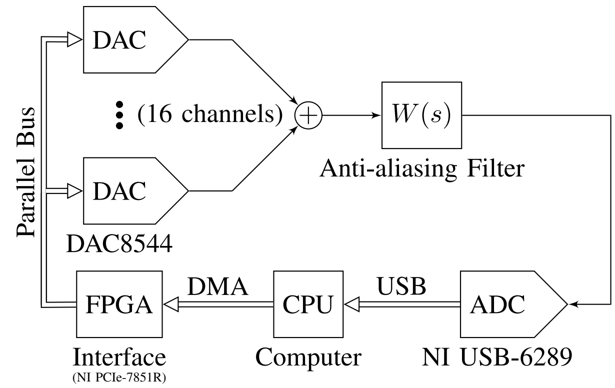


Fig. 12: Experimental set-up.

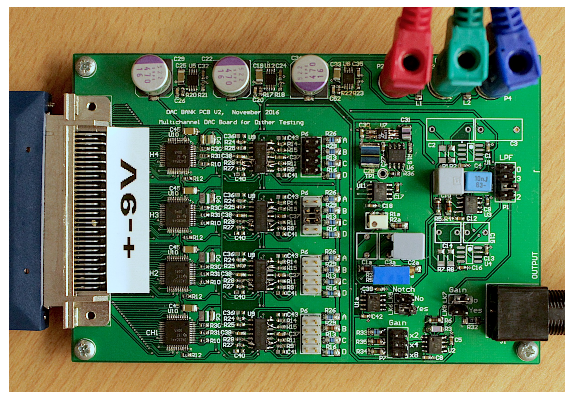


Fig. 13: Bank of DACs used in the experiments.

The DACs were connected via a parallel bus to an FPGA onboard a National Instruments PCIe-7851R interface card. The DAC driver was implemented in VHDL. LabVIEW, running on the computer (CPU), was used to generate and stream signals to the FPGA via direct memory access (DMA) over the peripheral component interconnect express (PCIe) bus. Using the sixteen DACs simultaneously, a sampling rate of 1 MS/s was achieved.

The output was measured using a National Instruments USB-6289, which contains an Analog Devices AD7674 18-bit successive approximation analogue-to-digital converter (ADC). A sampling rate of 625 kS/s was used. As described in Sec. II-C, the USB-6289 contains two first-order passive low-pass filters (13), with $f_c = 62.5$ kHz.

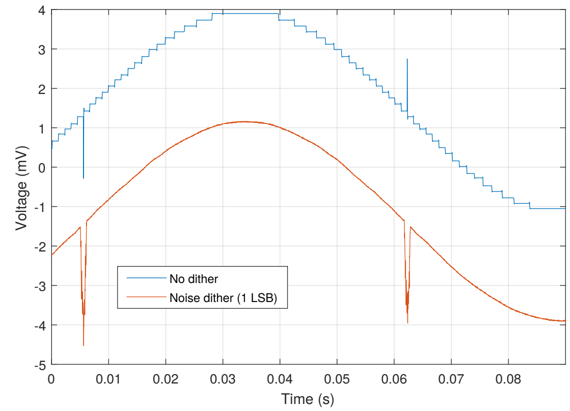
VI. Simulation and Experimental Results

A set of simulations and experiments were conducted to validate the model and evaluate the glitch mitigation method. Since the achievable sampling rate in the experimental set-up is 1 MS/s, the glitch duration in the model was set to $\tau = 1$ μs. Furthermore, the model was set up using the glitch areas found in Sec. II-C; that is, the area for the major glitches was set to $\hat{A}_i^+ = -60.6$ nVs for the rising input, and $\hat{A}_i^- = 51.9$ nVs for the falling input, at each major glitch i. The area of the minor glitches was set to ± 2.40 nVs.

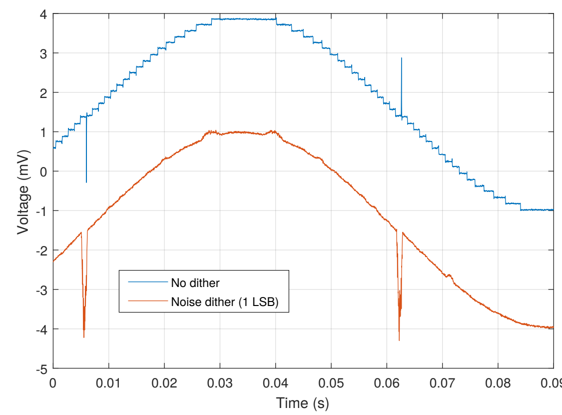
The reference signal $x(t)$ was set to be a 9 Hz sine-wave with an amplitude of 0.05% of the full range of the DAC. The stochastic dither signal $d(t)$ was produced using a pseudo-random number generator with uniformly distributed samples. The periodic dither signal $p(t)$ was set to be a 49 kHz triangle-wave (which has a uniform amplitude distribution).

The input to the ADC was filtered by the anti-aliasing filter (13). Additionally, a second-order low-pass Butterworth filter with cut-off frequency at 10 kHz was used to filter the measured response in order to achieve the desired averaging of the glitches.

Five pairs of configurations were trialled. The first pair was using a noise dither with a 1 least significant bit (LSB) range, that is, the dither was in the range $[-0.5, 0.5]$ LSB, or approximately 0.00153% of the full range of the DAC. The simulation result is shown in Fig. 14a and the experimental result is shown in Fig. 14b. The second pair was using a periodic dither with a 1 LSB peak-to-peak amplitude. The simulation and experimental results are shown in Fig. 15. The third was using a periodic dither with a 25 LSB (0.0381%) peak-to-peak amplitude, with results shown in Fig. 16. The fouth was using a noise dither with a 100 LSB (0.153%) range, with results shown in Fig. 17. The fifth was using a periodic dither with a 100 LSB peak-to-peak amplitude, with results shown in Fig. 18.



(a) Simulation result (stochastic)



(b) Experimental result (stochastic)

Fig. 14: Uniform noise with 1 LSB peak-to-peak amplitude.

VII. Discussion

The main benefit from the averaging effect due to dithering and filtering, is that a glitch can be converted from a large amplitude disturbance with a short duration, to a smaller amplitude disturbance with a longer duration. That is, a high-frequency disturbance is converted to a low-frequency disturbance. This is a major benefit in closed-loop control applications, as a control law, which will always have limited bandwidth [55], can attenuate these low-frequency disturbances, but not the high-frequency impulse-like disturbances.

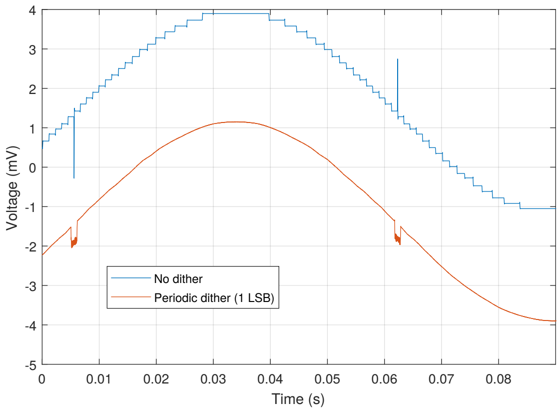
For symmetric glitches, it is apparent, from the example summarised in (60), that glitches of equal but opposing amplitudes tend to cancel each other out. The effective glitch response after dithering is very small. The actual shape of the glitch depends not only on the density function of the dither signal, but also the input signal $x(t)$. Increasing dither amplitude tends to reduce the effective glitch amplitude and increase the duration of the glitch, whereas an increasing rate of change (Lipschitz constant) for the input signal has the opposite effect. For asymmetric glitches, the effect of dither amplitude and input signal rate-of-change is the same, but the glitches do not cancel each other out, and a larger excursion for the effective glitch is therefore to be expected, as illustrated by the expression (61).

Considering the simulation and experimental results in Figs. 14, 15, 16, 17 and 18, it is apparent that the model provides a very good description of the glitches experienced in the DAC used in the experimental set-up. Using a small amplitude dither, as in Figs. 14 and 15, the minor glitches for each step are attenuated but the dither does not provide suppression of the major glitches. Note that the dither linearises the quantisation steps as expected [11]. A small amplitude dither is common in audio applications to mitigate quantisation error [56] but can be seen to aggravate the error due to glitches. Increasing the dither amplitude reduces the effect of the glitches significantly, as observed in Fig. 16. In this case the glitch has been effectively converted to a low-frequency disturbance. Further increasing the amplitude causes further suppression of the glitches, as observed in Figs. 17 and 18.

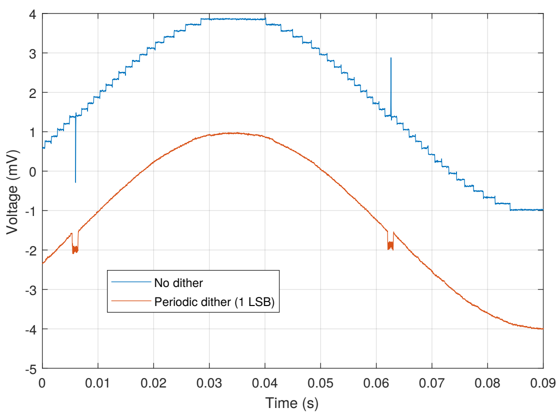
As the dither amplitude becomes larger, it becomes more difficult to attenuate at the output. In Fig. 18 some residual ripple can be seen, when using a periodic dither. Due to the broadband nature of white noise, a significant amount of dither power remains below the cut-off frequency when using a stochastic dither, as can be seen in Fig. 17. Since it is easier to remove the narrow-band ripple, rather than the broadband noise, a periodic dither is considered to be more practical.

VIII. Conclusions

A new glitch model was demonstrated to provide a good behavioural description of glitches produced by digital-to-analogue converters (DACs). It was further demonstrated both

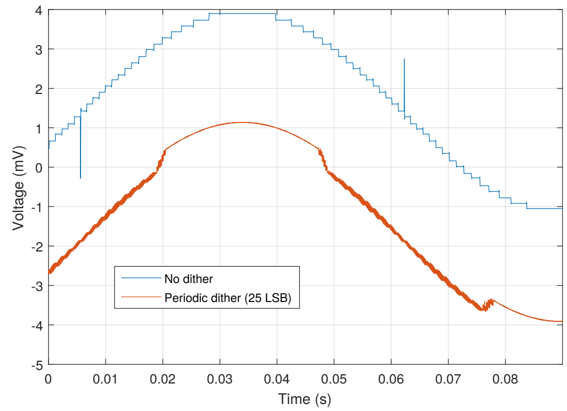


(a) Simulation result (periodic)

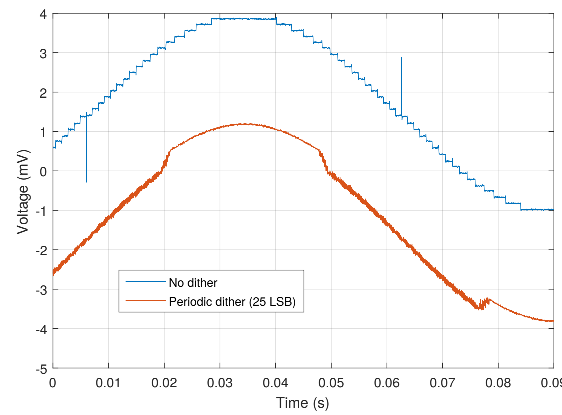


(b) Experimental result (periodic)

Fig. 15: Triangle-wave at 69 kHz, 1 LSB peak-to-peak.



(a) Simulation result (periodic)



(b) Experimental result (periodic)

Fig. 16: Triangle-wave at 69 kHz, 25 LSB peak-to-peak.

analytically and experimentally that dithering and low-pass filtering can be used to suppress glitches if the dither has sufficiently large amplitude. The improvement is due to the averaging effect on the non-linearity of the DAC. It is straightforward to retrofit the method to an existing, sufficiently fast DAC, as it requires only generation of a dither signal and a suitable analogue filter for attenuation of the dither in the output. The effect of dithering can be viewed as converting glitches from short-duration, high-amplitude disturbances to long-duration, low-amplitude disturbances, which may be further attenuated by an external feedback control law.

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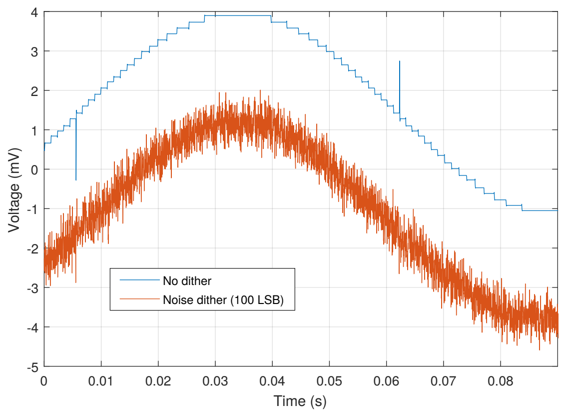
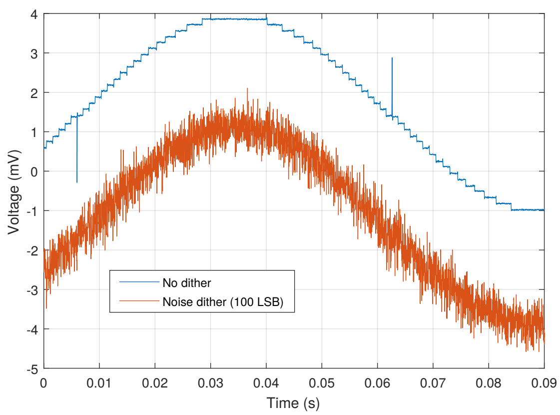
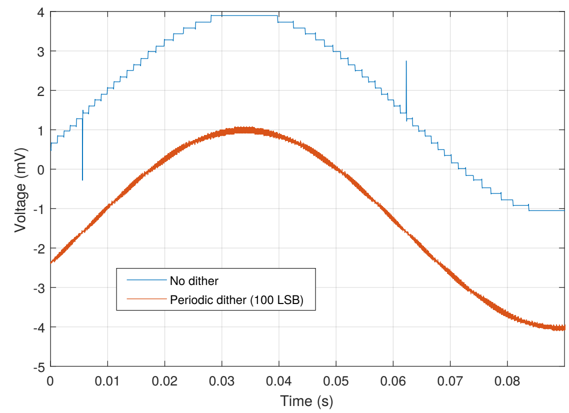


(a) Simulation result (stochastic)

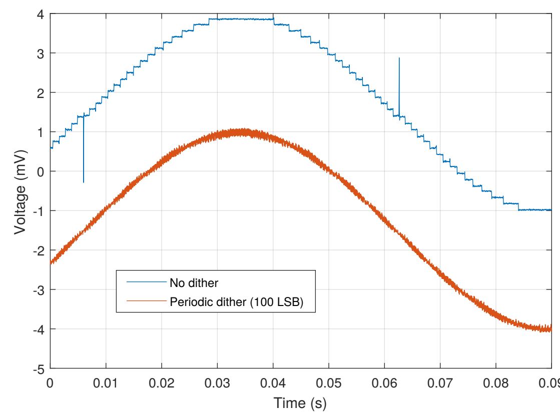


(b) Experimental result (stochastic)

Fig. 17: Uniform noise with 100 LSB peak-to-peak amplitude.



(a) Simulation result (periodic)



(b) Experimental result (periodic)

Fig. 18: Triangle-wave at 69 kHz, 100 LSB peak-to-peak.

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