PNP
What is in a Name?

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July 25, 2019

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Abstract and Summary

The name **PNP** was introduced by Eisenberg and Chen because it has important physical meaning beyond being the first letters of **P**oisson-**N**ernst-**P**lanck. **PNP** also means **P**ositive-**N**egative-**P**ositive, the signs of majority current carriers in different regions of a **PNP** bipolar transistor. **PNP** transistors are two diodes in series **PN** + **NP** that rectify by changing the shape of the electric field. Transistors can function as quite different types of nonlinear devices by changing the shape of the electric field. Those realities motivated Eisenberg and Chen to introduce the name **PNP**.

The pun "**PNP = P**oisson-**N**ernst-**P**lanck **= P**ositive-**N**egative-**P**ositive" has physical content. It suggests that **P**oisson-**N**ernst-**P**lanck systems like open ionic channels should not be assumed to have constant electric fields. The electric field should be studied and computed because its change of shape is likely to be important in the function of biological systems, as it is in semiconductor systems.

PNP is a shortened name for "**P**ositive-**N**egative-**P**ositive" or "**P**oisson-**N**ernst-**P**lanck equation". It was not meant to be just an abbreviation: names are important, beyond their logical meaning, as the world shows us everyday. The name **PNP** is no exception. The name was chosen to help understand the system it describes.

PNP was a pun introduced at the 1993 Biophysical Society (USA) meeting [1] by Eisenberg and Chen [2, 3] to emphasize the analogy between open ion channels and semiconductor devices. The **P**oisson equation is a version of Maxwell's first equation [4-7] that describes how charge creates electrical forces and thus electrical potential. The **N**ernst-**P**lanck equation [8-27] describes how electrical charges migrate (in the gradient of electric fields) and diffuse (in the gradient of concentration fields).[1] The combination **PNP** is often called the drift diffusion equation in the semiconductor literature [8, 11, 12, 14-17, 20]

PNP meaning "**P**ositive-**N**egative-**P**ositive" describes the spatial distribution of mobile charge produced (mostly) by the spatial distribution of doping in a semiconductor device, a bipolar transistor. Doping is a name for the ionizable impurities (dopants) introduced into pure semiconductors to create the quasiparticles holes **P** and electrons **N**. When dopants ionize, they leave behind a permanent charge (negative or positive) in a fixed spatial distribution, much like the permanent charge of ionized weak acids and bases. The ionized acid and base side chains of proteins, like glutamates **E** or lysines **K,** are one kind of the permanent charge of proteins.

Holes and electrons diffuse and migrate according to the **PNP** equations [8, 10, 28]. Note that the 'electrons' [29] of semiconductors are not the electrons [30, 31] discovered by JJ Thomson or the electrons found in atoms [32]. The 'electrons' of semiconductors are quasiparticles defined by properties of the conduction bands of semiconductors [17, 28, 33].

Eisenberg and Chen chose the name **PNP** to emphasize the analogy between doping of semiconductors and the permanent charge of channels, or ion exchange membranes [9, 20, 24, 25, 34-67].

Eisenberg and Chen were thinking of semiconductor devices because transistors and **PNP** equations have a wide range of nonlinear behavior. For example, **PNP** equations can describe an amplifier, limiter, multiplier, exponentiator, or logarithmic

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[1] Another paper is needed to describe the utility and evident limitations of **PNP**, as well as its antecedents and present uses.

converter depending on the range of voltages applied to the transistor through boundary conditions.

Eisenberg and Chen wondered if nonlinear devices, well described by the **PNP** equations in semiconductors, might also exist in biological systems, particularly protein channels and transporters [68]. These ideas were spelled out in an Abstract [69] presented at the Society of General Physiology Meeting 1992: "Exchange Diffusion, Single Filing, and Gating in Macroscopic Channels of One Conformation".

Nonlinearities of proteins were particularly interesting because they were thought to be the 'secret of life' by many physicists coming to biology soon after World War 2 [70-74]—at the same time that Shockley invented the transistors [75, 76]. Transistors create the complex nonlinearities of semiconductor devices.

We now know that the nonlinearities of biology exist on many scales [77]. Some of those nonlinearities arise on the cellular scale of neurons and dendrites [78, 79]. Some arise on the molecular scale of proteins. Some arise on the even smaller atomic scale of the open ionic channel [80, 81]. Eisenberg and Chen wondered which of the nonlinearities of channels and transporters might come from **PNP** equations like those describing semiconductor devices [69].

Most biologists sought other explanations for the nonlinearities of life. I hasten to add that those biologists seem to be right. Eisenberg and Chen's hope that nonlinear biological devices—e.g., channels or transporters—would emerge as analogs of transistor devices, described by **PNP** equations, has not been fulfilled, as far as I know, probably, I suspect, because the third terminal of transistors—so important to the history of technology and to human life as the source of amplification—has not yet been found (or recognized) in channels or transporters. Transistors connected as two terminal devices provide properties that have not been considered by biologists, as far as I know.

Eisenberg and Chen knew of the bipolar transistor **PNP** (along with its fraternal twin **NPN**) because bipolar transistors were the dominant form of solid state device, analog or digital, for much of their lives. Engineers today live quite a different life. Engineers today focus on digital technologies (usually **CMOS** and its cousins) and so knowledge of bipolar transistors is not widespread.[2]

The bipolar transistor [82] is made of two semiconductor diode rectifiers [83] **PN** and **NP** in series. Crystal rectifiers much like these were used in the early history of

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[2] It seemed wise to write this paper before the knowledge of bipolar **PNP** transistors and analog circuity disappears altogether.

radio broadcasting (around 1920) and remained of great interest to hobbyists for many years, including the young Eisenberg and his father. Crystal radios seem to “run on nothing”, using only the energy gathered by the antenna system (paraphrase of [84]). They demonstrate in a most practical way that the electric field of radio waves exists and has enough energy to power a (tiny) loudspeaker. Anyone who builds a crystal radio is likely to be entranced forever by the electric field and its propagation of power through empty space [85, 86]. Eisenberg was no exception.

PNP transistors and **PN** diodes are rectifiers that detect asymmetrical signals in radio waves. They ‘make current flow the right way’—that is, they make current flow in one direction as Edison thought it should in the war between Edison’s **DC** and Tesla’s **AC** systems of electrical power [87]. The resistance of rectifiers depends on the direction of current flow because the shape of the electric fields at the **PN** or **NP** junctions depends on the direction of current flow.

Rectifiers exist in biological membranes and so it was natural to analyze them the way crystal rectifiers were analyzed. One of the early papers on the crystal rectifier [88] served as the template for the constant field **GHK** theory of Goldman [89] and Hodgkin and Katz [90] of rectification in membranes.

The **GHK** theory has been used extensively in electrophysiology to describe the electric field in the proteins that make ionic channels [91-93], not just the electric field in membranes, although the theory contains no description of the structure or charge of the channel protein that determine most channel properties. It is unfortunate that such a widely used theory assumes a constant shape of the electric field independent of the structure, charge, concentration, or membrane potential associated with the channel protein. It is even more unfortunate that it computes a current or reversal potential independent of the structure or charge of the channel protein.

A central aspect of **PNP** physics is rectification. Rectification depends on the shape of the electric field. Rectifiers function by changing the shape of the field and so the shape of the field needs to be computed [94-98], not assumed. In fact, assuming the shape of the field will prevent understanding of how the device works.

This point was emphasized by Eisenberg in a series of reviews [80, 94, 99-103] because he felt the role of the electric field in biological systems (and chemical reactions) could not be understood if the field was assumed constant. Properties of biological systems or chemical reactions that arise from changes in the shape of the field cannot be robustly described in theories that assume a constant field. Constant field theories would not be transferable. Theories would have to change parameters as they tried to mimic the consequences of a changing field.

Transferrable theories (that use the same set of parameters in different conditions) are needed to create stable understanding or robust devices, in my view.

> It should clearly be understood that the shape of an electric field varies as experimental conditions are changed (as they are changed in almost all experiments). Voltage clamp apparatus maintains voltages only at one specific location. To maintain the shape of an electric field, voltage clamp must be applied at many locations (within an ionic channel or membrane) because the only way to maintain a potential, as conditions are changed, is to supply charge from an external device like a voltage clamp amplifier. That charge must come through 'wires' from an external source, because ion channels—like most proteins—are in themselves isolated devices, unable to create charge. These issues are discussed at embarrassing length in the reviews [80, 94, 99-103].

Isolated proteins need to be described as spatial distributions of permanent charge (to a first approximation) just as **PNP** transistors are described as distributions of doping (to a first approximation) and for the same reason. Their materials provide a permanent charge (as a first approximation) that arises from their chemical nature [104]. The second approximation is provided by the field dependent induced polarization charge of the dielectric. Further approximations require a fuller description of polarization [4, 86, 105].

The boundary condition describing the potential in isolated proteins like channels is an inhomogeneous Neumann condition defining the (normal) spatial derivative of the potential, that is to say, defining the permanent charge, not the potential itself, to a first approximation (Appendix eq. A25 of [104]). The chemical nature and structure of the amino acids and proteins determine the permanent charge and thus the inhomogeneous Neumann condition on the potential.

Proteins are almost always isolated from the outside world except at boundaries like baths connected to the outside world by the apparatus of electrochemical cells, i.e., 3M KCl bridges, AgAgCl electrodes, and amplifiers [22, 106]. Proteins have one unchanging spatial distribution of charge as conditions change (neglecting the second order effect of dielectric properties). Proteins cannot be described by a single field of potential as conditions change because the electric field changes. It is the permanent charge of the protein that does not change as conditions change.

Experiments maintaining a constant field in vacuum (i.e., one spatial distribution of potential) have been done. They are difficult to perform even in a **SQUID** (superconducting quantum interference device) [107].

Acknowledgement

PNP is just a name, no matter what it stands for. The science of the drift and diffusion of charge carriers, computed with a combination of electrodynamics (**P**oisson) and drift-diffusion (i.e., **N**ernst **P**lanck), is very much more than just a name. The community of scholars who have developed that science over more than a century deserve all the credit for what it has brought us, most remarkably, and unforeseen by almost all.

We have an information and audiovisual technology available easily to billions of people that allows human interactions, evolved in a village and tribal setting I suspect, to involve most of the human race in hours or even minutes. That technology is possible only because of the solid state semiconductor devices well described by the **PNP** equations in their various forms with physical or effective parameters.

It is a joy to thank that community of scholars, in general, and Karl Hess and Dave Ferry, in particular for all they and their community have done for me and all of us. It is sad that I could not include more discussion of the wonderful literature of semiconductors. But that would have over-burdened this essay which is, after all, more an historical confectionary, than a comprehensive review.

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