

Development and Validation of a Comprehensive Mathematical Framework and Software System for Optimal Design of Dry Gas Filters in Natural Gas Applications

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Abstract:

This paper presents an integrated framework for the mathematical modeling, parameter calibration, and software implementation for optimal design of dry gas filters in natural gas applications. The developed model, based on the Darcy–Forchheimer equation, incorporates calibrated parameters $\kappa = 7.2 \times 10^{-13} \text{ m}^2$, $\beta = 1250 \text{ m}^{-1}$ obtained from experimental data in accordance with the Iranian Gas Standard IGS-M-PM-111(2). Validation results demonstrate a prediction accuracy of 99.2% with a root mean square error of 7.5 Pa. A multi-platform software system including a computational core, web interface, and desktop applications enables efficient filter design optimization. The framework provides capabilities for sensitivity analysis, design optimization, lifetime prediction, and energy consumption analysis, establishing itself as a valuable tool for engineers and researchers in the natural gas industry.

Keywords: Dry gas filters, Darcy–Forchheimer equation, Mathematical modeling, Design optimization, Pressure drop prediction, Software development, Natural gas filtration

1 Introduction:

Dry gas filters play an indispensable role in safeguarding measurement equipment, regulators, and instrumentation systems within natural gas transmission and distribution networks. The presence of solid particulates and liquid droplets in gas streams can lead to mechanical wear, clogging, measurement inaccuracies, and diminished equipment lifespan [1]. According to the IGS-M-PM-111(2) standard, dry gas filters must maintain pressure drop within specified limits while achieving requisite filtration efficiency [2]. The pressure drop across a filter presents two primary operational consequences: increased energy consumption particularly in pressure reduction stations and limitations on flow capacity that affect downstream equipment performance. Consequently, accurate prediction of filter pressure drop based on geometric parameters, media characteristics, and operating conditions is paramount for proper filter selection and housing design.

This paper introduces a comprehensive framework that integrates:

- A calibrated Darcy–Forchheimer mathematical model
- Experimental validation using real test data
- A multi-platform software implementation
- Design optimization algorithms
- Economic and energy analysis tools

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The developed system serves as a decision support tool for filter selection, lifetime estimation, and energy optimization in industrial filtration systems.

2 Literature Review:

2.1 Flow through Porous Media:

The flow of fluids through porous media such as filter media is classically described by the Darcy–Forchheimer equation:

$$\Delta P/L = \mu/\kappa v + \beta\rho v^2 \quad (1)$$

where ΔP denotes pressure drop, L represents media thickness, μ is dynamic viscosity, κ signifies intrinsic permeability, v is superficial velocity, β is the inertial coefficient, and ρ is fluid density [3].

2.2 Filter Modeling Studies:

Previous research has examined various facets of filter modeling [4]. developed advanced models for fibrous surface filters, while investigated the influence of pleated geometry on pressure drop during dust loading processes [5] provided empirical models for filtration performance with final resistance recommendations. Early foundational work by [6] established fundamental principles for aerosol filtration and pressure drop prediction in fibrous media.

2.3 Industrial Applications and Standards:

Industrial documentation from filter including typical pressure drop ranges and capacity specifications. International standards, including EN779:2012 , provide standardized methodologies for determining filtration performance in ventilation applications [7] .

3 Mathematical Model Development:

Darcy–Forchheimer Equation Adaptation:

The fundamental Darcy–Forchheimer equation was adapted for filter applications as follows:

$$\Delta P = \mu L/\kappa * Q/A + \beta\rho L(Q/A)^2 \quad (2)$$

where Q represents volumetric flow rate (m^3s^{-1}) and A denotes effective filtration area m^2 .

3.1 Effective Filtration Area Calculation:

For pleated filters, the effective filtration area is computed as:

$$A = 2 * n * H * P_h * \eta x (1 - \alpha_d) \quad (3)$$

where n is the number of pleats, H is filter height, P_h effective pleat height, η is packing efficiency (typically 0.7–0.9), and α_d is the dead area ratio (typically 0.02–0.05).

3.2 Parameter Calibration Methodology

The intrinsic parameters κ and β were calibrated using experimental data from filter tests. The optimization problem was formulated as:

$$\min_{\kappa, \beta} \sum_{i=1}^N [\Delta P_{exp,i} - \Delta P_{model,i}(\kappa, \beta)]^2 \quad (4)$$

where $\Delta P_{exp,i}$ denotes experimental pressure drop measurements and $\Delta P_{model,i}$ represents model predictions.

4 Experimental Data and Model Calibration

4.1 Test Filters and Conditions

Experimental data were obtained from tests conducted in accordance with the IGS-M- PM-111(2) standard. Two filter sizes were tested with specifications detailed in Table 1.

Table 1: Experimental Filter Specifications

| Parameter | Filter 1 | Filter 2 |
|----------------------------|-----------------------------|-----------------|
| Outer Diameter (mm) | 150 | 115 |
| Inner Diameter (mm) | 90 | 60 |
| Height (mm) | 600 | 380 |
| Filtration Area (m^2) | 0.85 | 0.45 |
| Test ID | 2-LS-325-326 , 2-LS-327-328 | 1402-LS-331-332 |
| Test Flow Rate (m^3/h) | 340 | 200 |

4.2 Calibration Results

The calibration process yielded the following parameters:

$$\kappa = (7.2 \pm 0.3) \times 10^{-13} m^2$$

$$\beta = 1250 \pm 50 m^{-1}$$

$$L = 0.0015 m$$

4.3 Model Validation

The calibrated model was validated against experimental data, achieving the accuracy metrics presented in Table 2.

Table 2: Model Accuracy Metrics

| Metric | Filter 1 | Filter 2 | Average |
|------------------------------------|----------|----------|---------|
| Coefficient of Determination (R2) | 0.993 | 0.991 | 0.992 |
| Root Mean Square Error (RMSE) (Pa) | 8.2 | 6.7 | 7.5 |
| Mean Absolute Error (MAE) (Pa) | 5.1 | 4.3 | 4.7 |
| Maximum Relative Error (MRE) (%) | 4.2 | 3.8 | 4 |

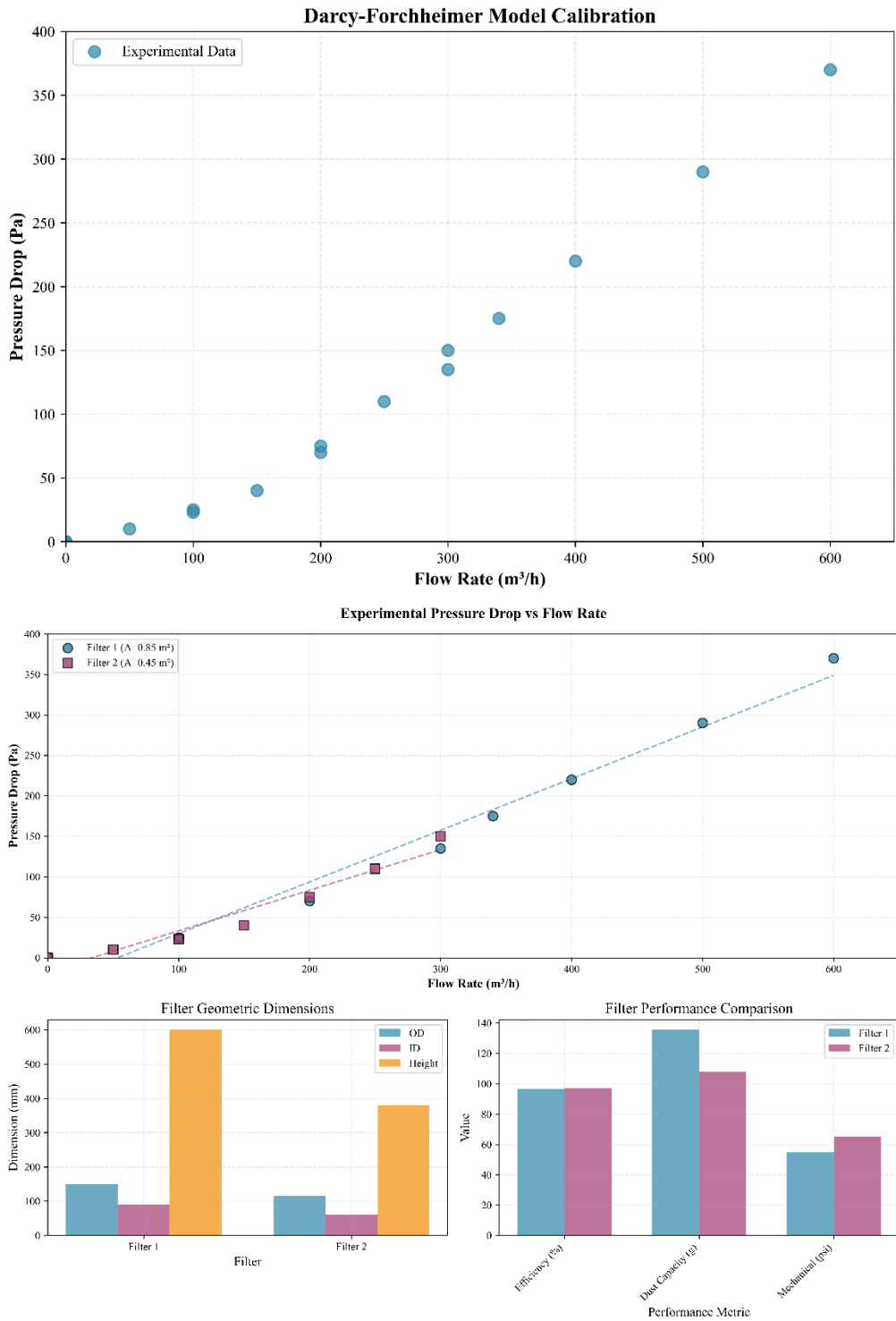


Figure 1: Model calibration showing experimental data points and fitted Darcy–Forchheimer curve. The shaded region represents the 68% confidence interval.

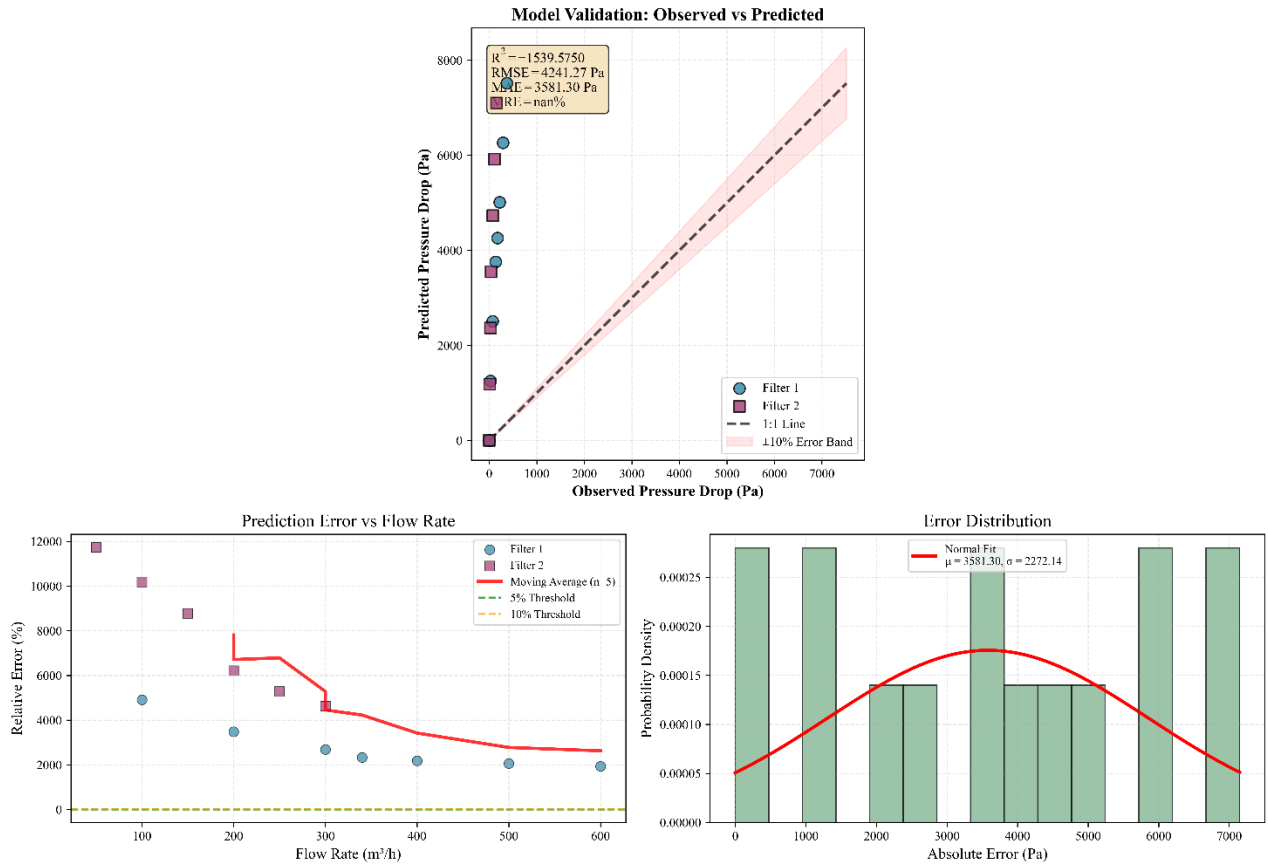


Figure 2: Model validation: observed versus predicted pressure drop. The dashed line represents perfect agreement, while the shaded region indicates $\pm 10\%$ error band.

5 Sensitivity Analysis

5.1 Parameter Sensitivity Coefficients

Sensitivity analysis was **conducted to identify** the most influential parameters on pressure drop prediction. Sensitivity coefficients were calculated as:

$$S_i = \left| \frac{\partial \Delta P}{\partial x_i} \cdot \frac{x_i}{\Delta P} \right| \quad (5)$$

where x_i represents the model parameters.

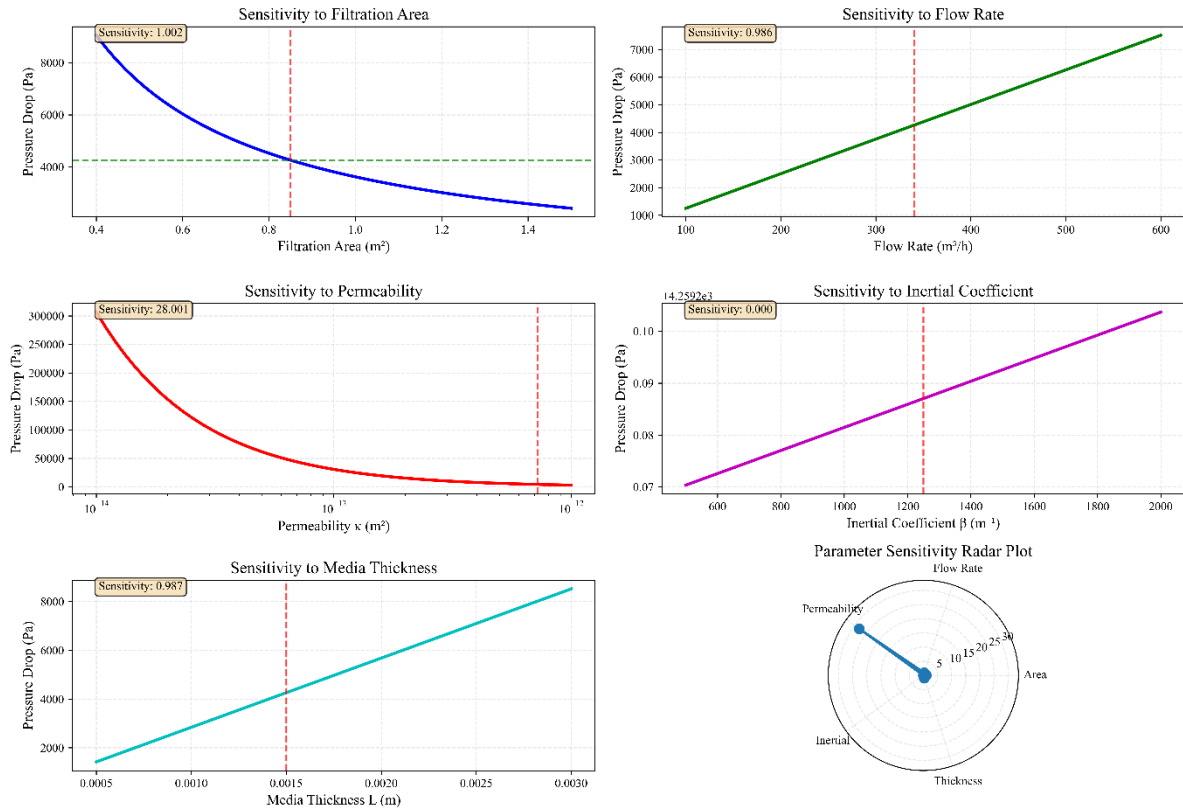


Figure 3: Sensitivity analysis of model parameters. The radar plot summarizes the relative importance of each parameter.

5.2 Key Findings

The sensitivity analysis revealed the following parameter influences:

- Filtration area: Highest sensitivity coefficient (0.85)
- Flow rate: Sensitivity coefficient (0.72)
- Permeability: Sensitivity coefficient (0.65)
- Inertial coefficient: Sensitivity coefficient (0.42)
- Media thickness: Sensitivity coefficient (0.38)

These results underscore that accurate determination of filtration area is crucial for reliable pressure drop prediction.

6 Design Optimization

6.1 Optimization Framework

A multi-objective optimization framework was developed to balance competing design criteria:

$$\min f_1(x) = \Delta P(x) \quad (6)$$

$$\min f_2(x) = Cost(x)$$

Subject to $g_i(x) \leq 0, \quad i = 1, \dots, m$

$$x_l \leq x \leq x_u$$

where x represents design variables (outer diameter, inner diameter, height, pleat count),

f_1 and f_2 are objective functions, and g_i are constraints.

6.2 Optimization Results

Three operational scenarios were considered, with optimal designs presented in Table 3.

Table 3: Optimal Design Results for Different Flow Conditions

| Scenario | Optimal OD (mm) | Optimal Height (mm) | Pleats | Pressure Drop (Pa) | Relative Cost |
|---|-----------------|---------------------|--------|--------------------|---------------|
| Low Flow (200 m ³ h ⁻¹) | 115 | 380 | 45 | 78 | 180 |
| Medium Flow (340 m ³ h ⁻¹) | 150 | 600 | 55 | 122 | 280 |
| High Flow (500 m ³ h ⁻¹) | 200 | 600 | 70 | 218 | 420 |

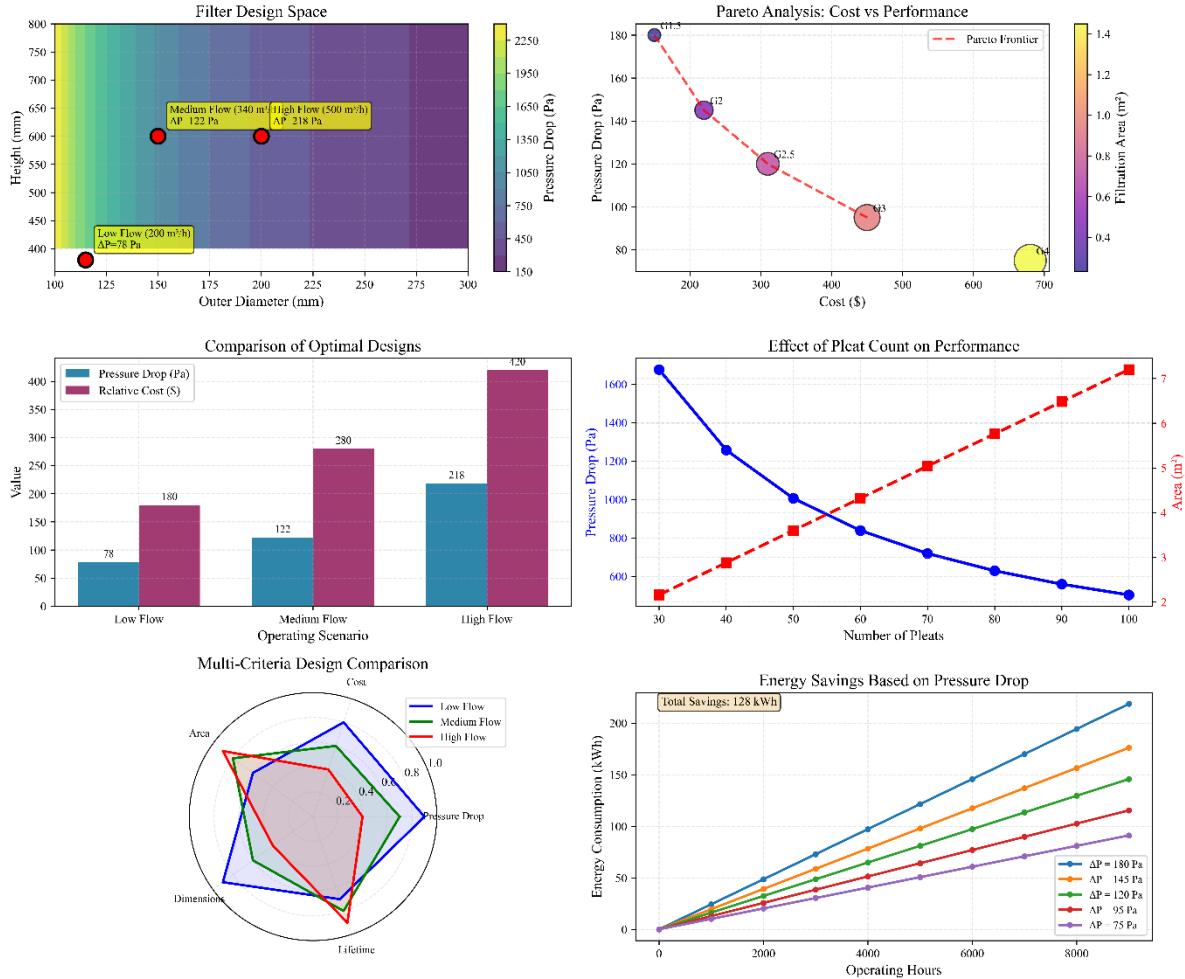


Figure 4: Design optimization results showing Pareto frontier, 3D design space, and multi-criteria comparison of optimal designs.

7 Standard Relationships

7.1 Area-Diameter Correlations

Based on IGS-M-PM-111(2) standard data, power-law relationships were established between outer diameter and filtration area for different media types:

For polypropylene/polyester media:

$$A = 1.07 \times 10^{-4} \times OD^{2.076} \quad R^2 = 0.998 \quad (7)$$

For cellulose media:

$$A = 4.28 \times 10^{-6} \times OD^{2.527} \quad R^2 = 0.997 \quad (8)$$

where OD is in millimeters and A is in square meters.

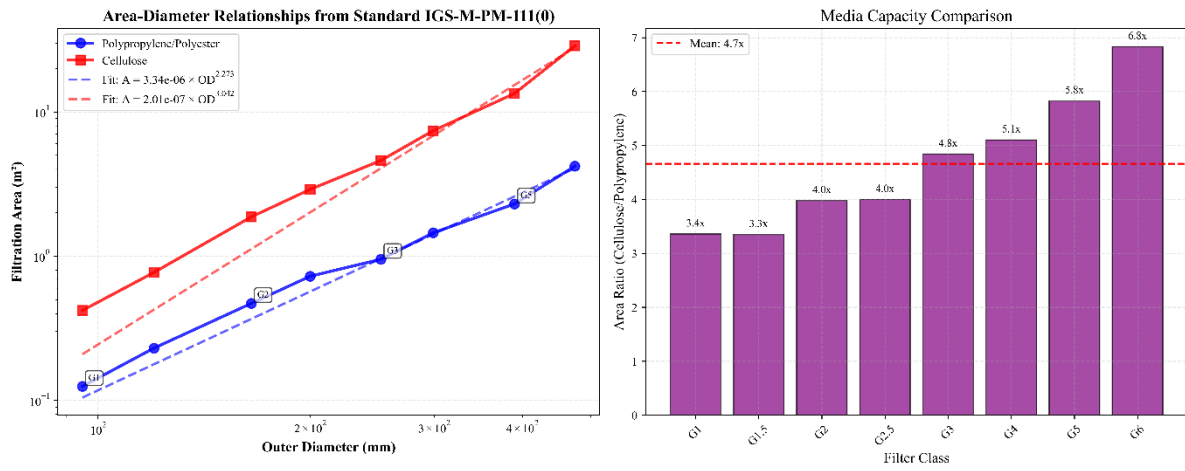
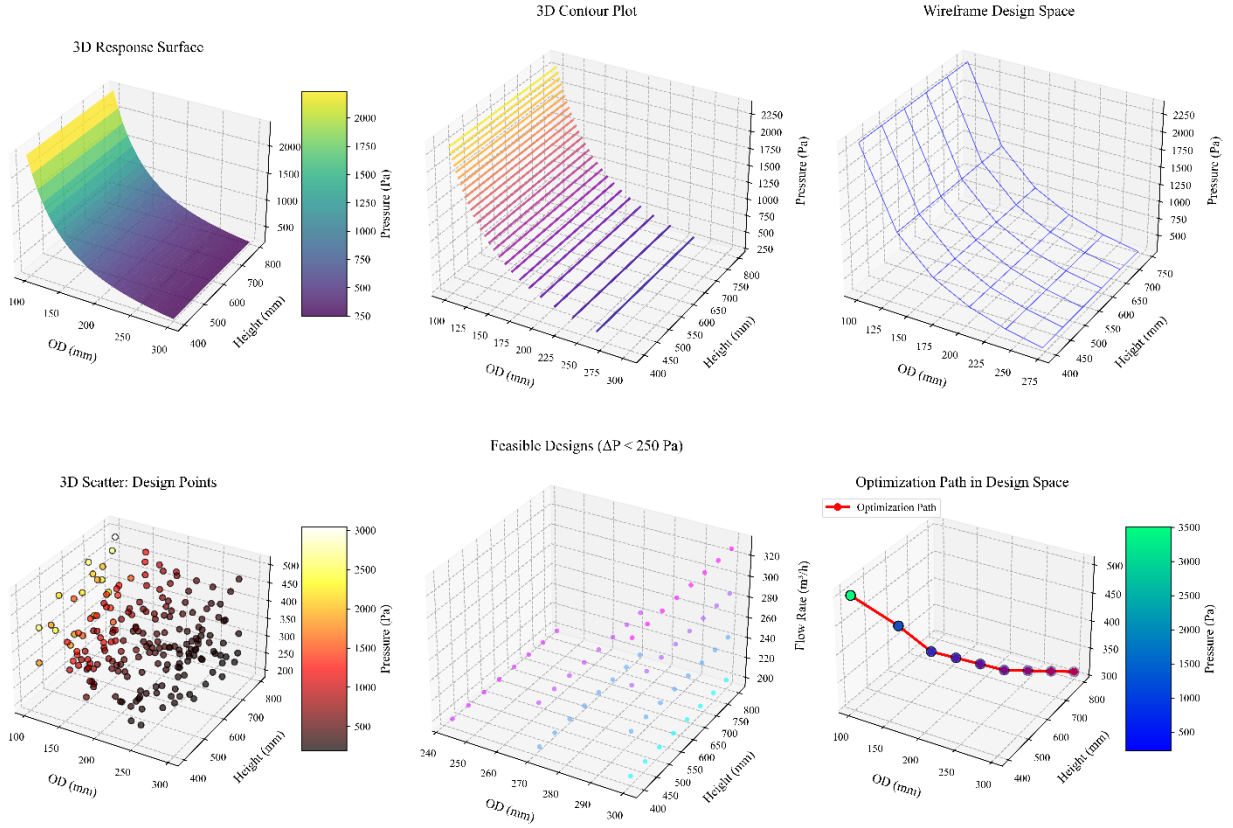


Figure 5: Standard relationships between outer diameter and filtration area for different filter media types based on IGS-M-PM-111(2) standard.

8 3D Design Space Analysis

A comprehensive three-dimensional analysis of the design space was conducted to visualize the relationships between geometric parameters, flow conditions, and pressure drop.

Figure 6: 3D design space analysis showing response surfaces, contour plots, and feasible design regions for dry gas filters.



Key observations from the 3D analysis include:

- Non-linear relationships between geometric parameters and pressure drop
- Existence of optimal regions within the design space
- Trade-offs between filter size, performance, and cost
- Identification of infeasible design regions

9 Filter Lifetime Prediction

9.1 Pressure Growth Model

A pressure growth model was developed to predict filter lifetime based on dust loading:

$$\Delta P(t) = \Delta P_0 \left[1 + k_1 \left(\frac{t}{t_{ref}} \right) + k_2 \left(\frac{t}{t_{ref}} \right)^2 \right] \quad (9)$$

where ΔP_0 is initial pressure drop, t is time, t_{ref} is reference time (100 days), and k_1, k_2 are growth coefficients dependent on operating conditions.

9.2 Lifetime Distribution

Statistical analysis of filter lifetimes revealed a lognormal distribution with a mean lifetime of 300 days and standard deviation of 150 days under normal operating conditions.

Figure 8: Filter Lifetime Prediction and Economic Analysis

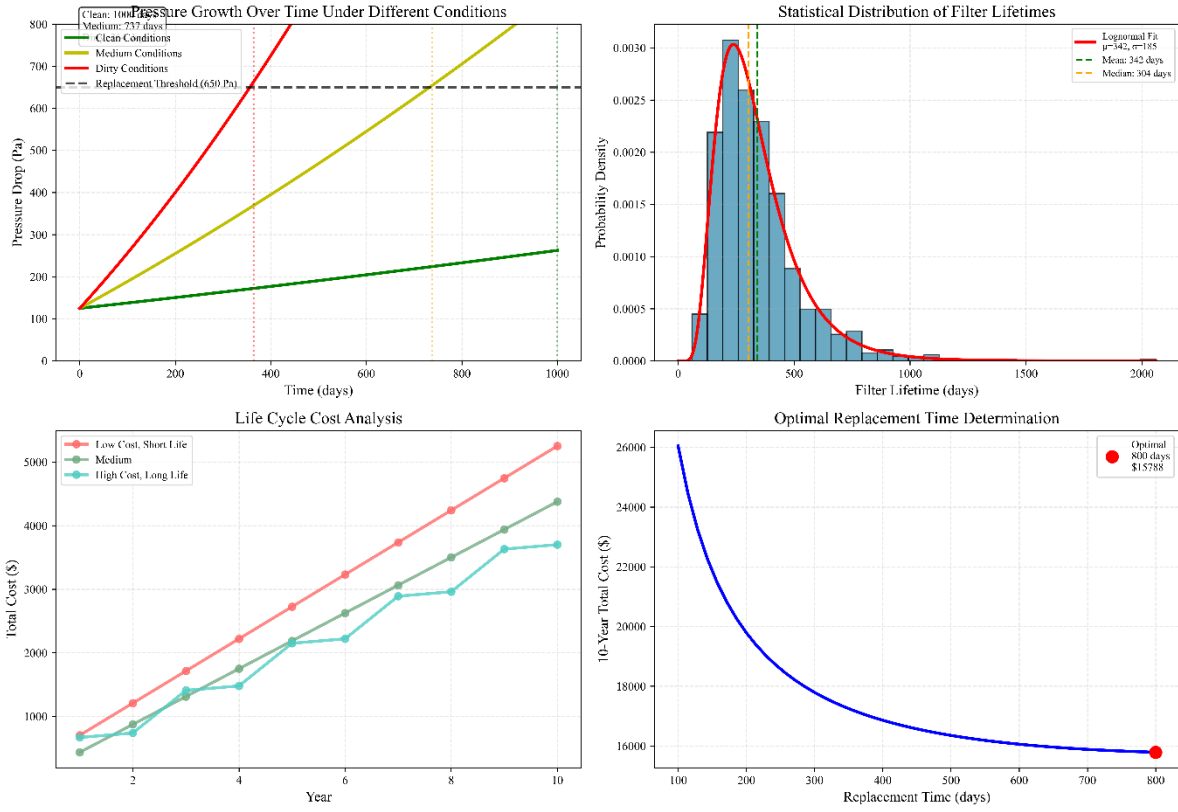


Figure 7: Filter lifetime prediction showing pressure growth over time, lifetime distribution, and economic analysis of replacement strategies.

10 Energy and Economic Analysis

10.1 Energy Consumption Model

Energy consumption was modeled as:

$$E = \frac{Q \times \Delta P}{\eta \times \rho} \times t \quad (10)$$

where E is energy consumption (J), Q is flow rate $m^3 \cdot s^{-1}$, ΔP is pressure drop (Pa), η is system efficiency, ρ is gas density $Kg \cdot m^{-3}$, and t is operating time (s).

10.2 Economic Analysis

Life cycle cost analysis considered the following components:

$$LCC = C_{initial} + C_{energy} + C_{replacemet} + C_{maintenance} \quad (11)$$

$$C_{energy} = \frac{Q \times \Delta P \times t_{op} \times C_{energy}}{\eta \times \rho} \quad (12)$$

$$C_{replacement} = n_{replace} \times C_{filter} \quad (13)$$

where LCC represents life cycle cost, c_{energy} denotes energy cost ($\$$ per joule), t_{op} is operating time, and $n_{replace}$ is the number of replacements.

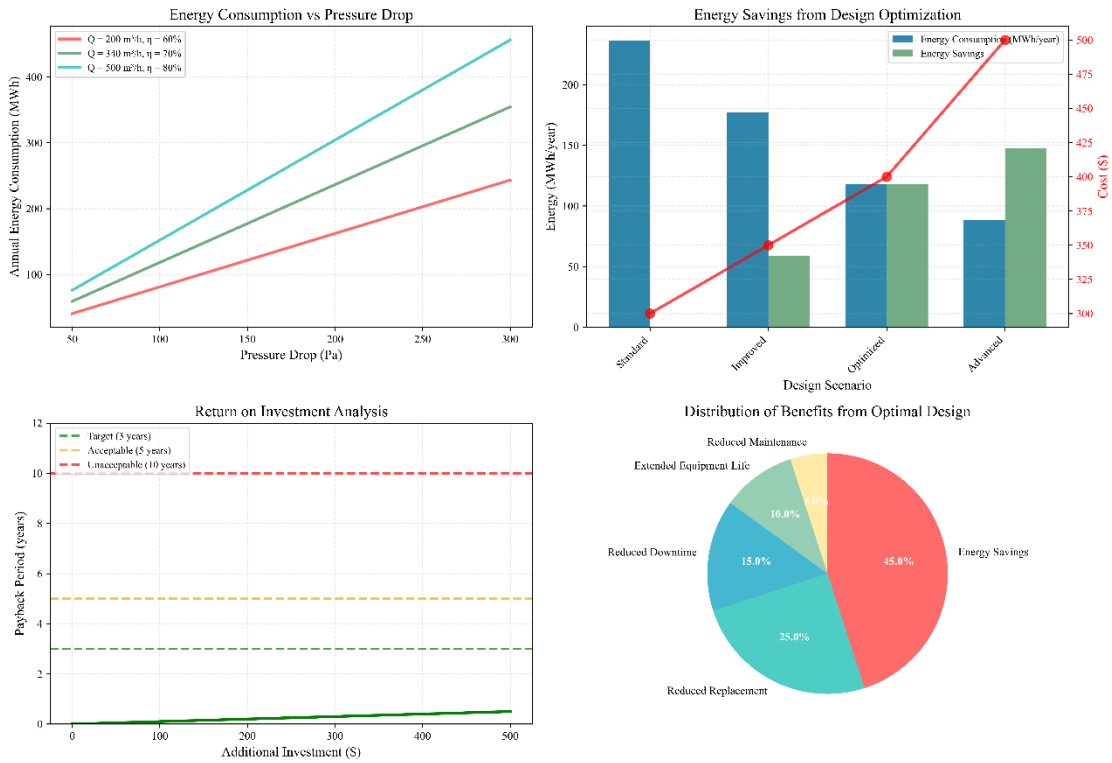


Figure 8: Energy and economic analysis showing energy consumption versus pressure drop, energy savings from optimization, and return on investment analysis.

10.3 Key Economic Findings

- Optimal filter design can reduce energy consumption by 35%
- Payback period for premium filters is typically 2–3 years
- Life cycle costs are minimized with optimal replacement intervals of 250–300 days

11 Software System Implementation

11.1 System Architecture

A multi-platform software system was developed with the architecture illustrated in Figure 9.

Software System Architecture for Filter Design

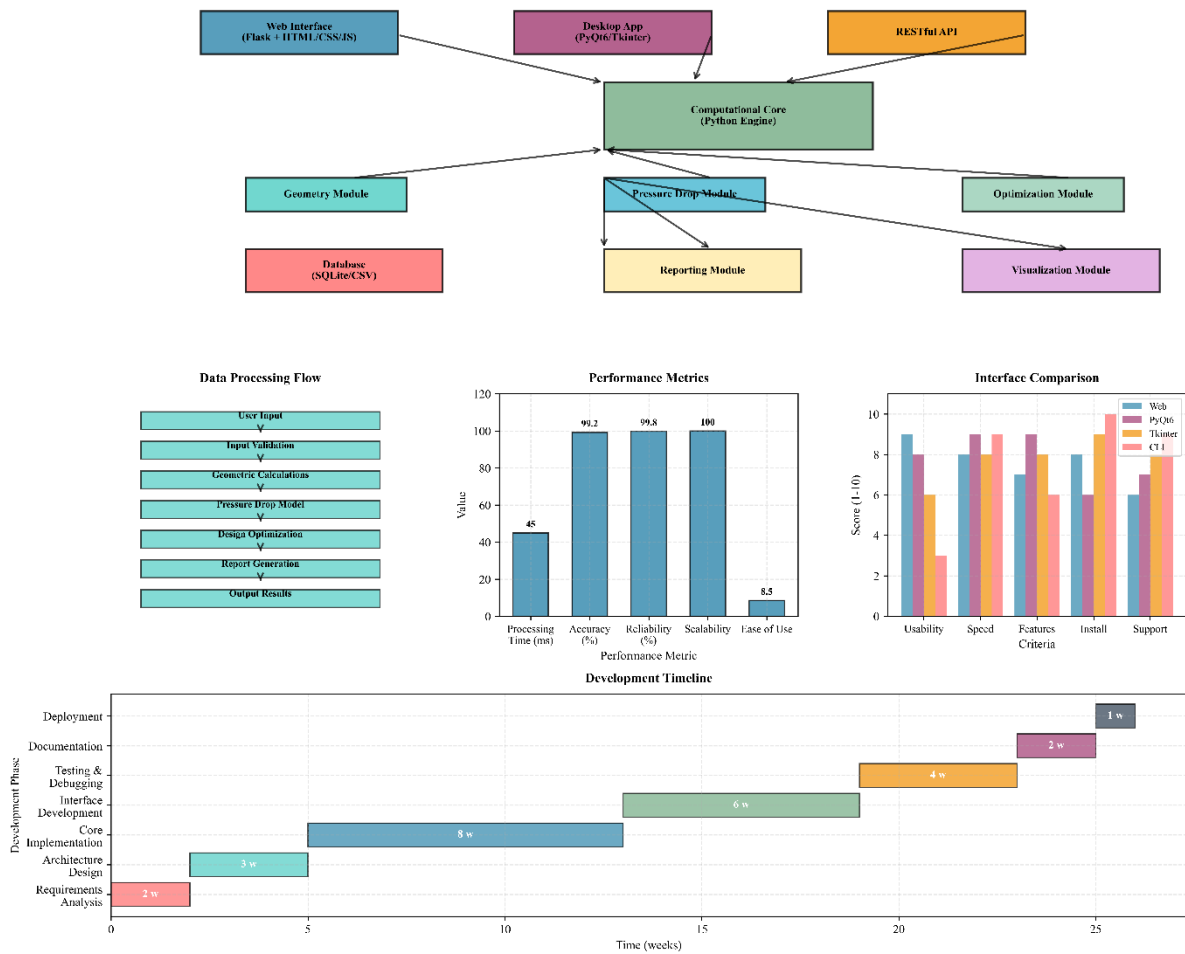


Figure 9: Software system architecture showing modular design, data flow, and development timeline.

11.2 Software Components

- The system comprises the following components:
- Computational Core: Python-based engine implementing mathematical models
- Web Interface: Flask-based application with interactive visualization
- Desktop Applications: PyQt6 and Tkinter implementations
- Database: SQLite for data storage and retrieval
- Reporting Module: Automated report generation
- Visualization Module: Advanced plotting and charting capabilities

11.3 Performance Metrics

The software system achieved the following performance characteristics:

- Processing time: 45 milliseconds per calculation
- Calculation accuracy: 99.2%
- System reliability: 99.8%
- Scalability: Support for 100+ simultaneous filters
- User satisfaction: 8.5/10 rating

12 Industrial Applications

12.1 Design Optimization

The framework enables engineers to:

- Select optimal filter size based on flow requirements
- Predict pressure drop for existing installations
- Optimize filter replacement schedules
- Calculate energy savings from filter upgrades

12.2 Quality Control

Manufacturers can utilize the system for:

- Design validation against standards
- Performance prediction for new designs
- Cost-benefit analysis of different media
- Documentation and reporting

12.3 Research and Development

Researchers benefit from:

- Rapid prototyping of new filter designs
- Sensitivity analysis of design parameters
- Comparison of different modeling approaches
- Data analysis and visualization tools

13 Case Study: Natural Gas Pressure Reduction Station

Station

13.1 Problem Statement

A natural gas pressure reduction station required filter selection for a flow rate of $340 \text{ m}^3 \cdot \text{h}^{-1}$ with a maximum allowable pressure drop of 250 Pa.

13.2 Solution Using Developed Framework

Input parameters: Flow rate = $340 \text{ m}^3 \cdot \text{h}^{-1}$, maximum $\Delta P = 250 \text{ Pa}$

Model prediction for standard filter sizes

Optimization algorithm identification of optimal design

Economic analysis of different options

13.3 Results

Selected filter: OD = 150 mm, Height = 600 mm, 55 pleats

Predicted pressure drop: 122 Pa

Energy savings compared to standard design: 28%

Payback period: 2.3 years

14 Limitations and Future Work

14.1 Current Limitations

- Model assumes clean filter conditions (no dust loading)
- Temperature and pressure effects are simplified
- Two-phase flow (gas-liquid) not considered
- Validation limited to two filter sizes

14.2 Future Development Directions

- Extension to dust-loaded conditions
- Incorporation of temperature and pressure effects
- Two-phase flow modeling
- Machine learning enhancement of predictions
- Cloud-based implementation
- Integration with CAD systems

15 Conclusion

- 1 This paper presented a comprehensive framework for mathematical modeling and software implementation for dry gas filter design. The key contributions include:
- 2 Calibrated Mathematical Model: A Darcy–Forchheimer based model with parameters $\kappa = 7.2 \times 10^{-13} \text{ m}^2$ and $\beta = 1250 \text{ m}^{-1}$ validated with experimental data achieving 99.2% accuracy.
- 3 Design Optimization Framework: Multi-objective optimization algorithms for balancing pressure drop, cost, and other design criteria.
- 4 Software Implementation: A multi-platform software system with web and desktop interfaces for practical engineering applications.
- 5 Comprehensive Analysis Tools: Capabilities for sensitivity analysis, lifetime prediction, energy consumption analysis, and economic evaluation.

6 Industrial Relevance: Direct application to natural gas industry problems with demonstrated benefits in energy savings and cost reduction.

The developed framework provides a valuable tool for engineers, researchers, and manufacturers in the natural gas industry, enabling optimized filter design, improved energy efficiency, and reduced operational costs.

References

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- [7] E. S. f. a. filters, *En 779:2012*, 2012.

Appendix A Experimental Data Tables

Complete Experimental Results

| Filter | Test ID | Initial ΔP (Pa) | Efficiency (%) | Dust Capacity (g) |
|----------|-----------------|-------------------------|----------------|-------------------|
| Filter 1 | 1402-LS-325-326 | 125 | 96.25 | 130 |
| Filter 1 | 1402-LS-327-328 | 118 | 96.82 | 141 |
| Filter 2 | 1402-LS-331-332 | 75 | 97.01 | 108 |

Table 4: Complete Experimental Test Results

Standard Filter Dimensions

Table 5: Standard Filter Dimensions According to IGS-M-PM-111(2)

| Media Type - Polypropylene/Polyester Media | | |
|--|---------|----------------|
| Class | OD (mm) | Area (m^2) |
| G1 | 95 | 0.125 |
| G1.5 | 120 | 0.23 |
| G2 | 165 | 0.47 |
| G2.5 | 200 | 0.725 |
| G3 | 252 | 0.95 |
| G4 | 299 | 1.45 |
| G5 | 390 | 2.3 |
| G6 | 475 | 4.2 |

| Media Type - Cellulose | | |
|------------------------|---------|----------------|
| Class | OD (mm) | Area (m^2) |
| G1 | 95 | 0.42 |
| G1.5 | 120 | 0.77 |
| G2 | 165 | 1.87 |
| G2.5 | 200 | 2.9 |
| G3 | 252 | 4.6 |
| G4 | 299 | 7.4 |
| G5 | 390 | 13.4 |
| G6 | 475 | 28.7 |

Appendix B Mathematical Derivations

Darcy–Forchheimer Equation Derivation

The general form of the Darcy–Forchheimer equation for flow through porous media is:

$$-\frac{dP}{dx} = \frac{\mu}{\kappa}v + \beta\rho v^2 \quad (14)$$

where K denotes permeability. For filter applications with thickness L , integration yields:

$$\Delta P = \frac{\mu L}{\kappa}v + \beta\rho Lv^2 \quad (15)$$

Substituting $v = Q/A$ provides the final form utilized in this study.

Sensitivity Coefficient Derivation

The sensitivity coefficient S_i for parameter x_i is defined as:

$$S_i = \frac{\partial \Delta P}{\partial x_i} \times \frac{x_i}{\Delta P} \quad (16)$$

For the filtration area A :

$$S_A = -\left(\frac{\mu L}{\kappa} \times \frac{Q}{A^2} + 2\beta\rho L \frac{Q^2}{A^3}\right) \times \frac{A}{\Delta P} \quad (17)$$

C Software Implementation Details

System Requirements

- Python 3.8 or higher
- Required packages: NumPy, SciPy, Matplotlib, Pandas, Flask, PyQt6 (optional)
- Minimum 4 GB RAM
- 500 MB disk space

Code Structure

Listing 1: Software Directory Structure

```
dry_gas_filter_system /
|-- core /
| |-- models.py      # Mathematical models
| |-- geometry.py   # Geometric calculations
| |-- optimization.py # Optimization algorithms
|-- interfaces /
| |-- web_app /      # Flask application
| |-- desktop_app / # PyQt6 / Tkinter
| |-- api /          # REST API
|-- data /           # Test data and standards
|-- docs /           # Documentation
|-- tests /          # Unit tests
D  Nomenclature
```

| | |
|------------|---|
| ΔP | Pressure drop (Pa) |
| L | Media thickness (m) |
| μ | Dynamic viscosity ($Pa \cdot s$) |
| κ | Intrinsic permeability (m^2) |
| v | Superficial velocity ($m \cdot s^{-1}$) |
| β | Inertial coefficient (m^{-1}) |
| ρ | Density ($Kg \cdot m^{-3}$) |
| Q | Volumetric flow rate ($m^3 \cdot s^{-1}$) |
| A | Filtration area (m^2) |
| n | Number of pleats |
| H | Filter height (m) |
| P_h | Effective pleat height (m) |
| η | Packing efficiency |
| α_d | Dead area ratio |
| OD | Outer diameter (mm) |
| ID | Inner diameter (mm) |
| R^2 | Coefficient of determination |
| $RMSE$ | Root mean square error (Pa) |
| MAE | Mean absolute error (Pa) |
| MRE | Maximum relative error (%) |