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Critical Buckling Load of Thin-Walled Plastic Cylinders in Axial and Radial Loading: Overview and FEA Case Study

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Abstract

This technical note discusses the theoretical static buckling load of thin-walled plastic cylinders under axial and radial loading. While the buckling of thin-walled cylinders is a well-researched topic, a variety of new design and manufacturing methods, as well as advances in polymer technologies, have brought plastic structures into prominence in recent years, necessitating a re-examination and exploration of their behavior under buckling loads. This work provides a background review of the problem, a discussion of the appropriate buckling equations, an extensive case study to demonstrate the concepts, and a brief review of some previously developed thin-walled structure reinforcement techniques. The presented work and results are intended to provide a helpful perspective, background review, and starting place for future research on the buckling behavior of thin plastic structures.

Keywords: Structural design, thin-walled structures, buckling failure, polymer testing

1. Introduction

Thin-walled elements such as domes, shells, plates, and membranes are widely used in engineering design, as they allow the construction of strong but relatively light-weight structures in many applications [1, 2]. The most common applications are to the design of load-bearing structures, such as buildings subjected to high winds and earthquakes [3, 4], and aircraft [5, 6], where strength, flexibility, and low mass are vital. Other applications include boats and ships, piping, pressure vessels, tanks, and machine components. While there are many advantages to employing thin-walled structures in design, including flexibility and mass reduction, their use is not without complications and trade-offs. The mass of the parts must typically be balanced against the strength, resulting in a system that is optimal for both mass and strength, but not for each individually. For load-bearing thin-walled structures, the impact of buckling is of paramount importance, as it is often the primary limiting factor for the safe design load. Typically, the buckling load is significantly lower than the compressive yield strength of the material. Buckling failures can happen very suddenly, with catastrophic results, so stable design of the structures is very important to achieve [7].

Figure 1 gives some examples of useful thin-walled structures which may be made from plastic (polymer or polymer-composite) materials, including trusses, machine components, and aircraft parts. Special consideration must be taken when designing structural components made from plastic materials, as they typically have much lower compressive strength and toughness when compared to metals [8–10]. In spite of this, plastic materials are sometimes the best choice for a variety of reasons, such as manufacturing cost [11–13], electrical and thermal resistance [14–17], vibration absorption [18–20], and recycling needs [21–23].

This technical note is structured as a hybrid article-review paper, providing a view of the problem combined from a variety of sources and a case study completed by the authors. It will be presented in several sections, beginning with the introduction (Section 1), followed by a discussion of the loading state

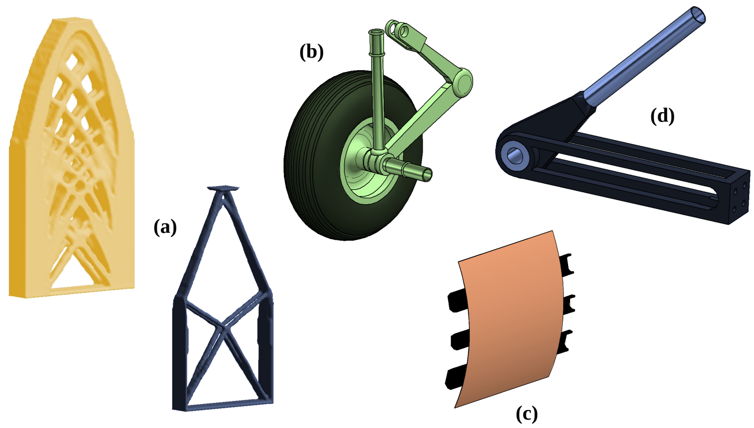

Figure 1: Examples of useful thin walled cylindrical and semi-cylindrical structures including (a) topologically-optimized trusses, (b) aircraft landing gear components, (c) machine structural components, and (d) aircraft skin panels

and important buckling equations (Section 2), a finite element case study using polycarbonate cylinders under the loading discussed in Section 2 for several cases (Section 3), a discussion of some previous work to reinforce thin-walled structures (Section 4), and finally discussion and conclusions about the presented work (Section 5).

2. Thin Cylinder Buckling

There are two basic modes of compressive loading for typical thin cylinders under which buckling may occur: Axial (such as column loading) and radial (such as pressure vessel loading) as shown in Figure 2. The axial loads may be uniformly applied (Figure 2a) or non-uniform (Figure 2b); practical examples of each are shown, where the wind turbine tower has an approximately uniform load and the traffic sign/light pole has a highly biased loading. Axial loading may be applied along the length of the cylindrical shell for some angle $\phi + \theta$ (Figure 2c), at one point or small area (Figure 2d), or uniformly along the entire surface. The case where load is applied at an angle is well-represented by a culvert under a road (Figure 2c), while the single location load case could be seen in a horizontal feed transfer system (Figure 2d) where the middle of the span experiences the most stress.

Loads could certainly be applied at an angle or in the reverse direction for any of these cases, but this would tend to reduce the load into components [24] so the shown cases will generally be the most extreme. When designing structural members, it is often the best choice to select the worst expected case for analysis when this will not produce a large design cost in terms of weight, production time, or material cost [25, 26]. Assuming no significant material defects, the cylinders are assumed to deform relative to the applied load. In the case of the axial loading, it is assumed that the load will be applied around the entire rim of the cylinder, whether this is uniform or not and regardless of the force direction. Note that a realistic structure may have several of these modes simultaneously and therefore may be under bending or torsion loading as well; however, this is not considered here as this focus of this work is on understanding and describing the basic modes of buckling for these cylinders.

For a generic smooth thin-walled cylinder in axial compression, structural collapse will take place at the point of buckling [27, 28], so the buckling load should be considered the ultimate strength of the structure. For the present study, the term "thin cylinder" is defined to comprise a wall thickness such that,

$$20t \leq D \tag{1}$$

where t is the wall thickness and D is the internal diameter of the cylinder. According to the NASA Space Vehicle Design Manual [27, 29], a reasonable buckling equation for a supported cylinder in axial compression is:

$$P_{cr} = k_x \frac{\pi^2 S}{l^2} \tag{2}$$

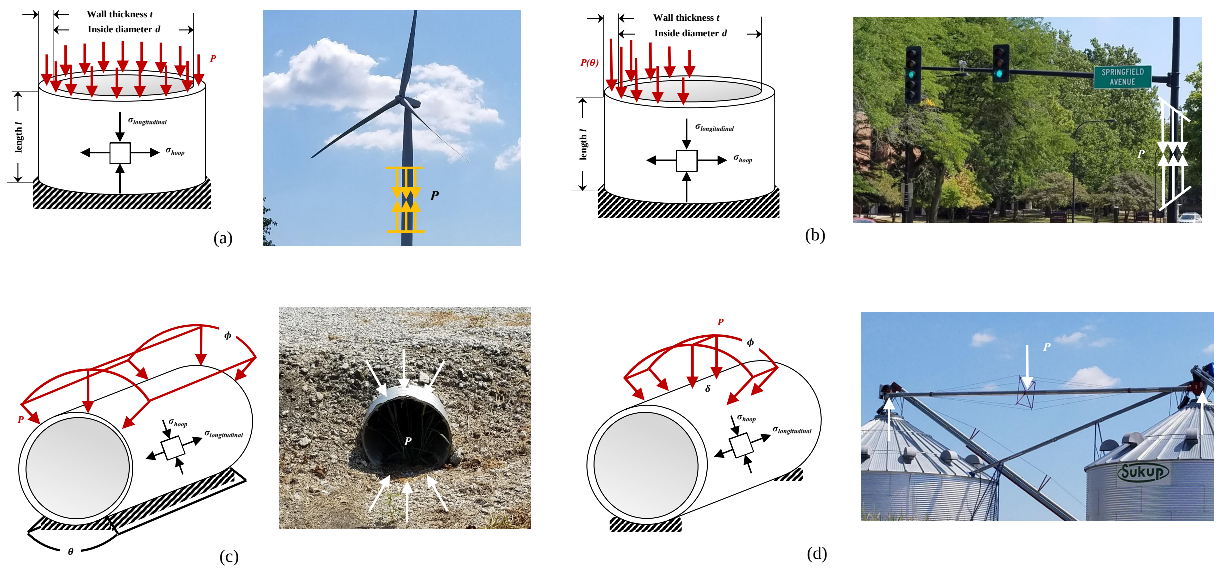

Figure 2: (a) uniform axial load (e.g., wind turbine tower), (b) non-uniform axial load (e.g., traffic light and sign pole), (c) angle-specific uniform radial loading (e.g., drainage culvert), and (d) angle-specific local radial loading (e.g., feed transfer system).

where P_{cr} describes the critical load, S describes the wall flexure stiffness per unit width, l describes the length of the cylinder, and k_x is the buckling coefficient. The buckling coefficient k_x is defined as,

$$k_x = m^2(1+\beta^2)^2 + \frac{12}{\pi^4}\frac{\gamma^2 Z^2}{m^2(1+\beta^2)^2} \tag{3}$$

while the wall flexure stiffness is,

$$S = \frac{Et^3}{12(1-\mu^2)} \tag{4}$$

where β describes the buckling aspect ratio, m is the number of buckle half-waves, γ is the correlation factor, Z is the curvature parameter, E is the elastic modulus of the material, t is the wall thickness, and μ is the Poisson ratio. The values of β, m, γ, Z, and t are determined by the exact geometry of the structure; likewise, the values of E and μ are determined by material choice. In the case of uniform loading, it is assumed that the single load will cause the buckling. This is not true for the non-uniform loading, as the structure will buckling at the point of highest loading; this will likely be at a much lower force than for the uniform loading due to the higher force being applied to a smaller area of the cylinder.

For the axial buckling cases, the deflection of the cylinder wall in this configuration is given by [7, 30, 31]:

$$w = w_1 sin^2\left(\frac{\pi\xi_v}{2\xi}\right) \tag{5}$$

where w is the deflection of the area under load, w_1 is the deflection amplitude, ξ_v is the variable of angle, and ξ is the angle of the applied load. In the case of uniform pressure over the entire cylinder, ξ will be 2π radians and will be

$$\xi = \phi + \theta \tag{6}$$

for the cases where the force is applied at an angle, both for cylinder-length loads and point loads. There are several methods for solving for the critical load under such conditions; the potential energy method [7, 30] best describes the effects relative to an angle of load (i.e. a non-uniform loading pattern). For the case described, the potential energy Π can be described as [7]:

$$\Pi = \frac{EI}{2r^3}\int_0^{\xi}(w+\ddot{w})^2 d\xi + \int_0^{\pi}\frac{N^2}{2EF} r d\xi - \int_0^{\xi} P_{cr} w r (w+\ddot{w})^2 d\xi \tag{7}$$

where N describes the hoop compressive force on the cylinder, P_{cr} is the external pressure accumulated during the buckling process, r is the radius of the cylinder in the loaded region, EF is the tensile stiffness of the hoop, and EI is the flexure stiffness. This equation should be solved relative the given boundary conditions and angles of force application. Note that the integration of force area would be used to distinguish between the cylinder-length and point-load cases. Most simple textbook solutions are formulated assuming a uniformly-applied pressure over the entire surface $[0, 2\pi]$ of the cylinder and true analytical solutions are difficult to find using these equations. Typically, finite element models [32–34] are used to find the true buckling loads; the background and equations are reproduced in this section for completeness and to give the reader a good basis for understanding the physical mechanisms at play.

3. Nonlinear FEA Case Study

To further explore the theoretical buckling behavior of polymeric thin-walled cylinders, a case study was completed using finite element analysis (FEA). This study was done in five parts, corresponding to each of the loading cases shown in Figure 2, with Case (d) completed with two different boundary conditions. For all cases, a nonlinear buckling analysis was done using Autodesk® Nastran® 2019 for the geometry shown in Figure 3a. A 2 mm quadrilateral shell mesh (Figure 3b) was used to model the 25 mm diameter cylinders, with two wall thicknesses of 0.5 mm and 1.0 mm. The material used for all cases was acrylonitrile butadiene styrene (ABS) and the analysis used the built-in Autodesk® material model (Table 1) at STP conditions. The material used was assumed to be isotropic (such as what would ideally be observed in molded ABS).

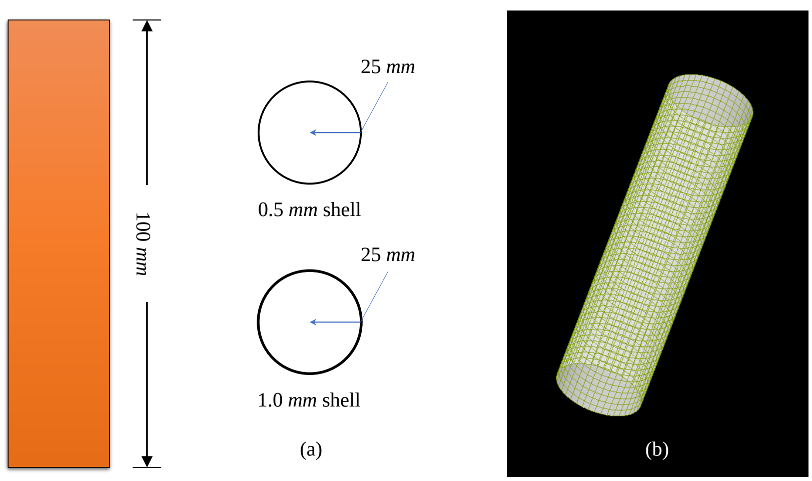

Figure 3: FEA cases: example contours

Table 1: Autodesk® non-linear ABS material model properties

Property	ABS	Units
Elastic modulus	2.24	GPa
Yield stress	20.0	MPa
Yield strain	0.0089	mm/mm
Ultimate strength	29.6	MPa
Poisson's ratio	0.38	
Material density	1050	kg/m^3

First, the cylinder (defined as previously described in Figure 3) was pinned at one end (no translation but rotation was allowed around all axes) and a load of 1 N applied symmetrically to the other. Upon running the nonlinear buckling analysis, the eigenvalues for the first three modes were collected and used to determine the critical load for the structure; these modes are shown in Figure 4. The first mode for each is theoretically the most likely case, as it will fail with a significantly smaller load than the other modes;

however, small material defects and small variations in force angle for a real case or physical experiment may drive other modes, so it is important to consider several. The full analysis required approximately 30 seconds of time using a Dell® Inspiron® desktop computer with an Intel i5-7400 processor (3.0 GHz) and 12 GB of RAM.

In order to determine whether the predicted failure was due to elastic buckling or yielding and then plastic buckling, the yielding load was calculated using the cross sectional area of the cylinders (Figure 3a) and the given yield stress of 20 MPa (Table 1). The thinner cylinder was predicted to yield at a force of 801 N, while the thicker was calculated to require 1634 N. It was noted that the thin cylinder (wall-to-diameter ratio of 50, well within the limit set by Equation 1) failed under elastic buckling for all three modes, but the thicker walled cylinder (wall-to-diameter ratio of 25) yielded before buckling plastically. Table 2 gives the critical load and failure mode for each of the shown buckling modes.

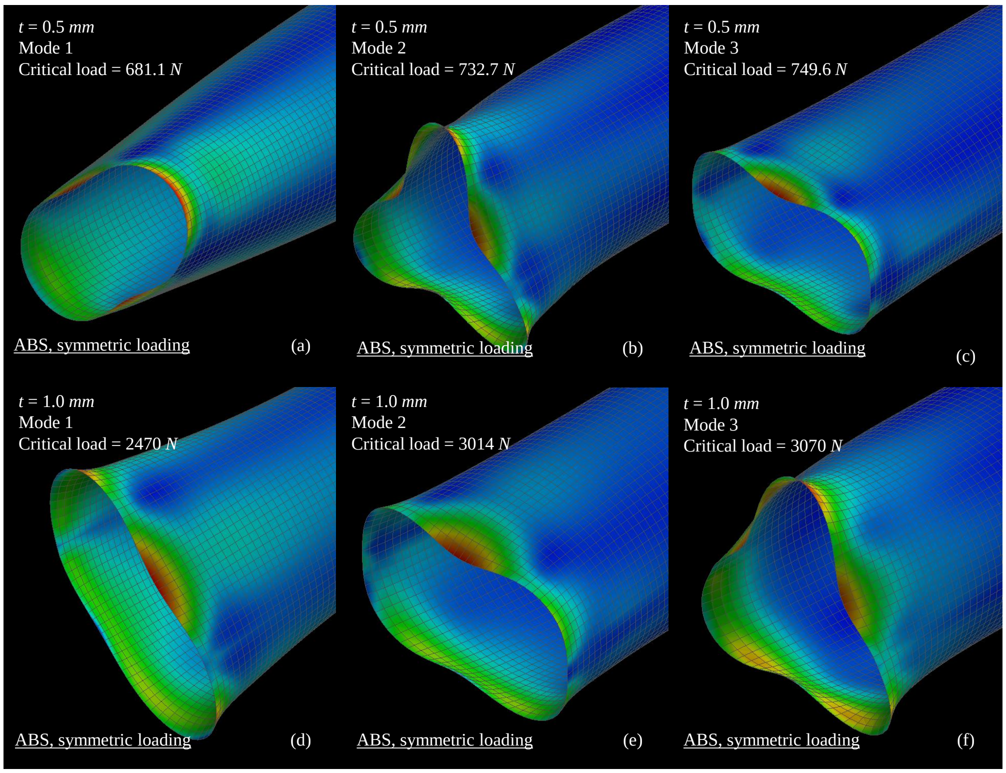

Figure 4: FEA models for symmetric axial loading cases. Shown are 0.5 mm wall (a) mode 1, (b) mode 2, (c) mode 3 and 1.0 mm wall (d) mode 1, (e) mode 2, and (f) mode 3

Table 2: Buckling modes, observed critical loads, and failure modes for the symmetric axial loading case

Mode	Critical Load (N) and Failure Mode			
	0.5 mm Wall	Failure	1.0 mm Wall	Failure
1	681.1	Elastic buckling	2470	Yield + plastic buckling
2	732.7	Elastic buckling	3014	Yield + plastic buckling
3	749.6	Elastic buckling	3070	Yield + plastic buckling

This analysis was then repeated for the case where the load was asymmetric; all detailed were identical except all the of the load was applied to only 180° of the cylinder edge. Figure 5 and Table 2 show the results for the first three modes for each of the two wall thicknesses. Analysis time was slower for this case, but still required under one minute to complete. As in the symmetric case, the thin cylinder buckled elastically before the the yield point. With an asymmetric load, the first two modes of the thicker cylinder was also predicted to fail by elastic buckling before the yield point of the material.

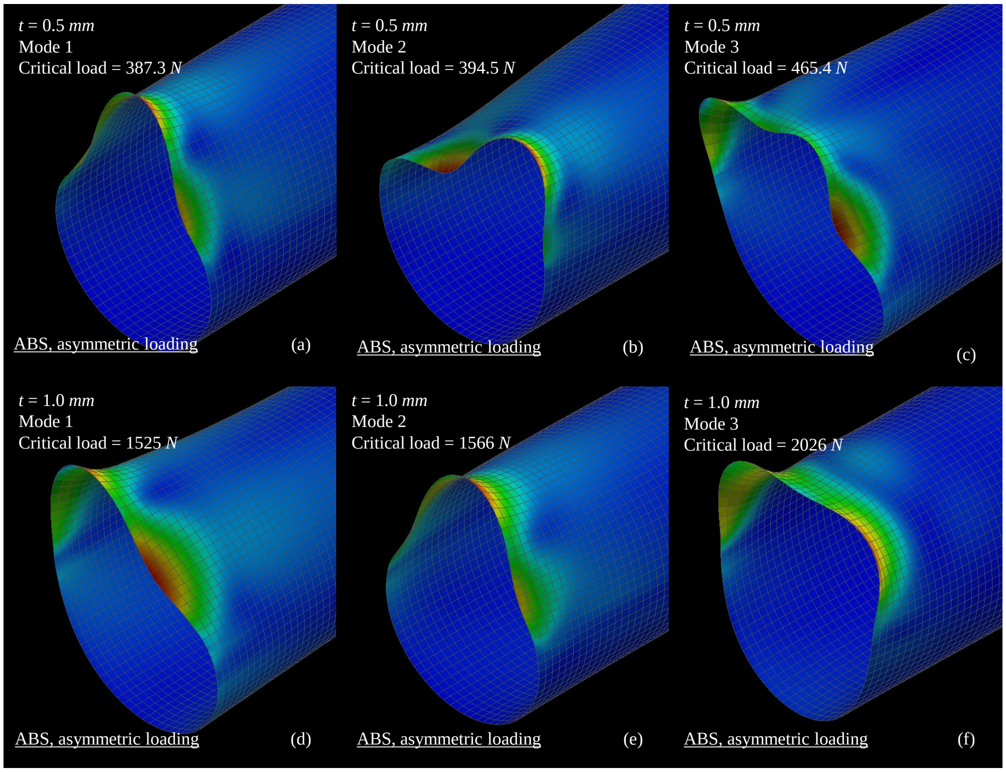

Figure 5: FEA models for symmetric axial loading cases. Shown are 0.5 mm wall (a) mode 1, (b) mode 2, (c) mode 3 and 1.0 mm wall (d) mode 1, (e) mode 2, and (f) mode 3

Table 3: Buckling modes, observed critical loads, and failure modes for the asymmetric axial loading case

Mode	Critical Load (N) and Failure Mode			
	0.5 mm Wall	Failure	1.0 mm Wall	Failure
1	387.3	Elastic buckling	1525	Elastic buckling
2	394.5	Elastic buckling	1566	Elastic buckling
3	465.4	Elastic buckling	2026	Yield + plastic buckling

After completion of the axial loading cases, three radial loading cases were analyzed. These included (R1) the loading shown in Figure 2c with $\theta = \phi = 60°$ and applied the full length of the cylinder and (R2 and R3) the loading shown in Figure 2d with the load applied at the center of the cylinder over a 60° angle and $\delta = 1$ mm. In the case of R1, the section defined by θ was pinned, while R2 and R3 used pinned and fixed constraints, respectively, applied to the ends of the cylinder. The results are shown in Table 4 for all three cases; analysis was completed identically to that of the axial cases, except that only one mode was calculated successfully for these loading cases. The buckling modes for each case are shown in Figure 6a and b (for R1), Figure 7a and b (for R2), and Figure 8a and b (for R3). Each of the analyses required about three minutes to complete using the same settings and assumptions as the axial loading cases.

After completion of the buckling studies and calculation of the critical loads, a simple linear static analysis was done on each case to determine if the failure was due to elastic buckling or yielding and then plastic buckling; it was found that the stress in the material at the critical load was significantly higher than the yielding point for all cases, so it was found that all the radial loading cases experienced material yielding before buckling. These results of these analyses, including maximum stress values for each case, can be seen in Figure 6c and d (R1), Figure 7c and d (R2), and Figure 8c and d (R3). There was some difference observed between the results of the pinned and fixed cases for the local radial loading, with the fixed ends having a slightly higher critical load.

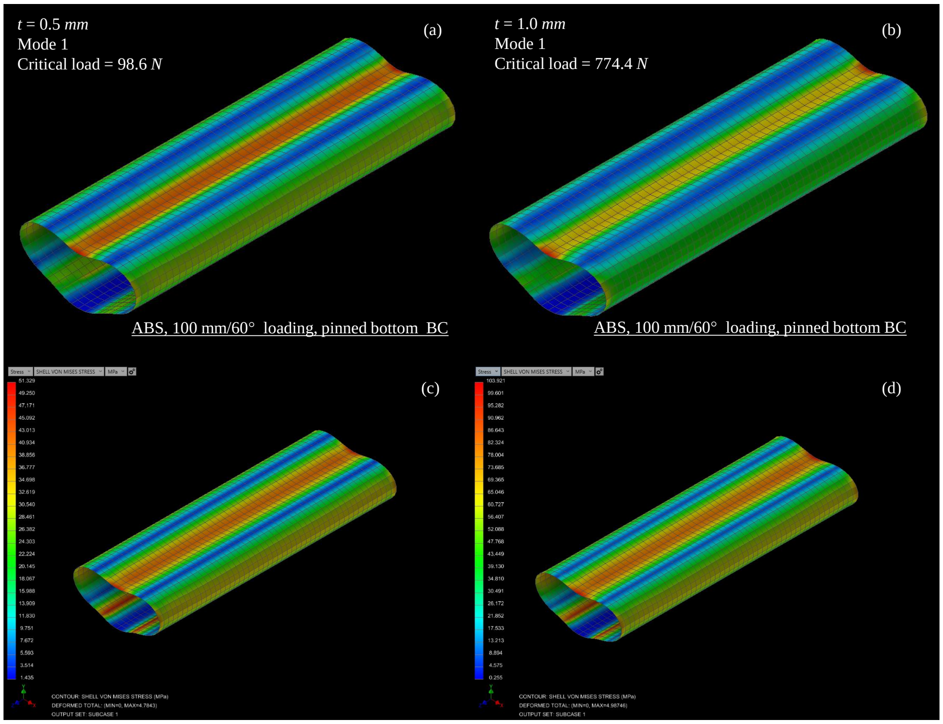

Figure 6: Buckling modes for the (a) thinner and (b) thicker-walled cylinders under the R1 radial loading case, with simple linear static analysis completed at the buckling load of the (c) thin and (d) thick cases to determine if the failure was due to elastic buckling or yielding and then plastic buckling

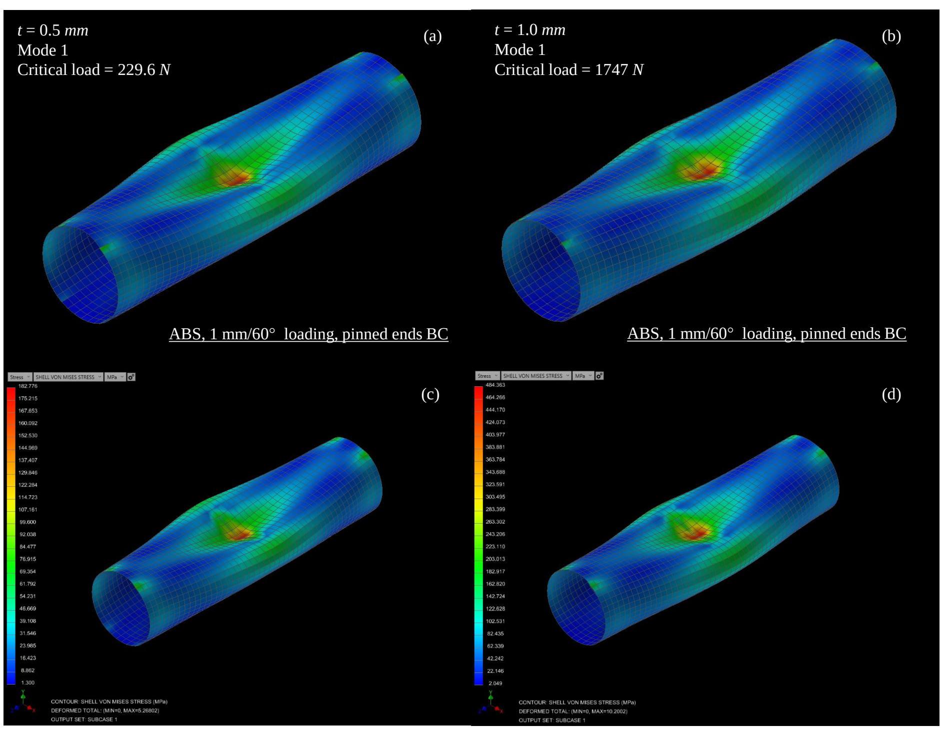

Figure 7: Buckling modes for the (a) thinner and (b) thicker-walled cylinders under the R2 radial loading case, with simple linear static analysis completed at the buckling load of the (c) thin and (d) thick cases to determine if the failure was due to elastic buckling or yielding and then plastic buckling

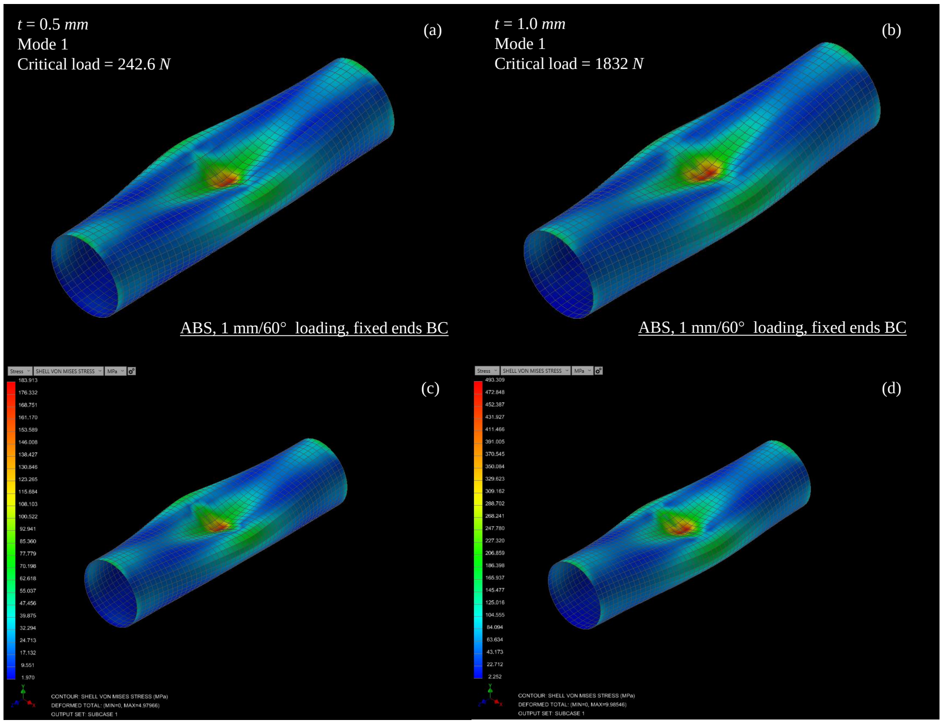

Figure 8: Buckling modes for the (a) thinner and (b) thicker-walled cylinders under the R3 radial loading case, with simple linear static analysis completed at the buckling load of the (c) thin and (d) thick cases to determine if the failure was due to elastic buckling or yielding and then plastic buckling

Table 4: Buckling modes, observed critical loads, and failure modes for the three radial loading cases

Radial Loading Case	Critical Load (N) and Failure Mode			
	0.5 *mm* Wall	Failure	1.0 *mm* Wall	Failure
R1	98.6	Yield + plastic buckling	774.4	Yield + plastic buckling
R2	229.6	Yield + plastic buckling	1747	Yield + plastic buckling
R3	242.6	Yield + plastic buckling	1832	Yield + plastic buckling

4. Thin Cylinder Reinforcement

One of the many solutions offered to improve the strength and buckling resistance of thin-walled parts is to fill them with expanding foam or composite material; if done properly, this can result in a part that is significantly stronger than an unfilled part, but that does not involve a significant increase in mass. Often, the structure can be made even lighter, as the addition of foam can allow the reduction of the wall thickness without sacrificing performance [35]. Foam-filled components are also more vibration-resistant and therefore often have longer fatigue lives due to improved energy absorbency by the foam [36]. Several studies [37–43] have explored the production and use of polymer and metal foams for structural applications. In particular, expanding polyurethane foam has been widely used to strengthen static structural components to address a variety of interesting problems. Heim *et al* [44] combining plastic components and foams to reduce the mass of vehicles, while increasing the safety, stiffness, and crushing resistance. Ashrafi *et al* [45], Tuwair *et al* [46], and Boccaccio *et al* [6] explored the use of foam to stiffen and strengthen honeycomb panels, particularly for aerospace applications. Several studies were also performed using polyurethane foam to strengthen various types of conical and cylindrical tubes, including those made from aluminum [47, 48], brass [49], and steel [45, 50]. Surprisingly, no previous studies were found that examined the foam filling and strength testing of structures made from plastics, even though many thin-walled plastic structures are manufactured and used.

5. Summary, Conclusions, and Future Work

This work explored the buckling behavior of thin-walled polymeric cylinders, first reviewing some literature to provide motivation for the importance of the problem, then examining several loading cases (with practical examples) and their basic equations. A case study was performed to further illuminate the behavior of these structures, demonstrating some interesting behavior. Finally, a brief review was completed to explore some of the previous work on strengthening thin-walled structures against crushing and buckling. This paper is meant to provide background and motivation for future work in this area, especially on providing a good perspective for experimental validation of the behavior. The buckling of thin polymeric material structures, especially structural members (not films), is a topic that has not been explored extensively in the literature this far and needs more attention.

The background review and case study from this work provided several interesting conclusions, all of which will be explored in future work. These include:

- Very little previous work has been done to study the buckling behavior of thin-walled cylindrical structures made from polymeric and polymer-composite materials, even though these are becoming more widely used in engineering design.
- The FEA results implied that the definition of thin-walled as being a wall thickness under 1/20 of the diameter may be too liberal for polymeric materials. This conclusion was driven by the fact that most of the 1/25 wall thickness produced yielding in the material before buckling, while the thinner one reliability produced elastic buckling. Perhaps it is appropriate to define thin-walled structures by their ability to elastically buckle, not by a specific wall thickness. This would be material-dependent and requires further study.
- The use of a shell model with quadrilateral mesh elements for the FEA was a good choice for these kinds of thin structures, as all the nonlinear buckling solutions converged reliably and quickly without a single failure.
- When examining these structures, it should be noted that the axial loading cases had several modes (at least three) each but the radial loading cases only had a single buckling mode.
- The impact of small material defects and microstructure discontinuities in the polymer material needs to be explored experimentally, as these may cause elastic buckling even of the yielding cases found here.
- The impact of anisotropy (especially from 3-D printing) needs to be examined with respect to buckling; this is a topic very little discussed in the literature, whether for molded or additively built materials

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