Efficiency of Strength Enhancement in Strengthened Singly Reinforced Concrete Sections: A Strain-Based Strength–Ductility Assessment

Abstract

This study examines how different strengthening modes influence the interaction between flexural resistance and strain-based ductility reserve in singly reinforced concrete sections. Rather than assessing strengthening effectiveness solely in terms of strength increase, the analysis evaluates how efficiently strength gains are achieved relative to changes in post-yield deformation capacity. A reference section, designed modestly above the Eurocode 2 minimum longitudinal reinforcement requirement, is analysed using strain-based sectional mechanics consistent with ultimate limit state assumptions. Three strengthening strategies—tension-side, compression-side, and balanced strengthening—are examined. Ductility reserve is quantified using a normalised strain-based metric, and a simple efficiency index is introduced to relate strength enhancement to changes in deformation capacity. The results show that strengthening mode governs both the strength–ductility interaction and the efficiency of strength enhancement. Tension-dominated strengthening achieves large strength gains but rapidly depletes deformation reserve, while compression-side strengthening substantially enhances ductility with limited strength increase. Balanced strengthening provides moderate strength enhancement while largely preserving deformation capacity. These findings demonstrate that sectional strengthening performance cannot be adequately characterised by strength-based assessment alone and highlight the value of explicitly considering deformation reserve when comparing strengthening strategies.

Keywords: Strengthening mode; Flexural resistance; Ductility reserve; Strain-based assessment; Reinforced concrete sections

1. Introduction

Strengthening of singly reinforced concrete beams has conventionally been framed as a problem of increasing flexural resistance while maintaining acceptable ductile failure modes, with deformation capacity typically evaluated indirectly through load–deflection response and observed failure behaviour rather than treated as an explicit design objective (Triantafillou, 1998; El-Gamal et al., 2019). Across strengthening techniques, analysis is commonly grounded in classical sectional mechanics—namely plane sections, strain compatibility (i.e., concrete, internal steel reinforcement, and externally added strengthening materials are assumed to deform together according to a linear strain distribution governed by a single section curvature), and force equilibrium—through which strengthening interventions are proportioned primarily to enhance moment capacity while preserving a failure sequence governed by tensile steel yielding followed by concrete crushing. In the seminal review by Triantafillou (1998), strengthening effectiveness is discussed predominantly in terms of strength enhancement and control of failure modes, with ductility inferred from moment–curvature behaviour rather than quantified through explicit deformation-capacity metrics. Contemporary design guidance, such as ACI PRC-440.2-23 (2023), continues this strength-oriented and failure-prevention-based philosophy by relying on sectional equilibrium, conservative strain limits, and reduction factors to ensure safe strength enhancement, without providing explicit measures describing how deformation capacity evolves relative to the achieved increase in flexural resistance. In the present study, strengthening is therefore idealised at the cross-section level and assumed to remain fully effective up to flexural failure, in order to isolate the intrinsic relationship between flexural resistance enhancement and deformation capacity in singly reinforced concrete beams. While this idealisation does not represent bond degradation or other member-level failure mechanisms, it is intentionally adopted to establish a mechanics-consistent, section-level basis for assessing how different strengthening modes consume or preserve post-yield deformation capacity.

Experimental studies show that strengthening of reinforced concrete (RC) beams influences not only ultimate strength but also cracking behaviour, stiffness development, and deformation capacity prior to failure. Tests on RC beams strengthened with externally bonded CFRP laminates report delayed crack initiation, changes in crack distribution, and increased post-cracking stiffness, even when gains in ultimate strength are limited (Li, Assih, & Delmas, 2001). Early experimental investigations further indicate that strength enhancement through externally bonded composite systems may be accompanied by reduced deformation capacity and energy dissipation, and that anchorage measures, while effective in delaying premature debonding, do not necessarily prevent this loss of ductility (Spadea et al., 2001). Comparative experiments demonstrate that the deformation response is strongly dependent on the strengthening technique: CFRP-strengthened beams often fail at relatively small deflections due to interface-controlled mechanisms, whereas RC jacketed beams exhibit flexural failure modes similar to reference specimens and show enhanced ductility and energy dissipation capacity (Tahsiri et al., 2015). Large-scale beam tests show that strengthening configurations can alter the governing failure mechanism, shifting behaviour from ductile flexure with steel yielding to interface-controlled debonding failures, which in turn constrain the deformation and energy dissipation capacity prior to collapse (Razaqpur, Cameron, & Mostafa, 2020). Experimental studies on beams strengthened with externally bonded composites also show interaction between internal steel reinforcement and external strengthening systems, as measured strains in internal stirrups are reduced and yielding may be precluded at peak load, indicating that strength gains are not necessarily accompanied by proportional participation of internal reinforcement (Gonzalez-Libreros et al., 2017a; Gonzalez-Libreros et al., 2017b). Analytical investigations of RC jacketing demonstrate that assuming monolithic behaviour between the original beam and the added jacket can significantly overestimate stiffness and deformation response, and that explicitly accounting for interfacial slip leads to reduced moment–curvature and load–deflection responses, even under ductile flexural failure conditions (Alhadid & Youssef, 2017, 2018).

State-of-the-art reviews and recent experimental studies confirm that the interaction between strength enhancement and deformation capacity depends strongly on the mechanical action and failure mode of the strengthening intervention, rather than on the strengthening technique alone. Megahed et al. (2023) report that flexural strengthening frequently alters stiffness evolution and failure sequencing, with premature activation of brittle mechanisms such as debonding, fibre rupture, or concrete cover separation limiting post-yield deformation. Similarly, Vahidpour et al. (2022) show that strength gains achieved through externally bonded composite reinforcement often coincide with reduced deformation capacity, as failure and post-yield response become controlled by brittle external reinforcement systems rather than ductile steel yielding. Experimental evidence indicates that strengthening systems can fundamentally alter the participation of existing internal reinforcement, such that assumed contributions based on isolated material capacities are not fully mobilised in strengthened members. Gonzalez-Libreros et al. (2017b) demonstrate that the presence of externally bonded FRP (fiber reinforced polymer) or FRCM (fiber reinforced cementitious matrix) reinforcement can limit the strain in internal steel stirrups and prevent them from achieving yielding, despite increased overall strength. Collectively, these findings show that interaction effects between internal and external reinforcement must be considered when comparing strengthening strategies. The variability in deformation response highlights the difficulty of comparing strengthening strategies solely on the basis of achieved flexural resistance.

Ductility is broadly understood as the capacity of reinforced concrete sections to sustain deformation beyond the elastic limit (a characteristic that becomes particularly ambiguous when conventional yielding mechanisms are altered by strengthening interventions), yet its definition and assessment remain non-unified, with variations in mathematical expressions, reference points, and evaluation objectives (Lopes, Lou, and Lopes, 2025). Lopes, Lou, and Lopes (2025) show that ductility assessment is strongly dependent on the reference state adopted and highlight that extending section-level ductility measures to RC elements and entire structures remains open. At the cross-sectional level, ductility is commonly evaluated using deformation measures relative to the yielding of tensile reinforcement, typically derived from moment–curvature relationships. However, alternative formulations have been proposed when conventional yielding mechanisms are altered, including energy- and deformability-based indices that relate global deformation to stored or dissipated energy (Oudah & El-Hacha, 2012). While such approaches provide useful insight into member-level response, they rely on load–deflection behaviour and energy measures that are not directly embedded within sectional equilibrium or strain compatibility assumptions. As a result, substantial variability persists in how yielding, ultimate state, and failure are defined, leading to divergent ductility indicators and, in some cases, contradictory conclusions regarding the influence of strengthening. These studies underline that ductility cannot be inferred solely from ultimate strength or reinforcement type, particularly at the section level.

In the context of singly reinforced concrete beams designed modestly above the Eurocode 2 (EC2) (EN 1992-1-1, 2004) minimum longitudinal reinforcement requirement, this issue is especially relevant. Such sections inherently possess a meaningful post-yield deformation reserve that contributes to structural robustness and warning prior to failure, yet this reserve may be either preserved or consumed depending on the strengthening mode adopted. Existing studies typically assess strengthening performance primarily through changes in flexural resistance, with deformation capacity discussed qualitatively or as a secondary outcome. While this approach is sufficient for ranking ultimate strength, it does not enable transparent comparison of how efficiently strength gains are achieved relative to the consumption or preservation of deformation capacity.

To address this gap, the present study adopts a strictly section-level, strain-based interpretation framework that explicitly relates changes in flexural resistance to corresponding changes in deformation capacity. Using conventional strain compatibility analysis, deformation capacity is quantified through the available post-yield strain reserve of the section, and its evolution is examined alongside strength enhancement for different idealised strengthening modes. Rather than introducing new response variables or design parameters, the framework relies exclusively on quantities inherent to classical sectional analysis, enabling a physically transparent and reproducible comparison. By expressing deformation change relative to strength gain through a simple, dimensionless efficiency measure, strengthening strategies can be evaluated not only by the magnitude of resistance they provide, but by how effectively they preserve or enhance post-yield deformation capacity in singly reinforced concrete sections.

2. Methodology

2.1 Overview of Analytical Framework

This study adopts a strain-based sectional analysis framework to investigate how different strengthening modes influence the interaction between flexural resistance and ductility reserve in singly reinforced concrete sections. The analysis is deliberately limited to sectional behaviour, allowing the mechanical effects of strengthening to be examined directly, without the influence of member-level geometry, loading conditions, or support constraints.

A common reference section is first defined in accordance with the Eurocode 2 provisions. Strengthening strategies are then introduced by modifying the sectional force-resisting components in a manner representative of commonly adopted strengthening interventions in practice. For each strengthened configuration, the resulting flexural resistance and strain-based ductility reserve are evaluated relative to the same baseline section (reference section). The strengthening strategies are defined by their mode of intervention, rather than by targeting identical strength gains, allowing the mechanical consequences of each strengthening approach to be isolated.

2.2 Reference Section Definition

The reference section consists of a rectangular, singly reinforced concrete section designed modestly above the Eurocode 2 minimum longitudinal reinforcement requirement. The longitudinal reinforcement ratio, ρ, is defined as the ratio of tensile steel area to the effective concrete section area, while ρ_{min} denotes the minimum longitudinal reinforcement ratio prescribed by Eurocode 2 to ensure adequate cracking control and ductile behaviour (EN 1992-1-1, 2004).

In this study, the reference section is designed with $\rho = 1.25*\rho_{min}$. This reinforcement level is selected to represent a realistic baseline section commonly encountered in practice, where reinforcement provided often exceeds the strict minimum requirement due to bar size availability, detailing constraints, and conservative design practice. Sections designed exactly at ρ_{min} are rarely adopted in construction. The selected reinforcement ratio therefore provides a practical and representative baseline while retaining substantial post-yield deformation capacity.

The reference section serves as the sole baseline against which all strengthening interventions are evaluated. No strengthened configuration is treated as a secondary reference; all changes in flexural resistance and ductility reserve are quantified relative to this original section.

2.3 Material Models and Constitutive Assumptions

Concrete and reinforcing steel are modelled using simplified uniaxial stress–strain relationships typically employed in reinforced concrete sectional analysis. These idealised models are sufficient to capture the key strain limits and yielding behaviour governing flexural resistance and ductility reserve.

Concrete in compression is represented by a nonlinear stress–strain relationship with a maximum compressive strain of $\varepsilon_{cu} = 0.0035$ in accordance with Eurocode 2. Tensile resistance of concrete is neglected in the sectional equilibrium calculations.

Reinforcing steel is modelled as an elastic–perfectly plastic material with yield strength f_{yk} and corresponding yield strain $\varepsilon_y = f_{yk}/E_s$, where E_s is the elastic modulus of steel.

Unless otherwise stated, characteristic material strengths adopted in the analysis are:
- Concrete compressive strength: $f_{ck} = 30$ MPa
- Steel yield strength: $f_{yk} = 500$ MPa

These values are representative of commonly used materials in strengthening applications and are not intended to reflect any specific structure.

2.4 Strengthening Strategies and Practical Interpretation

Three idealised strengthening strategies are examined, each representing a distinct mode of strength intervention commonly encountered in strengthening practice.

2.4.1 Strategy T – Tension-Dominated Strengthening

This strategy represents strengthening interventions that primarily increase the tensile resistance of the section, such as externally bonded or near-surface mounted reinforcement systems (e.g. steel plates or FRP laminates). In this approach, additional tensile capacity is introduced without any enhancement of the concrete compression zone. As a result, higher tensile forces must be balanced by the existing concrete in compression, leading to increased compressive strain demand and earlier attainment of the concrete crushing limit.

Two tension-dominated cases are considered:
- Strategy T-1, representing a moderate and practically relevant tensile strengthening level, resulting in an increase in flexural resistance of approximately 25% relative to the reference section.

- Strategy T-2, representing a higher-intensity tensile intervention, resulting in an increase in flexural resistance of approximately 50%. This case is included solely to illustrate the non-linear degradation of ductility reserve associated with aggressive tensile strengthening and is not intended as an optimised or recommended strengthening configuration.

2.4.2 Strategy B – Balanced Strengthening

The balanced strengthening strategy represents interventions in which both tensile and compressive capacities are enhanced in a coordinated manner. This is representative of strengthening schemes that combine tension-side capacity enhancement with measures that enhance compressive performance, such as section enlargement or confinement.

This strategy is intentionally configured to produce a moderate increase in flexural resistance while limiting upward movement of the neutral axis and maintaining a deformation pattern similar to that of the reference section.

2.4.3 Strategy C – Compression-Side Strengthening

Compression-side strengthening represents interventions that primarily improve compressive performance, either by increasing the effective compression zone or by enhancing concrete strain capacity, without directly increasing tensile reinforcement. Typical examples include concrete jacketing, compression-zone enlargement, and confinement systems.

This strategy is included to examine strengthening scenarios in which compressive enhancement alters the balance between strength gain and deformation capacity, allowing its efficiency to be assessed relative to tension-dominated and balanced interventions.

2.5 Sectional Analysis Procedure

Flexural resistance and ductility reserve are evaluated using a strain-based sectional analysis consistent with Eurocode 2 provisions for singly reinforced concrete sections.

For all sections, the ultimate limit state is defined by attainment of the maximum concrete compressive strain $\varepsilon_{cu} = 0.0035$ (EN 1992-1-1:2004, Table 3.1) at the extreme compression fibre. A linear strain distribution is assumed across the section depth (consistent with general sectional mechanics and the underlying assumptions of EN 1992-1-1:2004).

The rectangular concrete stress block and steel stress limits are adopted in the simplified Eurocode form (e.g. $0.85f_{ck}$ and $0.87f_{yk}$) to ensure consistency with standard sectional mechanics. These coefficients are used as mechanical idealisations rather than as part of a full Eurocode safety-format design, as the analysis is comparative and strain-controlled.

2.5.1 Reference Section Analysis

The reference section is analysed using a strain-based sectional analysis consistent with the ultimate limit state provisions of Eurocode 2 (EN 1992-1-1:2004). Concrete compression is represented using a simplified rectangular stress block, and sectional equilibrium is enforced between tensile steel and concrete compression. The adopted stress block parameters are consistent with the simplified design expressions commonly used in the Eurocode-based sectional analysis.

The depth of the neutral axis, x, is obtained directly from force equilibrium between tensile steel and concrete compression:

- Concrete compression force: $C = 0.85*f_{ck}*b*x$
- Tensile steel force: $T = 0.87*fyk*A_s$

Equilibrium (C = T) yields the neutral axis depth (x). The lever arm is evaluated as
$z = d - 0.4x$,
and the flexural resistance is computed as:

$M_{R,0}= 0.87*f_{yk}*A_s*z$

The tensile steel strain at the ultimate limit state is obtained from strain compatibility:

$\varepsilon_{s,ULS} = \varepsilon_{cu}*[(d – x)/(x)]$

Steel yielding is verified by comparison with the yield strain:

$\varepsilon_y = f_{yk}/E_s$.

The ductility reserve ratio (DRR), defined in Section 2.6, is evaluated as:

$DRR = (\varepsilon_{s,ULS} – \varepsilon_y)/\varepsilon_y$

This value represents the normalised tensile strain capacity available beyond first yield at the ultimate concrete strain limit.

For clarity, the tensile reinforcement area of the reference section is denoted as $A_{s,0}$, representing the longitudinal steel area provided in the unstrengthened baseline beam. The reference section is designed with a reinforcement ratio of $\rho = 1.25*\rho_{min}$.

The minimum longitudinal reinforcement area, $A_{s,min}$, is determined in accordance with Eurocode 2 (EN 1992-1-1:2004, Clause 9.2.1.1) as:

$$A_{s,min} = \max [0.26*(f_{ctm} / f_{yk})*b*d , 0.0013*b*d]$$

where $f_{ctm} = 0.30\ f_{ck}^{(2/3)}$ is the mean tensile strength of concrete. The reference section reinforcement area is therefore defined as:

$$A_{s,0} = 1.25*A_{s,min}$$

This choice reflects common design practice, in which reinforcement provided typically exceeds the strict code minimum due to detailing and constructability considerations, and ensures that the reference section possesses a realistic and stable post-yield deformation capacity.

2.5.2 Tension-Dominated Strengthening (Strategy T)

Tension-dominated strengthening is modelled by increasing the tensile reinforcement area while keeping concrete properties unchanged.

Two cases are considered:

- T-1: Moderate tensile strengthening ($As = 1.25*A_{s,0}$)
- T-2: Higher-intensity tensile strengthening ($As = 1.50*A_{s,0}$)

For each case, the neutral axis depth is recalculated using the same force equilibrium formulation, accounting for the increased steel area. Flexural resistance, tensile steel strain at the ultimate limit state, and ductility reserve ratio (DRR) are evaluated using the same strain-based procedure adopted for the reference section.

The normalised flexural resistance is expressed as:

$$\Delta M_R = M_{R,T} / M_{R,0}$$

where $M_{R,T}$ is the flexural resistance of the strengthened section and $M_{R,0}$ is that of the reference section.

Similarly, the normalised ductility reserve is defined as:

$$\Delta DRR = DRR_T / DRR_0$$

where DRR_T and DRR_0 denote the ductility reserve ratios of the strengthened and reference sections, respectively.

2.5.3 Balanced and Compression-Side Strengthening (Strategies B and C)

Balanced and compression-side strengthening strategies are evaluated using the same strain-based sectional analysis procedure adopted for the reference and tension-strengthened sections, with targeted modifications introduced to reflect changes in tensile and/or compressive capacity.

For Strategy B (Balanced Strengthening), both tensile and compressive capacities are enhanced in a coordinated manner. Tensile strengthening is modelled by increasing the longitudinal reinforcement area by 15% relative to the reference section ($A_s = 1.15*A_{s,0}$), while compressive capacity is enhanced by increasing the concrete characteristic strength to f_{ck} = 35 MPa. This combined modification is intended to represent a balanced strengthening approach in which gains in flexural resistance are achieved while limiting neutral axis migration, ensuring that steel yielding precedes concrete crushing while maintaining a strain distribution comparable to that of the reference section.

For Strategy C (Compression-Side Strengthening), only the compressive capacity of the section is enhanced by increasing the concrete characteristic strength to f_{ck} = 40 MPa, while the original tensile reinforcement area is retained. This strategy isolates the influence of compression-side enhancement on flexural resistance, steel strain development, and the available ductility reserve.

For both strategies, the neutral axis depth is recalculated to satisfy force equilibrium, and flexural resistance is evaluated at the attainment of the ultimate concrete compressive strain. The corresponding tensile steel strain is used to compute the ductility reserve ratio (DRR), enabling direct comparison with the reference and tension-strengthened configurations.

2.5.4 Reserve Efficiency Index

To enable comparison of strengthening strategies that exhibit fundamentally different strength–ductility interactions, a reserve efficiency index, η_R, is employed. To quantify the efficiency with which flexural strength is gained relative to changes in ductility reserve, the reserve efficiency index is defined as:

$\eta_R = \Delta DRR / \Delta M_R$

where:

- ΔDRR is defined as the ratio of the ductility reserve ratio of the strengthened section to that of the reference section (DRR/DRR_0), and
- ΔM_R is defined as the ratio of the flexural resistance of the strengthened section to that of the reference section ($M_R/M_{R,0}$).

The index is intentionally simple and is not proposed as a design parameter. Rather, it provides a compact and dimensionless measure of how efficiently deformation capacity is preserved or enhanced for a given relative increase in flexural resistance, enabling direct comparison between strengthening strategies governed by fundamentally different mechanical mechanisms. Values of η_R near unity indicate preservation of ductility reserve relative to strength gain; $\eta_R < 1$ reflects inefficient strength enhancement accompanied by disproportionate loss of deformation capacity, whereas $\eta_R > 1$ indicates strengthening modes that enhance deformation capacity more rapidly than flexural resistance.

2.5.5 Scope and Limitations of the Methodology

The methodology is restricted to strain-based sectional behaviour under monotonic bending (i.e., a steadily increasing bending moment applied in one direction until failure) and is intended to isolate the fundamental interaction between flexural resistance and post-yield ductility reserve associated with different strengthening modes, rather than to predict full member-level response. Strengthening interventions are idealised through their contribution to sectional force–strain equilibrium and are assumed to remain fully effective up to the attainment of the ultimate flexural limit state, consistent with classical Eurocode 2 strain compatibility analysis. Effects related to shear behaviour, bond–slip and debonding, anchorage failure, cyclic degradation, time-dependent material behaviour, geometric nonlinearity, and confinement enhancement beyond the adopted material models are intentionally excluded, as their inclusion would obscure the intrinsic section-level mechanisms governing strength–deformation interaction. Within this defined scope, deformation capacity is governed by steel yielding and concrete crushing, enabling a transparent and internally consistent comparison of strengthening strategies based on how efficiently flexural strength enhancement is achieved relative to the available ductility reserve.

2.6 Definition of Ductility Reserve Ratio (DRR)

In this study, a strain-based ductility reserve ratio (DRR) is adopted to quantify the available post-yield deformation capacity at the sectional level.

The ductility reserve ratio is defined as the ratio between the additional tensile strain capacity available beyond first yield and the steel yield strain:

$$DRR = (\varepsilon_{s,u} - \varepsilon_y)/\varepsilon_y$$

where:

- $\varepsilon_{s,u}$ is the steel strain at the attainment of the ultimate concrete compressive strain, and
- ε_y is the steel yield strain.

The DRR provides a strain-based measure of post-yield deformation capacity that is independent of member length, loading configuration, or boundary conditions and is therefore well-suited for sectional-level assessment. Normalisation by the steel yield strain is adopted to express deformation capacity relative to the onset of tension steel yielding, which governs the development of ductile flexural response in reinforced concrete sections.

2.7 Summary of Methodological Parameters

To ensure transparency and reproducibility of the sectional analysis, the principal geometric properties, material parameters, and strengthening configurations adopted in this study are compiled in Table 1. All strengthened sections are evaluated relative to the same reference configuration, and no strengthened case is treated as an independent baseline.

Table 1. Summary of sectional properties, material parameters, and strengthening strategies considered in the analysis (b = section width; d = effective depth).

Parameter	**Reference Section**	**Strategy T-1**	**Strategy T-2**	**Strategy B**	**Strategy C**
b (mm)	300	300	300	300	300
d (mm)	500	500	500	500	500
f_{ck} (MPa)	30	30	30	35	40
f_{yk} (MPa)	500	500	500	500	500
Tensile reinforcement	$A_{s,0}$ = 1.25*$A_{s,min}$	1.25*$A_{s,0}$	1.50*$A_{s,0}$	1.15*$A_{s,0}$	Baseline ($A_{s,0}$)

Change in flexural resistance	-	+25%	+50%	+15%	≈ 0% (marginal)
Dominant strengthening mode	-	Tension	Tension	Balanced	Compression

Table 1 consolidates the key modelling assumptions and parameter choices underlying the analytical framework, enabling direct comparison between strengthening strategies and supporting clear interpretation of the results presented in the following section.

3. Results and Discussion

The results are presented in terms of flexural resistance, strain-based ductility reserve, and a reserve efficiency index, with all strengthened sections evaluated relative to the same reference beam designed at 1.25 times the Eurocode 2 minimum longitudinal reinforcement requirement. The strengthening strategies are examined over a range of strength gains in order to isolate the influence of strengthening mode, rather than strength increase alone, on deformation capacity.

Two tension-dominated strengthening configurations are examined. Strategy T-1 represents a moderate tension-side intervention, while Strategy T-2 is included solely to illustrate the effect of increasing the intensity of tension-dominated strengthening on ductility reserve. Strategy T-2 is not intended as an optimised or practically targeted strengthening case, but rather to highlight the non-linear degradation of deformation capacity associated with aggressive tensile strengthening. Other strategies intentionally deliver different strength gains in order to expose the fundamentally different strength–ductility mechanisms associated with balanced and compression-side strengthening. All results are interpreted using absolute changes in ductility reserve together with the reserve efficiency index, which collectively capture both deformation capacity and strengthening efficiency.

3.1 Reference Section Response and Strengthening Outcomes

Table 2 summarizes the flexural resistance, strain-based ductility reserve metrics, and reserve efficiency index obtained for the reference section and for each strengthened section.

The reference section, designed at ρ/ρ_{min} = 1.25 (i.e., modestly above the Eurocode 2 minimum longitudinal reinforcement level), develops a flexural resistance of 60.63 kNm and exhibits a substantial ductility reserve ratio (DRR_0 = 41.19). This response therefore provides the benchmark against which the deformation consequences of strengthening interventions are assessed.

For strengthening configurations dominated by increased tensile capacity (Strategy T), the resulting increase in flexural resistance is accompanied by a pronounced reduction in ductility reserve. When flexural resistance is increased through tension-dominated strengthening, the neutral axis shifts upward, causing the concrete to reach its ultimate strain at lower levels of steel deformation. As a result, the available post-yield tensile strain capacity is reduced, indicating that strength gains achieved through tension-dominated strengthening mechanisms are accompanied by a disproportionate consumption of deformation reserve.

In contrast, when flexural resistance is increased through a balanced enhancement of tensile and compressive capacity (Strategy B), the ductility reserve is largely preserved relative to the reference section. The coordinated increase in tensile resistance and compressive capacity limits neutral-axis migration and maintains a strain distribution similar to that of the baseline beam. This response demonstrates that, at the sectional level, strengthening interventions may be configured to retain deformation capacity for practically relevant strength increases rather than consume it.

When strengthening is applied primarily on the compression side (Strategy C), the sectional response is characterized by a substantial increase in ductility reserve with only a marginal change in flexural resistance. This reflects the role of enhanced compressive capacity in delaying concrete crushing, thereby allowing greater post-yield tensile deformation to develop prior to failure. This response shows that ductility can be enhanced without a significant increase in flexural strength.

Taken together, these results demonstrate that the deformation consequences of strengthening interventions are governed not by the magnitude of the strength increase alone, but by the mechanism through which that strength is achieved. Strengthening strategies that deliver similar or even smaller increases in flexural resistance can exhibit markedly different effects on ductility reserve, underscoring the need to explicitly assess deformation capacity alongside strength when evaluating strengthening performance.

Table 2. Flexural resistance, ductility reserve metrics, and reserve efficiency index for the reference beam and strengthened beam sections evaluated relative to a common baseline. η_R is not defined for the baseline section and is therefore omitted.

Strategy/Case	M_R (kNm)	DRR	ΔM_R	ΔDRR	η_R
Baseline	60.63	41.19	1.0	1.0	-
T-1	75.54	32.47	1.2459	0.7883	0.6327
T-2	90.35	26.66	1.4902	0.647	0.4343
B	69.74	41.82	1.1502	1.0153	0.8827
C	60.83	55.72	1.0033	1.3528	1.3484

Strength gains are not uniform across strategies; this is intentional and reflects the fundamentally different mechanics of each strengthening mode.

3.2 Strength-Ductility Interaction and Reserve Efficiency

To enable a consistent comparison of how efficiently flexural strength gains are achieved with respect to deformation capacity, the results are interpreted in terms of relative changes in flexural resistance, ductility reserve ratio (DRR), and the reserve efficiency index (η_R). Each strengthening strategy was intentionally configured to produce a distinct level of strength intervention, allowing the interaction between strength enhancement and deformation capacity to be examined without targeting a specific or maximum flexural resistance.

The tension-dominated strengthening case designed to produce an approximately 25% increase in flexural resistance (Strategy T-1) increases the flexural resistance from 60.63 kNm to 75.54 kNm, corresponding to a 24.6% increase relative to the reference section (baseline beam). This increase is accompanied by a reduction in ductility reserve ratio from 41.19 to 32.47, representing a loss of approximately 21%. The resulting reserve efficiency, η_R = 0.6327, indicates that a substantial proportion of the available deformation capacity is consumed in achieving the prescribed strength increase. In relative terms, this corresponds to retaining only about 63% of the original ductility reserve for a 24.6% increase in flexural resistance. This response reflects the sensitivity of ductility reserve to tension-dominated strengthening when strength enhancement is achieved primarily through increased tensile force demand.

In contrast, the balanced strengthening strategy (Strategy B) was configured to represent a balanced strengthening intervention, resulting in a moderate increase in flexural resistance. The flexural resistance increases from 60.63 kNm to 69.74 kNm, corresponding to a 15.0% increase, while the ductility reserve ratio remains effectively unchanged (DRR = 41.82). The associated reserve efficiency, η_R = 0.8827, indicates that nearly the full ductility reserve of the reference section is retained relative to the achieved strength increase. This response demonstrates that a meaningful enhancement in flexural resistance can be obtained without materially compromising deformation capacity when strengthening is applied in a way that delays concrete crushing and preserves post-yield tensile deformation.

Compression-side strengthening (Strategy C) produces a response dominated by ductility enhancement rather than flexural strength gain. Relative to the reference section, the flexural resistance increases only marginally from 60.63 kNm to 60.83 kNm (corresponding to an increase of approximately 0.3%), indicating that this intervention does not materially alter sectional strength. In contrast, the ductility reserve ratio increases substantially from DRR_0 = 41.19 to DRR = 55.72, corresponding to an enhancement of approximately 35% relative to the reference beam. This behaviour reflects the role of enhanced compressive capacity in delaying the attainment of the ultimate concrete strain limit, thereby permitting greater post-yield tensile deformation to develop before crushing occurs. The resulting reserve efficiency, η_R = 1.3484, exceeds unity, indicating that deformation capacity is improved independently of strength gain. Strategy C therefore demonstrates that strengthening interventions applied on the compression side can be effective in enhancing ductility reserve without necessitating a corresponding increase in flexural resistance, highlighting a fundamentally different strength–ductility interaction compared to tension-dominated strengthening approaches.

The higher-intensity tension-dominated case (Strategy T-2) further illustrates the non-linear nature of the strength–ductility interaction. The resulting flexural resistance is 90.35 kNm, corresponding to a 49.0% increase relative to the reference section, and is accompanied by a pronounced reduction in ductility reserve (DRR = 26.66), equating to a loss of approximately 35%. The reserve efficiency drops to $\eta_R = 0.4343$, confirming that aggressive strength enhancement through tension-dominated mechanisms rapidly becomes inefficient from a deformation-capacity perspective.

Overall, the results demonstrate that the relationship between flexural resistance and ductility reserve is governed primarily by the mode of strengthening rather than by the magnitude of strength gain alone, a relationship that is not captured by strength-based assessment alone. Across the range of strength interventions examined, markedly different deformation outcomes emerge depending on the strengthening mode. Such behaviour would not be apparent from conventional strength-based checks, which do not explicitly track post-yield strain development or deformation reserve.

3.3 Practical Implications

From a broader perspective, these results highlight that sectional strengthening performance cannot be adequately characterised by strength increase alone. Strength-based assessment may indicate comparable or even favourable performance for tension-dominated strengthening strategies, while not explicitly quantifying the associated depletion of deformation reserve. The strain-based metrics and efficiency measures employed in this study provide a complementary lens through which strengthening interventions can be evaluated in terms of both strength and deformation capacity. At the sectional level, this enables strengthening configurations to be compared and screened not only for how much capacity they add, but how efficiently they add this capacity and preserve or enhance post-yield deformation capacity—an aspect directly relevant to robustness, redistribution, and damage tolerance.

4. Conclusions

This study investigated the influence of strengthening mode on the interaction between flexural resistance and strain-based ductility reserve in singly reinforced concrete sections. Rather than assessing strengthening effectiveness solely in terms of achievable strength enhancement, the analysis examined how different strengthening interventions modify the availability of post-yield deformation capacity and the efficiency of strength enhancement relative to changes in deformation capacity.

The reference section, designed modestly above the Eurocode 2 minimum longitudinal reinforcement requirement, was shown to possess a substantial ductility reserve. This provides a meaningful baseline for evaluating the deformation consequences of strengthening interventions.

Strengthening strategies dominated by tension-side intervention were found to produce substantial increases in flexural resistance but at the expense of ductility reserve. As flexural capacity is increased through this mode, deformation capacity is consumed disproportionately, particularly at higher strength targets. This behaviour indicates that ductility reserve is highly sensitive to tension-dominated strengthening, as the associated upward migration of the neutral axis causes the ultimate concrete strain to be reached at lower levels of steel deformation.

In contrast, the balanced strengthening strategy was intentionally configured to produce a moderate increase in flexural resistance, and this intervention largely preserved the ductility reserve of the reference section. This demonstrates that strength enhancement does not inherently necessitate ductility degradation, provided that strengthening is applied in a manner that avoids premature concrete crushing and preserves post-yield tensile deformation capacity.

Compression-side strengthening exhibits a fundamentally different strength–ductility interaction. When strengthening is applied on the compression side, the resulting response is characterized by only a marginal change in flexural resistance but a pronounced enhancement of ductility reserve. This demonstrates that ductility can be actively increased, rather than merely preserved, when strengthening delays the attainment of the ultimate concrete strain and permits greater post-yield tensile deformation to develop prior to crushing.

An important aspect of this study is the use of a simple reserve efficiency index (η_R) to provide a compact and mechanically transparent measure of how efficiently flexural strength gains are achieved relative to changes in ductility reserve. While the index is intentionally simple, it enables direct comparison between strengthening strategies that exhibit fundamentally different strength–ductility interactions. Unlike strength-based metrics alone, the index highlights interventions that may appear effective in terms of flexural resistance but are inefficient from a deformation-capacity perspective.

Overall, the results demonstrate that the relationship between flexural resistance and ductility reserve is governed primarily by the mode of strengthening, rather than by the magnitude of strength gain alone. By explicitly considering deformation reserve alongside flexural resistance, the study offers a transparent interpretive basis for comparing strengthening strategies beyond strength compliance, without departing from familiar strain-based sectional mechanics.

The findings are based on a strain-based sectional assessment of singly reinforced beams and are therefore intended to inform sectional strengthening behaviour rather than member-level response.

References

ACI Committee 440. (2023). Design and construction of externally bonded fiber-reinforced polymer (FRP) systems for strengthening concrete structures — Guide (ACI PRC-440.2-23). American Concrete Institute.

Alhadid, M. M. A., & Youssef, M. A. (2017). Analysis of reinforced concrete beams strengthened using concrete jackets. *Engineering Structures*, *132*, 172-187.

Alhadid, M. M. A., & Youssef, M. A. (2018). Assessment of the flexural behavior of reinforced concrete beams strengthened with concrete jackets. *Engineering Structures*, *167*, 108-120.

EN 1992-1-1 (2004): Eurocode 2 – Design of Concrete Structures – Part 1-1: General Rules and Rules for Buildings, European Committee for Standardization.

Gamal, S. E., Al-Nuaimi, A., Al-Saidy, A., & Al-Shanfari, K. (2019). Flexural behavior of RC beams strengthened with CFRP sheets using different strengthening techniques. *The Journal of Engineering Research*, *16*(1), 35-43.

Gonzalez-Libreros, J., D'Antino, T., Sneed, L. H., Pellegrino, C., & Giacomin, G. (2017a). Internal and external transversal reinforcement interaction in RC beams strengthened in shear with externally bonded composites. In *Proceedings of SMAR 2017, the fourth International Conference on Smart Monitoring, Assessment and Rehabilitation of Civil Structures* (pp. 1-8).

Gonzalez-Libreros, J. H., Sabau, C., Sneed, L. H., Pellegrino, C., & Sas, G. (2017b). State of research on shear strengthening of RC beams with FRCM composites. *Construction and Building Materials*, *149*, 444-458.

Li, A., Assih, J., & Delmas, Y. (2001). Shear strengthening of RC beams with externally bonded CFRP sheets. Journal of Structural Engineering, 127(4), 374-380.

Lopes, A. V., Lou, T., & Lopes, S. M. (2025). On the ductility of RC beam section: A revision and up to date of conclusions. *Engineering Structures*, *322*, 119105.

Megahed, F. A., Seleem, M. H., Badawy, A. A. M., & Sharaky, I. A. (2023). The flexural response of RC beams strengthened by EB/NSM techniques using FRP and metal materials: a state-of-the-art review. *Innovative Infrastructure Solutions*, *8*(11), 289.

Oudah, F., & El-Hacha, R. (2012). A new ductility model of reinforced concrete beams strengthened using fiber reinforced polymer reinforcement. *Composites Part B: Engineering*, *43*(8), 3338-3347.

Razaqpur, A. G., Cameron, R., & Mostafa, A. A. (2020). Strengthening of RC beams with externally bonded and anchored thick CFRP laminate. *Composite Structures*, *233*, 111574.

Spadea, G., Swamy, R. N., & Bencardino, F. (2001). Strength and ductility of RC beams repaired with bonded CFRP laminates. *Journal of Bridge Engineering*, *6*(5), 349-355.

Tahsiri, H., Sedehi, O., Khaloo, A., & Raisi, E. M. (2015). Experimental study of RC jacketed and CFRP strengthened RC beams. *Construction and Building Materials*, *95*, 476-485.

Triantafillou, T. C. (1998). Strengthening of structures with advanced FRPs. *Progress in Structural Engineering and Materials*, *1*(2), 126-134.

Vahidpour, M., Kheyroddin, A., & Kioumarsi, M. (2022). Experimental investigation on flexural capacity of reinforced concrete beams strengthened with 3D-fiberglass, CFRP and GFRP. *International Journal of Concrete Structures and Materials*, *16*(1), 18.

Statements & Declarations

Funding

The author declares that no funding was received.

Competing Interests

The author declares that no competing interests exist.

Author’s Contribution

The author solely conceived the study, developed the analytical framework, performed the sectional analyses, interpreted the results, and wrote the manuscript.

Data Availability

The data supporting the findings of this study are available within the article. Any additional derived data are available from the corresponding author upon reasonable request.

Ethics Approval

Not applicable.

Consent to Participate

Not applicable.

Consent to Publish

Not applicable.

Acknowledgements

Not applicable.