A RELATION BETWEEN LAMINATION PARAMETERS OF STIFFEST POST-BUCKLED COMPOSITE PLATE

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Abstract

The present paper deals with the anisotropic composite plates experiencing postbuckling. The plates have a symmetric lay-up and obey the geometrically non-linear von Karman law. A point-wise relation between the lamination parameters of the stiffest composite plates in postbuckling is derived and analyzed. It is shown that in the considered case not all lamination parameters are independent on each other (one of them is a linearly dependent one). The remaining linearly independent seven components of the lamination parameter vector are some non-linear functions of six parameters describing point-wisely the 2D principal strains and the principal curvatures. The analysis leads to a conclusion that the optimization w.r.t. the lamination parameters only is not sufficient for obtaining an optimal plate lay-up.

Keywords: composite plate, postbuckling, stiffness, lay-up, lamination parameters

1. INTRODUCTION

The number of papers dealing with the optimization of the composite plates in postbuckling is very high. As a rule, the papers are devoted to a numerical analysis of such problems, the corresponding reviews may be found in Turvey and Marshall 1995, Kassapoglou 2010, Gürdal et al. 1999, Bruyneel 2008, Ghiasi et al. 2009, 2010. In many of the papers the heuristic lamination parameter-based approaches are used (the lamination parameter approach is described, e.g., in Fukunaga and Sekine 1992). An example of such optimization approach is presented in the paper of IJsselmuiden et al. 2010.

There is a few papers analyzing the problems theoretically (Selyugin 2019 a, b, c). The first order lay-up optimality conditions for the stiffest plate in postbuckling have been obtained in (Selyugin, 2019a).

The present paper is devoted to the derivation and the analysis of the point-wise relations between the plate lamination parameters in the case of the postbuckling stiffness optimality. The basis of the development is the point-wise optimality conditions (Selyugin 2019a).

At the end of the paper there are some conclusions.

2. THEORETICAL CONSIDERATION

The assumptions of the present paper are: the symmetric point-wise arbitrary lay-up, the single-modal bifurcation point (Ohsaki and Ikeda 2007), symmetric buckling, the Cartesian orthogonal coordinates XYZ with respective displacements *u, v, w*; the plate with piece-wise-smooth boundary contour is located in the XY plane; the in-plane loading is applied with the value not much higher than the bifurcation load; the boundary conditions are the simple support or clamped. The classic laminated plate theory (Gibson 1994) is employed.

The functional of the kinematic variational principle is written as follows:

$$I=\int_S\left(\frac{1}{2}\vec{e}_0^T A\vec{e}_0+\frac{1}{2}\vec{\omega}^T D\vec{\omega}\right)dS-\int_{\Gamma_1}\left(\overline{N}_{x\nu}\delta u+\overline{N}_{y\nu}\delta v\right)d\Gamma \tag{1}$$

where the mid-plane (z=0) vector-columns are

$$\vec{e}_0=\left.\begin{pmatrix} e_{xx}\\ e_{yy}\\ 2e_{xy}\end{pmatrix}\right|_{z=0},\quad \vec{\omega}=\left.\begin{pmatrix} -\partial^2 w/\partial x^2\\ -\partial^2 w/\partial y^2\\ -2\partial^2 w/\partial x\partial y\end{pmatrix}\right|_{z=0} \tag{2}$$

and $e\ldots$ with subscripts are the corresponding Green strain components. The x and y force flows applied at the part Γ_1 of the contour are $\overline{N}_{x\nu},\overline{N}_{y\nu}$. The proper boundary conditions are applied at the contour also. Further we keep the notations of Selyugin 2019a,c.

For creating the best structural lay-up we consider the following optimization problem: to maximize the structural stiffness (1) making the point-wise variations of the plate ply orientation angles. The first order point-wise optimality conditions are written as follows (Selyugin 2019a), $i=1,\ldots,n$:

$$\begin{aligned}&\sin 2(\theta_i-\varphi)\left[\frac{U_2}{4U_3}\left(\varepsilon_1^2-\varepsilon_2^2\right)+\left(\varepsilon_1-\varepsilon_2\right)^2\cos 2(\theta_i-\varphi)\right]\left(z_i-z_{i-1}\right)+\\&+\sin 2(\theta_i-\psi)\left[\frac{U_2}{4U_3}\left(k_1^2-k_2^2\right)+\left(k_1-k_2\right)^2\cos 2(\theta_i-\psi)\right]\frac{1}{3}\left(z_i^3-z_{i-1}^3\right)=0\end{aligned} \tag{3}$$

where $\varphi,\psi,i,\theta_i,z_i,k_1,k_2,\varepsilon_1,\varepsilon_2,n,U_2,U_3$ respectively are the major principal mid-plane strain orientation angle, the major principal curvature orientation angle, the ply number ($z0$=0 corresponds to the mid-plane), the ply orientation angle, the ply coordinate, the major and the minor principal curvatures, the major and the minor principal 2D-strains, a half of the total ply (supposedly even) number. The two last quantities are the ply stiffness constants (Gibson 1994). The conditions (3) are derived from the analysis of the first variation of (1) w.r.t. θ_i. The variational principle (1) is a minimum-type one.

Following Fukunage and Sekine 1992, the lamination parameters of a plate with the symmetric lay-up are ($\bar{z}$ is a non-dimensional quantity corresponding to z divided by the plate half-thickness value):

$$\begin{cases} \xi_1 = \int_0^1 \cos 2\theta_i d\bar{z} \\ \xi_2 = \int_0^1 \cos 4\theta_i d\bar{z} \\ \xi_3 = \int_0^1 \sin 2\theta_i d\bar{z} \\ \xi_4 = \int_0^1 \sin 4\theta_i d\bar{z} \end{cases} \tag{4}$$

$$\begin{cases} \xi_9 = 3\int_0^1 \bar{z}^2 \cos 2\theta_i d\bar{z} \\ \xi_{10} = 3\int_0^1 \bar{z}^2 \cos 4\theta_i d\bar{z} \\ \xi_{11} = 3\int_0^1 \bar{z}^2 \sin 2\theta_i d\bar{z} \\ \xi_{12} = 3\int_0^1 \bar{z}^2 \sin 4\theta_i d\bar{z} \end{cases} \tag{5}$$

We introduce the following intermediate quantities:

$$\begin{cases} \alpha_1 = \frac{U_2}{4U_3}\left(\varepsilon_1^2 - \varepsilon_2^2\right) \\ \alpha_2 = \frac{1}{2}\left(\varepsilon_1 - \varepsilon_2\right)^2 \\ \beta_1 = \frac{U_2}{4U_3}\left(k_1^2 - k_2^2\right) \\ \beta_2 = \frac{1}{2}\left(k_1 - k_2\right)^2 \\ \gamma_i = \left(z_i^3 - z_{i-1}^3\right), i = 1,\ldots,n \end{cases} \tag{6}$$

Transforming (3) with the known trigonometric formulas and summing the optimality conditions for *i=1,...,n*, transverse to the plate lay-up, we obtain the following linear relation between the lamination parameters in optimum (h is the plate thickness):

$$\alpha_1\left(h/_{2n}\right)\cos(2\varphi)\sum_{i=1}^{n}\sin(2\theta_i)-\alpha_1\left(h/_{2n}\right)\sin(2\varphi)\sum_{i=1}^{n}\cos(2\theta_i)+\alpha_2\left(h/_{2n}\right)\cos(4\varphi)\sum_{i=1}^{n}\sin(4\theta_i)-$$
$$-\alpha_2\left(h/_{2n}\right)\sin(4\varphi)\sum_{i=1}^{n}\cos(4\theta_i)+\frac{1}{3}\beta_1\cos(2\psi)\sum_{i`=1}^{n}\left(\gamma_i\sin(2\theta_i)\right)-\frac{1}{3}\beta_1\sin(2\psi)\sum_{i`=1}^{n}\left(\gamma_i\cos(2\theta_i)\right)+$$
$$+\frac{1}{3}\beta_2\cos(4\psi)\sum_{i`=1}^{n}\left(\gamma_i\sin(4\theta_i)\right)-\frac{1}{3}\beta_2\sin(4\psi)\sum_{i=1}^{n}\left(\gamma_i\cos(4\theta_i)\right)=0$$

(7)

The relation (7) is, in fact, a scalar product of the vector-column, composed of the lamination parameters (4), (5):

$$\vec{\xi}=\left(\xi_1,\xi_2,\xi_3,\xi_4,\xi_9,\xi_{10},\xi_{11},\xi_{12}\right)^T \tag{8}$$

and the vector-column

$$\vec{d}=\begin{pmatrix} \left(h/_2\right)\alpha_1\cos(2\varphi) \\ -\left(h/_2\right)\alpha_1\sin(2\varphi) \\ \left(h/_2\right)\alpha_2\cos(4\varphi) \\ -\left(h/_2\right)\alpha_2\sin(4\varphi) \\ \frac{1}{3}\left(h/_2\right)^3\beta_1\cos(2\psi) \\ -\frac{1}{3}\left(h/_2\right)^3\beta_1\sin(2\psi) \\ \frac{1}{3}\left(h/_2\right)^3\beta_2\cos(4\psi) \\ -\frac{1}{3}\left(h/_2\right)^3\beta_2\sin(4\psi) \end{pmatrix} \tag{9}$$

where T means transposing.

Observing (7), we may say that the lamination parameters (after optimizing the plate w.r.t. them only) give some preliminary optimality information about the whole plate lay-up. The further optimization of the ply angles is necessary. The values of the principal 2D-strains and the principal curvatures, as well as the behavior of the principal curvature lines and the 2D-principal strain lines play an important role in the further optimization. The hyper-plane (7) for the lamination parameters may be also useful for a preliminary step of the lay-up optimization.

The vector (9) depends on six quantities $\varepsilon_1, \varepsilon_2, \varphi, k_1, k_2, \psi$ only. On the other hand, the lamination parameter vector (8) with the account of (7) depends on seven parameters, but it has eight components. This means that only seven components of the point-wise lamination parameter vector are linearly independent in the case of the plate with the stiffest postbuckling lay-up.

On the other hand, the seven linear-independent lamination parameters are the non-linear functions of the six parameters $\varepsilon_1, \varepsilon_2, \varphi, k_1, k_2, \psi$.

We underline again that the relations identified above are point-wise.

3. CONCLUSIONS

- It is shown that the lamination parameters (after possible optimization of the plate w.r.t. them only) may give some preliminary optimality information about the whole plate. The further optimization of the ply angles is necessary;
- Only seven components of the lamination parameter vector-column are linearly independent in the case of the plate with the stiffest postbuckling lay-up;
- The seven components are some non-linear functions of the directions and the values of the principal 2D-strains and the principal curvatures.

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