The *g*-Function: An ML-based Fragility Operator

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Abstract

Multi-hazard and multi-event fragility modeling has advanced rapidly in recent years, yet its widespread adoption remains limited by the computational burden of successive nonlinear simulations required to explicitly capture damage accumulation. Recent studies on earthquake–tsunami, fire-following-earthquake, and mainshock–aftershock sequences have revealed a recurring probabilistic structure governing joint fragility responses. This paper formalizes that structure through a unified *g*-function, a hazard-agnostic probabilistic operator that synthesizes multi-dimensional fragility surfaces directly from single-event fragility curves. Operating entirely in probability space, the *g*-function maps marginal exceedance probabilities to joint exceedance surfaces without direct dependence on hazard-specific intensity measures or physics. The formulation is demonstrated across three distinct hazard classes—earthquake–tsunami, fire-following-earthquake, and mainshock–aftershock—showing consistently high agreement with simulation-based benchmarks. The results establish the *g*-function as a general, scalable

framework for multi-hazard fragility modeling, enabling efficient portfolio-level and community-scale resilience assessments without reliance on high-performance computing.

Keywords: *g*-function; multi-hazard fragility surfaces; multi-event hazards; sequential damage; probabilistic fragility modeling

1. Introduction

Community resilience analysis increasingly requires explicit consideration of cascading and successive hazards, including earthquake–tsunami events, fire-following-earthquake (FFE) scenarios, and mainshock–aftershock (MS–AS) sequences (Harati and van de Lindt, 2025; Rashid and Nishio, 2024; Song et al., 2023; Xu et al., 2025). In these cases, damage induced by an initial hazard fundamentally alters the structural state prior to subsequent loading, rendering conventional single-event fragility functions inadequate (Kourehpaz et al., 2023). To address this limitation, a growing body of research has employed successive nonlinear simulations to generate multi-hazard or multi-event fragility surfaces that directly account for cumulative damage effects (Harati and van de Lindt, 2025).

While these physics-based frameworks provide high-fidelity representations of structural response, they are computationally intensive (Harati and van de Lindt, 2024). Generating near-continuous fragility surfaces through Monte Carlo simulation and successive nonlinear analyses often requires millions of simulations, limiting their applicability for regional risk assessment, portfolio analysis, and real-time decision support (Harati and van de Lindt, 2024; Harati and van de Lindt, 2024). To overcome this barrier, recent work introduced a data-driven synthesis approach capable of reconstructing earthquake–tsunami fragility surfaces from single-hazard fragility curves with high accuracy (Harati and van de Lindt, 2024b).

Across these developments, a recurring observation emerges: regardless of the hazard pairing, joint fragility surfaces exhibit a consistent probabilistic structure when expressed in terms of marginal exceedance probabilities. This paper extracts, formalizes, and generalizes that structure into a unified *g*-function. Rather than focusing on a specific hazard application, the contribution of this paper is to elevate the *g*-function itself as a general operator for multi-hazard and multi-event fragility synthesis.

2. Conceptual Origin of the *g*-Function

Fragility functions are commonly defined as the probability of exceeding a damage state (DS) conditional on a hazard intensity measure (IM). For a single hazard, this relationship may be expressed as

$$P_1 = P(\mathrm{DS} \mid IM_1), P_2 = P(\mathrm{DS} \mid IM_2),$$

where IM_1 and IM_2 correspond to two distinct hazards or events. In successive or cascading hazard scenarios, the quantity of interest is the joint exceedance probability

$$P_{12} = P(\mathrm{DS} \mid IM_1, IM_2)$$

Simulation-based multi-hazard fragility surfaces approximate P_{12} directly through successive nonlinear analyses. However, examination of these surfaces reveals that P_{12} can be consistently expressed as a function of the marginal probabilities P_1 and P_2, independent of the explicit IM definitions. This observation motivates the introduction of the *g*-function:

$$P_{12} = g(P_1, P_2).$$

The *g*-function operates entirely in probability space and exhibits the following fundamental properties:

I. Monotonicity: P_{12} increases monotonically with both P_1 and P_2.

II. Boundedness: $0 \leq P_{12} \leq 1$.

III. Consistency on boundaries:

$$g(P_1, 0) = P_1, g(0, P_2) = P_2$$

IV. Damage accumulation representation: The function incorporates interaction between hazards through a coupling mechanism.

In practical implementations, the *g*-function may be expressed in a parametric form that includes interaction coefficients governing the strength of hazard coupling, as demonstrated in prior earthquake–tsunami applications (Harati and van de Lindt, 2024a, 2024b). Importantly, the exact functional form is less critical than the conceptual abstraction: the *g*-function represents a probabilistic mapping between marginal and joint fragility responses.

3. Hazard-Agnostic Interpretation

A key contribution of this paper is the demonstration that the *g*-function is hazard-agnostic. Although earthquake–tsunami, FFE, and MS–AS scenarios differ substantially in their physical mechanisms, they share a common structural characteristic: damage accumulation through sequential loading.

In all cases:

- The structure experiences an initial hazard that modifies stiffness, strength, residual deformation, or boundary conditions.
- A subsequent hazard acts on the damaged configuration.
- Fragility curves encode this evolving state implicitly through exceedance probabilities.

By operating on probabilities rather than intensity measures, the *g*-function bypasses hazard-specific physics while retaining their cumulative effect. This explains why a *g*-function calibrated for earthquake–tsunami scenarios (Harati and van de Lindt, 2024b) can be applied—without retraining—to MS–AS sequences and FFE cases, as demonstrated in recent studies (Harati and van de Lindt, 2025; Harati and van de Lindt, 2025). This interpretation positions the *g*-function not as a surrogate model for physics-based simulation, but as a probabilistic operator that captures the essential structure of damage interaction across hazards.

4. Demonstration Across Multiple Hazard Classes

For near-field earthquake–tsunami scenarios, the *g*-function was shown to accurately synthesize three-dimensional fragility surfaces from independently generated earthquake and tsunami fragility curves (Harati and van de Lindt, 2024b). Validation against high-fidelity successive simulations yielded coefficients of determination exceeding 0.95 and low root mean square error values. In FFE applications, the same *g*-function structure was used to reconstruct fragility surfaces derived from sequential nonlinear earthquake and fire simulations of steel moment-resisting frames (Harati and van de Lindt, 2025). Despite the fundamentally different nature of thermal degradation compared to hydrodynamic loading, the synthesized surfaces closely matched simulation-based benchmarks. The *g*-function was further applied to MS–AS fragility surfaces for reinforced concrete frame archetypes subjected to real recorded ground motion sequences (Harati and van de Lindt, 2025). Without retraining or modification, the function reproduced joint exceedance probabilities with high accuracy, confirming cross-hazard generalization. Across all cases, the g-function remained invariant, reinforcing its role as a unifying probabilistic framework rather than a hazard-specific model.

5. Implications for Resilience Modeling

The introduction of a unified *g*-function has significant implications for resilience-informed decision making by enabling the generation of joint fragility surfaces without repeated nonlinear simulations, thereby substantially improving computational efficiency and making portfolio-level and community-scale analyses tractable without reliance on high-performance computing. In addition, the framework offers flexibility for mitigation studies, as retrofit strategies can be evaluated through horizontal shifting of marginal fragility curves with updated joint surfaces generated instantaneously, and it is directly compatible with community resilience platforms such as IN-CORE that rely on fragility functions as initial conditions for recovery modeling. By decoupling fragility synthesis from hazard-specific simulation workflows, the *g*-function facilitates rapid scenario exploration and supports time-sensitive planning and decision-making applications.

6. Limitations and Scope

The *g*-function framework is intended for sequential or cascading hazards where damage states are well-defined and consistent across events. It does not explicitly model concurrent hazards acting simultaneously, nor does it replace detailed physics-based simulation when local response mechanisms are the primary focus. Instead, the *g*-function should be viewed as a scalable synthesis tool that complements high-fidelity modeling.

7. Conclusions

This paper formalized the *g*-function as a unified, hazard-agnostic probabilistic operator for synthesizing multi-hazard and multi-event fragility surfaces. By operating entirely in probability space, the framework captures cumulative damage effects while eliminating dependence on hazard-specific intensity measures and computationally intensive successive simulations. Demonstrations across earthquake–tsunami, fire-following-earthquake, and mainshock–aftershock scenarios confirm the generality and robustness of the approach. The *g*-function provides a practical pathway toward scalable, physics-consistent fragility modeling for community resilience assessment.

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