CS 6640N - Int' Comp Sys(Rutledge)

Lecture 1 - Intro to Computer Systems

Mon Jan 13 2014

Course Overview

Computer Systems: A Programmer's Perspective (CSAPP) - Bryant and O'Hallaron

Goal: Learn how computer systems work, how they are organized, and how they execute programs. Understanding these things will help you be a better programmer, and help you write programs that make better use of the capabilities provided by the computer system.

Text Book

Computer Systems: A Programmer's Perspective (CSAPP) - Bryant and O'Hallaron

Class Topics

- Machine-level representation of programs
- Processor architecture
- Optimizing program performance
- Memory hierarchy
- Linking
- Exceptional control flow
- Virtual memory
- Concurrency

Class Assignments

- Data Lab: Manipulating bits
- Bomb Lab: Defusing a binary bomb
- Buffer Lab: Hacking a buffer bomb
- Shell Lab: Writing your own Unix shell
- Malloc Lab: Writing your own malloc package
- Proxy Lab: Writing your own Web proxy

Class Policies

- Grading: Assignments 60%, Midterm 15%, Final 25%
- Late Days: 2 late days without penalty, 15% per day after that
- Collaboration: Read the CMU policy. Understand the CMU policy. Live the CMU policy.

Introduction to Computer Systems

Information is Bits + Context

Hello.c

```
#include <stdio.h>

int main()
{
    printf("hello, world\n");
}
```

Compilation: `gcc -o hello hello.c`

Machine-level representation of hello program

```
0000000000400400 <main>:
  400400:   55                    push   %rbp
  400401:   48 89 e5              mov    %rsp,%rbp
  400404:   bf 84 05 40 00        mov    $0x400584,%edi
  400409:   e8 e2 fe ff ff        callq  4003f0 <puts@plt>
  40040e:   5d                    pop    %rbp
  40040f:   c3                    retq
```

Programs are Translated by Other Programs into Different Forms

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```

It Pays to Understand How Compilation Systems Work

Reasons for learning:

- Optimizing program performance
- Understanding link-time errors
- Avoiding security holes

Hardware Organization of a System

Hello.c

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```

Hello's Journey

Hello.c

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```

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Operating System Manages the Hardware

Hello.c

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Key Concepts

- Abstractions are good, but don't forget reality
- Most CS courses emphasize abstraction
- This course emphasizes the underlying implementation and its impact on program behavior

Course Perspective

- Most systems courses are built from the operating system's perspective
- This course is built from the programmer's perspective
- Our goal is to learn how the hardware and operating system interact with your programs in order to help you write better programs

Course Themes

- Amdahl's Law
- Concurrency and parallelism
- Cache memories
- Dynamic memory allocation
- Performance

Teaching Staff

- Instructor: Randy Bryant
- Instructor: Dave O'Hallaron
- Teaching Assistants: Khaled Harfoush, Karl Wurst, Jason Boles, Gaurav Kaul, Mike Reinhold, Mike Spear, Ellie Young, Yili Zheng

Class Resources

- Course Website: http://www.cs.cmu.edu/afs/cs/academic/class/15213-f13/www/index.html
- Piazza: https://piazza.com/cmu/fall2013/15213
- Autolab: https://autolab.cs.cmu.edu/15213-f13/

Getting Help

- Piazza
- Office hours
- Recitations

Class Rules

- Academic integrity
- Plagiarism
- Collaboration

Summary

- Computer systems are a key part of computer science
- Knowing them will help you be a better programmer
- Course is challenging but rewarding

Lecture 2 - Bits, Bytes and Integers

Wed Jan 15 2014

Representing and Manipulating Information

Information Storage

Hello.c

```
#include <stdio.h>

int main()
{
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}
```

Compilation: `gcc -o hello hello.c`

Machine-level representation of hello program

```
0000000000400400 <main>:
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```

Hexadecimal Notation

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```

Word Size

Hello.c

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```

Compilation: `gcc -o hello hello.c`

Machine-level representation of hello program

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```

Byte Ordering

Hello.c

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```

Compilation: `gcc -o hello hello.c`

Machine-level representation of hello program

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```

Representing Strings

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Compilation: `gcc -o hello hello.c`

Machine-level representation of hello program

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```

Representing Integer Data

Hello.c

```
#include <stdio.h>

int main()
{
    printf("hello, world\n");
}
```

Compilation: `gcc -o hello hello.c`

Machine-level representation of hello program

```
0000000000400400 <main>:
  400400:   55                    push   %rbp
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```

Summary

- Computers store and process information represented as bits
- Bits are grouped into bytes, words, and other units
- Different machines use different word sizes and byte ordering
- Hexadecimal notation is a convenient way to represent bit patterns
- Strings are represented as arrays of characters
- Integers are represented as fixed-size arrays of bytes

Bits, Bytes and Integers

Everything is Bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways, computers can represent and manipulate numbers, sets, strings, etc.
- Why bits? Electronic implementation is easy
- Can think of bits as numbers or as logical values

Boolean Algebra

- Developed by George Boole in 19th century
- Algebraic representation of logic
- Encode "True" as 1 and "False" as 0
- And: A&B = 1 when both A=1 and B=1
- Or: A|B = 1 when either A=1 or B=1
- Not: ~A = 1 when A=0
- Exclusive-Or (Xor): A^B = 1 when either A=1 or B=1, but not both

General Boolean Algebras

- Operate on bit vectors
- Operations applied bitwise

Bit-Level Operations in C

- Operations &, |, ~, ^ available in C
- Apply to any "integral" data type
- long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- ~0x41 → 0xBE
- ~0x00 → 0xFF
- 0x69 & 0x55 → 0x41
- 0x69 | 0x55 → 0x7D

Contrast: Logic Operations in C

- Contrast to Logical Operators
- &&, ||, !
- View 0 as "False"
- Anything nonzero as "True"
- Always return 0 or 1
- Early termination

Examples (Char data type)

- !0x41 → 0x00
- !0x00 → 0x01
- !!0x41 → 0x01
- 0x69 && 0x55 → 0x01
- 0x69 || 0x55 → 0x01

Shift Operations

- Left Shift: x << y
- Right Shift: x >> y
- Undefined Behavior: Shift amount < 0 or ≥ word size

Examples (Unsigned)

- 0x81 << 1 → 0x02
- 0x81 >> 1 → 0x40
- 0x81 >> 2 → 0x20

Examples (Signed)

- 0x81 << 1 → 0x02
- 0x81 >> 1 → 0xC0
- 0x81 >> 2 → 0xE0

Summary

- Boolean algebra is mathematical basis for digital logic
- Boolean algebra can be applied to any "integral" data type
- C supports bit-level and logical operations
- Right shift is unsigned or signed

Integer Representations

Integer Representations

- Unsigned and signed integers
- Representation of integers in binary
- Conversion between unsigned and signed integers
- Expanding and truncating integers
- Addition, negation, multiplication, and shifting

Encoding Integers

- Unsigned: B2U(X) = Σ_{i=0}^{w-1} x_i * 2^i
- Two's Complement: B2T(X) = -x_{w-1} * 2^{w-1} + Σ_{i=0}^{w-2} x_i * 2^i
- Sign bit: For two's complement, most significant bit indicates sign (0 for non-negative, 1 for negative)

Two's Complement Examples (4-bit)

- 0: 0000
- 1: 0001
- 2: 0010
- 3: 0011
- 4: 0100
- 5: 0101
- 6: 0110
- 7: 0111
- -8: 1000
- -7: 1001
- -6: 1010
- -5: 1011
- -4: 1100
- -3: 1101
- -2: 1110
- -1: 1111

Numeric Ranges

- Unsigned Values: UMin = 0, UMax = 2^w - 1
- Two's Complement Values: TMin = -2^{w-1}, TMax = 2^{w-1} - 1
- Other Values: Minus 1 = 111...1

Values for Different Word Sizes

	w=8	w=16	w=32	w=64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

- |TMin| = TMax + 1
- UMax = 2 * TMax + 1

Unsigned vs. Signed Numeric Values

- Equivalence: Same encodings for nonnegative values
- Uniqueness: Every bit pattern represents unique integer value
- Unsigned and Two's Complement are both unique

Conversion Visualized

- 2's Comp. → Unsigned: Ordering Inversion, Negative → Big Positive
- Unsigned → 2's Comp.: Ordering Inversion, Big Positive → Negative

Signed vs. Unsigned in C

- Constants: By default are considered to be signed integers
- Unsigned if have "U" as suffix: 0U, 4294967259U
- Casting: Explicit casting between signed and unsigned same as U2T and T2U
- Implicit casting also occurs via assignments and procedure calls

Casting Surprises

- Expression Evaluation: If there is a mix of unsigned and signed in single expression, signed values implicitly cast to unsigned
- Including comparison operations <, >, ==, <=, >=
- Examples for W = 32: TMIN, TMAX, UMAX
- Should be careful when mixing unsigned and signed in expressions

Code Security Example

```
/* Similar to GNU3 library code */
/* Read up to maxlen bytes from file fd.  Return number
   of bytes read, or 0 if too many bytes requested. */
int read_from_file(unsigned fd, char *buf, unsigned maxlen) {
    /* Byte count will be used with file I/O functions */
    unsigned cnt = read_count();
    char *buf = (char *) malloc(cnt);
    if (cnt > maxlen) /* Make sure don't read too many bytes */
        return 0;
    return read(fd, buf, cnt);
}
```

Summary

- Computer arithmetic must support many numerical schemes
- Unsigned integers: Just binary
- Two's complement integers: Most significant bit is sign bit
- Conversion between unsigned and signed: Same bit pattern, different interpretation
- Casting: Explicit or implicit
- Expanding, truncating: Extend sign bit, truncate high-order bits
- Addition, negation, multiplication, shifting: Bit-level operations

Integer Arithmetic

Unsigned Addition

- Operands: w bits
- True Sum: w+1 bits
- Discard Carry: w bits
- Standard Addition Function: Ignores carry output
- Implements Modular Arithmetic: s = UAdd_w(u, v) = (u + v) mod 2^w

Visualizing Unsigned Addition

- Wraps Around: If true sum ≥ 2^w, at most once

Two's Complement Addition

- Operands: w bits
- True Sum: w+1 bits
- Discard Carry: w bits
- Standard Addition Function: Ignores carry output
- Implements Modular Arithmetic: s = TAdd_w(u, v) = U2T((u + v) mod 2^w)

Visualizing 2's Complement Addition

- NOTE: Values printed in hex
- Wraps Around: If sum ≥ 2^{w-1}, becomes negative; if sum < -2^{w-1}, becomes positive

Multiplication

- Goal: Compute product of w-bit numbers x, y
- Either signed or unsigned
- But, exact results can be bigger than w bits
- Unsigned: up to 2w bits
- Result range: 0 ≤ x * y ≤ (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1
- Two's complement min (negative): up to 2w-1 bits
- Result range: x * y ≥ (-2^{w-1}) * (2^{w-1} - 1) = -2^{2w-2} + 2^{w-1}
- Two's complement max (positive): up to 2w bits, but only for (TMin_w)^2
- Result range: x * y ≤ (-2^{w-1})^2 = 2^{2w-2}
- Maintaining exact results: would need to keep expanding word size with each product computed
- Done in software by "arbitrary precision" arithmetic packages

Unsigned Multiplication in C

- Operands: w bits
- True Product: 2w bits
- Discard w bits: w bits
- Standard Multiplication Function: Ignores high order w bits
- Implements Modular Arithmetic: UMult_w(u, v) = (u * v) mod 2^w

Multiplication with Shift and Add

- Operation u << k gives u * 2^k
- Both signed and unsigned
- Examples: u << 3 == u * 8, u << 5 - u << 3 == u * 24
- Most machines shift and add faster than multiply
- Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

- Quotient of Unsigned by Power of 2: u / 2^k == u >> k
- Uses logical shift

Compiled Unsigned Division Code

- Uses logical shift for unsigned
- For Java Users: Logical shift written as >>>

Signed Power-of-2 Divide with Shift

- Quotient of Signed by Power of 2: x / 2^k == x >> k
- Uses arithmetic shift
- Rounds wrong direction when u < 0

Correct Power-of-2 Divide

- Quotient of Signed by Power of 2: x / 2^k == (x + (2^k - 1)) >> k
- Biases dividend toward 0

Compiled Signed Division Code

- Uses arithmetic shift for int
- For Java Users: Arithmetic shift written as >>

Arithmetic: Basic Rules

- Addition: Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: modular arithmetic
- Two's complement: modular arithmetic
- Multiplication: Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: modular arithmetic
- Two's complement: modular arithmetic
- Both signed and unsigned
- Examples: u << 3 == u * 8, u << 5 - u << 3 == u * 24
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Arithmetic: Basic Rules

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- Multiplication: Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: modular arithmetic
- Two's complement: modular arithmetic

Why Should I Use Unsigned?

- Don't use just because number nonnegative
- Easy to make mistakes
- Can be extremely subtle
- Do use when performing modular arithmetic
- Multi-precision arithmetic
- Do use when using bits to represent sets
- Logical right shift, no sign extension

Summary

- Addition, negation, multiplication, shifting: Bit-level operations
- Unsigned: modular arithmetic
- Two's complement: modular arithmetic
- Both signed and unsigned
- Examples: u << 3 == u * 8, u << 5 - u << 3 == u * 24
- Most machines shift and add faster than multiply
- Compiler generates this code automatically

Floating Point

Floating Point

- Representation of fractional values
- Sacrifice precision for dynamic range
- Representation of very small and very large numbers
- Historical, constantly evolving
- Mathematical properties of floating point operations
- How to add, multiply, etc. floating point values
- Clumsy operations
- Example: Converting between decimal strings and floating point
- Round off behavior
- Rounding, truncating, guard digits
- Future trends
- Suggestions for programmers

Fractional Binary Numbers

- Bits to right of "binary point" represent fractional powers of 2
- Represents rational number: Σ_{i=-n}^{m} b_i * 2^i

Fractional Binary Numbers: Examples

- Value: Representation
- 5 3/4: 101.11
- 2 7/8: 10.111
- 1 7/16: 1.0111
- Observations: Divide by 2 by shifting right, Multiply by 2 by shifting left, Numbers of form 0.111111...2 just below 1.0
- Use notation 1.0 - ε

Representable Numbers

- Limitation: Can only exactly represent numbers of the form x / 2^k
- Other numbers have repeating bit representations
- Value: Representation
- 1/3: 0.0101010101[01]...2
- 1/5: 0.0011001100[1100]...2
- 1/10: 0.0001100110[0110]...2

Floating Point Representation

- Numerical Form: (-1)^s * M * 2^E
- Sign bit s determines whether number is negative or positive
- Significand M normally a fractional value in range [1.0, 2.0)
- Exponent E weights value by power of two
- Encoding: MSB s is sign bit s, exp field encodes E (but is not equal to E), frac field encodes M (but is not equal to M)
- Sizes: Single precision: 8 exp bits, 23 frac bits, 32 bits total; Double precision: 11 exp bits, 52 frac bits, 64 bits total; Extended precision: 15 exp bits, 63 frac bits, 80 bits total

Normalized Encoding

- Condition: exp ≠ 000...0 and exp ≠ 111...1
- Exponent coded as biased value: E = Exp - Bias
- Exp: unsigned value of exp field
- Bias = 2^{k-1} - 1, where k is number of exponent bits
- Single precision: 127 (Exp: 1...254, E: -126...127)
- Double precision: 1023 (Exp: 1...2046, E: -1022...1023)
- Significand coded with implied leading 1: M = 1.xxx...x2
- xxx...x: bits of frac field
- Minimum when 000...0 (M = 1.0)
- Maximum when 111...1 (M = 2.0 - ε)
- Get extra leading bit for "free"

Normalized Encoding Example

- Value: Float F = 15213.0;
- 15213.0: 11101101101101.0
- Significand: 1.1101101101101 * 2^13
- M: 1.1101101101101
- frac: 11011011011010000000000
- Exponent: E = 13, Bias = 127, Exp = 140: 10001100
- Result: 0 10001100 11011011011010000000000

Denormalized Encoding

- Condition: exp = 000...0
- Exponent value: E = 1 - Bias (instead of E = 0 - Bias)
- Significand coded with implied leading 0: M = 0.xxx...x2
- xxx...x: bits of frac
- Cases: exp = 000...0, frac = 000...0: Represents zero value, Note distinct values: +0 and -0 (why?); exp = 000...0, frac ≠ 000...0: Numbers very close to 0.0, Lose precision as get smaller, "Gradual underflow"

Special Values

- Condition: exp = 111...1
- Case: exp = 111...1, frac = 000...0: Represents value ∞ (infinity), Operation that overflows, Both positive and negative, E.g., 1.0/0.0 = −1.0/−0.0 = +∞, 1.0/−0.0 = −∞
- Case: exp = 111...1, frac ≠ 000...0: Not-a-Number (NaN), Represents case when no numeric value can be determined, E.g., sqrt(−1), ∞ − ∞, ∞ * 0

Summary of Floating Point Real Number Encodings

- NaN
- Normalized
- Denormalized
- +∞
- −∞
- −0
- +0

Tiny Floating Point Example

- 8-bit Floating Point Representation: the sign bit is in the most significant bit, the next four bits are the exponent, with a bias of 7, the last three bits are the frac
- Same general form as IEEE Format: normalized, denormalized, representation of 0, NaN, infinity

Dynamic Range (Positive Only)

- s exp frac E Value
- 0 0000 000 -6 0
- 0 0000 001 -6 1/8 * 1/64 = 1/512
- 0 0000 010 -6 2/8 * 1/64 = 2/512
- 0 0000 110 -6 6/8 * 1/64 = 6/512
- 0 0000 111 -6 7/8 * 1/64 = 7/512
- 0 0001 000 -6 8/8 * 1/64 = 8/512
- 0 0001 001 -6 9/8 * 1/64 = 9/512
- 0 0110 110 -1 14/8 * 1/2 = 14/16
- 0 0110 111 -1 15/8 * 1/2 = 15/16
- 0 0111 000 0 8/8 * 1 = 1
- 0 0111 001 0 9/8 * 1 = 9/8
- 0 0111 010 0 10/8 * 1 = 10/8
- 0 1110 110 7 14/8 * 128 = 224
- 0 1110 111 7 15/8 * 128 = 240
- 0 1111 000 n/a inf

Distribution of Values

- 6-bit IEEE-like format
- e = 3 exponent bits
- f = 2 fraction bits
- Bias is 3

Distribution of Values (close-up view)

- 6-bit IEEE-like format
- e = 3 exponent bits
- f = 2 fraction bits
- Bias is 3

Interesting Numbers

- Description: exp, frac, Numeric Value
- Zero: 00...00, 00...00, 0.0
- Smallest Pos. Denorm.: 00...00, 00...01, 2^{-{bias+n-1}}
- Single ≈ 1.4 * 10^{-45}
- Double ≈ 4.9 * 10^{-324}
- Largest Denormalized: 00...00, 11...11, (1.0 − ε) * 2^{-{bias-1}}
- Single ≈ 1.18 * 10^{-38}
- Double ≈ 2.2 * 10^{-308}
- Smallest Pos. Norm.: 00...01, 00...00, 1.0 * 2^{-{bias-1}}
- Just larger than largest denormalized
- One: 01...11, 00...00, 1.0
- Largest Normalized: 11...10, 11...11, (2.0 − ε) * 2^{bias}
- Single ≈ 3.4 * 10^{38}
- Double ≈ 1.8 * 10^{308}

Special Properties of Encoding

- FP Zero Same as Integer Zero: All bits = 0
- Can (Almost) Use Unsigned Integer Comparison: Must first compare sign bits, Must consider −0 = 0, NaNs problematic, Will be greater than any other values, What should comparison yield?, Otherwise OK, Denorm vs. normalized, Normalized vs. infinity

Floating Point Operations: Basic Idea

- x +f y = Round(x + y)
- x *f y = Round(x * y)
- Basic idea: First compute exact result, Make it fit into desired precision, Possibly overflow if exponent too large, Possibly round to fit into frac

Rounding

- Rounding Modes (illustrate with $ rounding): $1.40, $1.60, $1.50, $2.50, −$1.50
- Towards zero: $1, $1, $1, $2, −$1
- Round down (−∞): $1, $1, $1, $2, −$2
- Round up (+∞): $2, $2, $2, $3, −$1
- Nearest Even (default): $1, $2, $2, $2, −$2
- What are the advantages of the modes?

Closer Look at Round-To-Even

- Default Rounding Mode: Hard to get any other kind without dropping into assembly, All others are statistically biased, Sum of set of positive numbers will consistently be over- or under-estimated
- Applying to Other Decimal Places / Bit Positions: When exactly halfway between two possible values, Round so that least significant digit is even, E.g., round to nearest hundredth: 7.8949999: 7.89 (less than half way), 7.8950001: 7.90 (greater than half way), 7.8950000: 7.90 (half way—round up), 7.8850000: 7.88 (half way—round down)

Rounding Binary Numbers

- Binary Fractional Numbers: "Even" when least significant bit is 0, "Half way" when bits to right of rounding position = 100...2
- Examples: Round to nearest 1/4 (2 bits right of binary point): 2 3/32: 10.00011: 10.00, 2 3/16: 10.00110: 10.01, 2 7/8: 10.11100: 11.00, 2 5/8: 10.10100: 10.10

FP Multiplication

- (−1)^{s1} * M1 * 2^{E1} * (−1)^{s2} * M2 * 2^{E2}
- Exact Result: (−1)^s * M * 2^E, Sign s: s1 ^ s2, Significand M: M1 * M2, Exponent E: E1 + E2
- Fixing: If M ≥ 2, shift M right, increment E, If E out of range, overflow, Round M to fit frac precision
- Implementation: Biggest chore is multiplying significands

FP Addition

- (−1)^{s1} * M1 * 2^{E1} + (−1)^{s2} * M2 * 2^{E2}
- Assume E1 > E2
- Exact Result: (−1)^s * M * 2^E, Sign s, significand M: Result of signed align & add, Exponent E: E1
- Fixing: If M ≥ 2, shift M right, increment E, if M < 1, shift M left k positions, decrement E by k, Overflow if E out of range, Round M to fit frac precision

Mathematical Properties of FP Add

- Compare to Those of Abelian Group: Closed under addition? Yes, But may generate infinity or NaN; Commutative? Yes; Associative? No, Overflow and inexactness of rounding; 0 is additive identity? Yes; Every element has additive inverse? Almost, Except for infinities & NaNs
- Monotonicity: a ≥ b ⇒ a +f c ≥ b +f c? Almost, Except for infinities & NaNs

Mathematical Properties of FP Mult

- Compare to Those of Commutative Ring: Closed under multiplication? Yes, But may generate infinity or NaN; Multiplication Commutative? Yes; Multiplication is associative? No, Possibility of overflow, inexactness of rounding; 1 is multiplicative identity? Yes; Multiplication distributes over addition? No, Possibility of overflow, inexactness of rounding
- Monotonicity: a ≥ b & c ≥ 0 ⇒ a *f c ≥ b *f c? Almost, Except for infinities & NaNs

Floating Point in C

- C Guarantees Two Levels: float: single precision, double: double precision
- Conversions/Casting: Casting between int, float, and double changes bit representation, double/float → int: Truncates fractional part, Like rounding toward zero, Not defined when out of range or NaN: Generally sets to TMin, int → double: Exact conversion, has enough precision, int → float: Will round according to rounding mode

Summary

- IEEE Floating Point has clear mathematical properties
- Represents numbers of form M * 2^E
- One can reason about operations independent of implementation
- As if computed with perfect precision and then rounded
- Not the same as real arithmetic
- Violates associativity/distributivity
- Makes life difficult for compilers & serious numerical applications programmers

Lecture 3 - Machine-Level Programming I: Basics

Fri Jan 17 2014

Machine-Level Programming I: Basics

Intel x86 Processors

- Dominant architecture for laptop, desktop, and server computers
- Machine evolution: 1978: 8086, 1982: 80286, 1985: i386, 1989: i486, 1993: Pentium, 1997: Pentium MMX, 1999: Pentium III, 2000: Pentium 4, 2004: Pentium 4E, 2006: Core 2, 2008: Core i7, 2011: Core i7 2nd Gen, 2012: Core i7 3rd Gen
- Added features as time goes on
- Complex instruction set computer (CISC)
- Many different instructions with many different formats
- But, only subset encountered with Linux programs
- Hard to match performance of Reduced Instruction Set Computers (RISC)
- But, Intel has done just that!
- In terms of speed, Intel did amazing job!

Intel x86 Evolution: Milestones

- Name: Date, Transistors, MHz
- 8086: 1978, 29K, 5-10
- First 16-bit Intel processor. Basis for IBM PC & DOS
- 1MB address space
- 386: 1985, 275K, 16-33
- First 32 bit Intel processor, referred to as IA32
- Added "flat addressing", capable of running Unix
- Pentium 4E: 2004, 125M, 2800-3800
- First 64-bit Intel x86 processor, referred to as x86-64
- Core 2: 2006, 291M, 1060-3500
- First multi-core Intel processor
- Core i7: 2008, 731M, 1700-3900
- Four cores (our machines)

Intel x86 Processors: Overview

- Machine Evolution: 386: 1985, 0.3M; Pentium: 1993, 3.1M; Pentium/MMX: 1997, 4.5M; PentiumPro: 1995, 6.5M; Pentium III: 1999, 8.2M; Pentium 4: 2000, 42M; Core 2 Duo: 2006, 291M; Core i7: 2008, 731M
- Added Features: 386: flat addressing; MMX: added graphics instructions; PentiumPro: conditional move instructions; Pentium III: added more graphics instructions; Pentium 4: added more graphics instructions; Core 2: multi-core; Core i7: multi-core, hyper-threading, special security instructions
- Linux/GCC Evolution: Linux 1.0: 1994, 1.1M lines (1.0); Linux 1.2: 1995, 1.5M lines (1.0); Linux 2.0: 1996, 2.5M lines (2.0); Linux 2.2: 1999, 3.0M lines (2.0); Linux 2.4: 2001, 4.5M lines (2.0); Linux 2.6: 2003, 5.2M lines (2.0); GCC 1.0: 1987, 110K lines; GCC 2.0: 1992, 155K lines; GCC 2.95: 1999, 1.5M lines; GCC 3.0: 2001, 1.9M lines; GCC 4.0: 2005, 2.1M lines