Modeling, Simulation and Control of a Robotic Arm

Nafiseh Ebrahimi

***Abstract*— The precise control upon each degree of freedom of a robotic arm is a great challenge in implementing industrial work. To simplify mechatronics motion control systems design, this paper proposes mathematical modeling, simulation and control of a given electric motor in terms of input volt, Vin and output motions. The proposed model can be used to select, design, test and validate both plant's and motion control design to meet desired output performance. It presents a basic example of PID control applied to a robotic manipulator arm.**

I. Introduction

The term robotics is practically defined as the study, design and use of robot systems for manufacturing. Robots are generally used to perform unsafe, hazardous, highly repetitive, and unpleasant tasks [1]. Most used actuator in mechatronics motion control applications is DC motors. Despite a lot of resources that propose different selections and design of control strategies to control motions in desired fashion most control system used are based on convention PID controller [2].

II. Description of the System

A. System Dynamics

Most used actuator in mechatronics motion applications is DC motor, therefore, motion control can be simplified to DC motor motion control. PMDC motor is an example of electromechanical system, having both mechanical of electric components, a simplified equivalent representation the armature controlled PMDC motor's two components is shown in Fig.1.

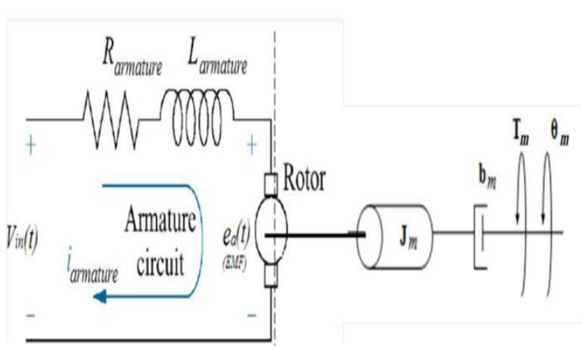


Fig.1. Equivalent representation of PMDC motor electromechanical component

mobile robot, and single joint robot arm, shown in Fig.2.

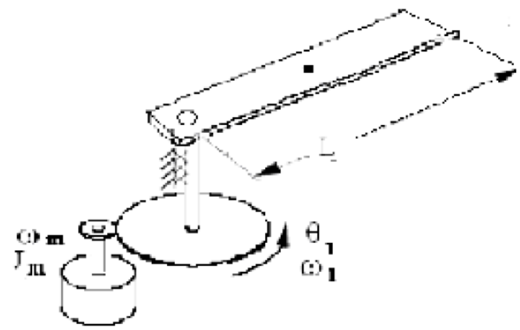


Fig.2 Single joint robot arm.

The motor torque is given by Eq (1). The generated EMF voltage, **e_a**, is given by Eq (2).

$$Tm = K_t i_a \quad (1)$$

$$e_a = K_b \omega_m = K_b \frac{d\theta_m}{dt} \quad (2)$$

Applying Kirchoff's law around the electrical loop, substituting, result in Eq(3) .Taking Laplace transform and rearranging, result in Eq(4):

$$L_a \frac{di}{dt} + R_a i = V - K_e \frac{d\theta_m}{dt} \quad (3)$$

$$I_a(s) = \frac{V(S) - K_e S \theta_m(s)}{R_a + L_a S} \quad (4)$$

Performing the energy balance; the sum of the torques must equal zero, yields Eq.(5), Taking Laplace transform and rearranging, result in Eq(6):

$$K_t * i - T_{Load} - J_m\left(\frac{d^2\theta}{dt^2}\right) - b_m\left(\frac{d\theta}{dt}\right) = 0 \quad (5)$$

$$K_t I(s) = (J_m s + b_m) s\,\theta(s) \quad (6)$$

substituting Eq.(4) in Eq.(6) and rearranging, to have the transfer function given by Eq.(7)in terms of the input voltage, V(s) and output motor shaft angle θ_m.

$$G_m = \frac{\Theta_m(s)}{V(s)} = \frac{K_t}{s[(R_a + L_m s)(J_a s + b_m)K_b K_{t]}} \quad (7)$$

the total equivalent inertia, J_{eq} and total equivalent damping, b_{eq} at the armature of the motor are given by:

$$b_{eq} = b_m + \frac{b_{load}}{n^2}, \qquad J_{eq} = J_m + \frac{J_{load}}{n^2}$$

Substituting J_{eq} and b_{eq} in Eq. (7), the total equivalent transfer function, relating input voltage V_{in} and Arm-load output angular position θ_{Load}, is given by Eq. (8). In this transfer function, gear ration (n) which is the transfer function of gear system is included.

$$G = \frac{\theta_{Load}(s)}{V_{in}(s)}$$

$$G = \frac{K_t * n}{L_a J_{eq} s^3 + (R_a J_{eq} + b_{eq} L_a) s^2 + (R_a b_{eq} + K_t K_b) s} \quad (8)$$

The robot arm system to be designed, has the following nominal values; arm mass, M= 8 Kg, arm length, L=0.4 m, and viscous damping constant, b = 0.09 N.sec/m. The following nominal values for the various parameters of eclectic motor used: Vin=12 Volts; Jm=0.02 kg·m²; bm =0.03; Kt =0.023 N-m/A; Kb =0.023 V-s/rad; Ra =1 Ohm; La=0.23 Henry; TLoad, gear ratio, for simplicity, n=1.

Substituting values of parameters into transfer function gives Eq. (9):

$$G = \frac{0.023}{0.02913\, S^3 + 0.1543\, S^2 + 0.1205\, S} \quad (9)$$

Regarding state space specification A, B, C and D matrix could be obtain as below Eq. (10):

$$\dot{X} = \begin{bmatrix} -5.2969 & -4.1366 & 0 \\ 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U \quad (10)$$

$$Y = [0 \quad 0 \quad 0.7896]\, X + [0] U$$

III. Open Loop Simulation

A. Stability

Root Locus diagram for mentioned open loop system is:

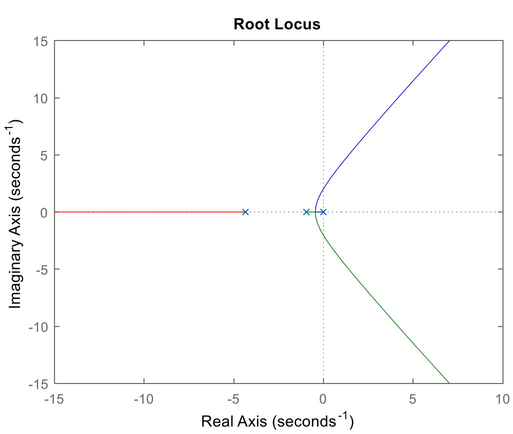


Fig.3. Root locus diagram

The loot locus is drawn for just open loop transfer function with gain K=1. However, if there is a variable gain (K) in the system for maintaining stability of the system K would have a range of K< 415.

On the other hand, stability can be considered investigating the eigenvalues of A (system state matrix). if the eigenvalues are positive, the system will not satisfy the condition of BIBO stability, and will therefore become *unstable*.

The eigenvalues of matrix A are: 0, -0.9521, -4.3449, therefore the output signal magnitude would not exceed a finite amount and it is stable.

B. System Performance Characteristics

Step response of open loop system to Vin= Step= 1V is:

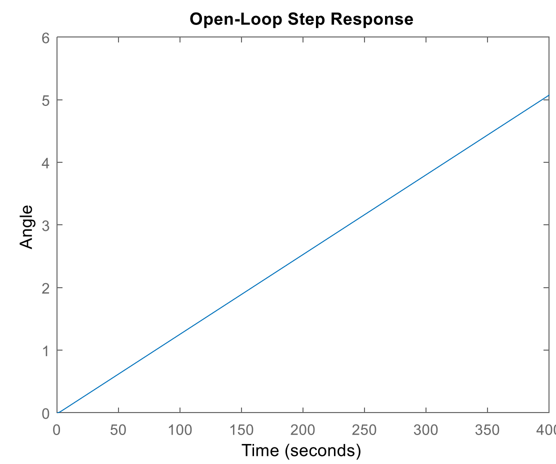


Fig.4. Open loop step respnse

It is obvious that this system with no feedback from output will continue to infinity. Using MATLAB, below system performance characteristics obtained (table 1).

Table1. Open loop system characteristics

Rise Time:	NaN
Settling Time	NaN
Settling Min	NaN
Settling Max	NaN
Overshoot	NaN
Undershoot	NaN
Peak	Inf
Peak Time	Inf

The system needs to be controlled with at least a feedback from output. Simple potentiometer could turn the feedback in the system. Therefore, we need to close the system.

IV. Closed Loop System (Part 1)

Closing the system by adding arm angular position sensor (potentiometer) gives closed loop transfer function Eq. (11):

$$T = \frac{0.023}{0.02913\,S^3 + 0.1543\,S^2 + 0.1205\,S + 0.001534} \qquad (11)$$

A negative closed loop feedback control system with forward controller is shown in Fig.5

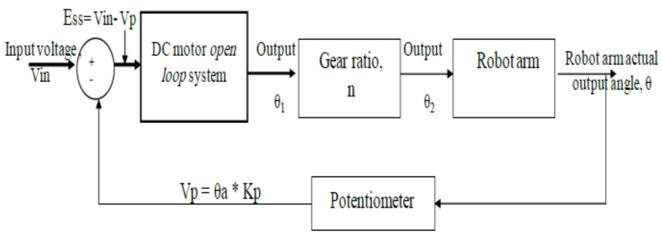


Fig.5. Negative feedback closed loop system block diagram

For the closed loop system K=12/180= 0.0667 is considered for feedback gain. Step response for closed loop system is depicted in Fig.6.

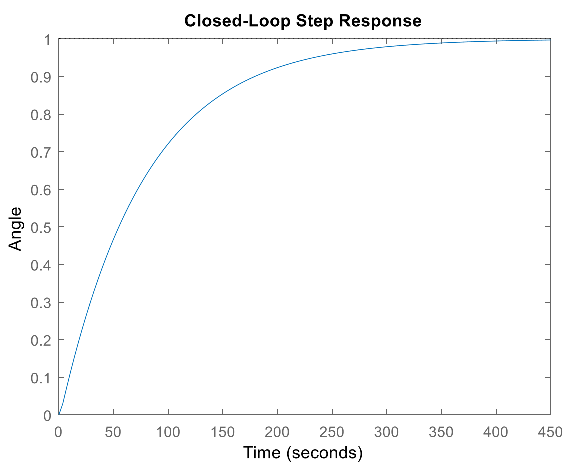


Fig.6. Negative feedback closed loop system step response

For the closed loop one, system performance characteristics are shown in table 2.

Table 2. Closed loop system characteristics

Rise Time:	169.82
Settling Time	303.68
Settling Min	13.54
Settling Max	14.99
Overshoot	0
Undershoot	0
Peak	14.99
Peak Time	565.92

A. *Design Specifications*

Our design goal is to design, model, simulate and analyze a control system so that a voltage input in the range of 0 to 12 volts corresponds linearly of a Robot arm output angle range of 0 to 180, that is to move the robot arm to the desired output angular position, θL, corresponding to the applied input voltage, Vin, with overshoot less than 5%, a settling time less than 2 second and zero steady state error. The error signal, e is the difference between the actual output robot arm position, θL, and desired output robot arm position. For 5% of overshoot we have:

$$\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.69 \qquad (12)$$

and for settling time of 2 seconds we have:

$$T_s = \frac{4}{\zeta\omega_n} = 2 \qquad (13)$$

Which yields

$\omega_n = 2.899$

For a second order system we have

$s^2 + 2\zeta\omega_n s + \omega_n{}^2 = 0 \Rightarrow$

$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

and the characteristics equation would be

$$s^2 + 4s + 8.4 = 0 \qquad (14)$$

but we do not want to use second order approximation for designing controller. Instead we are going to use PID command and PID Tuner of MATLAB which are far more powerful and precise in comparison to second order approximation.

For using PID command it is better to have an insight about the effect of different coefficients Kp, Ki, Kd on the system. The effects of each of controller parameters, Kp, Ki, and Kd on a closed-loop system are summarized in table 3.

Table 3. Effects of PID controller parameters

CL Response	Rise Time	Overshoot	Settling Time	S-S Error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	No change

Therefore, for the mentioned system which has settling time more than 300 seconds the most important coefficient would be Kd which decrease settling time. On the other hand, using Ki would worsen the condition of settling time. In terms of steady-state error, it is already zero and does not need any action. Similarly, overshoot is already zero and does not need to be considered.

We are going to investigate two types of controller on the closed loop system in the following.

B. P Controller

Using PID tuner MATLAB proposes Kp, Ki and Kd coefficients which are used in PID command to draw the step response of the controlled system.
We chose P type of controller in MATLAB, PID tuner and obtained following coefficient as first estimation:

Kp= 2.5927

Which yeilds settling time: 114.7 seconds and overshoot: 0% that setteling time is still too high and needs more modification. Step response for closed loop system with P controller is depicted in Fig.7.

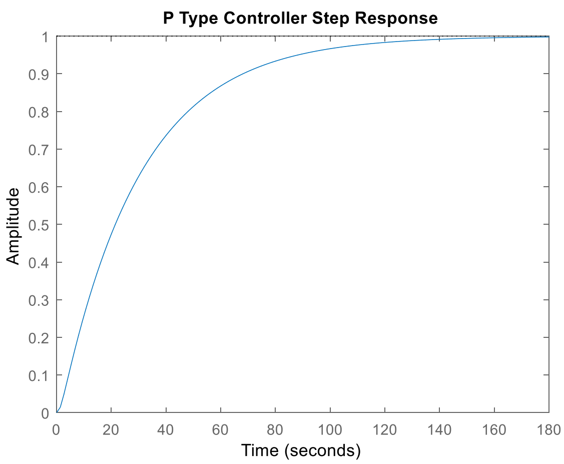


Fig.7. Closed loop system with P controller step response

System specifications for the controlled system is shown in table 4.

Table 4. P controlled system characteristics

Rise Time:	63.70
Settling Time	114.72
Settling Min	0.91
Settling Max	0.9993
Overshoot	0
Undershoot	0
Peak	0.9993
Peak Time	212.20

Trying larger Kp s like 100 and 1000 still does not reach desired specification. Then, we need to take into the picture other coefficients Ki and Kd. Ki is used for steady state error elimination while we do not have any. The main problem for the system is high settling time so as mentioned before we need to work with Kd to reduce this quantity. In next section we focus on PID controller design.

C. PID Controller

We chose PID type of controller in MATLAB, PID tuner and obtained following coefficient as first estimation:

Kp= 11.67
Ki= 2.19
Kd= 15.56

Which results Settling time: 64.58, overshoot: 39.56% that is not acceptable. Only by putting Ki=0 the results got better as setteling time: 26.59 sec, overshoot: 0 and steady state error:0 like previous time. After some manipulation on Kp and Kd coefficients Finally we chose the coeffients below as final controller:

Kp= 101
Ki= 0
Kd= 94

Step response for closed loop system with PID controller is depicted in Fig.8.

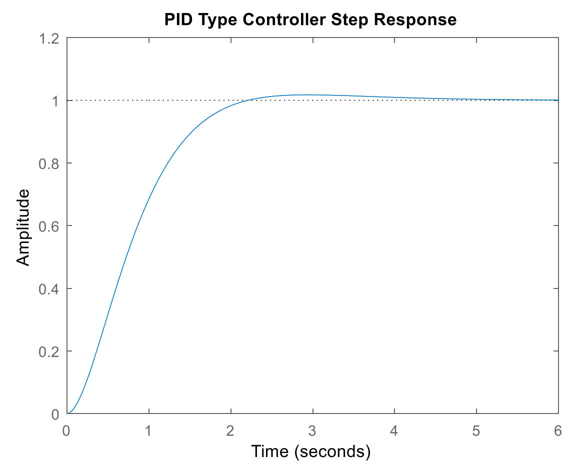


Fig.8. Closed loop system with PID controller step response

System specifications for the PID controlled system is listed in table 5.

Table 5. PID controlled system characteristics

Rise Time:	1.29
Settling Time	1.98
Settling Min	13.58
Settling Max	15.26
Overshoot	1.74
Undershoot	0
Peak	15.26
Peak Time	2.92

Steady state error is zero as well. Then all designing specifications are satisfied with final chosen Kp, Ki and Kd.

V. FULL STATE FEEDBACK CONTROL AND LINEAR QUADRATIC REGULATOR CONTROL

A. Pole Placement

pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane.[1] Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system. The system must be considered controllable in order to implement this method. This technique is widely used in systems with multiple inputs and multiple outputs, as in active suspension systems.
In order to implement pole placement method all state variable must be measurable. Moreover, system must be controllable. Using pole placement technique, we could get the closed loop poles to the desired location. Not only the "dominant poles", but "all poles" are forced to lie at specific desired locations. Our system state space model expressed in Eq. (10) is controllable because the rank of its controllability matrix is equal to the order of system (n). The scheme of full state feedback system is depicted in Fig.9.

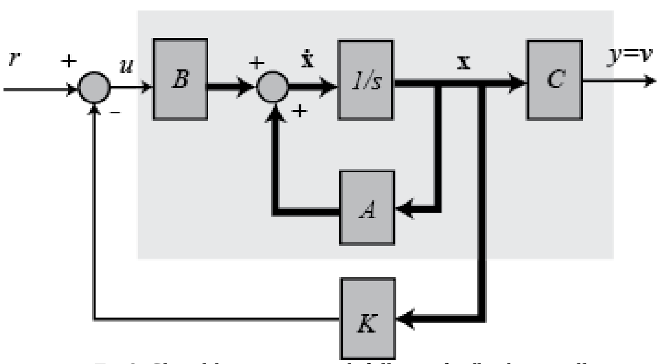


Fig.9. Closed-loop system with full state feedback controller

The control vector U is designed in the following state feedback form

$$U = -KX \qquad (15)$$

This leads to the following closed loop system

$$\dot{X} = (A - BK)X = A_{CL}X \qquad (16)$$

where

$$A_{CL} \equiv (A - BK) \qquad (17)$$

The gain matrix is designed in such a way that

$$|SI - (A - BK)| = (s - P_1)(s - P_2) \ldots (s - P_n) \qquad (18)$$

where $P_1, \ldots P_n$ are the desired pole locations.
Solving for K, the gain matrix K is obtained such that the state feedback control places the closed-loop poles at the loations of desired poles. That means the eigenvalues of *A* – *BK* are equal to the desired poles.
According to characteristics equation obtained from desired specification design (Eq.14) we have the following dominant poles:

$$P_{1,2} = -2.0 \pm 2.0976i$$

The third pole is chosen such that its magnitude be more than three times of the dominant poles. Fig.10 shows step response of the system after applying pole placement method.

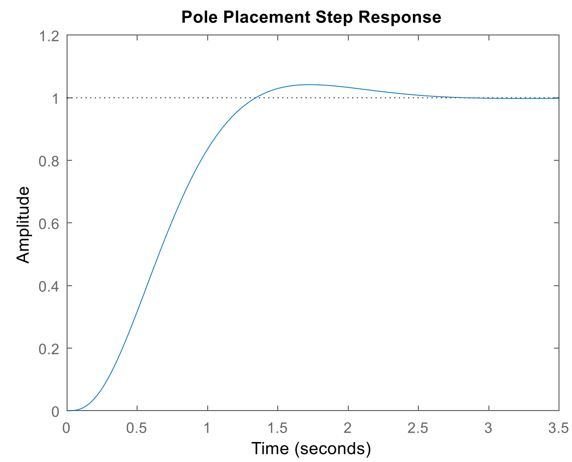


Fig.10. Full State Feedback Control step response

System specifications for the controlled system is listed in table 6.

Table 6. Full state feedback controlled system characteristics

Rise Time:	0.81
Settling Time	2.24
Settling Min	0.90
Settling Max	1.04
Overshoot	4.19
Undershoot	0
Peak	1.04
Peak Time	1.73

B. Linear quadratic regulator (LQR)

The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear–quadratic regulator (LQR), a feedback controller whose equations are given below. The LQR is an important part of the solution to the LQG (linear–quadratic–Gaussian) problem
For a time-continuous system, the state-feedback law $u = -Kx$ minimizes the quadratic cost function J(u).

$$J(u) = \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (19)$$

Subject to $\dot{X} = Ax + Bu$.

The goal is to design an optimal feedback K using the Linear Quadratic Regulator (LQR) problem, whose parameters are the state matrix A, input matrix B, state weighting matrix Q, and input weighting matrix R. Q is a symmetric positive semi-definite matrix and R is a symmetric positive-definite matrix. Matrices Q and R imposes constraints on states and control input, respectively.

The state feedback control gain matrix K is obtained as

$$K = R^{-1} B P \quad (20)$$

Where matrix P satisfies the following reduced matrix Riccati:

$$A^T P + PA - PBR^{-1} BP + Q = 0 \quad (21)$$

The weighting matrices are specified such that the closed loop system is able to track the reference signal with a control signal that does not significant violates the saturated actuator limits. The minimum value of J in Eq. (19) is obtained as

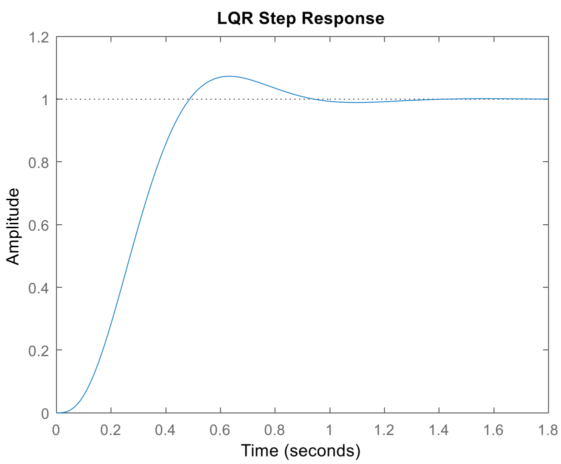
$$J_{min} = x^T(0) P x(0)$$

For our simulation we assumed Q and R as follows to design optimal control.

$$Q = 10^4 * C * C^T$$

$$R = 10^{-4}$$

In Fig.11 the step response of the system after applying LQR technique is displayed. We manipulated R to obtain acceptable system characteristics from the optimization.



System characteristics after implementation of LQR is shown in table 7.

Table 7. LQR controlled system characteristics

Rise Time:	0.29
Settling Time	2.85
Settling Min	0.90
Settling Max	1.07
Overshoot	7.29
Undershoot	0
Peak	1.07
Peak Time	0.63

VI. CONCLUSION

Our first approach to control the robotic manipulator arm is to derive a mathematical model for the system using physical governing equations on system. After obtaining the state space model and block diagram for the system, it is illustrated that this system is stable.

Using feedback to make a closed loop system and getting a preliminary insight from this system step response we designed several controllers to control the robotic arm system to meet system requirements. In order to meet the design specifications of the response which were 5% overshoot, settling time of 2 seconds, and zero steady-state error to step input, PD controller, and PID controller were designed using root locus and their performance was compared. Next, full state feedback control was implemented by assigning desired closed-loop poles. Finally, linear quadratic regulator was implemented to minimize the performance index.

Table 8 illustrates the characteristic of desired, open loop and all designed controller. System specifications for all type of applied controllers are illustrated in table 8. It can be inferred that best performance is for PID controller since it satisfies all of the desired specification the best and decries settling time significantly.

Moreover, PP (pole placement) controller satisfies the condition though its overshoot is more than PID and introduce a trivial amount of steady state error to the system as well. Also, LQR controller's overshoot exceeds the desired amount and this is unacceptable. In addition, P controller due to its unsatisfactory settling time is not acceptable.

Table 8. Comparison of system characteristics for different controllers

	OS%	T_s	e_{ss}
Desired system	5	2	0
Closed loop	0	303.68	0
P Controller	0	114.73	0
PID Controller	1.74	1.98	0
PP	4.19	2.34	$1.3\text{x}10^{-15}$
LQR	7.3	0.85	$1.11\text{x}10^{-16}$

VII. ACKNOWLEDGMENT

I would like to take this opportunity to express my profound gratitude and deep regard to Dr. Bing Dong for his guidance, valuable feedback and constant encouragement throughout the duration of the project. His valuable suggestions were of immense help throughout my project work.

VIII. REFERENCES

[1] G. O. Young, “Synthetic structure of industrial plastics (Book style with paper title and editor),” in Plastics, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 15–64.

[2] W.-K. Chen, “Linear Networks and Systems” (Book style). Belmont, CA: Wadsworth, 1993, pp. 123–135.

[3] R. Agrawal, “Modeling a Controller for an Articulated Robotic Arm”, Intelligent Control and Automation, 2012, 3, pp. 207-210

[4] F. A. Salem, “Modeling, Simulation and Control Issues for a Robot ARM; Education and Research”, I.J. Intelligent Systems and Applications, 2014, 04, pp.26-39

[5] T. A. Salih, O. I. Yehea, “A Novel Control System for Robotics Devices,” Journal of Theoretical & Applied Information Technology, Vol. 28, No. 1, 2011, pp. 48-53.

[6] M. Ruderman, J. Krettek, F. Hoffmann, T. Bertram, “Optimal State Space Control of DC Motor”, Proceedings of the 17th World Congress, The International Federation of Automatic Control Seoul, Korea, July 2008.