A Multi-Level Graded Thermal Spreader Under an Equal-Mass Constraint

A Numerical Study on Thickness Redistribution Near the Heat Source for Peak-Temperature Reduction and Degradation Resistance

Note: A patent application related to the concept presented in this work has been filed.

Abstract. This paper presents a numerical concept for a multi-level graded thermal spreader that redistributes thickness under an equal-mass constraint, so that thermally effective mass is concentrated near the heat source and gradually reduced toward the periphery. The final validated dataset consists of six benchmark cases defined by three base thicknesses (1.0, 0.5, and 0.4 mm) and two representative conductivities (k = 400 and k = 235 W/m·K), while the final tuning parameters are fixed at levels = 3, beta = 0.55, shrink = 1.0, t_boost = 8.0, radial_gamma = 1.0, directional_gamma = 0.25, gamma_hot = 0.2, and r0 = 0.01. Positive improvement was obtained in all six cases. The relative benefit increases as the base thickness decreases, and the lower-conductivity material shows a larger relative gain than the copper-like material. A controlled support comparison further shows that a cross-root shape alone provides only a partial improvement, whereas the larger performance gain arises from graded redistribution applied over the same shape. Within the scope of a two-dimensional equal-mass numerical model, the results provide numerical evidence for the concept and indicate potential relevance for thickness-constrained thermal applications.

Keywords: thermal spreader, multi-level grading, equal-mass constraint, lower-conductivity materials, degradation resistance, heat dissipation, hotspot.

1. Introduction and motivation

Thermal solutions for compact devices often rely on uniform plates, conventional spreaders, or global thickness increases whenever packaging allows. Such approaches are useful, but they do not necessarily use limited thermal mass efficiently when the total thickness is constrained. In thin platforms, any local increase in effective conductive area near the heat source can be more valuable than a homogeneous redistribution of material across the entire plate.

The central hypothesis of this study is that redistributing the same available thermal mass in a graded manner near the source can provide a clearer benefit than uniform allocation, particularly in thin devices and when using materials with lower conductivity than copper. Accordingly, the present work is not framed as a purely geometric proposal, but as a thermal design principle based on intelligent mass allocation under the same global mass budget.

In this study, the cross-root geometry is not claimed as the sole contribution. Rather, it is used as the best-performing numerical reference embodiment within the present scope. The core contribution is the principle of graded redistribution of thickness and effective conductance under a fixed total-mass constraint.

Related prior work has explored adjacent but distinct directions. Maranzana et al. studied the optimization of a uniform heat-spreader thickness, rather than a multi-level redistribution of thickness under fixed total mass. Topology-optimization and SIMP-based studies commonly optimize material layout or conductive paths within a design domain, rather than graded thickness allocation within a single homogeneous spreader. Other work on pyramidal compound substrates addresses layered or multi-material configurations, while mass-graded thermal-interface studies operate at a nanoscale interfacial-transport regime rather than at the scale of a macroscopic solid heat spreader. The present concept is therefore positioned as a single-material, multi-level graded thermal-spreader concept under a fixed mass or volume constraint, rather than as a uniform-thickness optimization, a pure material-layout optimization, a multilayer pyramidal substrate, or a nanoscale graded interface.

Scope of the study: this work is a two-dimensional numerical investigation of a multi-level graded thermal-spreader concept under an equal-mass constraint. It is not presented as a final experimental validation of a finished industrial product. Instead, it aims to test the physical soundness of the concept numerically and to measure its relative impact against a uniform baseline under the same total mass and the same external dimensions.

2. Physical basis and analytical reasoning

The paper starts from a direct physical argument: conductive thermal resistance decreases as the effective cross-sectional area near the heat source increases. Accordingly, increasing effective thickness or effective local mass near the center of heat generation gives the first spreading paths a greater ability to pick up heat early and distribute it before the thermal peak accumulates. Farther from the source, thickness can be reduced gradually without wasting mass in regions of lower thermal sensitivity.

The governing design logic is summarized by the following expressions:

$$q = -k\,A(x,y)\,\nabla T$$

$$R\_cond \sim L / (kA)$$

$$M\_target = constant, \quad t\_norm = t\_raw \times M\_target / \Sigma(t\_raw)$$

$$boost(level) = 1 + (t\_boost - 1) \times \beta\hat{}level$$

$$Improvement(\%) = (Baseline - Case) / Baseline \times 100$$

These expressions mean that the study does not increase the total thermal mass relative to the uniform case; it reallocates the same mass to thermally more important regions. This is the basis of fairness in the comparison between the uniform and graded configurations.

From a physical standpoint, increasing the effective area or thickness near the heat source reduces local conduction resistance in the earliest stages of heat spreading. This helps reduce early heat accumulation in the hotspot region. When the same total mass is redistributed gradually instead of spread uniformly over the entire plate, larger reductions in the thermal peak can be achieved without increasing total mass. Accordingly, the observed benefit in the numerical results is not a benefit of “more material”; it is a benefit of better thermal allocation under the same mass constraint.

3. Numerical model and implementation

The numerical model is a two-dimensional conduction solver based on a finite-volume / finite-difference formulation solved with sparse linear algebra. The external plate dimensions are 40 × 28 mm, the heat source is a centered 6 × 6 mm chip region, and the total heat input is 10 W.

Convective heat loss is imposed on both large faces through h_face = 100 $W/m^2 \cdot K$ and on the outer edges through h_edge = 8 $W/m^2 \cdot K$. The value h_face = 100 is not intended to represent only a single still-air scenario; instead, it is used as a fixed reference boundary condition representing relatively effective passive cooling or an enhanced equivalent surface heat rejection condition. Keeping the same value for all cases preserves fairness in the relative comparison between the uniform and graded configurations.

The final validated cases use a 300 × 210 grid. The key implementation routines include make_branch_level_map_symmetric for constructing the level map, build_thickness_map_equal_mass_graded_levels for generating the graded thickness field with total-mass normalization, build_cross_uniform_thickness_equal_mass for the controlled geometry-only support case, solve_T_sparse_full for solving the thermal field, and energy_balance_fvm for verifying energy balance.

4. Final fixed parameter set

For the six benchmark cases, all numerical and geometric parameters were fixed at the values listed in Table 1, while only the base thickness and the material conductivity were varied. This makes the final comparison easier to read and ensures that the observed changes arise from thickness and material effects rather than from shifting tuning parameters.

Parameter	Value
Number of levels, levels	3
Inter-level decay factor, beta	0.55
Geometric shrink factor, shrink	1.0
Thickness boost factor, t_boost	8.0
Radial weighting factor, radial_gamma	1.0
Directional weighting factor, directional_gamma	0.25
Hotspot weighting factor, gamma_hot	0.2
Hotspot decay length, r0	0.01 m
Plate dimensions	40 × 28 mm
Heat-source dimensions	6 × 6 mm
Total heat input	10 W
Ambient temperature, T_inf	25 °C
Face convection coefficient, h_face	100 $W/m^2 \cdot K$
Edge convection coefficient, h_edge	8 $W/m^2 \cdot K$
Numerical grid resolution	300 × 210
Central root width, trunk_width_m	6.2 mm
Branch width, branch_width_m	3.5 mm
Materials studied	k = 400 and k = 235 W/m·K
Base thickness values	1.0, 0.5, and 0.4 mm

Table 1. Final fixed parameter values used in the six validated benchmark cases.

5. Reference geometry and thickness distribution

The study adopts a graded cross-root multi-level geometry as its reference embodiment. This geometry is not the thesis of the paper by itself; it is the numerical platform through which the graded redistribution principle is demonstrated. All comparisons are conducted under the same external dimensions, the same total heat input, and the same total mass after normalization.

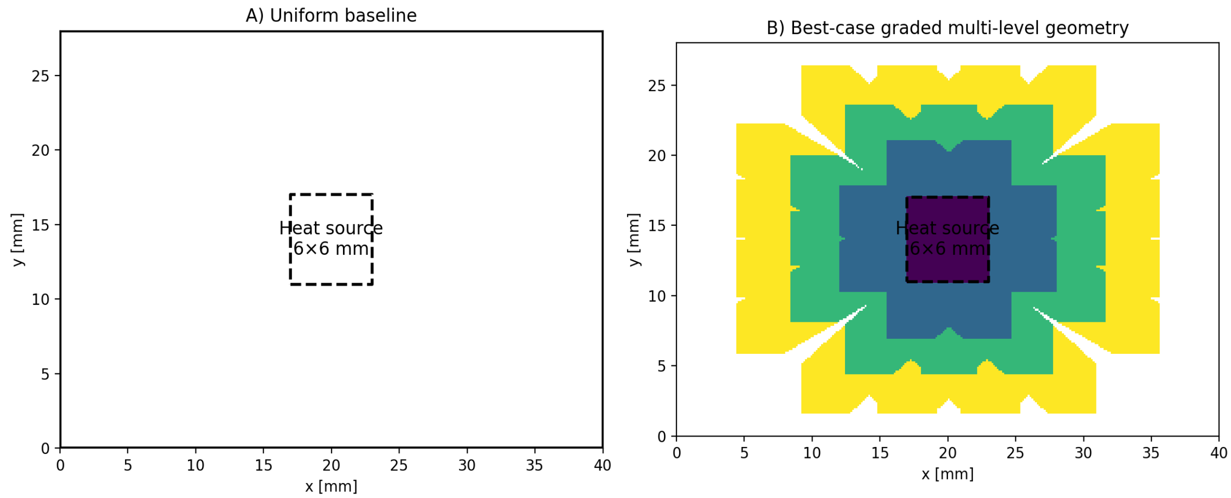

Figure 1. Schematic comparison between the uniform baseline plate and the graded cross-root multi-level reference geometry.

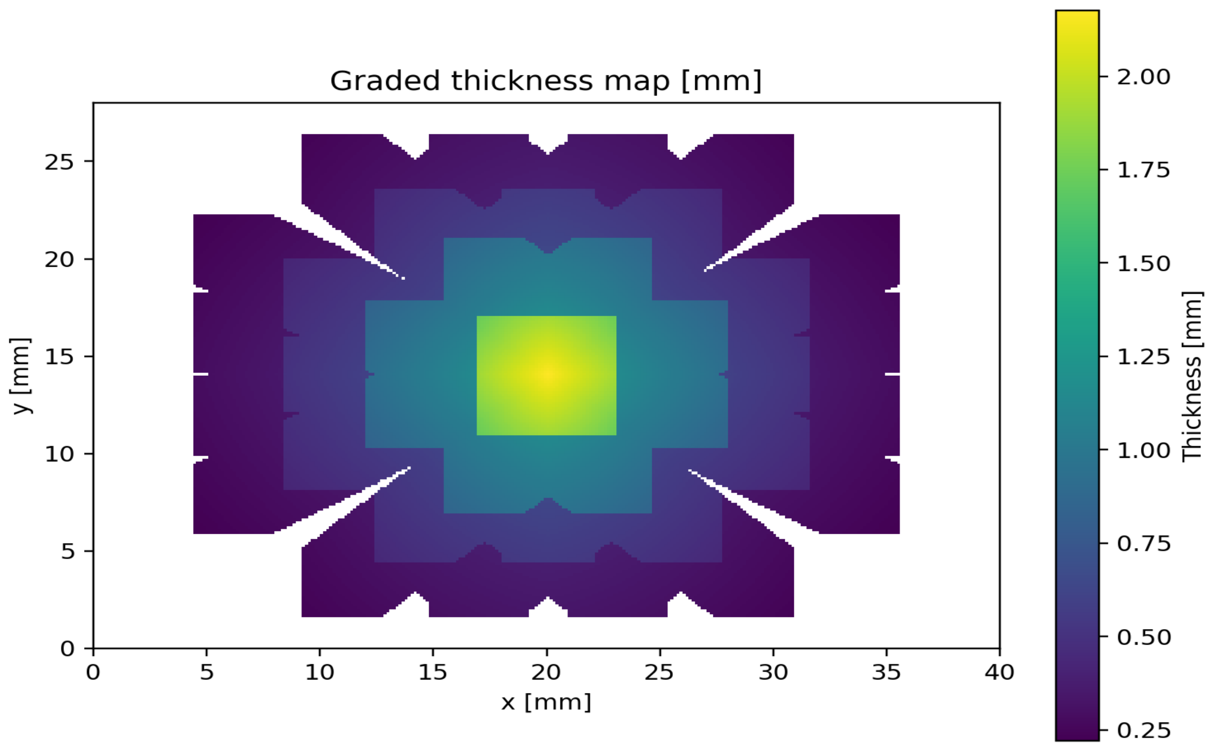

Figure 2. Example graded thickness distribution, showing locally elevated thickness near the heat source and a gradual reduction toward the periphery.

6. Final validated six-case results

The six final validated cases provide the cleanest summary of the study. Positive improvement is obtained in all cases, showing that the concept is neither material-specific nor dependent on a single thickness condition. More importantly, the relative improvement rises clearly as the base thickness is reduced from 1.0 mm to 0.5 mm and then to 0.4 mm, which indicates meaningful degradation resistance under thickness reduction.

Base thickness (mm)	k (W/m·K)	ΔTmax improvement (%)	Rth,avg improvement (%)	Energy-balance error
1.0	400	5.386	3.916	1.5961×10^{-9} %
1.0	235	8.676	6.450	1.0672×10^{-9} %
0.5	400	10.043	7.554	4.7608×10^{-10} %
0.5	235	15.000	11.524	1.5852×10^{-10} %
0.4	400	11.986	9.098	7.3168×10^{-11} %
0.4	235	17.460	13.553	4.2119×10^{-10} %

Table 2. Final validated six-case benchmark results under the fixed parameter set.

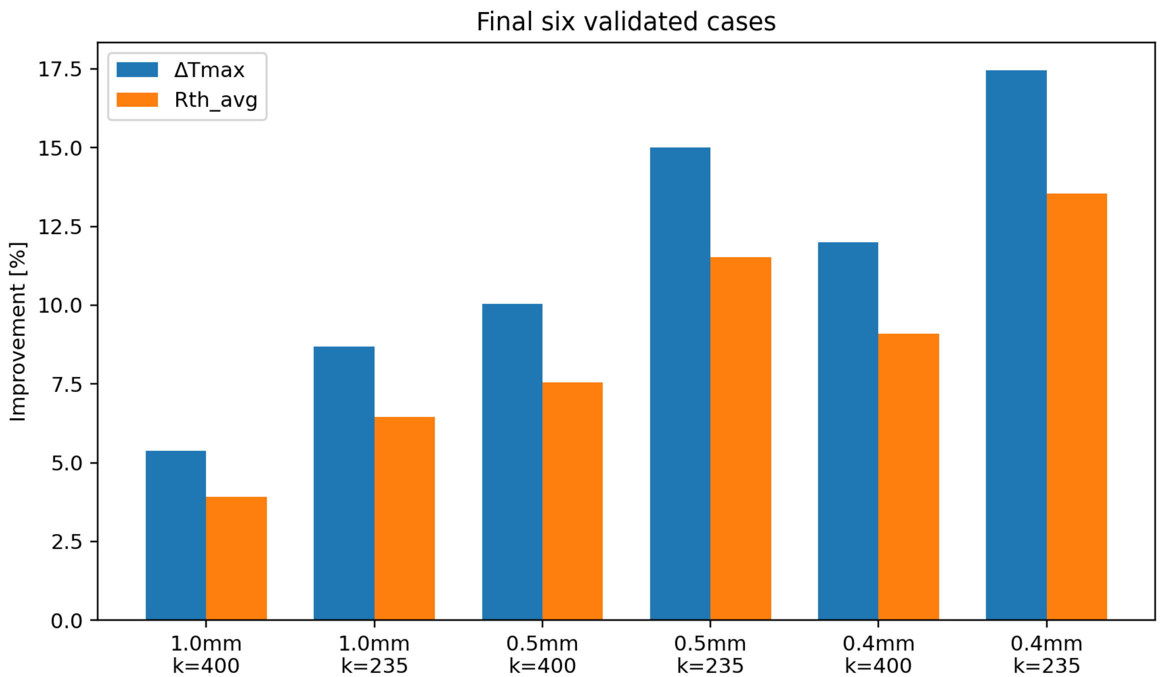

Figure 3. Summary of the six validated cases, showing improvement in both peak-temperature reduction (ΔTmax) and average thermal resistance (Rth,avg).

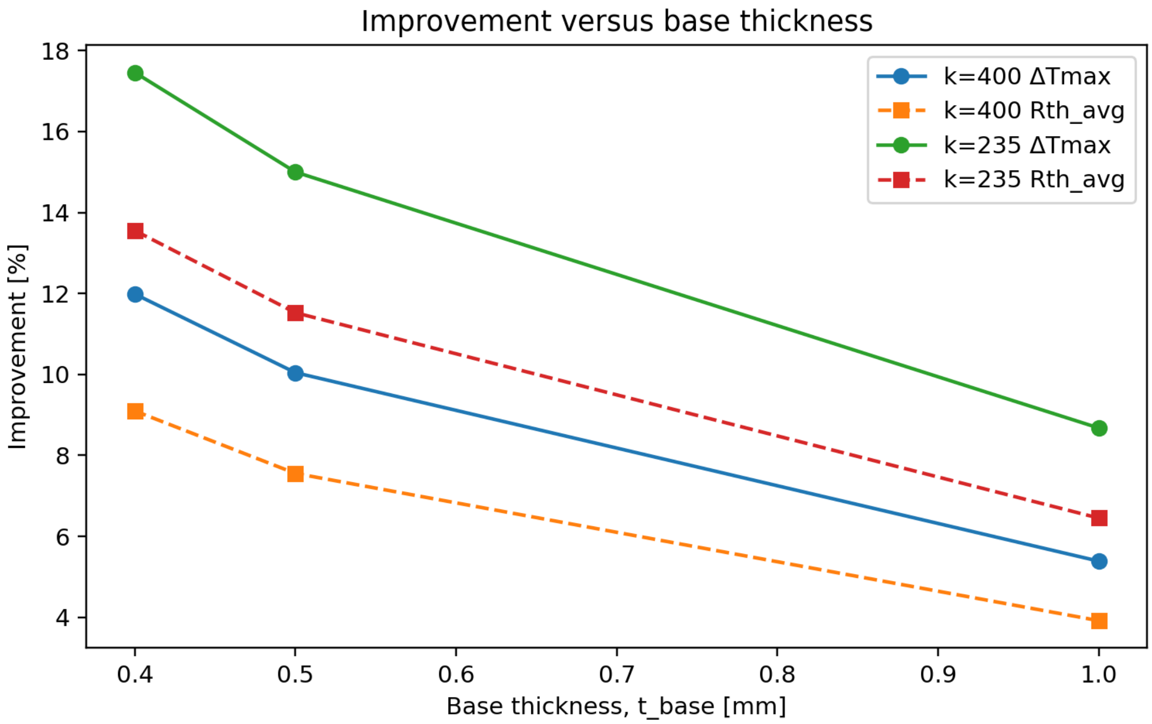

Figure 4. Relative improvement versus base thickness for both conductivity values.

7. Relevance to thin devices

One of the most important findings is that the relative benefit grows as the base thickness decreases. This does not mean that the final product must become externally thicker at the source location. Instead, it means that when the available thermal mass is limited, intelligent redistribution becomes more valuable than uniform allocation. In other words, the concept is especially relevant to thin-device conditions because it aims to reallocate the same available mass or effective conductance within the existing thickness envelope rather than simply adding more material.

8. Effectiveness in lower-conductivity materials

The lower-conductivity material (k = 235 W/m·K) achieved a larger relative improvement than the copper-like material (k = 400 W/m·K) at all three base thicknesses. This suggests industrial relevance because it implies that the concept is not only useful when the material is already highly conductive. Instead, its relative value can become even larger when conductivity is lower and the intelligent placement of limited thermal mass matters more.

At 1.0 mm, the improvement was 5.386% for k = 400, compared with 8.676% for k = 235. At 0.4 mm, the improvement rose to 11.986% for k = 400, compared with 17.460% for k = 235. This indicates that the concept is not merely a small refinement on top of an already excellent material; it can be even more valuable when the material is less conductive.

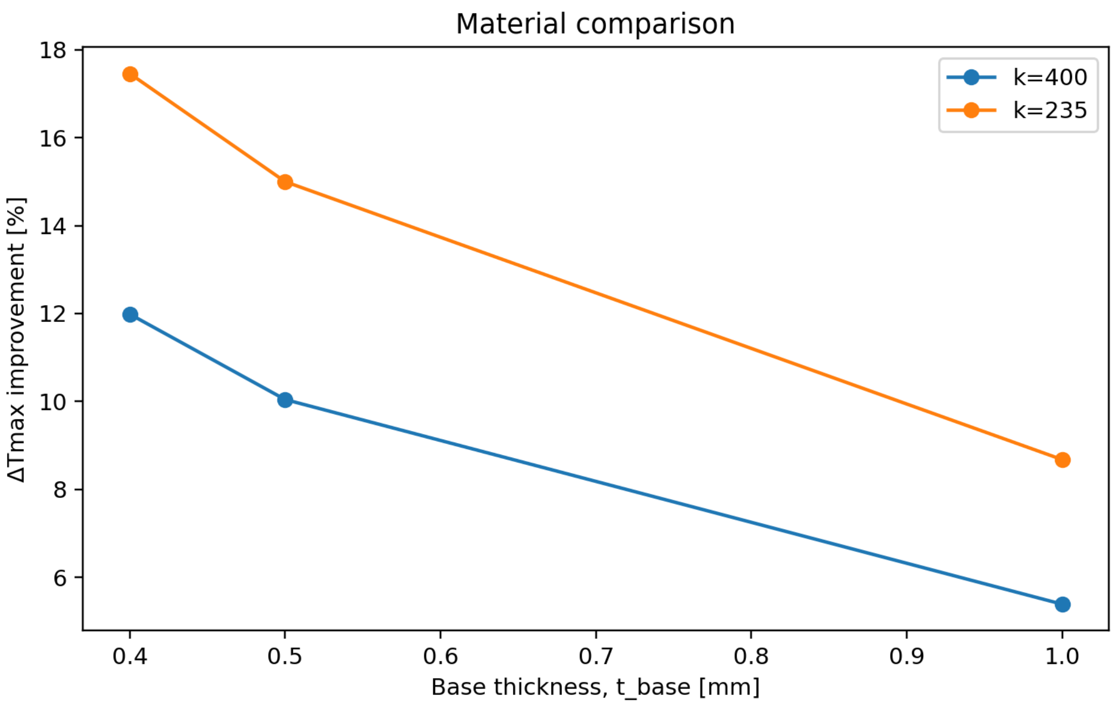

Figure 5. Material comparison showing that the lower-conductivity case (k = 235 W/m·K) exhibits a larger relative ΔTmax improvement across all three thicknesses.

9. Support comparison: shape effect versus graded redistribution

To separate the effect of geometry from the effect of grading, a controlled support comparison was performed for the most demanding benchmark condition: base thickness = 0.4 mm and k = 235 W/m·K. Three cases were compared: a uniform baseline, a cross-root geometry with uniform thickness along the active path, and the same cross-root geometry with graded multi-level redistribution.

The results show that the cross-root shape alone produces only a partial improvement. The graded version of the same shape performs substantially better, which demonstrates that the dominant benefit is not explained by shape alone; it arises from graded redistribution applied over that shape.

Case	dTmax (K)	Rth,avg (K/W)	ΔTmax improvement (%)
Baseline	69.352443	6.523530	–
Cross uniform	64.951689	6.164922	6.345
Cross graded	57.243642	5.639383	17.460

Table 3. Controlled support comparison used to isolate the effect of shape from the effect of graded redistribution for the 0.4 mm, k = 235 W/m·K benchmark.

In this controlled comparison, the cross-root shape alone reduced ΔTmax by 6.345%, whereas the graded cross-root configuration achieved a 17.460% reduction. Therefore, the extra gain contributed by grading over the same shape was 11.867% in ΔTmax, which strongly indicates that the dominant improvement cannot be attributed to shape alone.

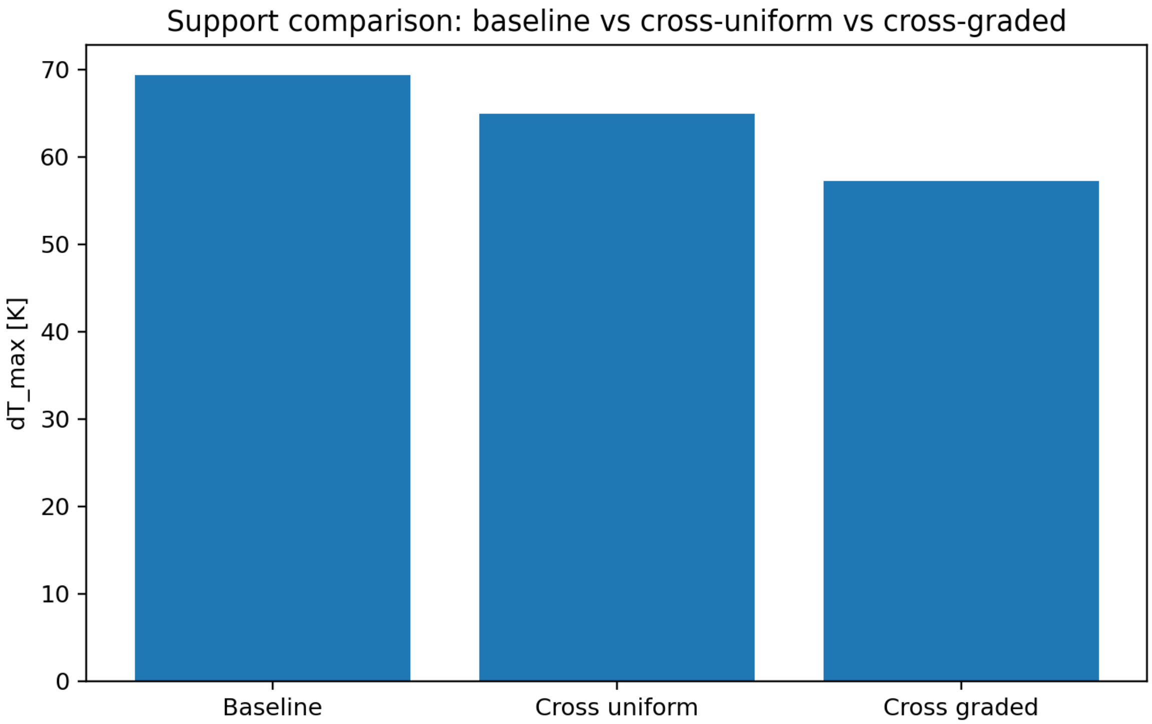

Figure 6. Support comparison between the baseline, cross-uniform, and cross-graded cases.

10. Industrial relevance

The final results do not imply that an industrial product must always adopt a rigid cross-shaped planform. Rather, they show that redistributing limited thermal mass near the heat source is particularly useful when the total thickness is constrained. This makes the concept promising for thin devices, since the relative benefit was shown to increase as base thickness decreased.

The better relative performance at k = 235 W/m·K also suggests that the concept may be valuable for lower-conductivity material systems, not only for copper-like ones. Moreover, the “thicker center” used in the model should not be interpreted narrowly as an unavoidable increase in external product thickness. In industrial terms, it can be understood as a reallocation of mass or effective conductance within the available envelope, potentially through internal graded structures, layered configurations, or other manufacturable equivalents.

11. Reproducibility and the role of code

The computational scripts are not merely implementation details; they are part of the demonstration itself. They generate the level maps, create equal-mass graded thickness fields, solve the thermal field, verify energy balance, and isolate the effect of shape from the effect of grading. At the same time, it is not necessary to embed the full code base in the body of the paper. A better balance is to identify the main routines, state the fixed benchmark parameters clearly, and provide the implementation as supplementary material or upon request.

12. Conclusions

- The graded multi-level concept achieved positive improvement in all six validated cases, confirming that the benefit is not restricted to a single thickness or a single conductivity value.

- The relative improvement increased systematically as the base thickness decreased from 1.0 mm to 0.4 mm, supporting the interpretation of degradation resistance and particular relevance to thickness-constrained platforms.
- The lower-conductivity material (k = 235 W/m·K) outperformed the copper-like material (k = 400 W/m·K) in relative improvement at all three thicknesses, indicating that the concept may be especially valuable when conductivity is lower.
- The controlled support comparison showed that a cross-root shape alone is not sufficient to explain the full benefit; graded redistribution added an extra 11.867% ΔTmax gain beyond the shape-only case.
- The governing equations, equal-mass normalization, sparse numerical solution, and very small energy-balance errors all support the conclusion that the results are physically and numerically consistent rather than accidental.
- Within the scope of a two-dimensional equal-mass study, the present work therefore provides numerical evidence of a multi-level graded thermal spreader concept and indicates potential relevance for extension to three-dimensional and experimental validation.

13. Limitations and future work

This study is numerical and conceptual in nature. It does not claim to be a final experimental validation of a ready-made industrial product. Nevertheless, the strength and consistency of the results make it a suitable foundation for future steps, including higher-fidelity three-dimensional analyses, the introduction of TIM layers and heterogeneous interfaces, sensitivity studies for additional boundary conditions, prototype fabrication and laboratory testing, and application-specific variants for smartphones, power modules, and other compact electronics.

References

1. Incropera, F. P., DeWitt, D. P., Bergman, T. L., and Lavine, A. S., Fundamentals of Heat and Mass Transfer, 8th ed., Wiley, 2020.
2. Holman, J. P., Heat Transfer, 10th ed., McGraw-Hill Education, 2009.
3. Patankar, S. V., Numerical Heat Transfer and Fluid Flow, CRC Press, 1980.
4. Versteeg, H. K., and Malalasekera, W., An Introduction to Computational Fluid Dynamics: The Finite Volume Method, 2nd ed., Pearson, 2007.
5. Yovanovich, M. M., and Marotta, E. E., “Thermal Spreading and Contact Resistances,” in Heat Transfer Handbook, Wiley, 2003, pp. 261–393.
6. Muzychka, Y. S., Yovanovich, M. M., and Culham, J. R., “Thermal Spreading Resistance in Compound and Orthotropic Systems,” Journal of Thermophysics and Heat Transfer, Vol. 18, No. 1, 2004.
7. Zhang, Z., Wang, X., and Yan, Y., “A review of the state-of-the-art in electronic cooling,” e-Prime – Advances in Electrical Engineering, Electronics and Energy, Vol. 1, 100009, 2021.
8. van Roekeghem, A., Carrete, J., Mingo, N., and Torres, S., "Thermal resistance of graded interfaces," Physical Review Applied, Vol. 11, No. 3, 2019, 034036. https://doi.org/10.1103/PhysRevApplied.11.034036
9. Ma, D., Ding, D., and Shiomi, J., "Facilitating thermal transport across Si/Ge interface via mass-graded interlayers: A molecular dynamics study," Journal of Applied Physics, Vol. 129, No. 17, 2021, 175302. https://doi.org/10.1063/5.0049793

10. Ghioldi, F., Hélie, J., and Piscaglia, F., "Multivariable optimization of pyramidal compound substrates for cooling of power-electronics in modern hybrid and electric propulsion systems," Applied Thermal Engineering, Vol. 233, 2023, 121368. https://doi.org/10.1016/j.applthermaleng.2023.121368

11. Tang, Z. L., Li, H., Zhao, D., et al., "Topology optimization for near-junction thermal spreading of electronics in ballistic-diffusive regime," Heliyon, Vol. 9, No. 7, 2023, e17984.

12. Maranzana, G., Perry, I., Maillet, D., and Raël, S., "Design optimization of a spreader heat sink for power electronics," International Journal of Thermal Sciences, Vol. 43, No. 1, 2004, pp. 21–29. https://doi.org/10.1016/S1290-0729(03)00107-8

13. Lim, H., and Lee, J., “Flat plate two-phase heat spreader on the thermal management of high-power electronics: a review,” Journal of Mechanical Science and Technology, Vol. 35, 2021, pp. 4801–4814.