# Regarding Transmission Loss when a Damping System is Installed

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## Abstract

While transmission loss has been studied in the past, understanding the properties of transmission loss when an attenuation system is attached to a flat plate is considered necessary in fields such as automobiles, where glass plates are attached to the vehicle body with rubber products. Therefore, we discussed the transmission loss of a flat plate when a damping system is attached to it. As a result, we found that the transmission loss was greatest across all frequencies when an infinite flat plate was attached to it.

## 1. Introduction

In the past, research on transmission loss has been conducted in fields such as architecture (Bies & Davies, 1977) (Halliwell & Warnock, 1985) (Kin, Z., X., & Ping, 2005) (Moore & Lyon, 1991) (Oba & Finette, 2002) (Priya & Sun, 1999) (Qibo, 2020) (Robin & Berry, 2018) (Santoni, Davy, Fausti, & Bonfiglio, 2020) (Tan & Sin, 2018). However, to the best of the author's knowledge, there are no studies that discuss the properties of transmission loss when a damping system is added. Understanding the properties of transmission loss when a damping system is attached to a flat plate is considered necessary in fields such as automobiles, where glass plates are attached to the vehicle body with rubber products.

Therefore, we will consider the transmission loss of a flat plate when a damping system is attached to it.

This will be discussed below.

## 2. Equation of Motion

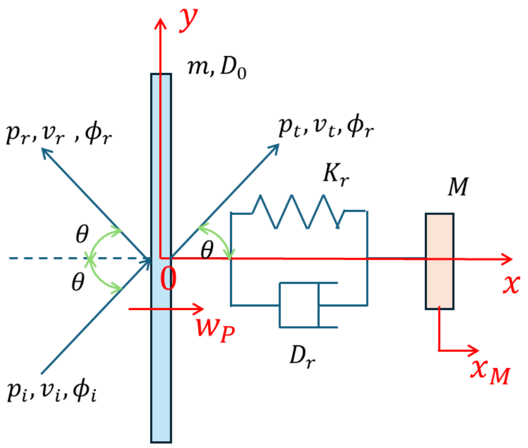

Fig. 1 Infinite flat plate with a damping system

Let us consider the transmission loss when a damping system is attached to an infinite flat plate, as shown in Fig. 1. Here, let $m$ be the mass per unit area of the flat plate, $D_0$ be the bending stiffness of the infinite flat plate, $M$ be the mass of the damping system, $K_r$ be the spring constant of the damping system, and $D_r$ be the damping coefficient of the damper in the damping system. Also, let $p_i$ be the sound pressure of the incident wave, $v_i$ be the particle velocity, $\phi_i$ be the velocity potential, and $\theta$ be the angle of incidence. Similarly, let $p_r$ be the sound pressure of the reflected wave, $v_r$ be the particle velocity, $\phi_r$ be the velocity potential, and $p_t$ be the sound pressure of the transmitted wave, $v_t$ be the particle velocity, and $\phi_t$ be the velocity potential. Let $w_P$ be the displacement of the infinite flat plate, and $x_M$ be the displacement of the mass of the damping system. Also, let $c$ be the speed of sound in air, and $\rho$ be the density of air. Finally, let $p_0$ be the amplitude of the sound pressure of the incident wave, $\omega$ be the angular frequency, $k = \frac{\omega}{c}$ be the wave number, $A_i$ be the amplitude of the velocity potential of the incident wave, $A_r$ be the amplitude of the velocity potential of the reflected wave, and $A_t$ be the amplitude of the velocity potential of the transmitted wave.

Then, the condition for the continuity of velocity at $x = y = 0$ is given by the following:

$$v_{ix} + v_{rx} = \frac{\partial w_P}{\partial t} = v_{tx} \tag{1}$$

Next, the condition for the continuity of the speed of sound is given by the following equation.

$$m\frac{\partial^2 w_P}{\partial t^2}+D_0\frac{\partial^4 w_P}{\partial y^4}+D_r\left(\frac{\partial w_P}{\partial t}-\frac{dx_M}{dt}\right)+K_r(w_P-x_M)=p_i+p_r-p_t \tag{2}$$

Furthermore, the equation of motion for the damped system is given by the following equation:

$$M\frac{d^2 x_M}{dt^2}+D_r\left(\frac{dx_M}{dt}-\frac{\partial w_P}{\partial t}\right)+K_r(x_M-w_P)=0 \tag{3}$$

Finally, the following equations hold for the velocity potential $\phi$, particle velocity $v$, and sound pressure $p$.

$$v=-\frac{\partial \phi}{\partial x} \tag{4}$$

$$p=\rho\frac{\partial \phi}{\partial t}+Const. \tag{5}$$

Here, let $A_i$ be the amplitude of $\phi_i$, $A_r$ be the amplitude of $\phi_r$, $A_t$ be the amplitude of $\phi_t$, $C_P$ be the amplitude of $w_P$, and $C_M$ be the amplitude of $x_M$. The quantities are set as follows, where $j$ is the imaginary unit.

$$\phi_i=A_i e^{j\{\omega t-k(x\cos\theta+y\sin\theta)\}} \tag{6}$$

$$\phi_r=A_r e^{j\{\omega t-k(-x\cos\theta+y\sin\theta)\}} \tag{7}$$

$$\phi_t=A_t e^{j\{\omega t-k(x\cos\theta+y\sin\theta)\}} \tag{8}$$

$$w_P=C_P e^{j\{\omega t-ky\sin\theta\}} \tag{9}$$

$$x_M=C_M e^{j\omega t} \tag{10}$$

Using these equations, the following equation can be obtained.

$$jkA_i\cos\theta-jkA_r\cos\theta=j\omega C_P=jkA_t\cos\theta \tag{11}$$

$$-m\omega^2 C_P+D_0k^4\sin^4\theta\cdot C_P+D_r(j\omega C_P-j\omega C_M)+K_r(C_P-C_M)=\rho j\omega(A_i+A_r-A_t) \tag{12}$$

$$-M\omega^2 C_M+D_r(j\omega C_M-j\omega C_P)+K_r(C_M-C_P)=0 \tag{13}$$

Therefore, the following relationship holds for $A_t$ and $A_i$. However, the acoustic impedance is set to $z=\rho c$.

$$A_t=\frac{2z}{\left\{[2z+\cos\theta\cdot F_1(\omega)]+jm\omega\cos\theta\left[1-\frac{D_0k^4\sin^4\theta}{m\omega^2}+F_2(\omega)\right]\right\}}A_i \tag{14}$$

Here,

$$F_1(\omega)=\frac{D_r(M\omega^2)^2}{(K_r-M\omega^2)^2+\omega^2D_r^2} \tag{15}$$

$$F_2(\omega)=\frac{K_rM(K_r-M\omega^2)+\omega^2D_r^2M\omega^2}{m\{(K_r-M\omega^2)^2+\omega^2D_r^2\}} \tag{16}$$

Therefore, the transmission loss $T_{loss-rd}$ is given by the following.

$$T_{loss-rd} = 10\log_{10}\frac{|A_i|^2}{|A_t|^2} \tag{17}$$

$$\therefore T_{loss-rd} = 10\log_{10}\left\{\left[1+\left(\frac{\cos\theta}{2z}\right)\cdot F_1(\omega)\right]^2+\left(\frac{m\omega\cos\theta}{2z}\right)^2\left[1-\frac{D_0k^4\sin^4\theta}{m\omega^2}+F_2(\omega)\right]^2\right\} \tag{18}$$

Furthermore, if $D_r = 0$, i.e., damping is eliminated in equation (18), the following equation is obtained. This is the equation for transmission loss when a mass-spring system is attached to an infinite plate.

$$T_{loss-r} = 10\log_{10}\left[1+\left(\frac{m\omega\cos\theta}{2z}\right)^2\left[1-\frac{D_0k^4\sin^4\theta}{m\omega^2}+\frac{K_rM}{m(K_r-M\omega^2)}\right]^2\right] \tag{19}$$

Furthermore, by setting $K_r = 0$, i.e., removing the mass-spring system, the following equation is obtained. This is the transmission loss equation considering the bending vibration of an infinite plate. The so-called coincidence frequency can be obtained from this equation.

$$T_{loss-c} = 10\log_{10}\left[1+\left(\frac{m\omega\cos\theta}{2z}\right)^2\left[1-\frac{D_0k^4\sin^4\theta}{m\omega^2}\right]^2\right] \tag{20}$$

Furthermore, if we ignore the bending vibration of an infinite plate, assuming $D_0 = 0$, we obtain the following equation. This is the transmission loss equation that represents the so-called mass law.

$$T_{loss} = 10\log_{10}\left[1+\left(\frac{m\omega\cos\theta}{2z}\right)^2\right] \tag{21}$$

## 3. Properties of Transmission Loss

Consider the properties of equation (18).

First, look at the denominators of equations (15) and (16). You can see that the minimum value occurs at the following frequencies. That is, the transmission loss becomes very large at the following frequencies.

$$\omega_{ar} = \sqrt{\frac{K_r}{M}} \tag{22}$$

$$\therefore f_{ar} = \frac{1}{2\pi}\sqrt{\frac{K_r}{M}} \tag{23}$$

Next, consider equation (15). At $\omega = 0$ and $\omega = \infty$, $F_1(\omega)$ has the following values.

$$F_1(0) = 0 \tag{24}$$

$$F_1(\infty) = D_r \tag{25}$$

Also, consider equation (16). At $\omega = 0$ and $\omega = \infty$, $F_2(\omega)$ has the following values.

$$F_2(0) = \frac{M}{m} \tag{26}$$

$$F_2(\infty) = \frac{D_r^2}{Mm} \tag{27}$$

Finally, consider the coincidence frequency. Coincidence frequencies often occur at high frequencies. Therefore, from equations (18) and (27), the coincidence effect occurs when the following equation is satisfied:

$$1 - \frac{D_0 k^4 \sin^4\theta}{m\omega^2} + \frac{D_r^2}{Mm} = 1 - \frac{D_0 \omega_{cd}^2 \sin^4\theta}{mc^4} + \frac{D_r^2}{Mm} = 0$$

Therefore, the coincidence frequency is given by the following:

$$\omega_{cd} = \frac{c^2}{\sin^2\theta}\sqrt{\frac{m}{D_0}\left(1 + \frac{D_r^2}{Mm}\right)} \tag{28}$$

$$\therefore f_{cd} = \frac{c^2}{2\pi\sin^2\theta}\sqrt{\frac{m}{D_0}\left(1 + \frac{D_r^2}{Mm}\right)} \tag{29}$$

Furthermore, when $D_r = 0$, the coincidence frequency of an infinite plate without a damping system can be found as follows:

$$f_c = \frac{c^2}{2\pi\sin^2\theta}\sqrt{\frac{m}{D_0}} \tag{30}$$

## 4. Numerical Calculation Example

Calculate by substituting the physical properties into the formulas obtained so far.

As an example, consider a state where a glass plate with a thickness of 3 (mm) is attached to a door with a mass of 20 (kg), separated by a run channel. The list of physical properties is summarized below.

Table 1Characteristics using calucuration

| Characteristics | Value |
|---|---|
| Young's modulus of Glass $E_G$ | 70(GPa) |
| Poisson ratio of Glass $\nu_G$ | 0.25 |
| Height of Glass $h_G$ | 3(mm) |
| Density of Glass $d_G$ | 2.5(g/cm$^3$) |
| Young’s modulus of Rubber $E_{rub}$ | 0.05(GPa) |
| Height of Rubber $h_{rub}$ | 2(mm) |
| Damping ratio of Rubber $\zeta_G$ | 0.02 |
| Mass of door $M$ | 20(kg) |

Note that $D_0, m$, and $K_r$ were obtained using the following formulas. $K_r$ is doubled because the run channel is installed with the glass in between, resulting in a parallel spring relative to the door.

$$D_0 = \frac{E_G h_G}{12(1-\nu_G^2)} \tag{31}$$

$$m = d_G h_G \tag{32}$$

$$K_r = 2E_{rub}h_{rub} \tag{33}$$

The calculation results for coincidence frequency, critical frequency, and frequency at which transmission loss is maximum are summarized in the table below.

Table 2 Kinds of Frequency

| Kinds of Frequency | Frequency (Hz) |
|---|---|
| Coincidence Frequency without Damping System ($\theta = 45$ (deg)) | $7.91 \times 10^3$ |
| Critical Frequency without Damping System | $3.96 \times 10^3$ |
| Anti-resonance Frequency with Mass-Spring System | $1.59 \times 10^1$ |
| Coincidence Frequency with Mass-Spring System ($\theta = 45$ (deg)) | $7.91 \times 10^3$ |
| Critical Frequency with Mass-Spring System | $3.96 \times 10^3$ |
| Anti-resonance Frequency with Damping System | $1.59 \times 10^1$ |
| Coincidence Frequency with Damping System ($\theta = 45$ (deg)) | $5.23 \times 10^4$ |
| Critical Frequency with Damping System | $2.61 \times 10^4$ |

First, the graph of transmission loss when the incident angle $\theta$ is 45 degrees is shown. The graph shows that considering the bending of the flat plate reveals a coincidence effect, that there are frequencies with high transmission loss at low frequencies when a mass-spring system or damping system is attached to the flat plate, and that transmission loss is high when a damping system is attached to the flat plate.

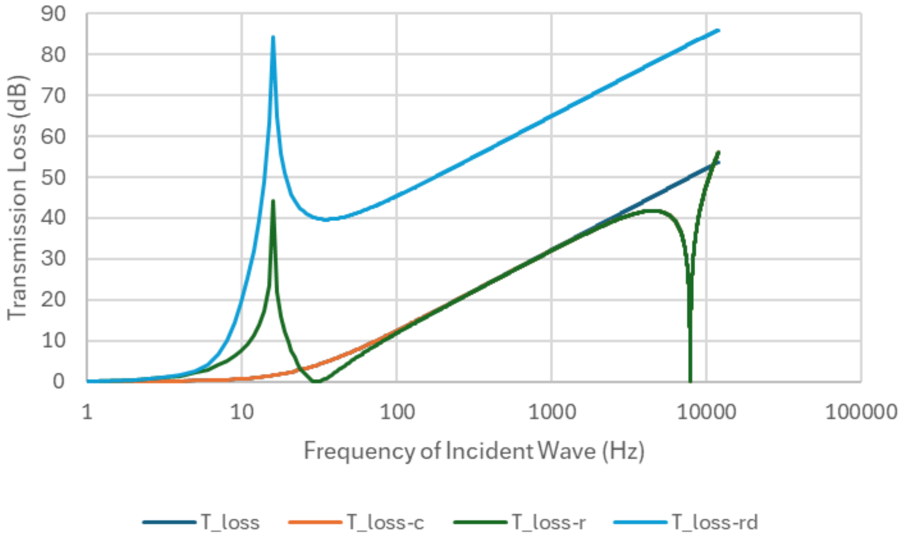

Fig. 2 Transmission Loss ($\boldsymbol{\theta} = \mathbf{45}$ (deg))

Next, considering the diffuse sound field, the graph of transmission loss obtained by numerical integration from the incident angle $\theta$ from 0 to 78 degrees is shown below. From the graph, it can be seen that once the coincidence effect occurs, at subsequent frequencies, the incident angle at which the coincidence effect occurs is taken into account during numerical integration, so the coincidence effect always occurs, and the transmission loss is always low. Furthermore, it can be seen that the transmission loss is highest across all frequencies when a damping system is attached to the flat plate.

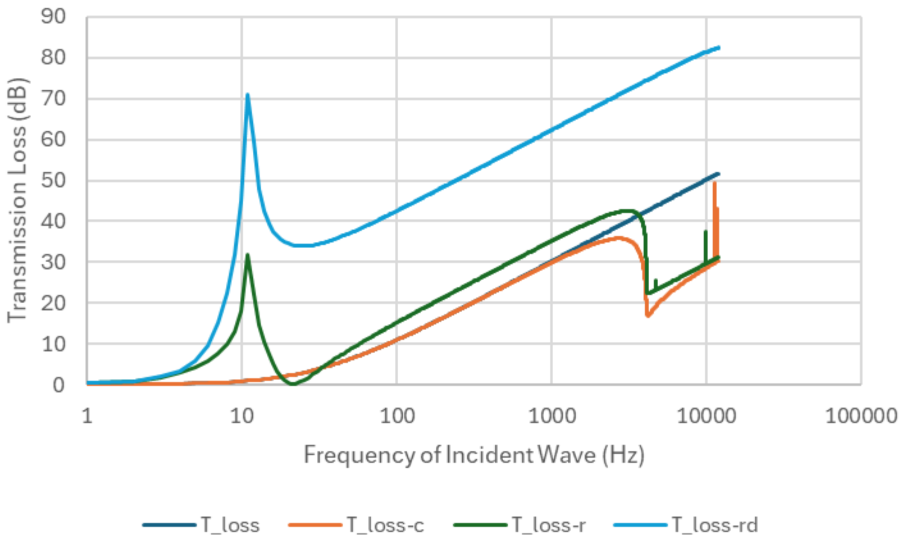

Fig. 3 Transmission Loss in Diffuse Sound Field

Finally, the graphs for the 1/3 octave band and O.A. value are shown. It can be seen that the transmission loss is highest across all frequency bands when a damping system is attached to an infinite flat plate. The O.A. value also shows that the transmission loss is greatest when the damping system is attached to an infinite flat plate.

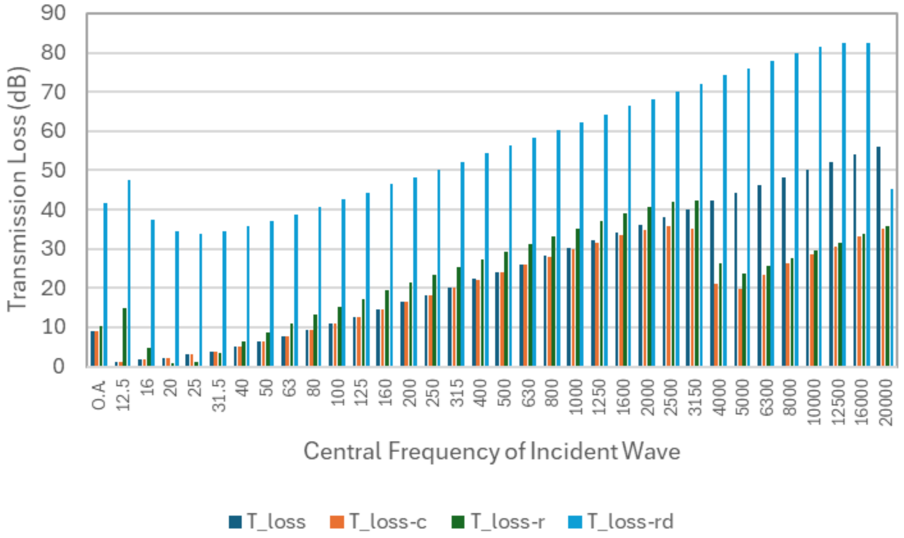

Fig. 4 Transmission Loss in Diffuse Sound Field (1/3 octave band)

## 5. Conclusion

While transmission loss has been studied in the past, understanding the properties of transmission loss when an attenuation system is attached to a flat plate is considered necessary in fields such as automobiles, where glass plates are attached to the vehicle body with rubber products. Therefore, we discussed the transmission loss of a flat plate when a damping system is attached to it. As a result, we found that the transmission loss was greatest across all frequencies when an infinite flat plate was attached to it.

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