Data Driven Kalman Filter via Dynamic Mode Decomposition

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Abstract

The Extended Kalman Filter (EKF) requires an analytical Jacobian of the system dynamics for uncertainty propagation, which is unavailable or expensive to compute in many practical settings. We propose a data-driven alternative in which the Jacobian is replaced by a local linear operator identified online from a sliding window of state observations using Dynamic Mode Decomposition (DMD). The nonlinear state prediction is retained in its original form via fourth-order Runge–Kutta integration, while the covariance matrix is propagated through the DMD operator, preserving the Gaussian structure of the uncertainty estimate. The proposed DMD KF algorithm is validated on the nonlinear inverted pendulum benchmark. Simulation results demonstrate that DMD KF achieves estimation accuracy comparable to the standard EKF without requiring any knowledge of the system Jacobian, and adapts online to changes in system dynamics.

Keywords: Extended Kalman Filter, Dynamic Mode Decomposition, data-driven estimation, uncertainty propagation, nonlinear systems, state estimation.

1 Introduction

The Kalman filter [1] is the optimal linear estimator under Gaussian noise assumptions. For nonlinear systems, the Extended Kalman Filter (EKF) [2] approximates the nonlinear dynamics by a first-order Taylor expansion about the current state estimate, yielding the analytical Jacobian $F_k = \partial f/\partial \mathbf{x}\big|_{\hat{\mathbf{x}}_k}$ for covariance propagation. While this approach is well-established, the computation of the Jacobian requires explicit knowledge of the system dynamics, which may be unavailable in black-box systems, or costly to derive for high-dimensional models.

Dynamic Mode Decomposition (DMD) [3, 4] is a data-driven method that identifies a best-fit linear operator mapping consecutive state snapshots. It has been applied to fluid dynamics, structural mechanics, and control [5, 6]. The connection between DMD and the Kalman filter has been explored in [7], where EKF is used to estimate the DMD coefficients online for the purpose of system identification and denoising. In that formulation, the DMD modes are augmented into the state vector and the EKF tracks their evolution.

The present work pursues a complementary direction. Rather than using EKF to identify a DMD model, we use a sliding-window DMD regression to provide a local linear approximation that serves as a data-driven substitute for the analytical Jacobian in the EKF covariance propagation step. The nonlinear state prediction is preserved through

Runge–Kutta integration, so the algorithm retains the full nonlinear fidelity of the EKF prediction while eliminating the requirement for an analytical Jacobian. To the best of our knowledge, this formulation has not been previously reported in the literature.

The paper is organized as follows. Section 2 states the problem and establishes notation. Section 3 derives the DMD KF algorithm. Section 4 presents simulation results on the inverted pendulum. Section 5 concludes.

2 Problem Formulation

Consider a discrete-time nonlinear dynamical system:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, u_k) + \mathbf{w}_k, \tag{1}$$

$$\mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}$ is the control input, $\mathbf{z}_k \in \mathbb{R}^m$ is the measurement, and $\mathbf{w}_k \sim \mathcal{N}(0, Q)$, $\mathbf{v}_k \sim \mathcal{N}(0, R)$ are independent Gaussian noise processes. The matrix $H \in \mathbb{R}^{m\times n}$ is the linear observation matrix.

The standard EKF propagates the covariance according to:

$$P_{k|k-1} = F_k \, P_{k-1|k-1} \, F_k^\top + Q, \tag{3}$$

where $F_k = \partial f/\partial \mathbf{x}\big|_{\hat{\mathbf{x}}_{k|k-1}} \in \mathbb{R}^{n\times n}$ is the Jacobian of f evaluated at the current state estimate. Computing F_k analytically requires explicit knowledge of f.

Goal. Replace F_k in (3) with a data-driven approximation $\hat{A}_k$ identified online from observed state trajectories, without modifying the nonlinear state prediction step.

3 Proposed Method: DMD KF

3.1 Sliding-Window DMD Regression

At each time step k, we maintain a buffer of W consecutive state estimates:

$$\mathcal{X}_k = \big[\hat{\mathbf{x}}_{k-W},\ \hat{\mathbf{x}}_{k-W+1},\ \ldots,\ \hat{\mathbf{x}}_{k-1}\big] \in \mathbb{R}^{n\times W}. \tag{4}$$

Define the shifted matrix $\mathcal{X}_k^+ = \big[\hat{\mathbf{x}}_{k-W+1},\ \ldots,\ \hat{\mathbf{x}}_k\big]$. The DMD regression solves the least-squares problem:

$$\hat{A}_k = \arg\min_A \big\|\mathcal{X}_k^+ - A\,\mathcal{X}_k\big\|_F^2, \tag{5}$$

whose closed-form solution with Tikhonov regularization $\alpha > 0$ is:

$$\boxed{\hat{A}_k = \mathcal{X}_k^+ \, \mathcal{X}_k^\top \big(\mathcal{X}_k \mathcal{X}_k^\top + \alpha I\big)^{-1}.} \tag{6}$$

The regularization parameter α prevents ill-conditioning when $W < n$ or when the state trajectories are nearly collinear.

3.2 DMD KF Algorithm

The DMD KF algorithm departs from the standard EKF in one location only: the Jacobian F_k in the covariance prediction step (3) is replaced by $\hat{A}_k$. All other steps are identical to the EKF.

Prediction step. The state is propagated nonlinearly using fourth-order Runge–Kutta (RK4):

$$\hat{\mathbf{x}}_{k|k-1} = \mathrm{RK4}\big(f,\, \hat{\mathbf{x}}_{k-1|k-1},\, u_{k-1},\, T_s\big), \tag{7}$$

while the covariance is propagated through the data-driven operator:

$$P_{k|k-1} = \hat{A}_k\, P_{k-1|k-1}\, \hat{A}_k^\top + Q. \tag{8}$$

Equation (8) preserves the Gaussian structure of the uncertainty estimate: since $\hat{A}_k$ is a linear operator, a Gaussian prior remains Gaussian after propagation.

Correction step. The measurement update follows the standard Kalman equations:

$$K_k = P_{k|k-1}\, H^\top \big(H P_{k|k-1} H^\top + R\big)^{-1}, \tag{9}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k\big(\mathbf{z}_k - H\hat{\mathbf{x}}_{k|k-1}\big), \tag{10}$$

$$P_{k|k} = (I - K_k H)\, P_{k|k-1}\, (I - K_k H)^\top + K_k R K_k^\top. \tag{11}$$

Equation (11) is the Joseph form of the covariance update, which guarantees positive semi-definiteness regardless of numerical errors in K_k.

Window update. After the correction step, the new estimate is appended to the buffer and the oldest entry is discarded:

$$\mathcal{X}_{k+1} \leftarrow \big[\hat{\mathbf{x}}_{k-W+1},\ \ldots,\ \hat{\mathbf{x}}_{k|k}\big]. \tag{12}$$

The full procedure is summarized in Algorithm 1.

Algorithm 1 DMD KF: Data-Driven Extended Kalman Filter

Initialization: $\hat{\mathbf{x}}_0$, P_0, buffer $\mathcal{X}_0$ (PRBS pre-fill)

for $k = 1, 2, \ldots, N$ **do**

  // DMD regression

  $\hat{A}_k \leftarrow \mathcal{X}_k^{+}\, \mathcal{X}_k^\top (\mathcal{X}_k \mathcal{X}_k^\top + \alpha I)^{-1}$

  // Prediction

  $\hat{\mathbf{x}}_{k|k-1} \leftarrow \mathrm{RK4}(f,\, \hat{\mathbf{x}}_{k-1|k-1},\, u_{k-1},\, T_s)$

  $P_{k|k-1} \leftarrow \hat{A}_k P_{k-1|k-1} \hat{A}_k^\top + Q$

  // Correction

  $K_k \leftarrow P_{k|k-1} H^\top (H P_{k|k-1} H^\top + R)^{-1}$

  $\hat{\mathbf{x}}_{k|k} \leftarrow \hat{\mathbf{x}}_{k|k-1} + K_k(\mathbf{z}_k - H\hat{\mathbf{x}}_{k|k-1})$

  $P_{k|k} \leftarrow (I - K_k H) P_{k|k-1} (I - K_k H)^\top + K_k R K_k^\top$

  // Slide window

  $\mathcal{X}_{k+1} \leftarrow [\hat{\mathbf{x}}_{k-W+1}, \ldots, \hat{\mathbf{x}}_{k|k}]$

end for

3.3 Comparison with Standard EKF

Table 1 summarizes the key differences between EKF and DMD KF.

It follows that DMD KF reduces to the standard EKF when $\hat{A}_k \to F_k$, i.e., when the window is large enough to accurately identify the local linearization.

Table 1: Comparison of EKF and DMD KF.

Property	EKF	DMD KF
State prediction	RK4: $f(\hat{\mathbf{x}}, u)$	RK4: $f(\hat{\mathbf{x}}, u)$
Jacobian source	Analytical $\partial f / \partial \mathbf{x}$	Data-driven $\hat{A}_k$
Model knowledge	Required	Not required
Covariance update	$F_k P F_k^\top + Q$	$\hat{A}_k P \hat{A}_k^\top + Q$
Online adaptation	No	Yes (sliding window)

4 Simulation Results

4.1 System Description

We validate the proposed algorithm on the nonlinear inverted pendulum on a cart, whose equations of motion are:

$$\ddot{x} = \frac{1}{D}\big(-m^2 L^2 g \cos\theta \sin\theta + mL^2(mL\dot{\theta}^2 \sin\theta - d\dot{x})\big), \tag{13}$$

$$\ddot{\theta} = \frac{1}{D}\big((m + M)mgL \sin\theta - mL\cos\theta(mL\dot{\theta}^2 \sin\theta - d\dot{x})\big), \tag{14}$$

where $D = mL^2(M+m(1-\cos^2\theta))$, x is the cart position, θ is the pendulum angle, m and M are the pendulum and cart masses, L is the rod length, g is gravitational acceleration, and d is the cart damping coefficient.

The state vector is $\mathbf{x} = [x,\, \dot{x},\, \theta,\, \dot{\theta}]^\top \in \mathbb{R}^4$. True trajectories are generated by RK4 integration with sampling period $T_s = 10^{-2}$ s over a 10 s horizon. Gaussian measurement noise with standard deviation $\sigma = 0.1$ is added to all four states.

4.2 Parameters

Table 2: Simulation and algorithm parameters.

Parameter	Symbol	Value
Pendulum mass	m	1 kg
Cart mass	M	1 kg
Rod length	L	2 m
Cart damping	d	$0.1\,\mathrm{N \cdot s/m}$
Sampling period	T_s	10^{-2} s
Initial angle	θ_0	75 deg
Noise std.	σ	0.1
DMD window	W	15
Regularization	α	10^{-6}
Process noise	Q	$10^{-5} I_4$

4.3 Results and Discussion

Figure 1 shows the state estimation results of the standard EKF. Figure 2 shows the corresponding results for the proposed DMD KF.

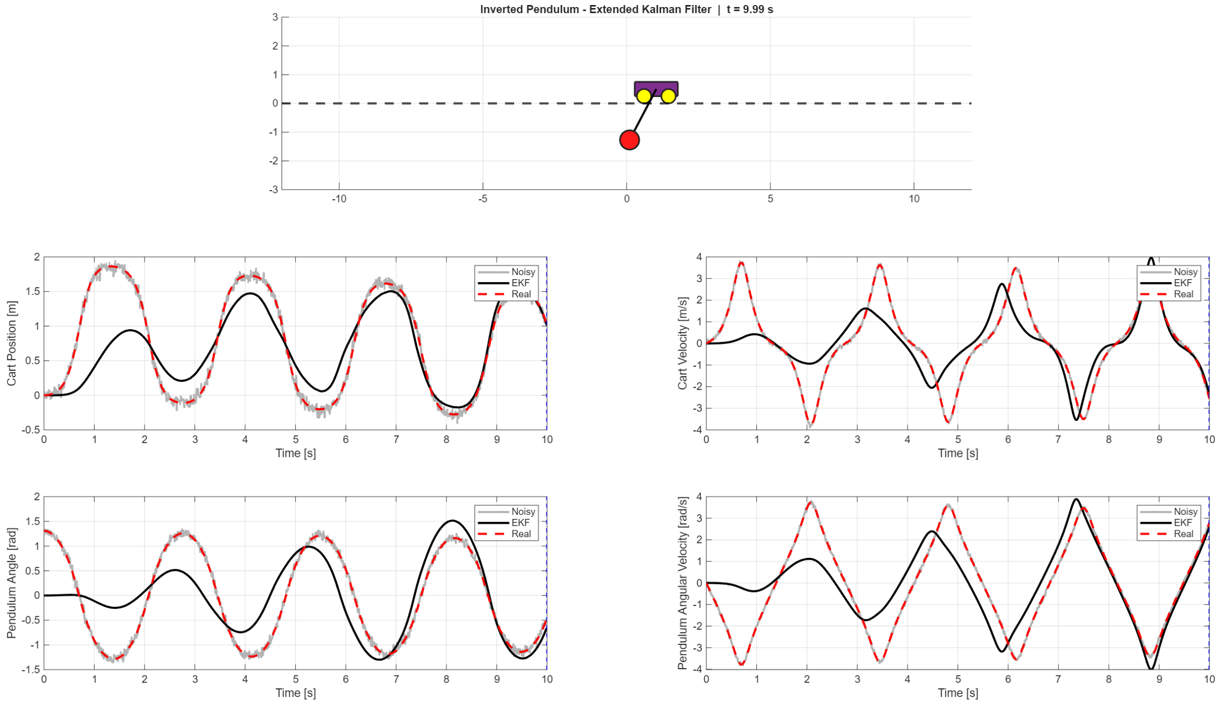

Figure 1: EKF on the inverted pendulum ($\theta_0 = 75\,\text{deg}$).

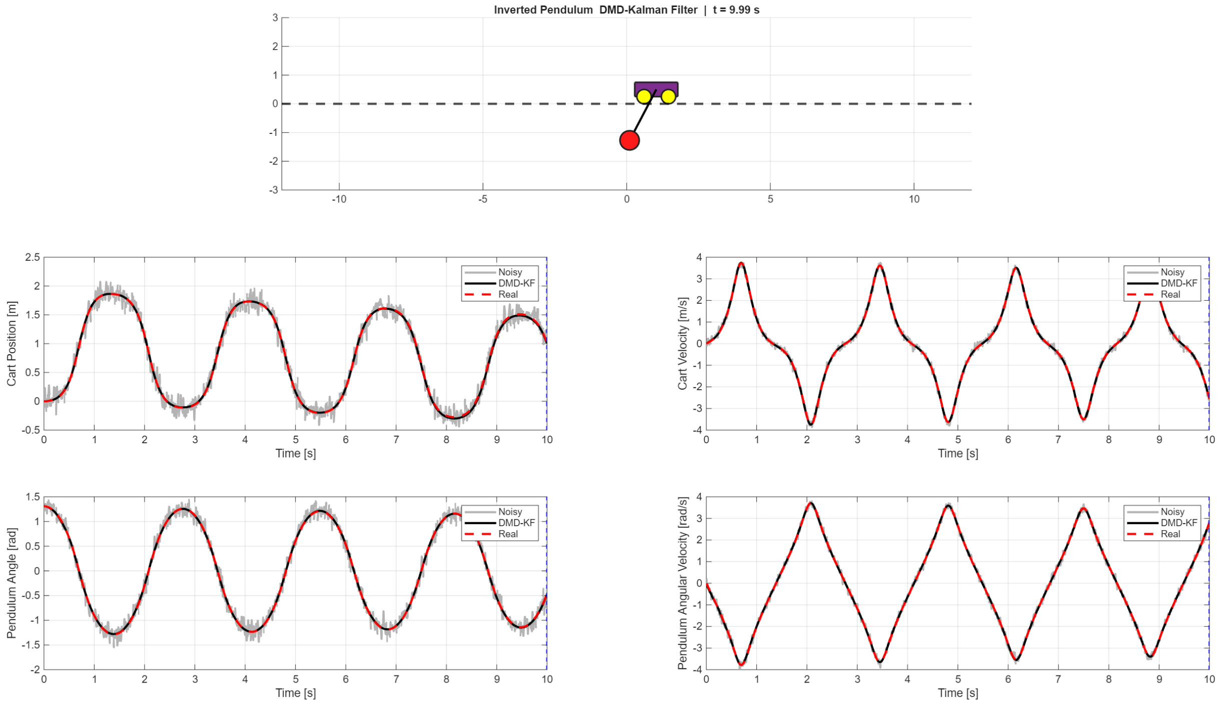

Figure 2: DMD KF on the inverted pendulum ($\theta_0 = 75\,\text{deg}$).

The initial angle $\theta_0 = 75\,\text{deg}$ places the system well outside the small-angle regime, where the analytical Jacobian of the EKF provides only a crude linearization. As visible in Figure 1, the standard EKF exhibits noticeable tracking errors during the initial transient, particularly in cart velocity and angular velocity. In contrast, DMD KF in Figure 2 achieves accurate tracking across all four states from the beginning of the simulation. This result reveals that a sliding window of length $W = 15$ is sufficient to capture the dominant local dynamics at each time step.

The PRBS pre-fill phase is essential: it excites the system across multiple directions in state space before the main estimation loop begins, ensuring that the initial DMD regression is not rank-deficient. The Tikhonov regularization parameter $\alpha = 10^{-6}$ prevents numerical instability during the brief transient in which the buffer has not yet fully refreshed.

5 Conclusions

This paper presented DMD KF, a data-driven formulation of the Extended Kalman Filter in which the analytical Jacobian required for covariance propagation is replaced by a local linear operator identified online via Dynamic Mode Decomposition. The main contributions are:

(a) A sliding-window DMD regression that provides a data-driven Jacobian substitute, updated at every time step with $\mathcal{O}(n^2W)$ cost.

(b) A hybrid prediction architecture in which the nonlinear state dynamics are integrated by RK4 while the covariance is propagated through the DMD operator, preserving Gaussian uncertainty structure.

(c) Empirical validation on the nonlinear inverted pendulum showing that DMD KF achieves estimation accuracy comparable to the standard EKF without any model knowledge, and adapts online to changes in system dynamics.

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