Regarding the Case of a 1-Dimensional Tube with a Vibration Source and a Hole in It

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Abstract

While research on automobile whistling and aspiration noise has been conducted in the past, we believe that understanding the properties of whistling and aspiration sounds using a 1-dimensional tube as a simple model is necessary in fields such as automobiles. Therefore, we discussed the case where there is a vibration source in a 1-dimensional tube with a hole. We derived the system of equations that must be satisfied and showed that the results derived using actual numerical examples are valid.

Nomenclature

S, S_H: Cross-sectional area of the 1-dimensional tube and area of the hole in the 1-dimensional tube

l: Position of the hole in the 1-dimensional tube

L: Length of the 1-dimensional tube

V: Volume of the 1-dimensional tube

p_1, v_1, ϕ_1: Sound pressure and particle velocity and velocity potential for $0 \le x \le l$

A_1, B_1: Amplitude of velocity potential for $0 \le x \le l$

p_2, v_2, ϕ_2: Sound pressure and particle velocity and velocity potential for $l \le x \le L$

A_2, B_2: Amplitude of velocity potential for $l \le x \le L$

p_H, v_H: Sound pressure and particle velocity at $x = l$

δ_H: Thickness of the 1-dimensional tube

ρ, c: Density of air and speed of sound in air

ω: Angular frequency

f: Frequency

k: Wavenumber

t: Time

j: Imaginary unit

p_E, v_E, δ_E: Sound pressure, particle velocity, and thickness of a virtual air wall.

1. Introduction

In the past, studies have been conducted on the whistling and suction sounds of automobile (Calvo, Diaz, & San Roman, 2005) (Chien-Hsiung, Lung-Ming , Chang-Hsien , Yen-Loung , & Jik-Chang , 2009) (George, 1990) (Jagtiani, 1972) (Jung & Oh, 1995) (Münder & Carbon, 2022) (Oettle & Sims-Williams, 2017) (Qatu, Abdelhamid, Pang, & Sheng, 2009) (Wang, Chen, & Zhang, 2021) (Zhang, Meng, Li, & Zheng, 2022). However, to the best of the author's knowledge, there are no studies that discuss the properties of whistling and aspiration noise using a 1-dimensional tube as a simple model Understanding the properties of whistling and aspiration noise using a 1-dimensional tube is considered essential in the automotive field.

Therefore, we will derive the system of equations that must be satisfied in the case of a vibration source in a 1-dimensional tube with a hole, and demonstrate that the derived results are valid based on actual numerical examples.

This will be discussed below.

2. In the Case of 1-Dimentional Tube with a Vibration Source and a Hole in It.

2.1. In the Case of an Air Wall

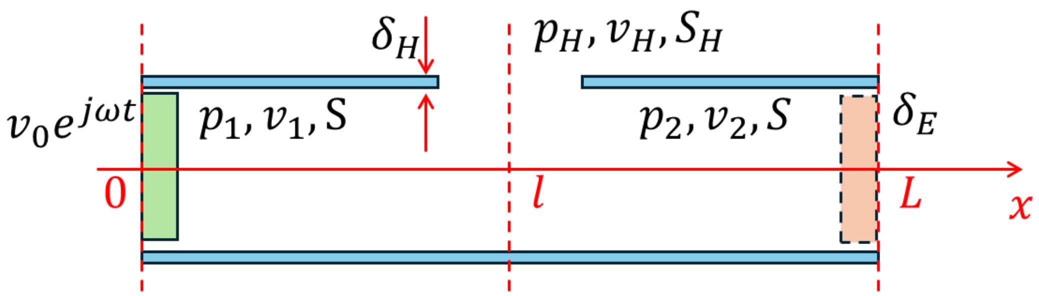

Fig. 1 1-Dimensional Tube which Have Virtual Air Wall

Fig. 1 shows a one-dimensional tube with a hole containing a vibration source and an air wall. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{1}$$

$$Sv_1(l,t) = Sv_2(l,t) + S_H v_H(t) \tag{2}$$

The equation of motion for the hole is as follows:

$$\rho\delta_H \frac{\partial v_H}{\partial t} = p_H(t) \tag{3}$$

The equation of motion for the air wall is as follows:

$$\rho\delta_E \frac{\partial}{\partial t} v_2(L,t) = p_2(L,t) \tag{4}$$

Here, we show the equations satisfied by the velocity potential, vibration source, hole, and air wall.

$$\phi_1(x,t) = A_1 e^{j(\omega t - kx)} + B_1 e^{j(\omega t + kx)} \tag{5}$$

$$\phi_2(x,t) = A_2 e^{j(\omega t - kx)} + B_2 e^{j(\omega t + kx)} \tag{6}$$

$$v_1(0,t) = v_0 e^{j\omega t} \tag{7}$$

$$v_H(t) = C_H e^{j\omega t} \tag{8}$$

$$p_H(t) = P_H e^{j\omega t} \tag{9}$$

We show the equations that the velocity potential, particle velocity, and sound pressure satisfy.

$$v = -\frac{\partial \phi}{\partial x} \tag{10}$$

$$p = \rho \frac{\partial \phi}{\partial t} + Const. \tag{11}$$

From these equations, the following six simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = P_H \tag{12}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = P_H \tag{13}$$

$$S\left(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\right) = S\left(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\right) + S_H C_H \tag{14}$$

$$jkA_1 - jkB_1 = v_0 \tag{15}$$

$$\rho j\omega \delta_H C_H = P_H \tag{16}$$

$$\delta_E\left(jkA_2 e^{-jkL} - jkB_2 e^{jkL}\right) = A_2 e^{-jkL} + B_2 e^{jkL} \tag{17}$$

The solution obtained from the system of six equations is as follows:

$$D = S_H \cos(kl) - Sk\delta_H \left\{ \sin(kl) - j\cos(kl) \frac{Ce^{jk(L-l)} - e^{-jk(L-l)}}{Ce^{jk(L-l)} + e^{-jk(L-l)}} \right\} \tag{18}$$

$$C = \frac{\delta_E jk + 1}{\delta_E jk - 1} \tag{19}$$

$$C_H = \frac{v_0 S}{D} \tag{20}$$

$$P_H = \rho j\omega \delta_H C_H \tag{21}$$

$$A_1 = \left\{ \delta_H C_H + \frac{v_0}{jk} e^{jkl} \right\} \frac{1}{2\cos(kl)} \tag{22}$$

$$B_1 = \left\{ \delta_H C_H - \frac{v_0}{jk} e^{-jkl} \right\} \frac{1}{2\cos(kl)} \tag{23}$$

$$A_2 = \delta_H C_H \frac{Ce^{jkL}}{Ce^{jk(L-l)} + e^{-jk(L-l)}} \tag{24}$$

$$B_2 = \delta_H C_H \frac{e^{-jkL}}{Ce^{jk(L-l)} + e^{-jk(L-l)}} \tag{25}$$

When equation (18) is 0, the resonance frequency can be found.

2.2. In the Case of a Single-Sided Open End

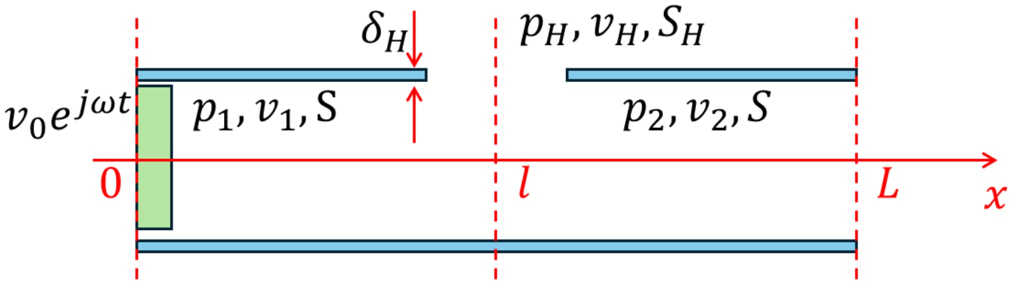

Fig. 2 1-Dimensional Tube which Right Side Opens

Fig. 2 shows a 1-dimensional tube with one end open. The following equations hold true based on the conditions for continuity of sound pressure and particle velocity.

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{26}$$

$$Sv_1(l,t) = Sv_2(l,t) + S_H v_H(t) \tag{27}$$

The equation of motion for the hole is as follows:

$$\rho \delta_H \frac{\partial v_H}{\partial t} = p_H(t) \tag{28}$$

Here, we show the equations satisfied by the velocity potential, vibration source, hole, and open end.

$$v_1(0,t) = v_0 e^{j\omega t} \tag{29}$$

$$v_H = C_H e^{j\omega t} \tag{30}$$

$$p_2(L,t) = 0 \tag{31}$$

From these equations, the following six simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = P_H \tag{32}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = P_H \tag{33}$$

$$S\left(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\right) = S\left(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\right) + S_H C_H \tag{34}$$

$$jkA_1 - jkB_1 = v_0 \tag{35}$$

$$A_2 e^{-jkL} + B_2 e^{jkL} = 0 \tag{36}$$

$$\rho j\omega \delta_H C_H = P_H \tag{37}$$

The solution obtained from the system of six equations is as follows:

$$D_{open} = S_H \cos(kl) - Sk\delta_H\{\sin(kl) - \cos(kl)\cot(k(L-l))\} \tag{38}$$

$$C_H = \frac{v_0 S}{D_{open}} \tag{39}$$

$$p_H = \rho j\omega\delta_H C_H \tag{40}$$

$$A_1 = \left\{\delta_H C_H + \frac{v_0}{jk}e^{jkl}\right\}\frac{1}{2\cos(kl)} \tag{41}$$

$$B_1 = \left\{\delta_H C_H - \frac{v_0}{jk}e^{-jkl}\right\}\frac{1}{2\cos(kl)} \tag{42}$$

$$A_2 = \delta_H C_H \frac{e^{jkL}}{2j\sin(k(L-l))} \tag{43}$$

$$B_2 = -\delta_H C_H \frac{e^{-jkL}}{2j\sin(k(L-l))} \tag{44}$$

These equations are equivalent to those obtained when $\delta_E = 0$ in the solution of Section 2.1.

2.3. In the Case of a Single-Sided Closed End

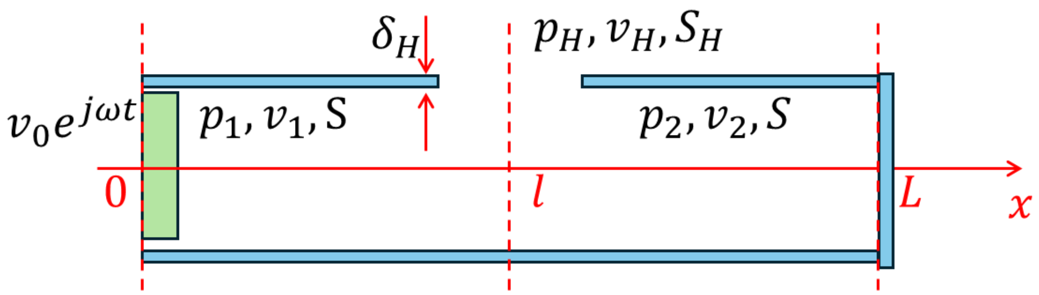

Fig. 3 1-Dimensional Tube which Right Side Closes

Fig. 3 shows a 1-dimensional tube with one end closed. The following equations hold true based on the conditions for continuity of sound pressure and particle velocity.

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{45}$$

$$Sv_1(l,t) = Sv_2(l,t) + S_H v_H \tag{46}$$

The equation of motion for the hole is as follows:

$$\rho\delta_H \frac{\partial v_H}{\partial t} = p_H(t) \tag{47}$$

Here, we show the equations satisfied by the velocity potential, vibration source, hole, and closed end.

$$v_1(0,t) = v_0 e^{j\omega t} \tag{48}$$

$$v_H = C_H e^{j\omega t} \tag{49}$$

$$v_2(L,t) = 0 \tag{50}$$

From these equations, the following six simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = P_H \tag{51}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = P_H \tag{52}$$

$$S\left(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\right) = S\left(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\right) + S_H C_H \tag{53}$$

$$jkA_1 - jkB_1 = v_0 \tag{54}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{55}$$

$$\rho j\omega \delta_H C_H = P_H \tag{56}$$

The solution obtained from the system of six equations is as follows:

$$D_{close} = S_H \cos(kl) - Sk\delta_H\{\sin(kl) + \cos(kl)\tan(k(L-l))\} \tag{57}$$

$$C_H = \frac{v_0 S}{D_{close}} \tag{58}$$

$$p_H = \rho j\omega \delta_H C_H \tag{59}$$

$$A_1 = \left\{\delta_H C_H + \frac{v_0}{jk} e^{jkl}\right\} \frac{1}{2\cos(kl)} \tag{60}$$

$$B_1 = \left\{\delta_H C_H - \frac{v_0}{jk} e^{-jkl}\right\} \frac{1}{2\cos(kl)} \tag{61}$$

$$A_2 = \delta_H C_H \frac{e^{jkL}}{2\cos(k(L-l))} \tag{62}$$

$$B_2 = \delta_H C_H \frac{e^{-jkL}}{2\cos(k(L-l))} \tag{63}$$

These equations are equivalent to those obtained when $\delta_E \to \infty$ in the solution of Section 2.1.

3. Numerical Examples

First, regarding the resonance frequencies of the open end in Section 2.2 and the closed end in Section 2.3, the difference between cases with and without holes is shown in Table 1 and Table 2.

Table 1 Resonant Frequency when Right Side Opens (Unit: Hz)

$(L = 1.0(\mathrm{m}),\ l = 0.4(\mathrm{m}),\ S = 1.0(\mathrm{m}^2),\ \delta_H = 0.001(\mathrm{m}),\ c = 343(\mathrm{m/s}))$

Order	$S_H = 0.0(\mathrm{m}^2)$	$S_H = 0.01(\mathrm{m}^2)$
1st	85.750	208.97
2nd	257.25	560.74
3rd	428.75	629.56
4th	600.25	843.76
5th	771.75	1043.4

Table 2 Resonant Frequency when Right Side Closes (Unit: Hz)

$(L = 1.0(\mathrm{m}),\ l = 0.4(\mathrm{m}),\ S = 1.0(\mathrm{m}^2),\ \delta_H = 0.001(\mathrm{m}),\ c = 343(\mathrm{m/s}))$

Order	$S_H = 0.0(\mathrm{m}^2)$	$S_H = 0.01(\mathrm{m}^2)$
1st	171.5	140.5
2nd	343.0	209.4
3rd	514.5	421.8
4th	686.0	626.2
5th	857.5	705.3

Table 1 shows that in the case of an open end, the presence of the hole shifts the resonance frequency to the higher frequency side. Table 2 shows that in the case of a closed end, the presence of the hole shifts the resonance frequency to the lower frequency side. These results are consistent with the properties of the actual phenomenon of aspiration noise at a closed end and the whistling noise at an open end, indicating that the model is valid.

Next, the table below shows how the resonance frequency changes when the value of δ_E is varied in the results from Section 2.1. The table also shows the results for both the open end and the closed end.

Table 3 Resonant Frequency by changing $\boldsymbol{\delta_E}$

($L = 1.0$(m), $l = 0.4$(m), $S = 1.0$(m²), $S_H = 0.01$(m²) $\delta_H = 0.001$(m), $c = 343$(m/s))

δ_E (m)	1st Resonant Frequency (Hz)
D_{open}	208.97
0.0	208.97
0.05	208.88
0.2	208.01
1.0	168.37
10.0	143.85
D_{close}	140.5

Table 3 shows that as δ_E increases from the open end to the closed end, the resonance frequency decreases, and in the limit, the results are the same as or converge to those of the open and closed ends. This indicates that the model is valid.

4. Conclusion

While research on automobile whistling and aspiration noise has been conducted in the past, we believe that understanding the properties of whistling and aspiration sounds using a 1-dimensional tube as a simple model is necessary in fields such as automobiles. Therefore, we discussed the case where there is a vibration source in a 1-dimensional tube with a hole. We derived the system of equations that must be satisfied and showed that the results derived using actual numerical examples are valid.

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