Regarding the Case where There are Vibration sources and Sound Pressure sources in Various 1-dimensional tubes with a Hole

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Abstract

While research on automobile whistling and aspiration noise has been conducted in the past, we believe that understanding the properties of whistling and aspiration sounds using a 1-dimensional tube as a simple model is necessary in fields such as automobiles. Therefore, we discussed the case where there is a vibration source and a sound pressure source in various 1-dimensional tubes with holes. We derived the system of equations that must be satisfied and discussed their properties.

Nomenclature

S, S_H: Cross-sectional area of the 1-dimensional tube and area of the hole in the 1-dimensional tube

l: Position of the hole in the 1-dimensional tube

L: Length of the 1-dimensional tube

V: Volume of the 1-dimensional tube

p_1, v_1, ϕ_1: Sound pressure and particle velocity and velocity potential for $0 \leq x \leq l$

A_1, B_1: Amplitude of velocity potential for $0 \leq x \leq l$

p_2, v_2, ϕ_2: Sound pressure and particle velocity and velocity potential for $l \leq x \leq L$

A_2, B_2: Amplitude of velocity potential for $l \leq x \leq L$

ρ, c: Density of air and speed of sound in air

ω: Angular frequency

f: Frequency

k: Wavenumber

t: Time

j: Imaginary unit

λ; Wavelength

1. Introduction

In the past, studies have been conducted on the whistling and suction sounds of automobile (Calvo, Diaz, & San Roman, 2005) (Chien-Hsiung, Lung-Ming , Chang-Hsien , Yen-Loung , & Jik-Chang , 2009) (George, 1990) (Jagtiani, 1972) (Jung & Oh, 1995) (Münder & Carbon, 2022) (Oettle & Sims-Williams, 2017) (Qatu, Abdelhamid, Pang, & Sheng, 2009) (Wang, Chen, & Zhang, 2021) (Zhang, Meng, Li, & Zheng, 2022)。 However, to the best of the author's knowledge, there are no studies that discuss the properties of whistling and aspiration noise using a 1-dimensional tube as a simple model. Understanding the properties of whistling and aspiration noise using a 1-dimensional tube is considered essential in the automotive field.

Therefore, we will discuss the derivation of the simultaneous equations that must be satisfied in various 1-dimensional tubes with holes, when there is a vibration source and a sound pressure source, and the properties of these equations.

This will be discussed below.

2. In the Case of a 1-Dimensional Tube with Holes and Closed Ends on Both Sides.

2.1. When There is a Source of Vibration

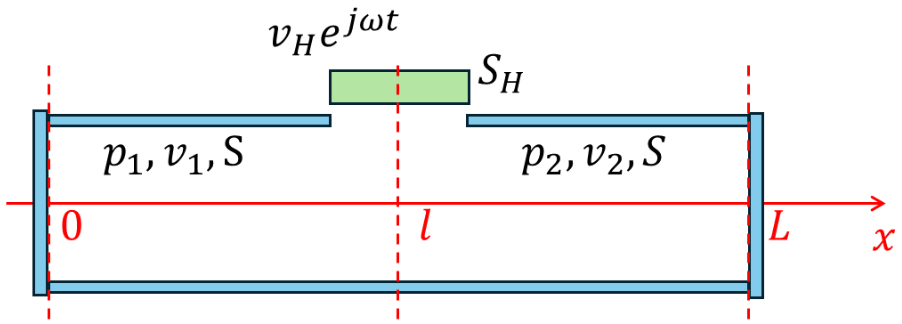

Fig. 1 1-Dimensional Tube which Have Vibration Source

Fig. 1 shows a case where the vibration source is in a 1-dimensional tube with holes in both closed ends. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) \tag{1}$$

$$Sv_1(l,t) = Sv_2(l,t) + S_H v_H e^{j\omega t} \tag{2}$$

Here, the velocity potential and boundary conditions are shown.

$$\phi_1(x,t) = A_1 e^{j(\omega t - kx)} + B_1 e^{j(\omega t + kx)} \tag{3}$$

$$\phi_2(x,t) = A_2 e^{j(\omega t - kx)} + B_2 e^{j(\omega t + kx)} \tag{4}$$

$$v_1(0,t) = 0 \tag{5}$$

$$v_2(L,t) = 0 \tag{6}$$

We show the equations that the velocity potential, particle velocity, and sound pressure satisfy.

$$v = -\frac{\partial \phi}{\partial x} \tag{7}$$

$$p = \rho \frac{\partial \phi}{\partial t} + C \tag{8}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = A_2 e^{-jkl} + B_2 e^{jkl} \tag{9}$$

$$S\left(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\right) = S\left(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\right) + S_H v_H \tag{10}$$

$$jkA_1 - jkB_1 = 0 \tag{11}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{12}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{S_H v_H}{2jkS} \frac{\cos(k(L-l))}{jk \sin(kL)} \tag{13}$$

$$B_1 = \frac{S_H v_H}{2jkS} \frac{\cos(k(L-l))}{j \sin(kL)} \tag{14}$$

$$A_2 = \frac{S_H v_H}{2jkS} \frac{\cos(kl)}{jk \sin(kL)} e^{jkL} \tag{15}$$

$$B_2 = \frac{S_H v_H}{2jkS} \frac{\cos(kl)}{jk \sin(kL)} e^{-jkL} \tag{16}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = \frac{\rho c S_H v_H}{S} \frac{\cos(kl)\cos(k(L-l))}{\sin(kL)} e^{j\omega t} \tag{17}$$

$$v_1(l,t) = \frac{S_H v_H}{S} \frac{\sin(kl)\cos(k(L-l))}{\sin(kL)} e^{j\omega t} \tag{18}$$

$$v_2(l,t) = -\frac{S_H v_H}{S} \frac{\cos(kl)\sin(k(L-l))}{\sin(kL)} e^{j\omega t} \tag{19}$$

From the pressure equation, the resonance conditions are as follows: In other words, it is determined only by the total length of the tube, regardless of the position l of the hole.

$$kL = n\pi \tag{20}$$

$$\therefore L = n\frac{\lambda}{2} \tag{21}$$

There are two types of anti-resonance conditions. In both cases, the standing wave becomes a node at the position of the hole.

$$kl = (2m-1)\frac{\pi}{2} \tag{22}$$

$$\therefore l = (2m - 1)\frac{\lambda}{4} \tag{23}$$

$$k(L - l) = (2m - 1)\frac{\pi}{2} \tag{24}$$

$$\therefore L - l = (2m - 1)\frac{\lambda}{4} \tag{25}$$

2.2. When There is a Sound Pressure Source

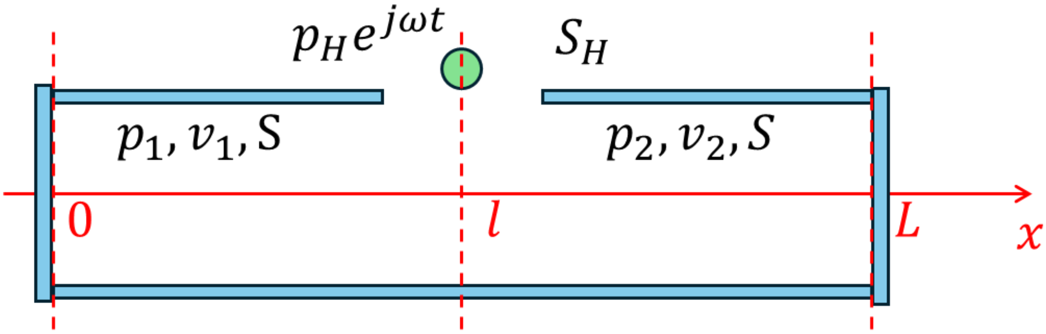

Fig. 2 1-Dimensional Tube which Have Sound Pressure Source

Fig. 2 shows a 1-dimensional tube with holes at both closed ends and a sound pressure source. From the condition of sound pressure continuity, the following equation holds:

$$p_1(l,t) = p_2(l,t) = p_H e^{j\omega t} \tag{26}$$

The boundary conditions are shown:

$$v_1(0,t) = 0 \tag{27}$$

$$v_2(L,t) = 0 \tag{28}$$

From these equations, the following system of four equations is obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \tag{29}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \tag{30}$$

$$jkA_1 - jkB_1 = 0 \tag{31}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{32}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{p_H}{2\rho j\omega \cos(kl)} \tag{33}$$

$$B_1 = \frac{p_H}{2\rho j\omega \cos(kl)} \tag{34}$$

$$A_2 = \frac{p_H}{2\rho j\omega \cos(k(L-l))} e^{jkL} \tag{35}$$

$$B_2 = \frac{p_H}{2\rho j\omega \cos(k(L-l))} e^{-jkL} \tag{36}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = p_H e^{j\omega t} \tag{37}$$

$$v_1(l,t) = \frac{p_H}{\rho c} \tan(kl)\, e^{j\omega t} \tag{38}$$

$$v_2(l,t) = -\frac{p_H}{\rho c} \tan(k(L-l))\, e^{j\omega t} \tag{39}$$

From the particle velocity equation, the equation for the total flow rate Q is as follows:

$$Q = Sv_1 - Sv_2 = \frac{Sp_H}{\rho c}\{\tan(kl) + \tan\big(k(L-l)\big)\}e^{j\omega t} \tag{40}$$

The resonance conditions are as follows. In all cases, standing waves form nodes at the location of the holes. Air is drawn in without limit, and the acoustic power is maximized.

$$kl = (2m-1)\frac{\pi}{2} \tag{41}$$

$$\therefore l = (2m-1)\frac{\lambda}{4} \tag{42}$$

$$k(L-l) = (2m-1)\frac{\pi}{2} \tag{43}$$

$$\therefore L - l = (2m-1)\frac{\lambda}{4} \tag{44}$$

Furthermore, the anti-resonance condition is as follows:

$$kl = -k(L-l) + n\pi$$

$$kL = n\pi \tag{45}$$

$$\therefore L = n\frac{\lambda}{2} \tag{46}$$

3. In the Case of a 1-Dimensional Tube with Holes, One End open and the Other End Closed.

3.1. When There is a Vibration Source in the Hole.

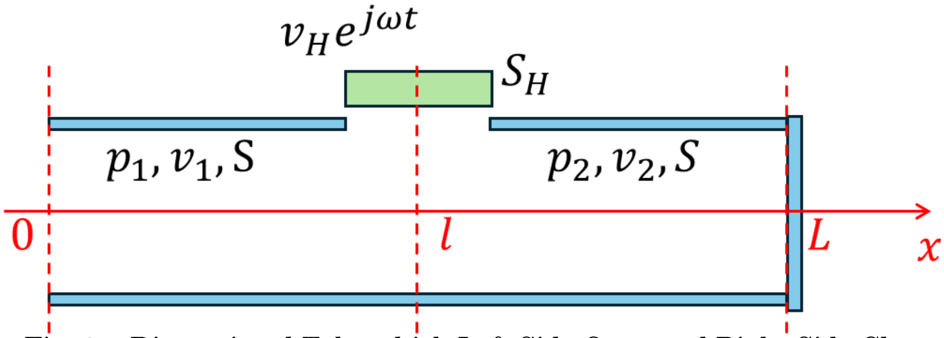

Fig. 3 1-Dimensional Tube which Left Side Open and Right Side Close and Have Vibration Source at the Hole

Fig. 3 shows a 1-dimensional tube with an open end and a closed end, where the vibration source is in the hole. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) \tag{47}$$

$$Sv_1(l,t) = Sv_2(l,t) + S_H v_H e^{j\omega t} \tag{48}$$

The boundary conditions are shown:

$$p_1(0,t) = 0 \tag{49}$$

$$v_2(L,t) = 0 \tag{50}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = A_2 e^{-jkl} + B_2 e^{jkl} \tag{51}$$

$$S\left(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\right) = S\left(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\right) + S_H v_H \tag{52}$$

$$A_1 + B_1 = 0 \tag{53}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{54}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{S_H v_H}{2jkS} \frac{\cos(k(L-l))}{\cos(kL)} \tag{55}$$

$$B_1 = -\frac{S_H v_H}{2jkS} \frac{\cos(k(L-l))}{\cos(kL)} \tag{56}$$

$$A_2 = \frac{S_H v_H}{2jkS} \frac{\sin(kl)}{j\cos(kL)} e^{jkL} \tag{57}$$

$$B_2 = \frac{S_H v_H}{2jkS} \frac{\sin(kl)}{j\cos(kL)} e^{-jkL} \tag{58}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = \frac{\rho c S_H v_H}{S} \frac{\sin(kl)\cos(k(L-l))}{\cos(kL)} e^{j\omega t} \tag{59}$$

$$v_1(l,t) = \frac{S_H v_H}{S} \frac{\cos(kl)\cos(k(L-l))}{\cos(kL)} e^{j\omega t} \tag{60}$$

$$v_2(l,t) = -\frac{S_H v_H}{S} \frac{\sin(kl)\sin(k(L-l))}{\cos(kL)} e^{j\omega t} \tag{61}$$

From the pressure equation, the resonance condition at $x = l$ is as follows: In other words, it is determined only by the total length of the tube, regardless of the position l of the hole.

$$kL = (2n-1)\frac{\pi}{2} \tag{62}$$

$$\therefore L = (2n-1)\frac{\lambda}{4} \tag{63}$$

There are two types of anti-resonance conditions at $x = l$. In both cases, the standing wave becomes a node at the position of the hole.

$$kl = m\pi \tag{64}$$

$$\therefore l = m\frac{\lambda}{2} \tag{65}$$

$$k(L-l) = (2m-1)\frac{\pi}{2} \tag{66}$$

$$\therefore L - l = (2m-1)\frac{\lambda}{4} \tag{67}$$

Here, the equations for pressure and particle velocity at $x = 0$ are as follows.

$$p(0,t) = 0 \tag{68}$$

$$v_1(l,t) = \frac{S_H v_H}{S} \frac{\cos(k(L-l))}{\cos(kL)} e^{j\omega t} \tag{69}$$

From the particle velocity equation, the resonance condition for particle velocity is the same as the resonance condition for pressure. The anti-resonance condition is as follows:

$$k(L-l) = (2m-1)\frac{\pi}{2} \tag{70}$$

$$L - l = (2m-1)\frac{\lambda}{4} \tag{71}$$

3.2. When There Is a Sound Pressure Source in the Hole

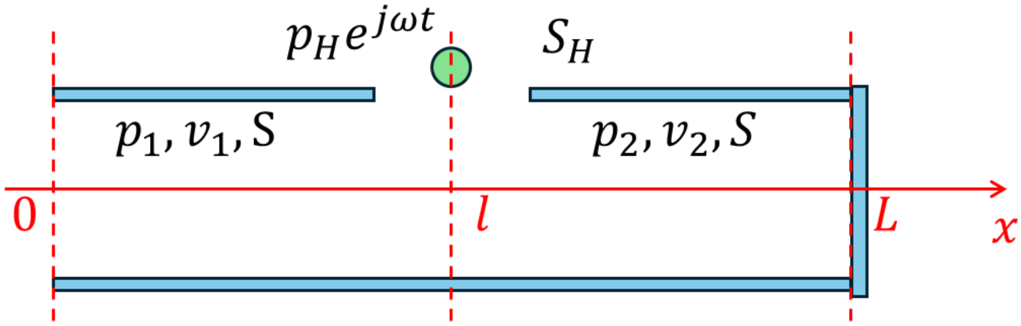

Fig. 4 1-Dimensional Tube which Left Side Open and Right Side Close and Have Sound Pressure Source at the Hole

Fig. 4 shows a 1-dimensional tube with an open end and a closed end, where the sound pressure source is located in the hole. From the condition of sound pressure continuity, the following equation holds:

$$p_1(l,t) = p_2(l,t) = p_H e^{j\omega t} \tag{72}$$

The boundary conditions are shown:

$$p_1(0,t) = 0 \tag{73}$$

$$v_2(L,t) = 0 \tag{74}$$

From these equations, the following system of four equations is obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \tag{75}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \tag{76}$$

$$A_1 + B_1 = 0 \tag{77}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{78}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{p_H}{2\rho\omega \sin(kl)} \tag{79}$$

$$B_1 = -\frac{p_H}{2\rho\omega \sin(kl)} \tag{80}$$

$$A_2 = \frac{p_H}{2\rho j\omega \cos(k(L-l))} e^{jkL} \tag{81}$$

$$B_2 = \frac{p_H}{2\rho j\omega \cos(k(L-l))} e^{-jkL} \tag{82}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = p_H e^{j\omega t} \tag{83}$$

$$v_1(l,t) = j\frac{p_H}{\rho c}\cot(kl)\,e^{j\omega t} \tag{84}$$

$$v_2(l,t) = j\frac{p_H}{\rho c}\tan(k(L-l))\,e^{j\omega t} \tag{85}$$

From the particle velocity equation, the equation for the total flow rate Q is as follows:

$$Q = Sv_1 - Sv_2 = j\frac{Sp_H}{\rho c}\{\cot(kl) - \tan\big(k(L-l)\big)\}e^{j\omega t}$$

$$= j\frac{Sp_H}{\rho c}\frac{\cos(kL)}{\sin(kl)\cos(k(L-l))}e^{j\omega t}\#(86)$$

The resonance conditions are as follows. In all cases, standing waves form nodes at the location of the holes. Air is drawn in without limit, and the acoustic power is maximized.

$$kl = m\pi \tag{87}$$

$$\therefore l = m\frac{\lambda}{2} \tag{88}$$

$$k(L-l) = (2m-1)\frac{\pi}{2} \tag{89}$$

$$\therefore L - l = (2m-1)\frac{\lambda}{4} \tag{90}$$

The anti-resonance condition is as follows:

$$kL = (2n-1)\frac{\pi}{2} \tag{91}$$

$$\therefore L = (2n-1)\frac{\lambda}{4} \tag{92}$$

Here, the equations for pressure and particle velocity at $x = 0$ are as follows:

$$p(0,t) = 0 \tag{93}$$

$$v_1(0,t) = j\frac{p_0}{\rho c}\frac{1}{\sin(kl)}e^{j\omega t} \tag{94}$$

From the particle velocity, the resonance condition is as follows:

$$kl = n\pi \tag{95}$$

$$\therefore l = n\frac{\lambda}{2} \tag{96}$$

3.3. When There Is a Vibration Source at the Open End

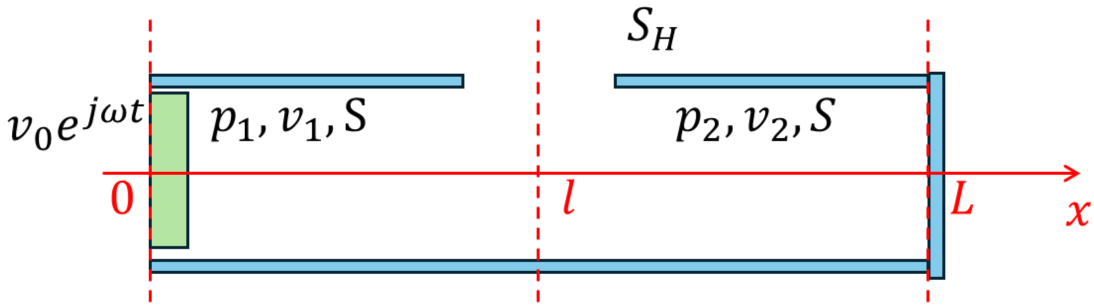

Fig. 5 1-Dimensional Tube which Right Side Close
and Have Vibration Source at the Left Side

Fig. 5 shows a 1-dimensional tube with an open end and a closed end on the other, where the vibration source is at the open end. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H = 0 \tag{97}$$

$$Sv_1(l,t) = Sv_2(l,t) + S_H v_H \tag{98}$$

The boundary conditions are shown below:

$$v_1(0,t) = v_0 e^{j\omega t} \tag{99}$$

$$v_2(L,t) = 0 \tag{100}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = 0 \tag{101}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = 0 \tag{102}$$

$$jkA_1 - jkB_1 = v_0 \tag{103}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{104}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = -\frac{v_0}{2jk}\frac{1}{\cos(kl)}e^{jkl} \tag{105}$$

$$B_1 = -\frac{v_0}{2jk}\frac{1}{\cos(kl)}e^{-jkl} \tag{106}$$

$$A_2 = 0 \tag{107}$$

$$B_2 = 0 \tag{108}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = 0 \tag{109}$$

$$v_1(l,t) = \frac{Sv_0}{S_H}\frac{1}{\cos(kl)} = v_H \tag{110}$$

$$v_2(l,t) = 0 \tag{111}$$

From the particle velocity equation, the resonance condition at $x = l$ is as follows: In other words, it is determined solely by the position l of the hole.

$$kl = (2n-1)\frac{\pi}{2} \tag{112}$$

$$\therefore l = (2n-1)\frac{\lambda}{4} \tag{113}$$

Here, the equations for pressure and particle velocity at x=0 are as follows:

$$p(0,t) = -\rho c v_0 e^{j\omega t} \tag{114}$$

$$v_1(l,t) = v_0 e^{j\omega t} \tag{115}$$

3.4. When There Is a Sound Pressure Source at the Open End

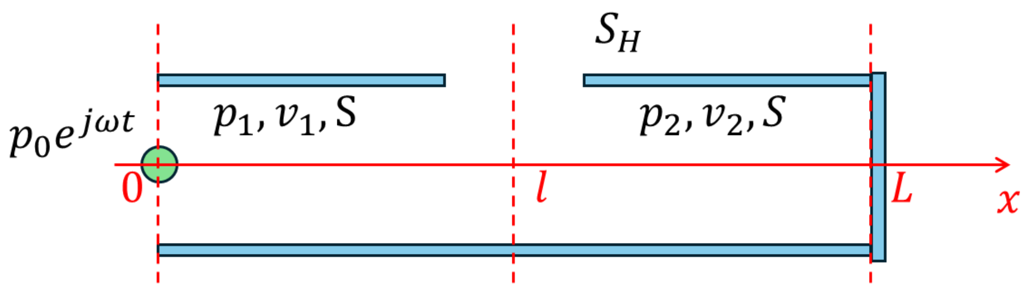

Fig. 6 1-Dimensional Tube which Right Side Close and Have Sound Pressure Source at the Left Side

Fig. **6** shows a 1-dimensional tube with an open end on one side and a closed end on the other, where there is a sound pressure source at the open end. From the condition of sound pressure continuity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = 0 \tag{116}$$

The boundary conditions are shown:

$$p_1(0,t) = p_0 e^{j\omega t} \tag{117}$$

$$v_2(L,t) = 0 \tag{118}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = 0 \tag{119}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = 0 \tag{120}$$

$$\rho j\omega A_1 + \rho j\omega\, B_1 = p_0 \tag{121}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{122}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{p_0}{2\rho\omega\sin(kl)} \tag{123}$$

$$B_1 = -\frac{p_0}{2\rho\omega\sin(kl)} \tag{124}$$

$$A_2 = 0 \tag{125}$$

$$B_2 = 0 \tag{126}$$

The equations for sound pressure and particle velocity when $x = l$ are as follows:

$$p(l, t) = 0 \tag{127}$$

$$v_1(l, t) = 0 \tag{128}$$

Here, the equations for pressure and particle velocity when $x = 0$ are as follows:

$$p(l, t) = p_0 e^{j\omega t} \tag{129}$$

$$v_1(l, t) = -\frac{p_0}{\rho c} e^{j\omega t} \tag{130}$$

3.5. When There Are Vibration Sources at the Open End and the Hole

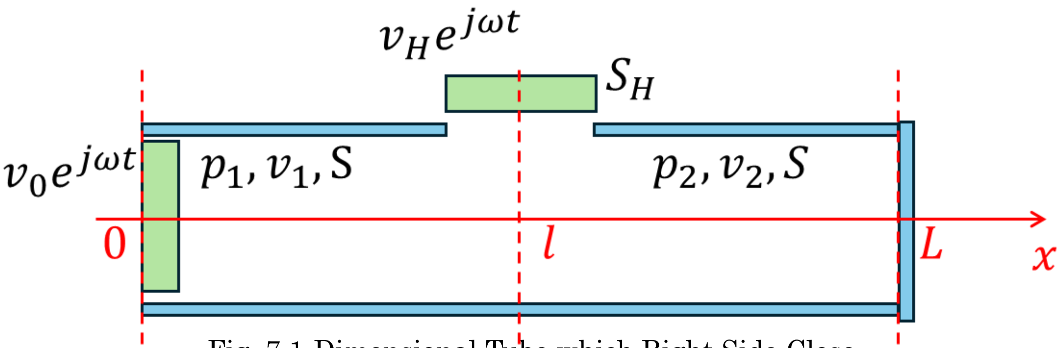

Fig. 7 1-Dimensional Tube which Right Side Close
and Have Vibration Source at the Left Side and the Hole

Fig. 7 shows a 1-dimensional tube with an open end on one side and a closed end on the other, where there are vibration sources at the open end and the hole. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l, t) = p_2(l, t) \tag{131}$$

$$Sv_1(l, t) = Sv_2(l, t) + S_H v_H e^{j\omega t} \tag{132}$$

The boundary conditions are shown:

$$v_1(0, t) = v_0 e^{j\omega t} \tag{133}$$

$$v_2(L, t) = 0 \tag{134}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1e^{-jkl} + B_1e^{jkl} = A_2e^{-jkl} + B_2e^{jkl} \tag{135}$$

$$S\left(jkA_1e^{-jkl} - jkB_1e^{jkl}\right) = S\left(jkA_2e^{-jkl} - jkB_2e^{jkl}\right) + S_Hv_H \tag{136}$$

$$jkA_1 - jkB_1 = v_0 \tag{137}$$

$$jkA_2e^{-jkL} - jkB_2e^{jkL} = 0 \tag{138}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{2Sv_0e^{j2kL} - S_Hv_H\left(e^{j2kL}e^{-jkl} + e^{jkl}\right)}{2jkS(e^{j2kL} - 1)} \tag{139}$$

$$B_1 = \frac{2Sv_0 - S_Hv_H\left(e^{j2kL}e^{-jkl} + e^{jkl}\right)}{2jkS(e^{j2kL} - 1)} \tag{140}$$

$$A_2 = \frac{\left(2Sv_0 - S_Hv_H\left(e^{jkl} + e^{-jkl}\right)\right)e^{j2L}}{2jkS(e^{j2kL} - 1)} \tag{141}$$

$$B_2 = \frac{2Sv_0 - S_Hv_H\left(e^{jkl} + e^{-jkl}\right)}{2jkS(e^{j2kL} - 1)} \tag{142}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = \left[\frac{\rho c}{\sin(kL)}\left\{v_0\cos\left(k(L-l)\right) - \frac{S_Hv_H}{S}\cos(kl)\sin(k(L-l)\,)\right\}\right]e^{j\omega t} \tag{143}$$

$$v_1(l,t) = \left[\frac{1}{\sin(kL)}\left\{v_0\sin\left(k(L-l)\right) - \frac{S_Hv_H}{S}\cos(kl)\cos(k(L-l)\,)\right\}\right]e^{j\omega t} \tag{144}$$

$$v_2(l,t) = \left[\frac{1}{\sin(kL)}\left\{v_0\sin\left(k(L-l)\right) - \frac{S_Hv_H}{S}\sin(kl)\sin(k(L-l)\,)\right\}\right]e^{j\omega t} \tag{145}$$

From the pressure equation, the resonance condition at $x = l$ is as follows. That is, it is determined only by the total length of the tube, regardless of the position l of the hole.

$$kL = n\pi \tag{146}$$

$$\therefore L = n\frac{\lambda}{2} \tag{147}$$

The anti-resonance condition at $x = l$ is given by the following:

$$\tan(k(L-l)) = \frac{Sv_0}{S_Hv_H}\frac{1}{\cos(kl)} \tag{148}$$

Here, the equations for pressure and particle velocity at $x = 0$ are as follows.

$$p(0,t) = \left[\frac{\rho c}{\sin(kL)}\left\{v_0\cos(kL) - \frac{S_Hv_H}{S}\cos\left(k(L-l)\right)\right\}\right]e^{j\omega t} \tag{149}$$

$$v_1(l,t) = v_0 \tag{150}$$

From the particle velocity equation, the resonance condition for particle velocity is the same as the resonance condition for pressure. The anti-resonance condition is as follows:

$$v_0 \cos(kL) = \frac{S_H v_H}{S} \cos\big(k(L-l)\big) \tag{151}$$

3.6. When There Are Sound Pressure Sources at the Open End and the Hole

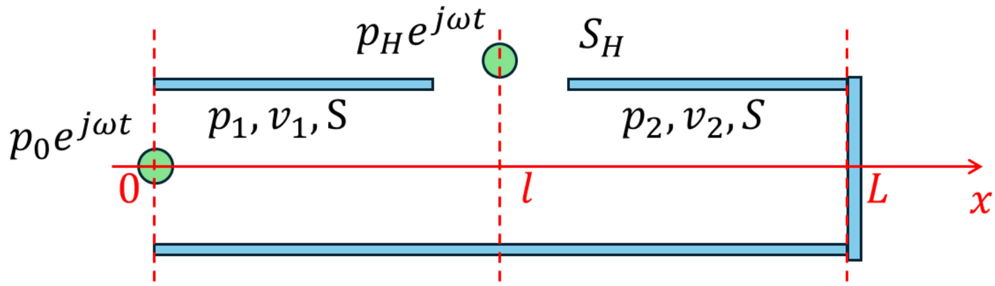

Fig. 8 1-Dimensional Tube which Right Side Close
and Have Sound Pressure Source at the Left Side and the Hole

Fig. 8 shows a one-dimensional tube with an open end on one side and a closed end on the other, where there is a sound pressure source at the hole. From the condition of sound pressure continuity, the following equation holds:

$$p_1(l,t) = p_2(l,t) = p_H e^{j\omega t} \tag{152}$$

The boundary conditions are shown:

$$p_1(0,t) = p_0 e^{j\omega t} \tag{153}$$

$$v_2(L,t) = 0 \tag{154}$$

From these equations, the following four simultaneous equations are obtained.

$$\rho j\omega A_1 e^{-jkl} + \rho j\omega B_1 e^{jkl} = p_H \tag{155}$$

$$\rho j\omega A_2 e^{-jkl} + \rho j\omega B_2 e^{jkl} = p_H \tag{156}$$

$$\rho j\omega A_1 + \rho j\omega B_1 = p_0 \tag{157}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{158}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{p_0 e^{jkl} - p_H}{2\rho\omega \sin(kl)} \tag{159}$$

$$B_1 = \frac{p_H - p_0 e^{-jkl}}{2\rho\omega \sin(kl)} \tag{160}$$

$$A_2 = \frac{p_H}{2\rho j\omega \cos(k(L-l))} e^{jkL} \tag{161}$$

$$B_2 = \frac{p_H}{2\rho j\omega \cos(k(L-l))} e^{-jkL} \tag{162}$$

Here, the equations for pressure and particle velocity when $x = l$ are as follows:

$$p(l,t) = p_H e^{j\omega t} \tag{163}$$

$$v_1(l,t) = \frac{k}{\rho\omega \sin(kl)} \{p_H \cos(kl) - p_0\} e^{j\omega t} \tag{164}$$

$$v_2(l,t) = \frac{k}{\rho\omega \cos(k(L-l))} \{p_H \sin\big(k(L-l)\big)\} e^{j\omega t} \tag{165}$$

From the particle velocity equation, the resonance conditions are as follows. In all cases, standing waves form nodes at the location of the hole. Air is drawn in without limit, and the acoustic power is maximized.

$$kl = m\pi \tag{166}$$

$$\therefore l = m\frac{\lambda}{2} \tag{167}$$

$$k(L-l) = (2m-1)\frac{\pi}{2} \tag{168}$$

$$\therefore L - l = (2m-1)\frac{\lambda}{4} \tag{169}$$

The anti-resonance condition is as follows:

$$kL = (2n-1)\frac{\pi}{2} \tag{170}$$

$$\therefore L = (2n-1)\frac{\lambda}{4} \tag{171}$$

Here, the equations for pressure and particle velocity at $x = 0$ are as follows:

$$p(0,t) = p_0 e^{j\omega t} \tag{172}$$

$$v_1(0,t) = \frac{1}{\rho c}\left\{\frac{p_0 \cos(kl) - p_H}{\sin(kl)}\right\} e^{j\omega t} \tag{173}$$

From the particle velocity, the resonance condition is as follows:

$$kl = n\pi \tag{174}$$

$$\therefore l = n\frac{\lambda}{2} \tag{175}$$

4. Conclusion

While research on automobile whistling and aspiration noise has been conducted in the past, we believe that understanding the properties of whistling and aspiration sounds using a 1-dimensional tube as a simple model is necessary in fields such as automobiles.

Therefore, we discussed the case where there is a vibration source and a sound pressure source in various 1-dimensional tubes with holes. We derived the system of equations that must be satisfied and discussed their properties.

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