2x2 Strongly Pay-off Salient Games

By

Somdeb Lahiri

ORCID: https://orcid.org/0000-0002-5247-3497

(Formerly with) PD Energy University, Gandhinagar (EU-G), India.

March 7, 2025.

This version: April 26, 2026.

Abstract

We provide a necessary and sufficient condition for a salient action profile to be a focal point for 2×2 pay-off salient games. In view of the necessary and sufficient condition for the existence of a focal point, we are able to provide an exact criteria that selects the focal point if it exists. We introduce the concepts of dominant salient action and dominant for 2×2 strongly pay-off salient games and provide a necessary and sufficient condition for a dominant salient action profile to be a dominant focal point. We also provide a procedure to implement a dominant focal point when one exists.

1. Introduction: The class of all 2×2 bi-matrix games is a very "rich" in content, but non-tautological theoretical propositions in such contexts are very difficult to formulate without invoking "randomizations". This was implicitly acknowledged by John von-Neumann in his seminal work on minimax theorem in 1928.

Antagonism or anti-coordination is what two-person zero sum games (as in Stahl (1999) and/or Ferguson (2000)) are all about. Prisoner's dilemma, due to Merrill Flood and Melvin Dresher (https://plato.stanford.edu/entries/prisoner-dilemma/) which is by far the most well-known 2×2 bi-matrix game, illustrates "instability" of Pareto optimal outcomes and "stability" of Pareto dominated outcomes. In section 5.4 of Luce and Raiffa (1957) there is an anecdote for the dilemma described Flood and Dresher (see Flood (1952)).

In this note we are concerned with "coordination problems" among agents that arise due to private information about pay-offs that are not shared with other participants engaged in interactive decision making. Thus, in the context of 2×2 bi-matrix games it is reasonable to assume that each player knows its own pay-off matrix. However, there may be 2×2 bi-matrix games, in which, unlike zero sum games or prisoner's dilemma, the decision makers are neither antagonistic to one another nor symmetrically positioned and hence may not be able to decipher information about the pay-off matrix of the "other" decision maker. Under such circumstances, decision making may need to be based entirely on the basis of one's own pay-off matrix. Alternatively and very frequently "coordination problems" arise due to the existence of multiple equilibria where one equilibrium is better than the other. Hence there is much to be gained by coordinating and choosing the better equilibrium. Coordination problems of this type was first discussed in Schelling (1960) and an elaborate discussion of coordination games is available in (the preface of) Cooper (1999).

In a comparatively recent paper, Leland and Schneider (2018) consider a sub-class of 2×2 bi-matrix games for each of which it is possible to resolve the kind of coordination problems that occur in Schelling (1960), with each decision maker relying solely on the information contained in its pay-off matrix. In such games, the pay-off matrix of the row player is such that the absolute value of the difference in its pay-offs in the first column is different from the absolute value of the difference in its pay-offs in the second column and the pay-off matrix of the column player is such that the absolute value of the difference in its pay-offs in the first row is different from the absolute value of the difference its pay-offs in the second row. In Leland and Schneider (2018) such games are referred to as "pay-off salient games". For pay-off salient games, Leland and Schneider (2018) recommends that the row player considers the column of its pay-off matrix that has the higher absolute value of the difference in its pay-offs and chooses the row in that column with the higher pay-off. We refer to such a choice as a salient action for the row player. Similarly, they recommend the column player considers the row of its pay-off matrix that has the higher absolute value of the difference in its pay-offs and chooses from that row the column with the higher pay-off. We refer to such a choice as a salient action for the column player. As observed in (2015), *a pay-off salient action for a decision maker in a pay-off salient game, is its unique expected pay-off maximizing action, assuming that the other decision maker will be choosing each of its two actions with "probability half"*. We refer to the pair consisting of the salient actions as a salient action profile. In Leland and Schneider (2018) a salient action profile that is an equilibrium as a focal point. Our first proposition in this paper is a necessary and sufficient condition for a salient action profile to be a focal point.

The first problem in Leland and Schneider (2018) is with the procedure suggested by them to choose a focal point when one exists. We provide an example (example 2) to show that their procedure does not unambiguously locate the unique focal point if there are multiple equilibria. In view of the necessary and sufficient condition for the existence of a focal point, we are able to provide an exact criteria that selects the focal point if it exists. The procedure suggests that a decision maker chooses its weakly dominant action, if any such is available. Otherwise, it chooses its salient action.

The second problem is with the concept of a focal point. There may be a unique equilibrium which is better for both decision makers than the salient action profile. Thus, the solution concept forgoes a better outcome simply because it is not a salient action profile and further the salient action profile is unstable. This is illustrated in our example 4. In order to circumvent such inadequacies, we introduce the concept of a dominant salient action. A dominant salient action is a best reply to the other decision maker′s weakly dominant strategy if any such exist; otherwise it is the salient action of the decision maker, the latter being as defined in Leland and Schneider (2018). We refer to the pair consisting of the dominant salient actions, as a dominant salient action profile. We refer to a dominant salient action profile that is an equilibrium as a dominant focal point.

Unambiguous choice of a dominant action profile requires, that there is a unique best response to a weakly dominant action. Thus, we consider "strongly pay-off salient matrix games", which satisfy the property that there is a unique best response against a weakly dominant action, if there be any, or in the absence of weakly dominant actions, it is a pay-off salient game as in Leland and Schneider (2018). Our main result for 2×2 strongly pay-off salient games is a necessary and sufficient condition for a dominant salient action profile to be a dominant profile. Both the proposition and its proof is very similar to the necessary and sufficient condition for a salient action profile to be a focal point, that we provide here.

The implementation of a dominant salient action action profile, requires that if there is a weakly dominant action available to a decision maker *at least* one such decision maker communicates the same to the other decision maker. Hence, the implementation of a dominant salient action profile *requires* "absence of malevolence" between the decision makers. It *does not require* "shared benevolence" towards each other.

Important observation: Note that for $\alpha\epsilon\{1, 2\}$, $\{1, 2\}\backslash\{\alpha\} = \{3- \alpha\}$.

2. The Framework of 2×2 Pay-off Salient Bi-matrix Games: Let the row player be denoted by 1, the column player by 2 and the pay-off matrix of player $h\epsilon\{1,2\}$ be $G_h = \begin{bmatrix} x_h(1,1) & x_h(1,2) \\ x_h(2,1) & x_h(2,2) \end{bmatrix}$.

The action set of the each player is {1, 2} and a pair $(i, j)\epsilon\{1, 2\}\times\{1, 2\}$,with 'i' denoting the action (row) chosen by the row player (1) and 'j' denoting the action (column) chosen by the column player (2), is referred to as an **action profile**.

The pair (G_1, G_2) is referred to as a **2×2 bi-matrix game**.

An action profile (i^*, j^*) is an **equilibrium** for (G_1, G_2) if for all $i, j\epsilon\{1, 2\}$: $x_1(i^*, j^*) \geq x_1(i, j^*)$ and $x_2(i^*, j^*) \geq x_2(i^*, j)$.

An equilibrium (i^*, j^*) for (G_1, G_2) is said to be a **strict equilibrium** if : $x_1(i^*, j^*) > x_1(3- i^*, j^*)$ and $x_2(i^*, j^*) > x_2(i^*, 3 - j^*)$.

An action i^* is said to be a **weakly dominant action for the row player** if for all $j\epsilon\{1, 2\}$: $x_1(i^*, j) \geq x_1(3-i^*, j)$.

An action j^* is said to be a **weakly dominant action for the column player** if for all $i\epsilon\{1, 2\}$: $x_2(i, j^*) \geq x_2(i, 3 - j^*)$.

It is easy to see that if $\max_{j\in\{1,2\}} |x_1(1,j) - x_1(2,j)| > \min_{j\in\{1,2\}} |x_1(1,j) - x_1(2,j)| = 0$, then $\operatorname{argmax}_{j\in\{1,2\}} |x_1(1,j) - x_1(2,j)|$ is a singleton $\{j_1^*\}$ and $\operatorname{argmax}_{i\in\{1,2\}} x_1(i, j_1^*)$ must also be a singleton, with its sole member being a weakly dominant action for the row player.

Similarly, if $\max_{i\in\{1,2\}} |x_2(i, 1) - x_2(i, 2)| > \min_{i\in\{1,2\}} |x_2(i, 1) - x_2(i, 2)| = 0$, then $\operatorname{argmax}_{i\in\{1,2\}} |x_2(i, 1) - x_2(i, 2)|$ is a singleton $\{i_2^*\}$ and $\operatorname{argmax}_{j\in\{1,2\}} x_2(i_2^*, j)$ must also be a singleton, with its sole member being a weakly dominant action for the column player.

The game (G_1, G_2) is said to be a **2×2 pay-off salient (bi-matrix) game** if $\operatorname{argmax}_{j\in\{1,2\}} |x_1(1,j) - x_1(2,j)|$ is a singleton $\{j_1^*\}$ and $\operatorname{argmax}_{i\in\{1,2\}} |x_2(i, i) - x_2(i, 2)|$ is a singleton $\{i_2^*\}$.

Thus $|x_1(1,j_1^*) - x_1(2,j_1^*)| > |x_1(1, 3 - j_1^*) - x_1(2, 3 - j_1^*)| \geq 0$ and $|x_2(i_2^*,1) - x_2(i_2^*,2)| > |x_2(3 - i_2^*, 1) - x_1(3- i_2^*, 2)| \geq 0$.

Since $|x_1(1, j_1^*) - x_1(2, j_1^*)| > 0$ and $|x_2(i_2^*,1) - x_2(i_2^*,2)| > 0$, let $\{i_1^*\} = \operatorname{argmax}_{i\in\{1,2\}} x_1(i, j_1^*)$ and $\{j_2^*\} = \operatorname{argmax}_{j\in\{1,2\}} x_2(i_2^*, j)$.

i_1^* is said to be the **salient action for the row player** and j_2^* is said to be the **salient action for the column player**.

The pair (i_1^*,j_2^*) is said to the **salient action profile** for (G_1, G_2).

Since $x_1(i_1^*, j_1^*) > x_1(3- i_1^*, j_1^*)$ *if a weakly dominant action exists for the row player it must be unique, equal to* i_1^* *and satisfy* $x_1(i_1^*, 3- j_1^*) \geq x_1(3- i_1^*, 3- j_1^*)$.

Similarly, since $x_2(i_2^*, j_2^*) > x_2(i_2^*, 3 - j_2^*)$ *if a weakly dominant action exists for the column player, it must be unique, equal to* j_2^* *and satisfy* $x_2(3- i_2^*, j_2^*) \geq x_2(3 - i_2^*, 3 - j_2^*)$.

Observation 1: An action is pay-off salient for a decision maker in a 2×2 pay-off salient game <u>if and only if</u> it is the decision maker's unique expected pay-off maximizing action assuming that the other decision maker chooses any one its two actions with equal probability. For, since $x_1(i_1^*, j_1^*) - x_1(3- i_1^*, j_1^*) > |x_1(i_1^*, 3-j_1^*) - x_1(3- i_1^*, 3-j_1^*)| \geq 0$, there are two possible cases. In the first case $x_1(i_1^*, 3-j_1^*) \geq x_1(3- i_1^*, 3-j_1^*)$, so that $|x_1(i_1^*, 3-j_1^*) - x_1(3- i_1^*, 3-j_1^*)| = x_1(i_1^*, 3-j_1^*) - x_1(3- i_1^*, 3-j_1^*) \geq 0$. Thus, $x_1(i_1^*, j_1^*) > x_1(3- i_1^*, j_1^*)$ and $x_1(i_1^*, 3-j_1^*) \geq x_1(3- i_1^*, 3-j_1^*)$ implies $\frac{1}{2}x_1(i_1^*, j_1^*) + \frac{1}{2}x_1(i_1^*, 3- j_1^*) > \frac{1}{2}x_1(3- i_1^*, j_1^*) + \frac{1}{2}x_1(3-i_1^*, 3- j_1^*)$. In the other case, $x_1(i_1^*, 3-j_1^*) < x_1(3- i_1^*, 3-j_1^*)$, so that $|x_1(i_1^*, 3-j_1^*) - x_1(3- i_1^*, 3-j_1^*)| = x_1(3- i_1^*, 3-j_1^*) - x_1(i_1^*, 3-j_1^*) > 0$. Thus, $x_1(i_1^*, j_1^*) - x_1(3- i_1^*, j_1^*) > x_1(3- i_1^*, 3-j_1^*) - x_1(i_1^*, 3-j_1^*)$ implies $x_1(i_1^*, j_1^*) + x_1(i_1^*, 3-j_1^*) > x_1(3- i_1^*, j_1^*) + x_1(3- i_1^*, 3-j_1^*)$ and hence $\frac{1}{2}x_1(i_1^*, j_1^*) + \frac{1}{2}x_1(i_1^*, 3- j_1^*) > \frac{1}{2}x_1(3-i_1^*, j_1^*) + \frac{1}{2}x_1(3-i_1^*, 3- j_1^*)$. Thus, the pay-off salient action for the row player maximizes its expected pay-off assuming that the column player chooses each of its actions with equal probability. Since in each case the inequality is strict, it follows that in a pay-off salient game, there is a unique expected pay-off maximizing action for the row player under the assumption that the column player chooses its actions with equal probability and this action is the unique salient action for the row player. A similar argument proves that in pay-off salient games an action is pay-off salient for the column player if and only if is its unique expected pay-off maximizing action, assuming that the row player chooses any one of its two actions with probability half. ▌

If in addition (i_1^*, j_2^*) is an equilibrium, then it is said to be a **focal point**.

We will refer to $\sigma_1 = |x_1(1, j_1^*) - x_1(2, j_1^*)|$ as the **higher column gap** (in G_1) and we will refer to $\sigma_2 = |x_2(i_2^*, 1) - x_2(i_2^*, 2)|$ as the **higher row gap** (in G_2).

$x_1(i_1^*, j_1^*)$ is said to be the **salient pay-off for the row-player** and $x_2(i_2^*, j_2^*)$ is said to be the **salient pay-off for the column-player**.

The following example of a 2×2 pay-off salient bi-matrix game shows that it may not have an equilibrium and hence its salient action profile is not an equilibrium.

Example 1: $x_1(1, 1) > x_1(2, 1)$, $x_1(1, 2) < x_1(2, 2)$, $x_2(1, 1) < x_2(1, 2)$, $x_2(2, 1) > x_2(2, 2)$.

Suppose, $x_1(1, 1) - x_1(2, 1) > x_1(2, 2) - x_1(1, 2) > 0$ and $x_2(1, 2) - x_2(1, 1) > x_2(2, 1) - x_2(2, 2) > 0$.

Thus, $(i_1^*, j_1^*) = (1, 1)$ and $(i_2^*, j_2^*) = (1, 2)$.

However, (G_1, G_2) has no equilibrium action profile.

Clearly (G_1, G_2) does not have any focal point.

3. The Leland-Schneider Method: We present below the method for implementing a salient action profile suggested by Leland and Schneider (2018) and discuss a problem concerning the procedure.

Let $i^{LS} = i_1^*$ if $\sigma_1 \geq \sigma_2$ and $i^{LS} \epsilon \operatorname*{argmax}_{i \epsilon \{1, 2\}} x_1(i, j_2^*)$ if $\sigma_1 < \sigma_2$.

Let $j^{LS} = j_2^*$ if $\sigma_2 \geq \sigma_1$ and $j^{LS} \epsilon \operatorname*{argmax}_{j \epsilon \{1, 2\}} x_2(i_1^*, j)$ if $\sigma_2 < \sigma_1$.

Example 2: $x_1(1, 1) > x_1(2, 1)$, $x_1(1, 2) > x_1(2, 2)$, $x_2(1, 1) = x_2(1, 2)$, $x_2(2, 2) > x_2(2, 1)$.

Suppose, $x_1(1, 1) - x_1(2, 1) > x_1(1, 2) - x_1(2, 2) > x_2(2, 2) - x_2(2, 1) > 0$.

$(i_1^*, j_1^*) = (1, 1)$ and $(i_2^*, j_2^*) = (2, 2)$.

$(i_1^*, j_2^*) = (1, 2)$ is a focal point. However, (1, 1) is another equilibrium.

$\sigma_1 = x_1(1, 1) - x_1(2, 1) > x_2(2, 2) - x_2(2, 1) = \sigma_2 > 0$.

According to the Leland-Schneider method, $i^{LS} = 1$ and $j^{LS} \epsilon \{1, 2\}$.

Thus, the Leland-Schneider method may not select the focal point if there are multiple equilibria.

4. A Complete Characterization of Focal Point: The following proposition provides a necessary and sufficient condition for a salient action profile to be a focal point.

Proposition 1: (i_1^*, j_2^*) is a focal point <u>if and only if</u> either $j_2^* = j_1^*$ and $i_1^* = i_2^*$ (in which case (i_1^*, j_2^*) is a strict equilibrium) <u>or</u> $i_1^* = i_2^*$, $j_1^* \neq j_2^*$ and i_1^* is a weakly dominant action for the row player <u>or</u> $j_1^* = j_2^*$, $i_1^* \neq i_2^*$ and j_2^* is a weakly dominant action for the column player <u>or</u> $i_1^* \neq i_2^*$, $j_1^* \neq j_2^*$, i_1^* is a weakly dominant action for the row player and j_2^* is a weakly dominant action for the column player.

Proof: (i_1^*, j_2^*) is a focal point if and only if (i_1^*, j_2^*) is an equilibrium.

Suppose either $j_2^* = j_1^*$ and $i_1^* = i_2^*$ <u>or</u> $i_1^* = i_2^*$, $j_1^* \neq j_2^*$ and i_1^* is a weakly dominant action for the row player <u>or</u> $j_1^* = j_2^*$, $i_1^* \neq i_2^*$ and j_2^* is a weakly dominant action for the column player <u>or</u> $i_1^* \neq i_2^*$, $j_1^* \neq j_2^*$, i_1^* is a weakly dominant action for the row player and j_2^* is a weakly dominant action for the column player.

Case 1: $(i_1^*, j_1^*) = (i_2^*, j_2^*)$.

In this case, by the definition of (i_h^*, j_h^*) for $h \in \{1, 2\}$: $x_1(i_1^*, j_2^*) = x_1(i_1^*, j_1^*) > x_1(3- i_1^*, j_1^*) = x_1(3 - i_1^*, j_2^*)$ and $x_2(i_1^*, j_2^*) = x_2(i_2^*, j_2^*) > x_2(i_2^*, 3- j_2^*)$.

Thus (i_1^*, j_2^*) is a strict equilibrium.

Case 2: $j_1^* \neq j_2^*$, $i_1^* = i_2^*$ and i_1^* is a weakly dominant action for the row player.

$x_2(i_1^*, j_2^*) = x_2(i_2^*, j_2^*) > x_2(i_2^*, 3 - j_2^*)$, by the definition of (i_2^*, j_2^*).

$x_1(i_1^*, j_2^*) \geq x_1(3- i_1^*, j_2^*)$ since i_1^* is a weakly dominant action for the row player.

Thus (i_1^*, j_2^*) is an equilibrium.

Case 3: $i_1^* \neq i_2^*$, $j_1^* = j_2^*$ and j_2^* is a weakly dominant action for the column player.

Thus, $x_1(i_1^*, j_2^*) = x_1(i_1^*, j_1^*) > x_1(3 - i_1^*, j_2^*) = x_1(3 - i_1^*, j_1^*)$, by the definition of (i_1^*, j_1^*) and $x_2(i_1^*, j_2^*) \geq x_2(i_1^*, 3 - j_2^*)$ for $j \in \{1, 2\} \backslash \{j_2^*\}$ since i_1^* is a weakly dominant action for the row player.

Thus (i_1^*, j_2^*) is an equilibrium.

Case 4: $i_1^* \neq i_2^*$, $j_1^* \neq j_2^*$, i_1^* is a weakly dominant action for the row player and j_2^* is a weakly dominant action for the column player.

Thus, $x_1(i_1^*, j_2^*) \geq x_1(3 - i_1^*, j_2^*)$ since i_1^* is a weakly dominant action for the row player and $x_2(i_1^*, j_2^*) \geq x_1(i_1^*, 3- j_2^*)$ since j_2^* is a weakly dominant action for the column player.

Thus (i_1^*, j_2^*) is an equilibrium.

To prove the converse suppose (i_1^*, j_2^*) is an equilibrium. If $(i_1^*, j_2^*) = (i_2^*, j_1^*)$, then form the definitions of (i_1^*, j_1^*) and (i_2^*, j_2^*) it follows that (i_1^*, j_2^*) is a strict equilibrium.

Hence suppose $(i_1^*, j_2^*) \neq (i_2^*, j_1^*)$.

Suppose $j_1^* \neq j_2^*$ and $i_1^* \neq i_2^*$. Thus, $x_2(i_1^*, j_2^*) \geq x_2(i_1^*, j_1^*)$. Further, $x_2(i_2^*, j_2^*) > x_2(i_2^*, j_1^*)$ by definition of (i_2^*, j_2^*). Thus, j_2^* is a weakly dominant action for the column player.

Also, $x_1(i_1^*, j_2^*) \geq x_1(i_2^*, j_2^*)$. Further, $x_1(i_1^*, j_1^*) > x_1(i_2^*, j_1^*)$ by definition of (i_1^*, j_1^*).

Thus, i_1^* is a weakly dominant action for the row player.

Suppose, $j_1^* \neq j_2^*$ and $i_1^* = i_2^*$.

$x_1(i_1^*, j_2^*) \geq x_1(3-i_1^*, j_2^*)$.

$x_1(i_1^*, j_1^*) > x_1(3-i_1^*, j_1^*)$ by definition of (i_1^*, j_1^*).

Thus, i_1^* is a weakly dominant action for the row player.

Suppose, $j_1^* = j_2^*$ and $i_1^* \neq i_2^*$.

$x_2(i_1^*, j_2^*) \geq x_2(i_1^*, 3\text{-}j_2^*)$.

Further, $x_2(i_2^*, j_2^*) > x_2(i_2^*, 3\text{-}j_2^*)$ by definition of (i_2^*, j_2^*).

Thus, j_2^* is a weakly dominant action for the column player. Q.E.D.

5. An alternative procedure for focal point selection: In this section we suggest an alternative method for implementing a salient action profile that circumvents the ambiguity associated with the procedure suggested in Leland and Schneider (2018).

Proposition 2: If i^* is a weakly dominant action for the row player then $i^* = i_1^*$. If j^* is a weakly dominant action for the column player then $j^* = j_2^*$.

Proof: Since $x_1(i_1^*, j_1^*) > x_1(3\text{-}\ i_1^*, j_1^*)$, if i^* is a weakly dominant strategy then $i^* = i_1^*$. Since $x_2(i_2^*, j_2^*) > x_2(i_2^*.\ 3\text{-}\ j_2^*)$, if j^* is a weakly dominant strategy then $j^* = j_1^*$. Q.E.D.

Thus, the focal point can be uniquely selected by the following procedure.

If neither has a weakly dominant action, then let each choose their unique salient action.

If the row player has a weakly dominant action but the column player does not have a weakly dominant action, then let the row player choose its weakly dominant action and the column player choose its salient action.

If the column player has a weakly dominant action but the row player does not have a weakly dominant action, then let the column player choose its weakly dominant action and the row player choose its salient action.

If both have weakly dominant actions, then let each choose its weakly dominant action.

The outcome of this procedure must be a focal point.

The procedure may be briefly summarized as follows: If a player has a weakly dominant action, then it chooses it; if not it chooses its salient action.

Example 3: For h$\in\{1, 2\}$, $G_h = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. Here $(i_1^*, j_1^*) = (i_2^*, j_2^*)$ = (1, 1) and (1, 1) is a focal point. Neither player has a "weakly dominant strategy. The procedure we suggested requires the players to choose their salient actions, i.e., the row player chooses the first row and the column player chooses the first column.

6. Dominant Focal Points for 2×2 Strongly Pay-off Salient Bi-matrix **Games:** The next example illustrates the inadequacy of focal point as defined in Leland and Schneider (2018).

Example 4: Let $G_1 = \begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$.

For G_1, the column with the higher column gap is the second column so that (i_1^*, j_1^*) = (2, 2).

For G_2, the row with the higher row gap is the second row so that (i_2^*, j_2^*) = (2, 1).

Thus, the salient action profile is (2, 1).

Since $x_1(1, 1) = 5 > 4 = x_1(2, 1)$, (2, 1) is not an equilibrium and hence not a focal point.

The salient pay-off for the row player is 4 and the salient pay-off for the column player is 3.

The unique equilibrium for (G_1, G_2) is (1, 1) yielding a pay-off of 5 to each player and hence (1, 1) is better for both players than the salient action profile.

Thus (G_1, G_2) has an extremely desirable equilibrium although it does not have a focal point.

Note than column 1 is a weakly dominant action and if the column player chooses column 1, then the best response for the row-player is row 1.

The 2×2 bi-matrix game (G_1, G_2) is said to be a **2×2 strongly pay-off salient (bi-matrix game)** if :

(i) $x_1(1, j) \neq x_1(2, j)$ for some $j \in \{1, 2\}$ and $x_2(i, 1) \neq x_2(i, 2)$ for some $i \in \{1, 2\}$;

(ii) either the column player has a weakly dominant action j and $|x_1(1, j) - x_1(2, j)| > 0$ or $\underset{j \in \{1, 2\}}{\operatorname{argmax}} |x_1(1, j) - x_1(2, j)|$ is *a singleton*;

(iii) either the row player has a weakly dominant action i and $|x_2(i, 1) - x_2(i, 2)| > 0$ or $\underset{i \in \{1, 2\}}{\operatorname{argmax}} |x_2(i, 1) - x_2(i, 2)|$ is *a singleton.*

(i) ensures that if a player has a weakly dominant action then it is unique.

Let j_1^{**} be the unique weakly dominant action for the column player if any such exist; otherwise, let $\{j_1^{**}\} = \underset{j \in \{1, 2\}}{\operatorname{argmax}} |x_1(1, j) - x_1(2, j)|$.

Let i_2^{**} be the unique weakly dominant action of the row player if any such exist; otherwise, let $\{i_2^{**}\} = \underset{i \in \{1, 2\}}{\operatorname{argmax}} |x_2(i, 1) - x_2(i, 2)|$.

Let $\{i_1^{**}\} = \underset{i \in \{1,2\}}{\operatorname{argmax}} x_1(i, j_1^{**})$ and $\{j_2^{**}\} = \underset{j \in \{1, 2\}}{\operatorname{argmax}} x_2(i_2^{**}, j)$.

It is easy to verify that if j_1^{**} is a weakly dominant action for the column player then $j_2^{**} = j_1^{**}$ and if i_2^{**} is a weakly dominant action for the row player then $i_1^{**} = i_2^{**}$.

We will refer to i_1^{**} as the **dominant salient action for the row player** and to j_2^{*} as the **dominant salient action for the column player**.

We will refer to (i_1^{**}, j_2^{**}) as the **dominant salient action profile**.

If a dominant salient action profile is an equilibrium then we will refer to it as the **dominant focal point**.

In example 4 above, (1, 1) is a dominant salient action profile and also an equilibrium. Hence, it is a dominant focal point.

Example 5: Let G$_1$ = $\begin{bmatrix} 6 & 2 \\ 4 & 3 \end{bmatrix}$ and G$_2$ = $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.

Neither player has a weakly dominant strategy.

It is easily verified that j_1^{**} = 1, i_2^{**} = 1, i_1^{**} = 1 and j_2^{**} = 1.

Thus, (1, 1) is a dominant salient action profile which is also an equilibrium. Thus (1, 1) is a dominant focal point. Note that $(i_1^{**}, j_1^{**}) = (i_2^{**}, j_2^{**})$ = (1, 1).

7. A Complete Characterization of Dominant Focal Point: The main result of this paper is the following necessary and sufficient condition for a dominant salient action profile to be a dominant focal point.

Proposition 3: (i_1^{**}, j_2^{**}) is a dominant focal point <u>if and only if</u> either $j_2^{**} = j_1^{**}$ and $i_1^{**} = i_2^{**}$ (in which case (i_1^{**}, j_2^{**}) is a strict equilibrium) <u>or</u> $i_1^{**} = i_2^{**}$, $j_1^{**} \neq j_2^{**}$ and i_1^{**} is a weakly dominant action for the row player <u>or</u> $j_1^{**} = j_2^{**}$, $i_1^{**} \neq i_2^{**}$ and j_2^{**} is a weakly dominant action for the column player <u>or</u> $i_1^{**} \neq i_2^{**}$, $j_1^{**} \neq j_2^{**}$, i_1^{**} is a weakly dominant action for the row player and j_2^{*} is a weakly dominant action for the column player.

The proof of proposition 3 is analogous to the proof of proposition 1.

In view of proposition 3, a procedure to implement a dominant focal point if it exists is the following: If a decision-maker has a weakly dominant action it chooses it and shares this information with the other decision-maker, the latter responding with its dominant salient action. If not, then they proceed under the assumption that neither has a weakly dominant action and each chooses its salient action.

Note: This paper is based almost entirely on pages 30-34 of the pdf file available at the following link:

https://digitalcommons.chapman.edu/cgi/viewcontent.cgi?article=1156&context=esi_pubs. This reflects our personal preference about where to start reading a scientific document. A "like-minded" reader may wish to ignore the first three paragraphs in the introductory section of this paper ending with Cooper (1999), if not our introductory section.

The material cited in the paper were accessed by us and are noted in the reference section. We doubt our ability to create a comprehensive list of citations comparable to the one in Leland and Schneider (2018). Hence, with the help of Dr. Suresh Kumar Pillai (Chief Librarian and Information Officer, PDEU), we have extracted those citations in Leland and Schneider (2018) that do not appear in our "references" and created a section entitled "additional bibliographic material". We would like to put on record our sincere gratitude to Dr. Pillai for help in creating the bibliographic material.

References

1. Cooper, R. W. (1999): Coordination Games: Complementarities and Macroeconomics. Cambridge University Press.

2. Ferguson, T. S. (2000): Game Theory: Part II (Class notes for Math 167, Fall 2000).

3. Flood, M. M. (1952): Some Experimental Games. US Air Force Project, Rand Research Memorandum (RM-789-1), The RAND Corporation.

4. Leland, J.W. and Schneider, M. (2015): Salience and Strategy of Choice in 2×2 Games. Games, Volume 6, pages 521-529. DOI: 10.3390/g6040521

5. Leland, J.W. and Schneider, M. (2018): A Theory of Focal Points in 2×2 Games (Appendix: A Model of Focal Points with Endogenous Salient Pay-offs). *Journal of Economic Psychology*, Volume 65, Pages 75-89. DOI: https://psycnet.apa.org/doi/10.1016/j.joep.2018.03.002

6. Luce, R. D., and Raiffa, H. (1957). Games and decisions: Introduction and critical survey. Wiley.

7. Schelling, T. C. (1960): The Strategy of Conflict. Harvard University Press.

8. Stahl, S. (1999): A Gentle Introduction to GAME THEORY. Volume 13 in Mathematical World Series, American Mathematical Society (Indian Edition).

Additional Bibliographic Material:

Agranov, M. & Schotter, A. (2012). Ignorance is bliss: An experimental study of the use of ambiguity and vagueness in the coordination games with asymmetric payoffs. American Economic Journal: Microeconomics. 4, 77-103.

Bacharach, M., & Bernasconi, M. (1997). The variable frame theory of focal points: An experimental study. Games and Economic Behavior. 19, 1-45.

Bordalo, P., Gennaioli, N., & Shleifer, A. (2012). Salience theory of choice under risk. Quarterly Journal ofEconomics. 127, 1243-1285.

Bosch-Domenech, A., & Vriend, N. (2013). On the role of non-equilibrium focal points as coordination devices. Journal ofEconomic Behavior and Organization, 94, 52-67.

Camerer, C., Ho, T. & Chong, J.K. (2004). A cognitive hierarchy model of games. The Quarterly Journal of Economics. 119, 816-898.

Colman, A. M., (2006). Thomas C. Schelling's psychological decision theory: Introduction to a special issue. Journal ofEconomic Psychology. 27, 603-608.

Colman, A. M., Pulford, B. D., & Lawrence, C. L. (2014). Explaining strategic coordination: Cognitive hierarchy theory, strong Stackelberg reasoning, and team reasoning. Decision, 1, 35.

Cooper, R.W., DeJong, D.V., Forsythe, R. & Ross, T.W. (1990). Selection criteria in coordination games: some experimental results. American Economic Review. 80, 218-233.

Crawford, V.P., Gneezy, U., & Rottenstreich, Y. (2008). The power of focal points is limited: Even minute payoff asymmetry may yield large coordination failures. American Economic Review. 98, 1443-1458.

Di Guida, S., & Devetag, G. (2013). Feature-based choice and similarity perception in normal-form games: An experimental study. Games. 4, 776-794.

Fisher, G., & Rangel, A. (2014). Intertemporal discount rates are mediated by relative attention. Manuscript.

Gold, N., & Sugden, R. (2006), Beyond individual choice2 Teams and frames in game theory.Princeton University Press, Princeton, NJ.

González-Vallejo, C. (2002). Making trade-offs: A probabilistic and context-sensitive model of choice behavior. Psychological Review, 109, 137-155.

Harsanyi, J.C. & Selten, R. (1988). A general theory ofequilibrium selection in games. MIT Press. Cambridge, MA.

Haruvy, E. & Stahl, D. (2004). Deductive versus inductive equilibrium selection: Experimental results. Journal of Economic Behavior and Organization. 53, 319-331.

Koszegi, B. & Szeidl, A. (2013). A model of focusing in economic choice. Quarterly Journal of Economics. 128, 53-104.

Leland, J. (1994). Generalized similarity Judgments: An alternative explanation for choice anomalies. Journal ofRisk and Uncertainty. 9, 151-172.

Leland, J. (1998). Similarity judgments in choice under uncertainty: A reinterpretation ofregret theory. Management Science. 44, 1-14.

Leland, J. (2002). Similarity judgments and anomalies in intertemporal choice. Economic Inquiry. 40, 574-581.

Leland, J. (2013). Equilibrium selection, similarity judgments, and the 'Nothing to Gain/Nothing to Lose' effect. Journal ofBehavioral Decision Making. 26, 418-428.

Loomes, G. & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. The Economic Journal. 92, 805-824.

Mehta, J., Starmer, C. & Sugden, R. (1994). The nature of salience: An experimental investigation ofpure coordination games. American Economic Review. 84, 658-73.

Reyna, V. & Brainerd, C. (1991). Fuzzy-trace theory and framing effects in choice: Gist extraction, truncation, and conversion. Journal of Behavior and Decision Making, 4, 249-262.

Rubinstein, A. (1988). Similarity and decision-making under risk (Is there a utility theory resolution to the Allais paradox?). Journal of Economic Theory. 46, 145-153.

Rubinstein, A. (2003). Is it "Economics and Psychology"? The case of hyperbolic discounting International Economic Review. 44, 1207-1216.

MA Scholten, M. & Read, D. (2010). The psychology of intertemporal trade offs. Psychological Review. 117, 925-944.

Shubik, M. (2012). What is a solution to a matrix game? Cowles Discussion Paper. No. 1866.

Sugden, R. (2011). Salience, inductive reasoning and the emergence of conventions. Journal of Economic Behavior and Organization, 79, 35-47.

Stahl, D.O., & Wilson, P.W. (1994). Experimental evidence on players' models of other players. Journal of Economic Behavior & Organization, 25, 309-327.

Stahl, D.O., & Wilson, P.W. (1995). On players' models of other players: Theory and experimental evidence. Games and Economic Behavior, 10, 218-254.

Tversky, A. (1969). The Intransitivity of Preferences. Psychological Review, 76, 31-48.

Van Huyck, J.B., Battalio, R.C. & Beil, R.O. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. American Economic Review. 80, 234-248.