Physics-Informed Residual Learning for Directional Wave Spectrum Reconstruction

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***Abstract*—This study investigates the problem of reconstructing the two-dimensional ocean wave spectrum using sparse input data using a Physics-informed Surrogate model equipped with residual learning. Analytical models such as the Fourier-based expansion, NDBC approximation, and the Maximum Entropy Method (MEM) provide a physically grounded solution but are limited either in flexibility or are computationally expensive. Pure machine learning models offer great flexibility but lack the physical consistency needed to handle diverse wave climates.**

To overcome these limitations, this study proposes a hybrid approach that combines the analytical baseline reconstruction with physics-informed data-driven residual learning. The baseline analytical reconstruction is constructed using the Fourier expansion from the one-dimensional spectrum and directional moments. The neural network is trained to learn the residual corrections of this baseline's outputs. Additional physics-informed constraints are incorporated to enforce energy conservation and directional moments consistency to better guide the model during the training process.

The experiments conducted on the ERA5 reanalysis datasets show that the hybrid combination of physics-informed data-driven models with residual learning techniques performed significantly better, improving the reconstruction accuracy compared to pure machine learning models. The proposed model outperformed standard machine learning approaches, even those that were equipped with residual learning without physics loss constraints, achieving substantial reductions in reconstruction error. However, analytical methods like MEM and NDBC remain the most accurate overall in diverse sea conditions. Generalization experiments on unseen North Sea data show that machine learning models are sensitive to domain shift, whereas analytical methods demonstrate great robustness.

Keywords: Directional wave spectrum; Spectral reconstruction; Physics-informed neural networks; Residual learning; Fourier expansion; Maximum entropy method; Ocean wave modelling; ERA5 reanalysis; Deep learning; Wave energy distribution

I. Introduction

The energy contained in the ocean surface waves is described by the directional wave spectrum S(f, θ). A clear view of this spectrum is important for wave forecasting, offshore marine engineering, and the growing use of ocean renewable energy applications. However, the complete 2D spectrum is not always easily accessible. It can be challenging to obtain due to its large size and varying spatial context.

One such way to obtain the spectrum is to use the numerical model outputs provided by reanalysis data [4]. This is, however, a very time-consuming task due to the frequency and spatial grids, as well as the time period the spectrum needs to be extracted for. Hence, in many scenarios, only the reduced spectral information is available and can be obtained using directional buoys or processing numerical analysis data [7]. This data consists of the 1D spectrum and the directional moments a1, b1, a2, and b2. The reconstruction of the full 2D spectrum using these inputs, however, is a high-dimensional ill-posed inverse problem.

Analytical methods like Fourier expansion or the NDBC approach which uses Longuet-Higgins Fourier Coefficients [9] are very efficient, but capture only limited spectral structures, while the Maximum Entropy Method (MEM), which is based on probability and widely used in this domain (Lygre & Krogstad 1986), is highly accurate but is computationally expensive and time-consuming due to its iterative nature in finding the optimal distribution with the most randomness. The pure machine learning models are known for their data-driven nature but lack the physical consistency of the real world and struggle with generalization [14]. Hence, the gap in this literature is that the existing approaches do not effectively combine the hybrid nature of a physical structure of the wave spectra and the data-driven flexibility of machine learning in order to follow the physical constraints of the system, improve the reconstruction accuracy, and remain computationally efficient.

The proposed approach introduced a physics-informed surrogate model that uses the residual learning framework. It trains the neural network with the physics-based baseline of the Fourier expansion to learn the residual corrections rather than the full 2D spectrum. Additional physics-informed loss functions like energy conservation, shape, and directional moment loss are used to better guide the model in making predictions more effectively [13] [6].

This research proposed a hybrid reconstruction framework that combines data-driven learning, residual learning, and physics-informed constraints, demonstrating that residual learning significantly improves reconstruction accuracy, outperforming both pure ML and baseline analytical methods like the Fourier expansion and NDBC approach (uses Longuet-Higgins Fourier Coefficients [9]). The proposed approach was evaluated on ERA5 reanalysis data [4] and compared with the Fourier expansion methods, NDBC reconstruction, MEM, and standard neural networks. This approach achieves a lower reconstruction error compared to the baseline methods, demonstrating the effectiveness of the hybrid approach of combining physics with machine learning.

II. Related Work

Early work on the wave spectrum theory was introduced by Pierson & Moskovitz, who developed spectral models for fully developed seas [12]. This was improved by Hasselmann [3] with the introduction of the JONSWAP spectrum by modeling fetch-limited wave growth [3]. These models represent the frequency distribution of the wave energy but do not represent the directional spreading.

The reconstruction of the directional spreading function using the Fourier coefficient was introduced by Longuet-Higgins [9] to model the directional distributions. Standard reconstruction methods use the first and second harmonic coefficients to represent this distribution [7]. The Maximum Entropy Method introduced by Lygre & Krogstad reconstructs the spectra using an iterative process to find the distribution that satisfies given constraints with maximum randomness (Lygre & Krogstad 1986). This approach has a high accuracy but is computationally expensive due to its iterative process.

Numerical wave models, such as those of the WAM model, attempt to simulate the evolution of waves using energy balance equations. The operational models, like the WAVEWATCH III, generate global-scale wave forecasts (Tolman 2009), and reanalysis datasets like ERA5, [4], provide global wave spectra and are consistent across long-term datasets. These datasets enable researchers to build data-driven modelling approaches, since these methods have proven to be computationally expensive to operate.

Recent work in the literature explores the use of deep learning for spectral reconstruction. Some of the existing approaches include CNN-based by [18], Deep learning wave models like DELWAVE [5], and the global wave spectrum model GWSM4C [17]. These methods have demonstrated that ML can capture the data-driven side of the complex spectral patterns but struggle with the lack of physical constraints and limited generalization across regions.

The introduction of Physics-Informed Neural Networks (PINNs) [13], which use physics constraints in the loss function of the model during training. Extensions to this methodology, such as those of [6] and [1], combine data-driven models with physical constraints, highlighting improved generalization and better physical consistency.

Operator learning models like the Fourier Neural Operator models by [8] and DeepONet by [10] learn nonlinear mappings between functions in the Fourier domain. These models have demonstrated their ability to be well-suited for high-dimensional, efficient systems, but they do rely strongly on the training data distribution.

III. Method

The goal of the proposed approach is to use the 1D spectrum S(f) and the directional moments (a1, b1, a2, b2) to reconstruct the full 2D directional wave spectrum S(f, θ) over the set frequency and direction bins. Hence, the problem is mapping from sparse spectral inputs to a high-dimensional output.

The baseline directional spectrum D(θ) is constructed using the Fourier expansion of the first and second harmonic components defined using a1, b1, a2, and b2. The baseline spectrum is computed as shown below, which helps achieve a physically consistent reconstruction.

$$S(f, \theta)_{baseline} = S(f) \cdot D(\theta)$$

The residual framework operates as follows. Instead of computing the full spectrum, the model learns the residual corrections defined as Δ S. This reduces the learning complexity, making the model focus on the deviations from the baseline.

$$\Delta S = S_{true} - S_{baseline}$$

For the neural network, a ResNet 1D architecture was used, or a 1D convolutional residual network. The output of the model is the flattened residual spectrum ΔS, which are the nonlinear corrections to the baseline spectrum. During training, the loss function consisted of the data-driven loss and the physics-based constraints.

The physics loss is composed of the spectral loss, which measures the difference between the predicted and the true spectra using Mean Squared Error (MSE), energy conservation loss to ensure integrated energy of the predicted spectrum matches the input S(f), and the directional moment loss applied to the directional moments to preserve the structure. Additionally, clamping is done to ensure non-negativity of spectral energy.

IV. Experimental Setup

The experiments conducted used the ERA5 wave reanalysis dataset, which provides the full 2D directional wave spectra in discretized frequency-direction grids. The datasets were extracted for a selected region and time period. These datasets were used as the training data due to their physical consistency.

The inputs selected were the 1D frequency spectrum S(f) and the directional moments a1, b1, a2, b2, and the outputs are the 2D directional spectrum S(f, θ), discretized into frequency-direction bins flattened for model training.

The dataset was divided into training, testing, and validation to keep the training and evaluation separate. Additional evaluation was performed on a dataset extracted using the North Sea bounds to assess the model's generalization in complex sea conditions.

The baseline methods evaluated were the First and Second harmonic Fourier reconstruction methods, the NDBC reconstruction method, and the Maximum Entropy Method. The machine learning models evaluated were standard MLP models, residual learning models, and physics-informed models.

The primary evaluation metrics used during the experiments were the Relative L2 error, which measures the overall measure of accuracy of the reconstruction by computing the normalized Euclidean distance between the predicted and target spectra, and the coefficient of determination (R-squared) to evaluate the variance of the predicted spectra from the true spectra, and to also indicate the degree of correlation between the two. Additionally, the MSE and the MAE metrics were also carried out. For the visualization, heat maps were used to analyze a 3-dimensional view of the 2d spectrum, allowing a qualitative assessment of the spectral energy spreading and overall structure.

V. RESULTS

TABLE I. BASELINE RESULTS

Method	L2 Error (Std)	Train R²	Eval R²	Eval MSE	Correlation
First Harmonic	0.67253	0.5472	0.631	3.921e-02	0.8035
Second Harmonic	0.50038	0.7538	0.863	1.451e-02	0.9372
MEM	0.0857	0.9919	0.987	1.37e-03	0.9935
NDBC	0.31230	0.9172	0.839	1.705e-02	0.9269

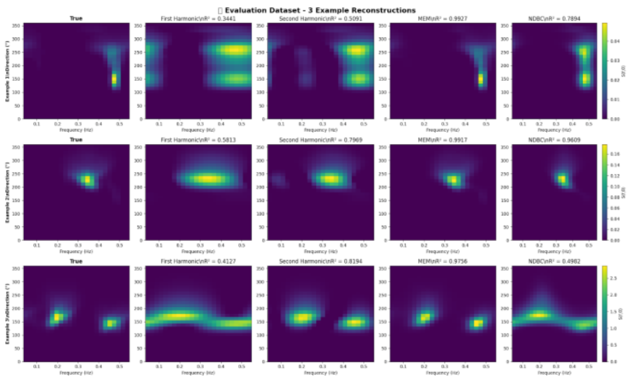

Fig 1: ERA5 Baseline Evaluation

TABLE II. MLP MODEL RESULTS

Model	Best Val Loss	L2 Error	R² Train	Eval MSE	Eval RMSE	Eval R²
MLP	0.008	0.62	0.7811	—	—	-0.507
Physics-Informed MLP	0.007	0.78	0.8021	0.167	0.408	-0.572
MLP Residuals	0.004	0.51	0.8734	0.050	0.225	0.522
Physics-Informed Residuals	0.003	0.28	0.9150	0.046	0.216	0.560

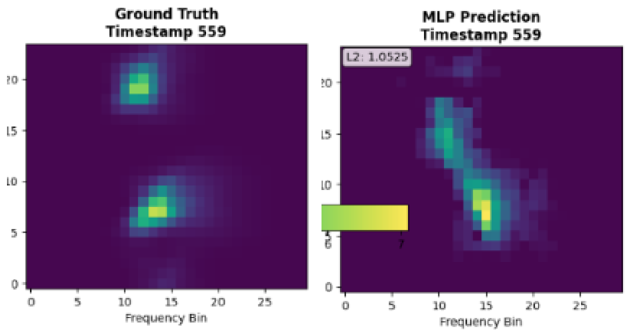

Fig 2: ERA5 MLP Evaluation

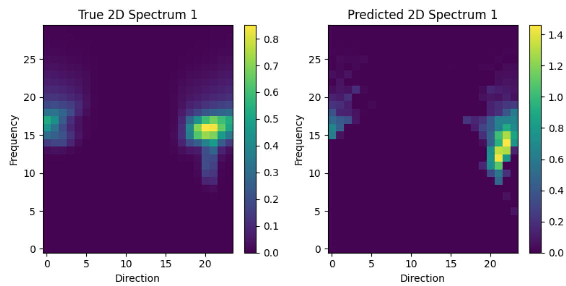

Fig 3: ERA5 MLP Residual Evaluation

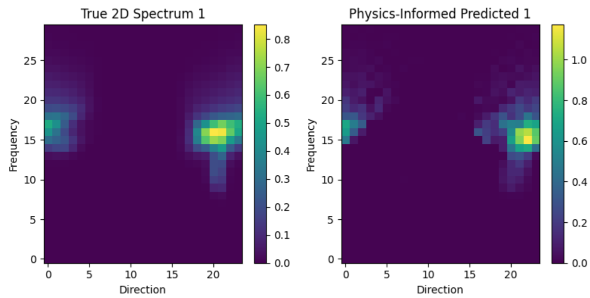

Fig 4: ERA5 MLP Physics Residual Evaluation

TABLE III. RESNET MODAL RESULTS

Model	Best Val Loss	L2 Error	R² Train	Eval MSE	Eval MAE	Eval R²
ResNet1 Physics	0.136	0.928	0.931	0.023	0.043	0.776
ResNet1 Residuals	0.002	0.354	0.945	0.018	0.023	0.828
ResNet1 Physics Residuals	0.001	0.199	0.967	0.018	0.020	0.826

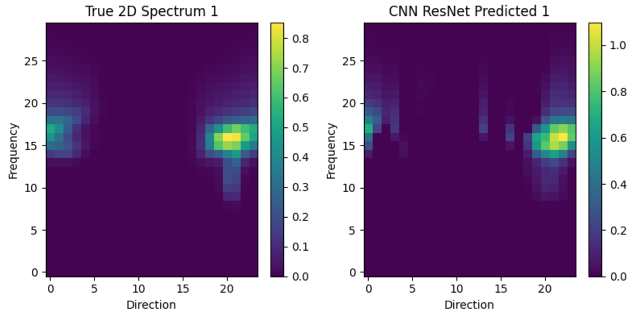

Fig 5: ERA5 ResNet1 Physics Residual Evaluation

Evaluation on North Sea

TABLE IV. BASELINE MODELS NORTH SEA RESULTS

Method / Harmonic	Eval R^2	Eval MSE	Correlation
First Harmonic	0.551659	8.421527e-01	0.757278
Second Harmonic	0.775299	4.220731e-01	0.896598
MEM	0.995470	8.508271e-03	0.997733
NDBC	0.961214	7.285458e-02	0.982182

TABLE V. RESNET NORTH SEA RESULTS

Model	Eval MSE	Eval MAE	Eval R^2
ResNet1 (Physics)	1.323697	0.161439	0.29529
ResNet1 (Residuals)	0.756218	0.118291	0.59740
ResNet1 (Res + Phys)	1.184137	0.127278	0.36959

VI. DISCUSSION

The first harmonic baseline method shows an L2 error of 0.67, indicating that using only the first-order directional moments is insufficient in capturing the full spectral structure. The second-harmonic method reduces L2 to 0.5, demonstrating that increasing the harmonics improves the model's predictions. The NDBC method further improves the L2 score to 0,31, suggesting that its formula manipulation captures additional features beyond the standard methods. And finally, the MEM method achieves a near-perfect score of L2 = 0.086 and R-squared of 0.99, demonstrating its robustness using moment constraints directly.

The pure baseline MLP shows a decent reconstruction of the spectrum with an L2 score of 0.63, but a negative R-squared, indicating poor generalization. The physics MLP performs worse with an L2 score of 0.79, suggesting that enforcing the physics constraints alone causes the losses to contradict themselves, i.e., data loss and physics loss. These results indicate that directly learning the pattern of the full high-dimensional spectrum is a difficult task, and pure ML models fail to capture sufficient spectral patterns.

Introducing residual learning into the model's flow significantly improved the performance in the MLP model, reducing the L2 score to 0.52. Hence, this proves that using residual learning reduces the learning complexity of the problem, leveraging the baseline to capture the dominant spectral structure, allowing the model to focus on the nonlinear deviations. The improvement of both the L2 and R-squared metrics indicates that residual learning helps both in accuracy improvement and generalization.

Adding a physics constraint to the residual MLP improves it further, reducing the L2 to 0.28 and R-squared to 0.56. This demonstrates that the physics constraints become effective when combined with residual learning, complementing each other. The combined constraints help to enforce energy constraints, preserve directional moments, thus improving the physical realism and reducing the reconstruction error.

The ResNet 1D architecture, when equipped with physics constraints show a high L2 error of 0.93 despite a high R-squared, indicating strong overfitting, not able to generalize well. The residual learning improved the model's performance with an L2 error of 0.35 and R squares of 0.83. The best results were shown with the hybrid integration of physics constraints with residual learning achieving an L2 score of 0.2 and the highest training R-squares of 0.97. These results show that improving the architecture alone does not guarantee better results, and that the performance gains are primarily improved by the combination of physics and residual learning.

The MEM model outperforms all the machine learning models due to its constraint optimization nature using the known moments. Also, all machine learning models show a significant performance drop during evaluation in the North Sea, even with the hybrid physics and residual learning combination. The analytical models show more robustness, with the NDBC method having an R-squared of 0.96. This indicates the data-driven and physical approach followed by the models is sensitive to the training data distribution, degrading performance under a domain shift to complex wave climates.

VII. LIMITATIONS

The primary limitation seen in this research is that the models' limited generizability across different wave climates. Their performance decreases significantly under unseen conditions like multi-modal and wind-dominated sea states, due to their sensitivity to the training distribution. Additionally, the physics constraints do improve the consistency of the system but do reduce its adaptability with reduced performance in unseen conditions. This shows that there is a need for context-aware physics constraints to adapt to the correct sea states.

The models depend on directional moments a1, b1, a2, and b2 as inputs, which are typically obtained using directional wave buoys. Hence, this approach has limited applicability in regions where there are only bulk parameters.

Based on the results obtained from the experiments, the proposed model does have a limited performance since it does not outperform the MEM model, because it learns to approximate the mapping under physics constraints, while the MEM directly solves the constrained problem.

The model was trained using ERA5 reanalysis data, which is model-generated and may be subject to biases that would affect its real-world applicability. Additionally, the data extraction process is very time-consuming, where one month of hourly data could take more than five hours.

VIII. CONCLUSION

This study researched the problem of reconstructing the two-dimensional directional wave spectrum using sparse spectral inputs. The proposed model was a physics-informed surrogate model that uses the residual learning framework. It uses a Fourier baseline reconstruction where the neural network learns the residual correction.

The results depict that the pure machine learning models are unable to accurately reconstruct the spectra. Integrating residual learning significantly improves the model, and adding physics constraints on top of that enhances the physical consistency, improving the accuracy even further. The proposed method achieves a lower reconstruction error compared to standard machine learning models. However, the MEM method, due to its constraint optimization techniques, remains the most accurate but takes a longer period of time.

The proposed hybrid models are, however, sensitive to a domain shift across different sea conditions, making them not able to generalize well, while analytical models like the NDBC approach or MEM show robustness in unseen conditions.

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