# Regarding the Frequency Response Function of a 1-Dimensional Tube with a Hole

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Abstract

While research on automobile whistling and aspiration noise has been conducted in the past, we believe that understanding the properties of whistling and aspiration sounds using a 1-dimensional tube as a simple model is necessary in fields such as automobiles. Therefore, we determined the frequency response function for a 1-dimensional tube with a hole in it.

Nomenclature

$S, S_H$: Cross-sectional area of the 1-dimensional tube and area of the hole in the 1-dimensional tube

$l$: Position of the hole in the 1-dimensional tube

$L$: Length of the 1-dimensional tube

$V$: Volume of the 1-dimensional tube

$p_1, v_1, \phi_1$: Sound pressure and particle velocity and velocity potential for $0 \leq x \leq l$

$A_1, B_1$: Amplitude of velocity potential for $0 \leq x \leq l$

$p_2, v_2, \phi_2$: Sound pressure and particle velocity and velocity potential for $l \leq x \leq L$

$A_2, B_2$: Amplitude of velocity potential for $l \leq x \leq L$

$\rho, c$: Density of air and speed of sound in air

$\omega$: Angular frequency

$f$: Frequency

$k$: Wavenumber

$t$: Time

$j$: Imaginary unit

$\lambda$; Wavelength

$p_H, v_H,$: Sound pressure and particle velocity at the hole at position $x = l$

$\delta_H$: Thickness of the 1-dimensional tube

$P_{total}$: External sound pressure given at the position of the hole at $x = l$

## 1. Introduction

In the past, studies have been conducted on the whistling and suction sounds of automobile (Calvo, Diaz, & San Roman, 2005) (Chien-Hsiung, Lung-Ming , Chang-Hsien , Yen-Loung , & Jik-Chang , 2009) (George, 1990) (Jagtiani, 1972) (Jung & Oh, 1995) (Münder & Carbon, 2022) (Oettle & Sims-Williams, 2017) (Qatu, Abdelhamid, Pang, & Sheng, 2009) (Wang, Chen, & Zhang, 2021) (Zhang, Meng, Li, & Zheng, 2022)。 However, to the best of the author's knowledge, there are no studies that discuss the properties of whistling and aspiration noise using a 1-dimensional tube as a simple model. Understanding the properties of whistling and aspiration noise using a 1-dimensional tube is considered essential in the automotive field.

Therefore, we will determine the frequency response function of a 1-dimensional tube with a hole in it.

This will be discussed below.

## 2. In the Case of a 1-Dimensional Tube with Holes and Closed Ends on Both Sides.

### 2.1. When There is a Source of Vibration

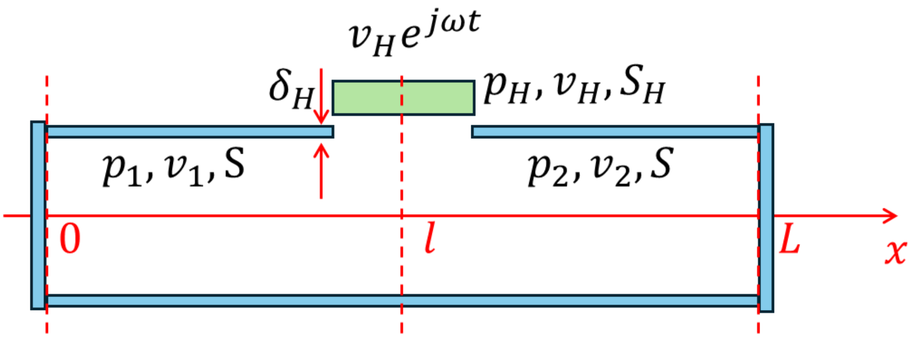


Fig. 1 1-Dimensional Tube which Have Vibration Source

Fig. 1 shows a case where the vibration source is in a 1-dimensional tube with holes in both closed ends. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{1}$$

$$-Sv_1(l,t) + Sv_2(l,t) = S_H v_H e^{j\omega t} \tag{2}$$

Here, the velocity potential and boundary conditions are shown.

$$\phi_1(x,t) = A_1 e^{j(\omega t - kx)} + B_1 e^{j(\omega t + kx)} \tag{3}$$

$$\phi_2(x,t) = A_2 e^{j(\omega t - kx)} + B_2 e^{j(\omega t + kx)} \tag{4}$$

$$v_1(0,t) = 0 \tag{5}$$

$$v_2(L,t) = 0 \tag{6}$$

We show the equations that the velocity potential, particle velocity, and sound pressure satisfy.

$$v = -\frac{\partial \phi}{\partial x} \tag{7}$$

$$p = \rho \frac{\partial \phi}{\partial t} + C \tag{8}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = \frac{p_H}{\rho j \omega} \tag{9}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = \frac{p_H}{\rho j \omega} \tag{10}$$

$$jkA_1 - jkB_1 = 0 \tag{11}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{12}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{p_H}{\rho j \omega} \frac{1}{2\cos(kl)} \tag{13}$$

$$B_1 = \frac{p_H}{\rho j \omega} \frac{1}{2\cos(kl)} \tag{14}$$

$$A_2 = \frac{p_H}{\rho j \omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{jkL} \tag{15}$$

$$B_2 = \frac{p_H}{\rho j \omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{-jkL} \tag{16}$$

Therefore, $p_1(x,t)$ and $v_1(x,t)$ can be found as follows.

$$p_1(x,t) = p_H \frac{\cos(kx)}{\cos(kl)} e^{j\omega t} \tag{17}$$

$$v_1 = \frac{p_H}{j\rho c} \frac{\sin(kx)}{\cos(kl)} e^{j\omega t} \tag{18}$$

Here, the equations for pressure and particle velocity when $x = 0, l$ are as follows:

$$p_1(0,t) = p_H \frac{1}{\cos(kl)} e^{j\omega t} \tag{19}$$

$$v_1(0,t) = 0 \tag{20}$$

$$p_1(l,t) = p_H e^{j\omega t} \tag{21}$$

$$v_1(l,t) = \frac{p_H}{j\rho c} \tan(kl)\, e^{j\omega t} \tag{22}$$

Here, from the continuity equation for particle velocity at the hole position, the following equation can be obtained.

$$-S\big(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\big) + S\big(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\big) = S_H v_H \tag{23}$$

Therefore, the relationship between $p_H$ and $v_H$ can be found as follows.

$$p_H = \frac{-\rho j\omega S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}} v_H \tag{24}$$

Now, at the location of the hole, the following equation of motion holds true.

$$\rho\delta_H \frac{\partial v_H}{\partial t} = P_{total} - p_H \tag{25}$$

Therefore, the frequency response functions of $P_{total}$ and $v_H$ can be obtained as follows.

$$v_H = \frac{1}{\rho j\omega\left\{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}\right\}} P_{total} \tag{26}$$

The frequency response functions of $P_{total}$ and $p_H$ are obtained as follows.

$$p_H = \frac{\frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}}{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}} P_{total} \tag{27}$$

## 2.2. When There is a Sound Pressure Source

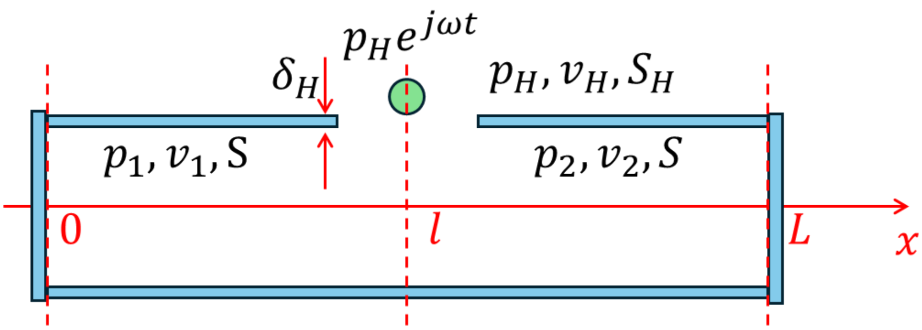


Fig. 2 1-Dimensional Tube which Have Pressure Source

Fig. 2 shows a 1-dimensional tube with holes at both closed ends and a sound pressure source. From the condition of sound pressure continuity, the following equation holds:

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{28}$$

$$-Sv_1(l,t) + Sv_2(l,t) = S_H v_H e^{j\omega t} \tag{29}$$

The boundary conditions are shown:

$$v_1(0,t) = 0 \tag{30}$$

$$v_2(L,t) = 0 \tag{31}$$

From these equations, the following system of four equations is obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \tag{32}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \tag{33}$$

$$jkA_1 - jkB_1 = 0 \tag{34}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{35}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \frac{p_H}{\rho j\omega}\frac{1}{2\cos(kl)} \tag{36}$$

$$B_1 = \frac{p_H}{\rho j\omega}\frac{1}{2\cos(kl)} \tag{37}$$

$$A_2 = \frac{p_H}{\rho j\omega}\frac{1}{2\cos\big(k(L-l)\big)} e^{jkL} \tag{38}$$

$$B_2 = \frac{p_H}{\rho j\omega}\frac{1}{2\cos\big(k(L-l)\big)} e^{-jkL} \tag{39}$$

Therefore, $p_1(x,t)$ and $v_1(x,t)$ can be found as follows.

$$p_1(x,t) = p_H \frac{\cos(kx)}{\cos(kl)} e^{j\omega t} \tag{40}$$

$$v_1 = \frac{p_H}{j\rho c}\frac{\sin(kx)}{\cos(kl)} e^{j\omega t} \tag{41}$$

Here, the equations for pressure and particle velocity when $x = 0, l$ are as follows:

$$p_1(0,t) = p_H \frac{1}{\cos(kl)} e^{j\omega t} \tag{42}$$

$$v_1(0,t) = 0 \tag{43}$$

$$p_1(l,t) = p_H e^{j\omega t} \tag{44}$$

$$v_1(l,t) = \frac{p_H}{j\rho c}\tan(kl)\, e^{j\omega t} \tag{45}$$

Here, from the continuity equation for particle velocity at the hole position, the following equation can be obtained.

$$-S\big(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\big) + S\big(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\big) = S_H v_H \tag{46}$$

Therefore, the relationship between $p_H$ and $v_H$ can be found as follows.

$$p_H = \frac{-\rho j\omega S_H}{Sk\big\{\tan\big(k(L-l)\big) + \tan(kl)\big\}} v_H \tag{47}$$

Now, at the location of the hole, the following equation of motion holds true.

$$\rho\delta_H \frac{\partial v_H}{\partial t} = P_{total} - p_H \tag{48}$$

Therefore, the frequency response functions of $P_{total}$ and $v_H$ can be obtained as follows.

$$v_H = \frac{1}{\rho j\omega\left\{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}\right\}} P_{total} \quad (49)$$

The frequency response functions of $P_{total}$ and $p_H$ are obtained as follows.

$$p_H = \frac{1}{1 - \frac{\delta_H Sk\{\tan(k(L-l)) + \tan(kl)\}}{S_H}} P_{total} \quad (50)$$

## 3. In the Case of a 1-Dimensional Tube with Holes, One End open and the Other End Closed.

### 3.1. When There Are Vibration Sources at the Open End and the Hole

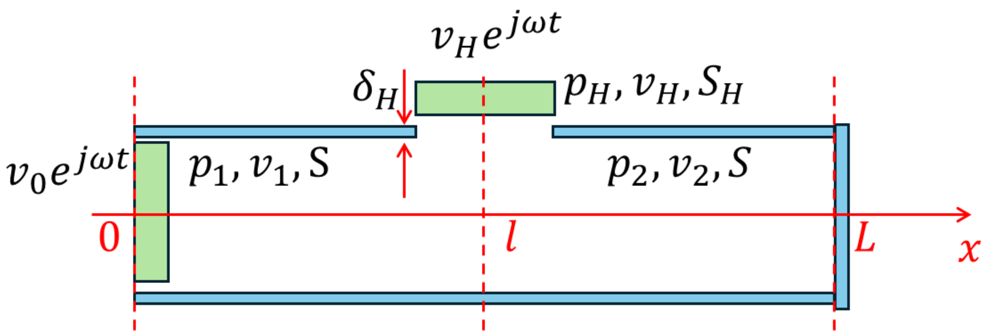


Fig. 3 1-Dimensional Tube which Left Side Open and Right Side Close and Have Vibration Source at Left Side and the Hole

Fig. 3 shows a 1-dimensional tube with an open end and a closed end, where the vibration source is in the open end and the hole. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H(t) \quad (51)$$

$$-Sv_1(l,t) + Sv_2(l,t) = S_H v_H e^{j\omega t} \quad (52)$$

The boundary conditions are shown:

$$v_1(0,t) = v_0 e^{j\omega t} \quad (53)$$

$$v_2(L,t) = 0 \quad (54)$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \quad (55)$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \quad (56)$$

$$jkA_1 - jkB_1 = v_0 \quad (57)$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \quad (58)$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \left\{\frac{p_H}{\rho j\omega} + \frac{v_0}{jk} e^{-jkl}\right\} \frac{1}{2\cos(kl)} \tag{59}$$

$$B_1 = \left\{\frac{p_H}{\rho j\omega} - \frac{v_0}{jk} e^{-jkl}\right\} \frac{1}{2\cos(kl)} \tag{60}$$

$$A_2 = \frac{p_H}{\rho j\omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{jkL} \tag{61}$$

$$B_2 = \frac{p_H}{\rho j\omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{-jkL} \tag{62}$$

Therefore, $p_1(x,t)$ and $v_1(x,t)$ can be found as follows.

$$p_1(x,t) = p_H \frac{\cos(kx)}{\cos(kl)} e^{j\omega t} + j\rho c v_0 \frac{\sin\big(k(l-x)\big)}{\cos(kl)} e^{j\omega t} \tag{63}$$

$$v_1 = \frac{p_H}{j\rho c} \frac{\sin(kx)}{\cos(kl)} e^{j\omega t} + v_0 \frac{\cos\big(k(l-x)\big)}{\cos(kl)} e^{j\omega t} \tag{64}$$

Here, the equations for pressure and particle velocity when $x = 0, l$ are as follows:

$$p(0,t) = p_H \frac{1}{\cos(kl)} e^{j\omega t} + j\rho c v_0 \tan(kl)\, e^{j\omega t} \tag{65}$$

$$v_1(0,t) = v_0 e^{j\omega t} \tag{66}$$

$$p(l,t) = p_H e^{j\omega t} \tag{67}$$

$$v_1(l,t) = \frac{p_H}{j\rho c} \tan(kl)\, e^{j\omega t} + v_0 \frac{1}{\cos(kl)} e^{j\omega t} \tag{68}$$

Here, from the continuity equation for particle velocity at the hole position, the following equation can be obtained.

$$-S\big(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\big) + S\big(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\big) = S_H v_H \tag{69}$$

Therefore, the relationship between $p_H$ and $v_H$ can be found as follows.

$$p_H = \frac{-\rho j\omega S_H}{Sk\big\{\tan\big(k(L-l)\big) + \tan(kl)\big\}} v_H - \Delta P_{v_0} \tag{70}$$

Here, the following relationship holds true.

$$\Delta P_{v_0} = \frac{j\rho c v_0}{\cos(kl)\big\{\tan\big(k(L-l)\big) + \tan(kl)\big\}} \tag{71}$$

Now, at the location of the hole, the following equation of motion holds true.

$$\rho \delta_H \frac{\partial v_H}{\partial t} = P_{total} - p_H \tag{72}$$

Therefore, the frequency response functions of $P_{total}$ and $v_H$ can be obtained as follows.

$$v_H = \frac{1}{\rho j\omega\left\{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}\right\}}\left(P_{total} + \Delta P_{v_0}\right) \tag{73}$$

The frequency response functions of $P_{total}$ and $p_H$ are obtained as follows.

$$p_H = \frac{\frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}}{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}}\left(P_{total} + \Delta P_{v_0}\right) - \Delta P_{v_0} \tag{74}$$

### 3.2. When There Are Sound Pressure Sources at the Open End and the Hole

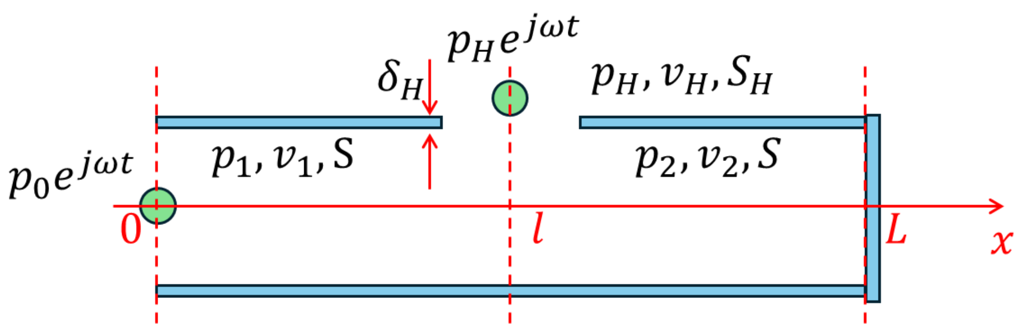


Fig. 4 1-Dimensional Tube which Left Side Open and Right Side Close and Have Pressure Source at the Left Side and the Hole

Fig. 4 shows a 1-dimensional tube with an open end and a closed end, where the sound pressure source is in the open end and the hole. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{75}$$

$$-Sv_1(l,t) + Sv_2(l,t) = S_H v_H e^{j\omega t} \tag{76}$$

The boundary conditions are shown:

$$p_1(0,t) = p_0 e^{j\omega t} \tag{77}$$

$$v_2(L,t) = 0 \tag{78}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \tag{79}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \tag{80}$$

$$\rho j\omega A_1 + \rho j\omega B_1 = p_0 \tag{81}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{82}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \left\{-\frac{p_H}{\rho j\omega} + \frac{p_0}{\rho j\omega}e^{-jkl}\right\}\frac{1}{2\sin(kl)} \tag{83}$$

$$B_1 = \left\{\frac{p_H}{\rho j\omega} - \frac{p_0}{\rho j\omega}e^{-jkl}\right\}\frac{1}{2\sin(kl)} \tag{84}$$

$$A_2 = \frac{p_H}{\rho j\omega}\frac{1}{2\cos\big(k(L-l)\big)}e^{jkL} \tag{85}$$

$$B_2 = \frac{p_H}{\rho j\omega}\frac{1}{2\cos\big(k(L-l)\big)}e^{-jkL} \tag{86}$$

Therefore, $p_1(x,t)$ and $v_1(x,t)$ can be found as follows.

$$p_1(x,t) = p_H\frac{\sin(kx)}{\sin(kl)}e^{j\omega t} + p_0\frac{\sin\big(k(l-x)\big)}{\sin(kl)}e^{j\omega t} \tag{87}$$

$$v_1 = -\frac{p_H}{j\rho c}\frac{\cos(kx)}{\sin(kl)}e^{j\omega t} + \frac{p_0}{j\rho c}\frac{\cos\big(k(l-x)\big)}{\sin(kl)}e^{j\omega t} \tag{88}$$

Here, the equations for pressure and particle velocity when $x = 0, l$ are as follows:

$$p(0,t) = p_0e^{j\omega t} \tag{89}$$

$$v_1(0,t) = -\frac{p_H}{j\rho c}\frac{1}{\sin(kl)}e^{j\omega t} + \frac{p_0}{j\rho c}\frac{1}{\tan(kl)}e^{j\omega t} \tag{90}$$

$$p(l,t) = p_He^{j\omega t} \tag{91}$$

$$v_1(l,t) = -\frac{p_H}{j\rho c}\frac{1}{\tan(kl)}e^{j\omega t} + \frac{p_0}{j\rho c}\frac{1}{\sin(kl)}e^{j\omega t} \tag{92}$$

Here, from the continuity equation for particle velocity at the hole position, the following equation can be obtained.

$$-S\big(jkA_1e^{-jkl} - jkB_1e^{jkl}\big) + S\big(jkA_2e^{-jkl} - jkB_2e^{jkl}\big) = S_Hv_H \tag{93}$$

Therefore, the relationship between $p_H$ and $v_H$ can be found as follows.

$$p_H = \frac{-\rho j\omega S_H}{Sk\big\{\tan\big(k(L-l)\big) - \cot(kl)\big\}}v_H - \Delta P_{p_0} \tag{94}$$

Here, the following relationship holds true.

$$\Delta P_{p_0} = \frac{p_0}{\sin(kl)\big\{\tan\big(k(L-l)\big) - \cot(kl)\big\}} \tag{95}$$

Now, at the location of the hole, the following equation of motion holds true.

$$\rho\delta_H\frac{\partial v_H}{\partial t} = P_{total} - p_H \tag{96}$$

Therefore, the frequency response functions of $P_{total}$ and $v_H$ can be obtained as follows.

$$v_H = \frac{1}{\rho j\omega\left\{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) - \cot(kl)\}}\right\}}\left(P_{total} + \Delta P_{p_0}\right) \tag{97}$$

The frequency response functions of $P_{total}$ and $p_H$ are obtained as follows.

$$p_H = \frac{\frac{S_H}{Sk\{\tan(k(L-l)) - \cot(kl)\}}}{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) - \cot(kl)\}}}\left(P_{total} + \Delta P_{p_0}\right) - \Delta P_{p_0} \tag{98}$$

Alternatively, it can be found as follows.

$$p_H = \frac{1}{1 - \frac{\delta_H Sk\{\tan(k(L-l)) - \cot(kl)\}}{S_H}}\left(P_{total} - \frac{\delta_H Sk}{S_H}\frac{p_0}{\sin(kl)}\right) \tag{99}$$

### 3.3. When There Is a Vibration Source at the Open End and a Sound Pressure Source in the Hole.

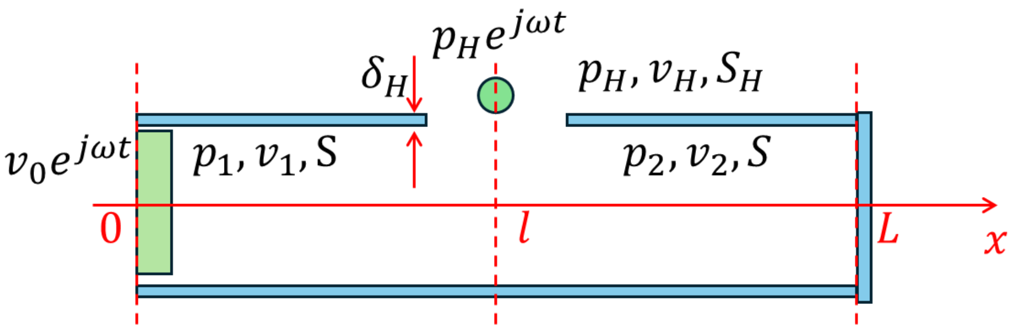


Fig. 5 1-Dimensional Tube which Right Side Close and Have Vibration Source at the Left Side and Sound Pressure Source at the Hole

Fig. 5 shows a 1-dimensional tube with an open end and a closed end, where the vibration source is in the open end and the sound pressure source is in the hole. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{100}$$

$$-Sv_1(l,t) + Sv_2(l,t) = S_H v_H e^{j\omega t} \tag{101}$$

The boundary conditions are shown:

$$v_1(0,t) = v_0 e^{j\omega t} \tag{102}$$

$$v_2(L,t) = 0 \tag{103}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \tag{104}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \tag{105}$$

$$jkA_1 - jkB_1 = v_0 \tag{106}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{107}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \left\{\frac{p_H}{\rho j\omega} + \frac{v_0}{jk} e^{-jkl}\right\} \frac{1}{2\cos(kl)} \tag{108}$$

$$B_1 = \left\{\frac{p_H}{\rho j\omega} - \frac{v_0}{jk} e^{-jkl}\right\} \frac{1}{2\cos(kl)} \tag{109}$$

$$A_2 = \frac{p_H}{\rho j\omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{jkL} \tag{110}$$

$$B_2 = \frac{p_H}{\rho j\omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{-jkL} \tag{111}$$

Therefore, $p_1(x,t)$ and $v_1(x,t)$ can be found as follows.

$$p_1(x,t) = p_H \frac{\cos(kx)}{\cos(kl)} e^{j\omega t} + j\rho c v_0 \frac{\sin\big(k(l-x)\big)}{\cos(kl)} e^{j\omega t} \tag{112}$$

$$v_1 = \frac{p_H}{j\rho c} \frac{\sin(kx)}{\cos(kl)} e^{j\omega t} + v_0 \frac{\cos\big(k(l-x)\big)}{\cos(kl)} e^{j\omega t} \tag{113}$$

Here, the equations for pressure and particle velocity when $x = 0, l$ are as follows:

$$p(0,t) = p_H \frac{1}{\cos(kl)} e^{j\omega t} + j\rho c v_0 \tan(kl)\, e^{j\omega t} \tag{114}$$

$$v_1(0,t) = v_0 e^{j\omega t} \tag{115}$$

$$p(l,t) = p_H e^{j\omega t} \tag{116}$$

$$v_1(l,t) = \frac{p_H}{j\rho c} \tan(kl)\, e^{j\omega t} + v_0 \frac{1}{\cos(kl)} e^{j\omega t} \tag{117}$$

Here, from the continuity equation for particle velocity at the hole position, the following equation can be obtained.

$$-S\big(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\big) + S\big(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\big) = S_H v_H \tag{118}$$

Therefore, the relationship between $p_H$ and $v_H$ can be found as follows.

$$p_H = \frac{-\rho j\omega S_H}{Sk\big\{\tan\big(k(L-l)\big) + \tan(kl)\big\}} v_H - \Delta P_{v_0} \tag{119}$$

Here, the following relationship holds true.

$$\Delta P_{v_0} = \frac{j\rho c v_0}{\cos(kl)\big\{\tan\big(k(L-l)\big) + \tan(kl)\big\}} \tag{120}$$

Now, at the location of the hole, the following equation of motion holds true.

$$\rho\delta_H \frac{\partial v_H}{\partial t} = P_{total} - p_H \tag{121}$$

Therefore, the frequency response functions of $P_{total}$ and $v_H$ can be obtained as follows.

$$v_H = \frac{1}{\rho j\omega\left\{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}\right\}}\left(P_{total} + \Delta P_{v_0}\right) \tag{122}$$

The frequency response functions of $P_{total}$ and $p_H$ are obtained as follows.

$$p_H = \frac{\frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}}{\delta_H - \frac{S_H}{Sk\{\tan(k(L-l)) + \tan(kl)\}}}\left(P_{total} + \Delta P_{v_0}\right) - \Delta P_{v_0} \tag{123}$$

3.4. When There Is a Sound Pressure Source at the Open End and a Vibration Source in the Hole.

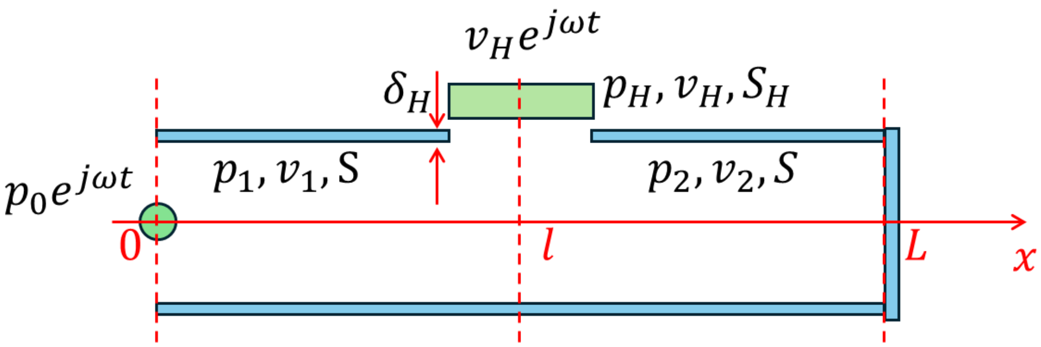


Fig. 6 1-Dimensional Tube which Right Side Close
and Have Sound Pressure Source at the Left Side and Vibration Source at the Hole

Fig. 6 shows a 1-dimensional tube with an open end and a closed end, where the sound pressure source is in the open end and the vibration source is in the hole. From the conditions for continuity of sound pressure and particle velocity, the following equations hold:

$$p_1(l,t) = p_2(l,t) = p_H(t) \tag{124}$$

$$-Sv_1(l,t) + Sv_2(l,t) = S_H v_H e^{j\omega t} \tag{125}$$

The boundary conditions are shown:

$$p_1(0,t) = p_0 e^{j\omega t} \tag{126}$$

$$v_2(L,t) = 0 \tag{127}$$

From these equations, the following four simultaneous equations are obtained.

$$A_1 e^{-jkl} + B_1 e^{jkl} = p_H \tag{128}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = p_H \tag{129}$$

$$\rho j\omega A_1 + \rho j\omega B_1 = p_0 \tag{130}$$

$$jkA_2 e^{-jkL} - jkB_2 e^{jkL} = 0 \tag{131}$$

The solution obtained from the system of four equations is as follows:

$$A_1 = \left\{-\frac{p_H}{\rho j\omega} + \frac{p_0}{\rho j\omega} e^{-jkl}\right\} \frac{1}{2\sin(kl)} \tag{132}$$

$$B_1 = \left\{\frac{p_H}{\rho j\omega} - \frac{p_0}{\rho j\omega} e^{-jkl}\right\} \frac{1}{2\sin(kl)} \tag{133}$$

$$A_2 = \frac{p_H}{\rho j\omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{jkL} \tag{134}$$

$$B_2 = \frac{p_H}{\rho j\omega} \frac{1}{2\cos\big(k(L-l)\big)} e^{-jkL} \tag{135}$$

Therefore, $p_1(x,t)$ and $v_1(x,t)$ can be found as follows.

$$p_1(x,t) = p_H \frac{\sin(kx)}{\sin(kl)} e^{j\omega t} + p_0 \frac{\sin\big(k(l-x)\big)}{\sin(kl)} e^{j\omega t} \tag{136}$$

$$v_1 = -\frac{p_H}{j\rho c} \frac{\cos(kx)}{\sin(kl)} e^{j\omega t} + \frac{p_0}{j\rho c} \frac{\cos\big(k(l-x)\big)}{\sin(kl)} e^{j\omega t} \tag{137}$$

Here, the equations for pressure and particle velocity when $x = 0, l$ are as follows:

$$p(0,t) = p_0 e^{j\omega t} \tag{138}$$

$$v_1(0,t) = -\frac{p_H}{j\rho c} \frac{1}{\sin(kl)} e^{j\omega t} + \frac{p_0}{j\rho c} \frac{1}{\tan(kl)} e^{j\omega t} \tag{139}$$

$$p(l,t) = p_H e^{j\omega t} \tag{140}$$

$$v_1(l,t) = -\frac{p_H}{j\rho c} \frac{1}{\tan(kl)} e^{j\omega t} + \frac{p_0}{j\rho c} \frac{1}{\sin(kl)} e^{j\omega t} \tag{141}$$

Here, from the continuity equation for particle velocity at the hole position, the following equation can be obtained.

$$-S\big(jkA_1 e^{-jkl} - jkB_1 e^{jkl}\big) + S\big(jkA_2 e^{-jkl} - jkB_2 e^{jkl}\big) = S_H v_H \tag{142}$$

Therefore, the relationship between $p_H$ and $v_H$ can be found as follows.

$$p_H = \frac{-\rho j\omega S_H}{Sk\big\{\tan\big(k(L-l)\big) - \cot(kl)\big\}} v_H - \Delta P_{p_0} \tag{143}$$

Here, the following relationship holds true.

$$\Delta P_{p_0} = \frac{p_0}{\sin(kl)\big\{\tan\big(k(L-l)\big) - \cot(kl)\big\}} \tag{144}$$

Now, at the location of the hole, the following equation of motion holds true.

$$\rho\delta_H \frac{\partial v_H}{\partial t} = P_{total} - p_H \tag{145}$$

Therefore, the frequency response functions of $P_{total}$ and $v_H$ can be obtained as follows.

$$v_H = \frac{1}{\rho j\omega\left\{\delta_H - \frac{S_H}{Sk\{\tan\left(k(L-l)\right) - \cot(kl)\}}\right\}}\left(P_{total} + \Delta P_{p_0}\right) \tag{146}$$

The frequency response functions of $P_{total}$ and $p_H$ are obtained as follows.

$$p_H = \frac{\frac{S_H}{Sk\{\tan\left(k(L-l)\right) - \cot(kl)\}}}{\delta_H - \frac{S_H}{Sk\{\tan\left(k(L-l)\right) - \cot(kl)\}}}\left(P_{total} + \Delta P_{p_0}\right) - \Delta P_{p_0} \tag{147}$$

Alternatively, it can be found as follows.

$$p_H = \frac{1}{1 - \frac{\delta_H Sk\{\tan\left(k(L-l)\right) - \cot(kl)\}}{S_H}}\left(P_{total} - \frac{\delta_H Sk}{S_H}\frac{p_0}{\sin(kl)}\right) \tag{148}$$

## 4. Conclusion

While research on automobile whistling and aspiration noise has been conducted in the past, we believe that understanding the properties of whistling and aspiration sounds using a 1-dimensional tube as a simple model is necessary in fields such as automobiles. Therefore, we determined the frequency response function for a 1-dimensional tube with a hole in it.

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