# On Complex Vibration Intensity and Vortices

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Abstract

The mathematical properties of complex vibration intensity, which is an extension of vibration intensity to complex numbers, were clarified from the constitutive equations of elastic bodies and acoustics in order to obtain knowledge that allows structural design to take vibration intensity into account in the preliminary stage of structural analysis. The mathematical properties obtained were applied to a dynamic absorber, and the properties of the dynamic absorber as revealed by the complex vibration intensity were explained. Furthermore, the coupling of vibration and acoustics was discussed using the relationship between complex vibration intensity and complex acoustic intensity. Furthermore, we discussed the significance of eliminating complex vibration intensity and complex acoustic intensity using external forces and external sound sources. Finally, the mathematical properties of complex vibration intensity were applied to structural design, and the findings that can be applied to structural design were presented.

## 1. Introduction.

Vibration intensity, an analysis focusing on energy flow, is a method proposed by Noiseux (Noiseux, 1970) and Pavic (Pavic, 1976) (Garvic & Pavic, 1993) to determine vibration energy flow. Vibration intensity is defined as the product of stress and vibration velocity at a point in an elastic body. Therefore, there are no SEA-like assumptions in the analysis method, and as long as the stress and vibration velocity are available, it is possible to show the energy flow by determining the vibration intensity of the system, even if the system has a long wavelength or strong nonlinearities.

There exists a literature on attempts to control the energy flow of a system by focusing on vibration intensity (Liu, Lee, & Lu, 2006) (Yamazaki & Nakamura, 2015) . However, it is difficult to say that there is a method that can control the energy flow of the system as intended to the extent that it can be applied in actual design situations. This is because, since the vibration intensity is obtained by post-processing the results of the system response analysis, finding a system that achieves the desired energy flow is an inverse problem that is very difficult to solve.

Therefore, there is a demand to clarify the mathematical properties of vibration intensity and to obtain knowledge that can be used in design without requiring vibration intensity. Complex vibration intensity, which extends vibration intensity to complex numbers, has also been proposed (Alfredsson, 1997) (Jolly & Pascal, 2006). However, the mathematical properties have not been elucidated.

This literature clarifies the properties of vibration intensity using complex vibration intensity, complex acoustic intensity, and the constitutive equations for elastic bodies and acoustics so that they can be used in design without requiring vibration intensity. The obtained characteristics of vibration intensity are also applied to dynamic absorbers. Furthermore, the coupling of vibration and acoustics will be discussed using the relationship between complex vibration intensity and complex acoustic intensity. We will also discuss the meaning of eliminating complex vibration intensity and complex acoustic intensity using external forces and external sound sources. Finally, we will propose knowledge that can be applied to the design of structures based on the characteristics of vibration intensity.

## 2. Divergence and Rotation of Vectors

Consider the divergence and rotation of a vector $\vec{I} = \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix}$ , i.e., the following two values

$$\mathrm{div}\vec{I} = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} \tag{1}$$

$$\mathrm{rot}\vec{I} = \begin{pmatrix} \dfrac{\partial I_z}{\partial y} - \dfrac{\partial I_y}{\partial z} \\ \dfrac{\partial I_x}{\partial z} - \dfrac{\partial I_z}{\partial x} \\ \dfrac{\partial I_y}{\partial x} - \dfrac{\partial I_x}{\partial y} \end{pmatrix} \tag{2}$$

The following can be said about $\mathrm{div}\vec{I}$ (Hidaka, Ankyu, & Tachibana, 1987).

- $\mathrm{div}\vec{I} = 0$ is solenoidal, in this case, the vector field has no upwelling point and no sucking point.
- $\mathrm{div}\vec{I} > 0$ is nonsolenoidal and the vector field may have an upwelling point.
- $\mathrm{div}\vec{I} < 0$ is nonsolenoidal and the vector field may have a sucking point.

In addition, the following can be said about $\mathrm{rot}\vec{I}$.

- $\mathrm{rot}\vec{I} = \vec{0}$ is irrotational and the vector field has no vortex.
- $\mathrm{rot}\vec{I} \neq \vec{0}$ is rotational and the vector field may flow in a closed path with vortices.

3. On the Complex Vibration Intensity of Elastic Materials

From now on, instead of the so-called vibration intensity, the complex vibration intensity $\vec{\imath}_c$ defined by the following equation will be used. This is a vector. Note that $\sigma_e$ means stress of elastic body, $V_e$ means velocity of elastic body, and $*$ on the right shoulder means complex conjugation.

$$\vec{\imath}_c = \sigma_e V_e^* \tag{3}$$

The real part of the complex vibration intensity $\vec{\imath}_c$ is the vibration active intensity $\vec{\imath}$ and the imaginary part is the vibration reactive intensity $\vec{q}$ . The vibration active intensity $\vec{\imath}$ is the so-called vibration intensity per unit area, which is equal in definition and units to the sound intensity. However, $j$ shall be in imaginary units.

$$\vec{\imath}_c = \vec{\imath} + j\vec{q} \tag{4}$$

4. Divergence and Rotation of Vibration Active Intensity and Vibration Reactive Intensity

4.1. About Divergence

The equation of motion of an elastic body is given by However, $u, v, w$ shall be the displacement in the direction $x, y, z$ .

$$\begin{cases} \rho \dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial z} \\ \rho \dfrac{\partial^2 v}{\partial t^2} = \dfrac{\partial \tau_{yx}}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z} \\ \rho \dfrac{\partial^2 w}{\partial t^2} = \dfrac{\partial \tau_{zx}}{\partial x} + \dfrac{\partial \tau_{zy}}{\partial y} + \dfrac{\partial \sigma_{zz}}{\partial z} \end{cases} \tag{5}$$

Multiply the expression (5) by $\dot{u}^*, \dot{v}^*, \dot{w}^*$ , respectively.

$$\begin{cases} \rho \dfrac{\partial^2 u}{\partial t^2} \dot{u}^* = \dfrac{\partial \sigma_{xx}}{\partial x} \dot{u}^* + \dfrac{\partial \tau_{xy}}{\partial y} \dot{u}^* + \dfrac{\partial \tau_{xz}}{\partial z} \dot{u}^* \\ \rho \dfrac{\partial^2 v}{\partial t^2} \dot{v}^* = \dfrac{\partial \tau_{yx}}{\partial x} \dot{v}^* + \dfrac{\partial \sigma_{yy}}{\partial y} \dot{v}^* + \dfrac{\partial \tau_{yz}}{\partial z} \dot{v}^* \\ \rho \dfrac{\partial^2 w}{\partial t^2} \dot{w}^* = \dfrac{\partial \tau_{zx}}{\partial x} \dot{w}^* + \dfrac{\partial \tau_{zy}}{\partial y} \dot{w}^* + \dfrac{\partial \sigma_{zz}}{\partial z} \dot{w}^* \end{cases} \tag{6}$$

Consider the derivative.

$$\begin{cases} \rho \dfrac{\partial}{\partial t}\left(\dfrac{1}{2}\dot{u}\dot{u}^*\right) = \dfrac{\partial(\sigma_{xx}\dot{u}^*)}{\partial x} + \dfrac{\partial\left(\tau_{xy}\dot{u}^*\right)}{\partial y} + \dfrac{\partial(\tau_{xz}\dot{u}^*)}{\partial z} - \sigma_{xx}\dfrac{\partial \dot{u}^*}{\partial x} - \tau_{xy}\dfrac{\partial \dot{u}^*}{\partial y} - \tau_{xz}\dfrac{\partial \dot{u}^*}{\partial z} \\ \rho \dfrac{\partial}{\partial t}\left(\dfrac{1}{2}\dot{v}\dot{v}^*\right) = \dfrac{\partial\left(\tau_{yx}\dot{v}^*\right)}{\partial x} + \dfrac{\partial\left(\sigma_{yy}\dot{v}^*\right)}{\partial y} + \dfrac{\partial\left(\tau_{yz}\dot{v}^*\right)}{\partial x} - \tau_{yx}\dfrac{\partial \dot{v}^*}{\partial x} - \sigma_{yy}\dfrac{\partial \dot{v}^*}{\partial y} - \tau_{yz}\dfrac{\partial \dot{v}^*}{\partial z} \\ \rho \dfrac{\partial}{\partial t}\left(\dfrac{1}{2}\dot{w}\dot{w}^*\right) = \dfrac{\partial(\tau_{zx}\dot{w}^*)}{\partial x} + \dfrac{\partial\left(\tau_{zy}\dot{w}^*\right)}{\partial y} + \dfrac{\partial(\sigma_{zz}\dot{w}^*)}{\partial x} - \tau_{zx}\dfrac{\partial \dot{w}^*}{\partial x} - \tau_{zy}\dfrac{\partial \dot{w}^*}{\partial y} - \sigma_{zz}\dfrac{\partial \dot{w}^*}{\partial z} \end{cases} \tag{7}$$

Add the equations (7) to each side and consider the time derivative.

$$j\omega\rho\left(\frac{1}{2}\dot{u}\dot{u}^* + \frac{1}{2}\dot{v}\dot{v}^* + \frac{1}{2}\dot{w}\dot{w}^*\right)$$

$$= \frac{\partial}{\partial x}\left(\sigma_{xx}\dot{u}^* + \tau_{yx}\dot{v}^* + \tau_{zx}\dot{w}^*\right) + \frac{\partial}{\partial y}\left(\tau_{xy}\dot{u}^* + \sigma_{yy}\dot{v}^* + \tau_{zy}\dot{w}^*\right)$$

$$+ \frac{\partial}{\partial z}\left(\tau_{xz}\dot{u}^* + \tau_{yz}\dot{v}^* + \sigma_{zz}\dot{w}^*\right) \quad (8)$$

$$+j\omega\left(\sigma_{xx}\frac{\partial u^*}{\partial x} + \tau_{xy}\frac{\partial u^*}{\partial y} + \tau_{xz}\frac{\partial u^*}{\partial z} + \tau_{yx}\frac{\partial v^*}{\partial x} + \sigma_{yy}\frac{\partial v^*}{\partial y} + \tau_{yz}\frac{\partial v^*}{\partial z}\right.$$

$$\left. + \tau_{zx}\frac{\partial w^*}{\partial x} + \tau_{zy}\frac{\partial w^*}{\partial y} + \sigma_{zz}\frac{\partial w^*}{\partial z}\right)$$

Thus, the following equation is obtained

$$j\omega T = \mathrm{div}\vec{\imath}_c + j\omega U \quad (9)$$

However, $T$ :kinetic energy density,

$U$ Elastic energy (potential energy) density

$$\vec{\imath}_c = \begin{pmatrix} \sigma_{xx}\dot{u}^* + \tau_{yx}\dot{v}^* + \tau_{zx}\dot{w}^* \\ \tau_{xy}\dot{u}^* + \sigma_{yy}\dot{v}^* + \tau_{zy}\dot{w}^* \\ \tau_{xz}\dot{u}^* + \tau_{yz}\dot{v}^* + \sigma_{zz}\dot{w}^* \end{pmatrix} \text{Complex Vibration Intensity} \quad (10)$$

Furthermore, from the equation (4) , the equation is transformed.

$$j\omega L = \mathrm{div}\vec{\imath}_c = \mathrm{div}\vec{\imath} + j\mathrm{div}\vec{q} \quad (11)$$

However, $L = T - U$ :Lagrangian density (12)

$T, U$ is a real number because it is energy. Comparing both sides, we obtain the following equation

$$\mathrm{div}\vec{\imath} = 0 \quad (13)$$

$$\mathrm{div}\vec{q} = \omega L \quad (14)$$

### 4.2. About Rotation

When the complex vibrational intensity creates a vortex, the energy per unit length along the closed curve through the vortex is given by where $\vec{t}$ is the tangential unit vector of the closed curve.

$$\left(\oint_c \vec{\imath}_c \cdot \vec{t} ds\right)\delta t \quad (15)$$

This is equal to the energy that produces the rotation. Since we are considering a 3-dimensional space, in terms of velocity, which is a 3-dimensional vector, the kinetic energy density that produces rotation is given by the following equation.

$$\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right)$$

$$\text{However,}\vec{V_e}=\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} \tag{16}$$

Therefore, the vortex becomes smaller as the energy goes outward, so the following equation holds. However, $\vec{n}$ is the unit vector in the outward normal direction of the surface bounded by the closed curve.

$$\left(\oint_c \vec{\imath}_c\cdot\vec{t}ds\right)\delta t=-\delta\int_S\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right)\cdot\vec{n}dS \tag{17}$$

Equation (17) into an equation to account for the time derivative.

$$\oint_c \vec{\imath}_c\cdot\vec{t}ds=-\frac{d}{dt}\int_S\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right)\cdot\vec{n}dS$$

$$\therefore\oint_c \vec{\imath}_c\cdot\vec{t}ds=-j\omega\int_S\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right)\cdot\vec{n}dS \tag{18}$$

From Stokes' theorem, transforming the left-hand side of the equation (18) , we obtain the following equation

$$\int_S \mathrm{rot}\vec{\imath}_c\cdot\vec{n}dS=-j\omega\int_S\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right)\cdot\vec{n}dS \tag{19}$$

Therefore, the following equation holds.

$$\mathrm{rot}\vec{\imath}_c=-j\omega\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right) \tag{20}$$

From the equation (4) we obtain the following equation

$$\mathrm{rot}\vec{\imath}+j\mathrm{rot}\vec{q}=-j\omega\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right) \tag{21}$$

$\vec{V_e}\times\vec{V_e}^*$ is only the imaginary part, so comparing both sides, the following equation holds.

$$\mathrm{rot}\vec{\imath}=-j\omega\frac{1}{2}\rho\left(\vec{V_e}\times\vec{V_e}^*\right) \tag{22}$$

$$\mathrm{rot}\vec{q}=\vec{0} \tag{23}$$

Next, the following matrix is introduced. Where $E$ is Young's modulus and $\nu$ is Poisson's ratio.

$$[\sigma]=\begin{bmatrix}\sigma_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_{zz}\end{bmatrix} \tag{24}$$

$$[G]^{-1} = \begin{bmatrix} \frac{1-2\nu}{2G(1-\nu)} & \frac{1}{G} & \frac{1}{G} \\ \frac{1}{G} & \frac{1-2\nu}{2G(1-\nu)} & \frac{1}{G} \\ \frac{1}{G} & \frac{1}{G} & \frac{1-2\nu}{2G(1-\nu)} \end{bmatrix} \tag{25}$$

$$[c^2]^{-1} = \begin{bmatrix} \frac{1}{c_L^2} & \frac{1}{c_T^2} & \frac{1}{c_T^2} \\ \frac{1}{c_T^2} & \frac{1}{c_L^2} & \frac{1}{c_T^2} \\ \frac{1}{c_T^2} & \frac{1}{c_T^2} & \frac{1}{c_L^2} \end{bmatrix} \tag{26}$$

$$c_L = \sqrt{\frac{2G(1-\nu)}{\rho(1-2\nu)}}, c_T = \sqrt{\frac{G}{\rho}},\ G = \frac{E}{2(1+\nu)} \tag{27}$$

The following matrix is also referred to as the elastic energy (potential energy) density matrix.

$$[U] = [\sigma][\sigma]^*[G]^{-1} \tag{28}$$

From the above,

$$\begin{aligned} -j\omega\frac{1}{2}\rho(\vec{V}\times\vec{V}^*) &= -j\omega\frac{1}{2}\rho[\sigma]^{-*}[\sigma]^{-1}\left([\sigma]^*\vec{V}_e\times[\sigma]\vec{V}_e^{\ *}\right) \\ &= -j\omega\frac{1}{2}\rho[\sigma]^{-*}[\sigma]^{-1}(\vec{\imath}_c^{*}\times\vec{\imath}_c) \\ &= -j\omega\frac{1}{2}\rho[\sigma]^{-*}[\sigma]^{-1}\{(\vec{\imath}-j\vec{q})\times(\vec{\imath}+j\vec{q})\} \\ &= -j\omega\frac{1}{2}\rho[\sigma]^{-*}[\sigma]^{-1}\cdot 2j(\vec{\imath}\times\vec{q}) \\ &= \omega\rho[G]^{-1}[G][\sigma]^{-*}[\sigma]^{-1}(\vec{\imath}\times\vec{q}) \\ &= \omega[c^2]^{-1}[U]^{-1}(\vec{\imath}\times\vec{q}) \end{aligned} \tag{29}$$

Therefore, the following equation holds.

$$\begin{aligned} \mathrm{rot}\vec{\imath} &= -j\omega\frac{1}{2}\rho\left(\vec{V}_e\times\vec{V}_e^{\ *}\right) \\ &= \omega[c^2]^{-1}[U]^{-1}(\vec{\imath}\times\vec{q}) \end{aligned} \tag{30}$$

$$\mathrm{rot}\vec{q} = \vec{0} \tag{31}$$

Now, for the expression (22) , multiplying both sides bydiv , we obtain the following expression from $\mathrm{div}(\mathrm{rot}\vec{\imath}) = 0$ at any vector.

$$-j\omega\frac{1}{2}\rho\,\mathrm{div}\left(\vec{V}_e\times\vec{V}_e^{\ *}\right) = 0 \tag{32}$$

In equation (19) , let the surface be a closed surface $S'$ and consider the closed space formed by the closed surface. Applying Gauss's divergence theorem for the right-hand side,

$$\int_{S'} \mathrm{rot}\vec{\imath}_c \cdot \vec{n} dS = -j\omega \frac{1}{2}\rho \int_V \mathrm{div}\left(\vec{V}_e \times \vec{V}_e^{\,*}\right) dV \tag{33}$$

Therefore, from the equation (32) , the following equation holds.

$$\int_{S'} \mathrm{rot}\vec{\imath}_c \cdot \vec{n} dS = 0 \tag{34}$$

Note that this expression (34) is only an integral expression that is valid on the closed surface $S'$ and not necessarily $\mathrm{rot}\vec{\imath}_c = \vec{0}$ on the entire closed surface $S$ from here. For surfaces on a closed surface $S$ , $\mathrm{rot}\vec{\imath}_c$ may also have a value.

## 4.3. For complex sound intensity

Note that complex acoustic intensity is also considered. Let $\vec{\imath}_{ac} = pV_a^*$ ($p$ is sound pressure and $V_a$ is particle velocity), $\vec{\imath}_a$ be the acoustic active intensity and $\vec{q}_a$ be the acoustic reactive intensity. The equation of motion satisfied by the sound pressure and particle velocity is given by

$$\begin{cases} \rho \dfrac{\partial^2 u}{\partial t^2} = -\dfrac{\partial p}{\partial x} \\ \rho \dfrac{\partial^2 v}{\partial t^2} = -\dfrac{\partial p}{\partial y} \\ \rho \dfrac{\partial^2 w}{\partial t^2} = -\dfrac{\partial p}{\partial z} \end{cases} \tag{35}$$

The equation of continuity is also given by However, the volume modulus $K$ is used equal to $\rho c^2$ .

$$\frac{\partial \dot{u}}{\partial x} + \frac{\partial \dot{v}}{\partial y} + \frac{\partial \dot{w}}{\partial z} = -\frac{1}{\rho c^2}\frac{\partial p}{\partial t} \tag{36}$$

Consider it in the same way as complex vibrational intensity.

$$\begin{aligned} &\rho \frac{\partial}{\partial t}\left(\frac{1}{2}\dot{u}\dot{u}^* + \frac{1}{2}\dot{v}\dot{v} + \frac{1}{2}\dot{w}\dot{w}^*\right) \\ &= -\mathrm{div}\vec{\imath}_{ac} + p\left(\frac{\partial \dot{u}^*}{\partial x} + \frac{\partial \dot{v}^*}{\partial y} + \frac{\partial \dot{w}^*}{\partial z}\right) = -\mathrm{div}\vec{\imath}_{ac} - \frac{1}{\rho c^2} p \frac{\partial p^*}{\partial t} \end{aligned} \tag{37}$$

Considering the time derivative,

$$j\omega T = -\mathrm{div}\vec{\imath}_{ac} + j\omega \frac{1}{\rho c^2} pp^* = -\mathrm{div}\vec{\imath}_{ac} + j\omega U \tag{38}$$

$$\therefore j\omega L = -\mathrm{div}\vec{\imath}_{ac} = -\mathrm{div}\vec{\imath}_a - j\mathrm{div}\vec{q}_a \tag{39}$$

Therefore, a side-by-side comparison yields the following equation. Also, $\mathrm{rot}\vec{\imath}_a$ and

$\mathrm{rot}\vec{q}_a$ are considered as in the case of complex vibrational intensities, and the same equations are obtained. However, $\vec{V}_a$ is the particle velocity vector. Also, $[U]^{-1}$ and $[c^2]^{-1}$ are scalars. $c_a$ is the speed of sound.

$$\mathrm{div}\vec{\imath}_a = 0 \tag{40}$$

$$\mathrm{div}\vec{q}_a = -\omega L \tag{41}$$

$$\begin{aligned}\mathrm{rot}\vec{\imath}_a &= -j\omega\frac{1}{2}\rho\left(\vec{V}_a \times \vec{V}_a^{\ *}\right) \\ &= \frac{\omega}{c_a^2}\frac{\vec{\imath}_a \times \vec{q}_a}{U}\end{aligned} \tag{42}$$

$$\mathrm{rot}\vec{q}_a = \vec{0} \tag{43}$$

The equation (40) to the equation (43) obtained by considering the complex acoustic intensity is identical to the reference (Hidaka, Ankyu, & Tachibana, 1987) . However, the reference substitutes $p = P(\vec{r})e^{j(\omega t - \varphi(\vec{r}))}$ into Euler's equation and other equations to obtain the particle velocity equation, from which $\vec{\imath}_{ac}$ is obtained, and div and rot are obtained directly.

Furthermore, considering it similarly to the complex vibration intensity, the following equation holds for a closed surface $S'$:

$$\int_{S'} \mathrm{rot}\vec{\imath}_a \cdot \vec{n} dS = 0 \tag{44}$$

Note that this equation (44) is an integral that holds only for the closed surface $S'$, and does not necessarily mean that $\mathrm{rot}\vec{\imath}_a = \vec{0}$ for the entire closed surface $S$. On a surface on the closed surface $S$, $\mathrm{rot}\vec{\imath}_a$a may have a value.

## 5. Properties of Vibration Active Intensity and Vibration Reactive Intensity

### 5.1. Vibration active intensity in the absence of input or damping$\vec{\imath}$ properties

In the case of no input and damping on a surface that is not a closed surface, the equations that satisfy the vibration active intensity$\vec{\imath}$ are from equations (13) , (22)

$$\mathrm{div}\vec{\imath} = 0 \tag{45}$$

$$\begin{aligned}\mathrm{rot}\vec{\imath} &= -j\omega\frac{1}{2}\rho\left(\vec{V}_e \times \vec{V}_e^{\ *}\right) \\ &= \omega[c^2]^{-1}[U]^{-1}(\vec{\imath} \times \vec{q})\end{aligned} \tag{46}$$

In the case of vibration active intensity $\vec{\imath}$ , there is no input or damping, $\mathrm{div}\vec{\imath} = 0$ , so there are no upwelling or suction points. It is also generally $\mathrm{rot}\vec{\imath} \neq \vec{0}$ and vortices exist. $\vec{\imath}$ flows from the antinode of the eigenmode to the node because it flows from the point of higher energy to the point of lower energy. However, since there is no upwelling or suction point, no $\vec{\imath}$ is generated or dissipated from the antinode or node. At the antinode position of the mode, $\vec{V}_e \times \vec{V}_e^{\,*}$ has a value, and at the node position of the mode, $[U]^{-1}$ has a value. Therefore, it creates a vortex as if it springs out at the antinode and sucks in at the node.

If a vortex forms at the antinode of the mode, the vibration active intensity stays there, which, combined with the eigenmode shape, causes the vibration to increase. If a vortex forms at the nodal position of the mode and the vibration active intensity stays there, the vibration does not increase and is not a problem because it is at the nodal position.

One way to eliminate the vortex is to design it to be $\vec{V}_e \times \vec{V}_e^{\,*} = \vec{0}$ . If this is achieved, $\mathrm{div}\vec{\imath} = 0$ and $\mathrm{rot}\vec{\imath} = \vec{0}$ , then the vector field created by the oscillating active intensity $\vec{\imath}$ is a Laplace field. The Laplace field has the property that the maximum and minimum values exist at the boundary. Therefore, the vibrational active intensity $\vec{\imath}$ flows from a place of high energy to a place of low energy without creating vortices.

## 5.2. Vibration active intensity in the presence of inputs and damping$\vec{\imath}$ properties

Consider the case where inputs and damping exist. Let $\vec{\imath}_f$ be the real part of the complex vibration intensity $\vec{\imath}_{fc}$ due to the input $\vec{f}$ and $\vec{q}_f$ be the imaginary part, and let $\vec{\imath}_D$ be the real part of the complex vibration intensity $\vec{\imath}_{Dc}$ due to damping and $\vec{q}_D$ be the imaginary part. In this case, the following equation holds. Where $\otimes$ is the product of the components of the vectors. $c_D$ is the proportional damping coefficient.

$$\vec{\imath}_f = \mathrm{Re}\left(\vec{f} \otimes \vec{V}^*\right),\ \vec{q}_f = \mathrm{Im}\left(\vec{f} \otimes \vec{V}^*\right) \tag{47}$$

$$\vec{\imath}_D = \mathrm{Re}\left(c_D\vec{V} \otimes \vec{V}^*\right),\ \vec{q}_D = \mathrm{Im}\left(c_D\vec{V} \otimes \vec{V}^*\right) = \vec{0} \tag{48}$$

First, we consider the divergence equation for complex vibrational intensities. The equation is as follows from (11)

$$\mathrm{div}\vec{\imath}_c = j\omega L \tag{49}$$

Both sides are volume integrated.

$$\int_V \mathrm{div}\vec{\imath}_c dV = \int_V j\omega L dV \tag{50}$$

For the left-hand side, we transform it using Gauss's divergence theorem.

$$\int_S \vec{\imath}_c \cdot \vec{n} dS = \int_V j\omega L dV \tag{51}$$

If there is input or damping, the complex vibration intensity of the input $\vec{\imath}_{fc}$ and the complex vibration intensity of the damping $\vec{\imath}_{Dc}$ move in and out of the surface, so the following equation holds.

$$\int_S \vec{\imath}_c \cdot \vec{n} dS = \int_V j\omega L dV + \int_S \vec{\imath}_{fc} \cdot \vec{n} dS - \int_S \vec{\imath}_{Dc} \cdot \vec{n} dS \tag{52}$$

Apply Gauss's divergence theorem to both sides and transform the equation.

$$\int_S \mathrm{div}\vec{\imath}_c dV = \int_V j\omega L dV + \int_S \mathrm{div}\vec{\imath}_{fc} dV - \int_S \mathrm{div}\vec{\imath}_{Dc} dV \tag{53}$$

Therefore, the following equation holds.

$$\mathrm{div}\vec{\imath}_c = j\omega L + \mathrm{div}\vec{\imath}_{fc} - \mathrm{div}\vec{\imath}_{Dc} \tag{54}$$

Next, we consider the complex vibrational intensities from the equation of rotation. From the equation (18) , we have the following equation

$$\oint_c \vec{\imath}_c \cdot \vec{t} ds = -j\omega \int_S \frac{1}{2}\rho\left(\vec{V}_e \times \vec{V}_e^{\,*}\right) \cdot \vec{n} dS \tag{55}$$

If there is input or damping, the complex vibration intensity of the input $\vec{\imath}_{fc}$ and the complex vibration intensity of the damping $\vec{\imath}_{Dc}$ also have energy in and out per unit length along the closed curve, so the following equation holds.

$$\oint_c \vec{\imath}_c \cdot \vec{t} ds = -j\omega \int_S \frac{1}{2}\rho\left(\vec{V}_e \times \vec{V}_e^{\,*}\right) \cdot \vec{n} dS + \oint_c \vec{\imath}_{fc} \cdot \vec{t} ds - \oint_c \vec{\imath}_{Dc} \cdot \vec{t} ds \tag{56}$$

Apply Stokes' theorem to both sides and transform the equation.

$$\begin{aligned} \int_S \mathrm{rot}\vec{\imath}_c \cdot \vec{n} dS = -j\omega \int_S \frac{1}{2}\rho\left(\vec{V}_e \times \vec{V}_e^{\,*}\right) \cdot \vec{n} dS + \int_S \mathrm{rot}\vec{\imath}_{fc} \cdot \vec{n} dS \\ - \int_S \mathrm{rot}\vec{\imath}_{Dc} \cdot \vec{n} dS \end{aligned} \tag{57}$$

Therefore, the following equation holds.

$$\begin{aligned} \mathrm{rot}\vec{\imath}_c &= -j\omega \frac{1}{2}\rho\left(\vec{V}_e \times \vec{V}_e^{\,*}\right) + \mathrm{rot}\vec{\imath}_{fc} - \mathrm{rot}\vec{\imath}_{Dc} \\ &= \omega[c^2]^{-1}[U]^{-1}(\vec{\imath} \times \vec{q}) + \mathrm{rot}\vec{\imath}_{fc} - \mathrm{rot}\vec{\imath}_{Dc} \end{aligned} \tag{58}$$

Thus, if there is input and damping on a surface that is not a closed surface, the equation that satisfies the vibration active intensity$\vec{\imath}$ is the following, taking out the real part of the two equations (54) and (58) obtained.

$$\mathrm{div}\vec{\imath} = \mathrm{div}\vec{\imath}_f - \mathrm{div}\vec{\imath}_D \tag{59}$$

$$\begin{aligned} \mathrm{rot}\vec{\imath} &= -j\omega \frac{1}{2}\rho\left(\vec{V}_e \times \vec{V}_e^{\,*}\right) + \mathrm{rot}\vec{\imath}_f - \mathrm{rot}\vec{\imath}_D \\ &= \omega[c^2]^{-1}[U]^{-1}(\vec{\imath} \times \vec{q}) + \mathrm{rot}\vec{\imath}_f - \mathrm{rot}\vec{\imath}_D \end{aligned} \tag{60}$$

If we look at the equation (59) , we can see that the input position is at the gushing point and the damping position is at the sucking point. Looking at the right hand side of the equation(60) , the first term means the vortex generated by the eigenmodes, the second term means the vortex generated by the input, and the third term means the vortex generated by the damping. In order to eliminate vortices in the vibration active intensity $\vec{\iota}$ , we can also use damping as $\mathrm{rot}\vec{\iota} = \vec{0}$ , since in the case of a car, the vibration active intensity $\vec{\iota}_f$ due to input cannot be eliminated. At the same time, it can be made into a Laplace field if it is $\mathrm{div}\vec{\iota} = 0$ . However, if the damping is made too large, vortices will be created by $\mathrm{rot}\vec{\iota}_D$ . Alternatively, applying an external force can eliminate the divergence or rotation of the vibrational active intensity $\vec{\iota}$.

Now, in the equation (57), consider the case where the surface is a closed surface $S'$ . Applying Gauss's divergence theorem for the first term on the right-hand side, we obtain the following equation.

$$\int_{S'} \mathrm{rot}\vec{\iota}_c \cdot \vec{n}dS = -j\omega\frac{1}{2}\rho\int_V \mathrm{div}\left(\vec{V}_e \times \vec{V}_e^{\,*}\right)dV + \int_{S'} \mathrm{rot}\vec{\iota}_{fc} \cdot \vec{n}dS - \int_{S'} \mathrm{rot}\vec{\iota}_{Dc} \cdot \vec{n}dS \tag{61}$$

Thus, from the equation (32) we obtain the following equation

$$\int_{S'} \mathrm{rot}\vec{\iota}_c \cdot \vec{n}dS = \int_{S'} \mathrm{rot}\vec{\iota}_{fc} \cdot \vec{n}dS - \int_{S'} \mathrm{rot}\vec{\iota}_{Dc} \cdot \vec{n}dS \tag{62}$$

Therefore, the following equation holds on a closed surface.

$$\mathrm{rot}\vec{\iota}_c = \mathrm{rot}\vec{\iota}_{fc} - \mathrm{rot}\vec{\iota}_{Dc} \tag{63}$$

Thus, in the case of input and damping on a closed surface, the equation satisfying the vibration active intensity$\vec{\iota}$ is as follows, taking out the real part of the two equations (54) , (63) obtained.

$$\mathrm{div}\vec{\iota} = \mathrm{div}\vec{\iota}_f - \mathrm{div}\vec{\iota}_D \tag{64}$$

$$\mathrm{rot}\vec{\iota} = \mathrm{rot}\vec{\iota}_f - \mathrm{rot}\vec{\iota}_D \tag{65}$$

From the equation of div, for the input position can be seen to be the gushing point and the damping position is the sucking point. In order to eliminate the vortex at $\vec{\iota}$ in the case of the car, the intensity $\vec{\iota}_f$ due to the input cannot be eliminated, so we can use the damping as $\mathrm{rot}\vec{\iota} = \vec{0}$ . At the same time, if $\mathrm{div}\vec{\iota} = 0$ , it can be made into a Laplace field. However, if the attenuation is made too large, the vortex will be created by $\mathrm{rot}\vec{\iota}_D$ . Alternatively, applying an external force can eliminate the divergence or rotation of the vibrational active intensity $\vec{\iota}$ on a closed surface.

### 5.3. Vibration reactive intensity in the absence of input or damping $\vec{q}$ properties

In the absence of input and damping, the equations satisfying vibration reactive intensity $\vec{q}$ are from equations (14) , (23)

$$\mathrm{div}\vec{q} = \omega L \tag{66}$$

$$\mathrm{rot}\vec{q} = \vec{0} \tag{67}$$

Then $\vec{q}$, if there is no input or damping, it is $\text{div}\vec{q} \neq 0$ and there is an upwelling or suction point. Also, it is $\text{rot}\vec{q} = \vec{0}$ and there is no vortex.

Now, consider a partial extraction for $L$ in the $x$ direction.

$$\begin{aligned} L &= T - U \\ &= \frac{1}{2}\rho\dot{u}\dot{u}^* - \sigma_{xx}\frac{\partial u^*}{\partial x} \\ &= \frac{1}{2}\rho(j\omega u)(-j\omega u^*) - \sigma_{xx}\frac{\partial u^*}{\partial x} \end{aligned} \quad (68)$$

$$\therefore L = \frac{1}{2}\rho\omega^2 uu^* - \sigma_{xx}\frac{\partial u^*}{\partial x} \quad (69)$$

$u$ is at its maximum, $\frac{\partial u^*}{\partial x}$ becomes $0$. In other words, it is the antinode position of the mode, which is $L = T - U > 0$, at this time. Conversely, where $u$ becomes $0$, $\frac{\partial u^*}{\partial x}$ has value. This is the point of maximum inclination in the sinusoidal wave, which is the nodal position of the mode and is $L = T - U < 0$. The same is true for the other directions.

$\vec{q}$ also flows from a place of high energy to a place of low energy, so it flows from the antinode of the eigenmode to the node. Therefore, an outflow point is created at the eigenmode's antinode position, and a suction point is created at the node position. In other words, by looking at $\vec{q}$, we can clearly identify the eigenmode's antinodal and nodal positions.

## 5.4. Vibration reactive intensity in the presence of inputs and damping$\vec{q}$ properties and the causes of the formation of eigenmodes.

In the case of input and damping on surfaces that are not closed, the equation that satisfies the vibration reactive intensity $\vec{q}$ is the following, taking out the imaginary part of the equations (54), (58) For oscillatory reactive intensity $\vec{q}$, the same equation holds for closed surfaces.

$$\text{div}\vec{q} = \omega L + \text{div}\vec{q}_f \quad (70)$$

$$\text{rot}\vec{q} = \text{rot}\vec{q}_f \quad (71)$$

If there is the input and attenuation at $\vec{q}$, the input location is the point and vortex of the upwelling, while the attenuation has no effect. Therefore, it may be possible to probe the input location by looking at $\vec{q}$.

From the above, $\vec{q}$ exists when there is an eigenmode (or standing wave) in the vicinity of the input position or in the presence of an eigenmode (or standing wave), and it is not consumed by the structure, only moving from the antinode of the mode to the nodal position. Conversely, an eigenmode exists because there is a vibrational reactive intensity $\vec{q}$ . In particular, if $\vec{V_e} \times \vec{V_e}^*$ has a value where $L > 0$ , i.e., where the kinetic energy density is dominant, it can be said that the vibration active intensity creates a vortex, which stagnates and increases the vibration and becomes the antinode of the eigenmode.

Also, $\vec{q}$ cannot be consumed in decay. In other words, it cannot be $\mathrm{div}\vec{q} = 0$ or $\mathrm{rot}\vec{q} = \vec{0}$ . Therefore, if it is generated, it will continue to exist without disappearing as long as it is on that frequency. Alternatively, applying an external force can eliminate the divergence and rotation of the vibrational reactive intensity $\vec{q}$.

### 5.5. Consideration of Complex Vibration Intensities with Low-Degree-of-Freedom Models

#### 5.5.1. For 1-DOF systems

Fig.1 . The equations of motion are as follows

$$m_1 \ddot{x_1} + c_1 \dot{x_1} + k_1 x_1 = f_1 \tag{72}$$

If $f_1 = F_1 e^{j\omega t}$, $x_1$ becomes $X_1 e^{j\omega t}$, the mobility is as follows.

$$\frac{\dot{X}_1}{F_1} = \frac{c_1\omega^2 + j\omega(k_1 - m_1\omega^2)}{(k_1 - m_1\omega^2)^2 + c_1^2\omega^2} \tag{73}$$

The complex vibration intensities are as follows

$$F_1 \dot{X}_1^* = |F_1|^2 \left(\frac{\dot{X}_1}{F_1}\right)^* = |F_1|^2 \frac{c_1\omega^2 - j\omega(k_1 - m_1\omega^2)}{(k_1 - m_1\omega^2)^2 + c_1^2\omega^2} \tag{74}$$

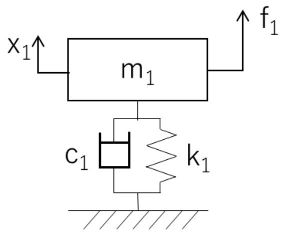


Fig.1 1-DOF system

Thus, the vibration active intensity $i_1$ and vibration reactive intensity $q_1$ are as follows

$$i_1 = \frac{c_1\omega^2}{(k_1 - m_1\omega^2)^2 + c_1^2\omega^2}|F_1|^2 \tag{75}$$

$$q_1 = \frac{-\omega(k_1 - m_1\omega^2)}{(k_1 - m_1\omega^2)^2 + c_1^2\omega^2}|F_1|^2 \tag{76}$$

when there is no damping, i.e., no damping ($c_1 = 0$,), the vibration active intensity $i_1$ is 0 . If no damping exists in the structure, there is no vibration active intensity. Also, the vibration reactive intensity $q_1$ has a value as long as the spring-mass system exists, even if there is no damping. In other words, if no eigenmodes exist, $q_1$ becomes 0 .

In Section 5.1, we mentioned that the design is to be $\vec{V_e} \times \vec{V_e}^* = \vec{0}$ , which is valid when $\vec{V}$ is only the real part or only the imaginary part. The imaginary part only means that there is no damping in the structure, which is difficult to achieve. To have only the real part means that the eigenmodes of a certain frequency are eliminated, which is feasible.

### 5.5.2. For 2-DOF systems

Next, consider the 2-DOF system shown inFig.2 . The equations of motion are as follows

$$m_1\ddot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2) = f_1 \tag{77}$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \tag{78}$$

This can be nondimensionalized as follows.

$$\ddot{\tilde{x}}_1 + 2\mu\zeta\dot{\tilde{x}}_1 + (1 + \mu p^2)\tilde{x}_1 - 2\mu\zeta\dot{\tilde{x}}_2 - \mu p^2\tilde{x}_2 = \tilde{f}_1 \tag{79}$$

$$\ddot{\tilde{x}}_1 + 2\zeta\dot{\tilde{x}}_2 + p^2\tilde{x}_2 - 2\zeta\dot{\tilde{x}}_1 - p^2\tilde{x}_1 = 0 \tag{80}$$

$$\text{However,}\mu = \frac{m_2}{m_1}, \zeta = \frac{c_2}{2\mu\sqrt{m_1k_1}},\ p^2 = \frac{k_2}{\mu k_1},\ \tilde{f}_1 = \frac{f_1}{k_1x_0},\ \tilde{x}_1 = \frac{x_1}{x_0}\ ,\tilde{x}_2 = \frac{x_2}{x_0} \tag{81}$$

From the above equations, the conductance and mobility are calculated, and the vibration active intensity $i_1, i_2, i_f$ and vibration reactive intensity $q_1, q_2, q_f$ of the mass 1, mass 2, and input are calculated as follows. However, $\lambda = \frac{\omega}{\omega_0}$, $\omega_0 = \sqrt{\frac{k_1}{m_1}}$ .

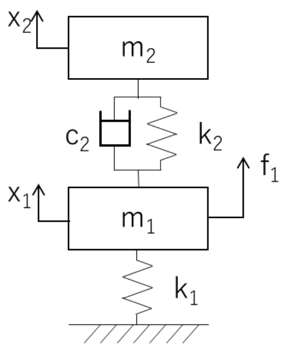


Fig.2 2-DOF system

$$i_1 = \frac{2\mu\zeta\lambda\{p^2(p^2 - \lambda^2) - p^2\lambda^2 + (2\zeta\lambda)^2\}}{\{(1 - \lambda^2)(p^2 - \lambda^2) - \mu p^2\lambda^2\}^2 + (2\zeta\lambda)^2\{1 - (1 + \mu)\lambda^2\}^2} \tag{82}$$

$$q_1 = \frac{-\mu\lambda\{p^4(p^2-\lambda^2)+(2\zeta\lambda)^2(p^2+\lambda^2)\}}{\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}^2+(2\zeta\lambda)^2\{1-(1+\mu)\lambda^2\}^2} \tag{83}$$

$$i_2 = \frac{2\zeta\lambda^2\{p^2+(2\zeta\lambda)^2\}}{\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}^2+(2\zeta\lambda)^2\{1-(1+\mu)\lambda^2\}^2} \tag{84}$$

$$q_2 = \frac{-\lambda(p^2-\lambda^2)\{p^4+(2\zeta\lambda)^2\}}{\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}^2+(2\zeta\lambda)^2\{1-(1+\mu)\lambda^2\}^2} \tag{85}$$

$$i_f = \frac{2\zeta\lambda^2[(p^2-\lambda^2)\{1-(1+\mu)\lambda^2\}-\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}]}{\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}^2+(2\zeta\lambda)^2\{1-(1+\mu)\lambda^2\}^2} \tag{86}$$

$$q_f = \frac{-\lambda[(p^2-\lambda^2)\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}+(2\zeta\lambda)^2\{1-(1+\mu)\lambda^2\}]}{\{(1-\lambda^2)(p^2-\lambda^2)-\mu p^2\lambda^2\}^2+(2\zeta\lambda)^2\{1-(1+\mu)\lambda^2\}^2} \tag{87}$$

First, when $\zeta = 0$ , i.e., when there is no damping, it is $i_1 = 0, i_2 = 0, i_f = 0$ . This is the same as the one-degree-of-freedom system. Furthermore, when $\lambda^2 = p^2$ is used, it becomes $q_1 = 0, q_2 = 0, q_f = 0$ . In other words, there are no eigenmodes. If it is an undamped dynamic absorber, it is as designed.

Next, when $\zeta \neq 0$, if $\lambda^2 = p^2$ , then $q_1 < 0,\ q_2 = 0$ , so mass point 1 is the node. When $\zeta$ is small, the numerator of $i_1$ is $2\mu\zeta p\{-p^4 + (2\zeta p)^2\} = -2\mu\zeta p^5 < 0$ and the denominator is positive, then $i_1 < 0$, and $i_2 > 0$ as well, and the vibration active intensity flows to mass 1. However, since it is a node, the vibration does not increase. Also, when $\lambda^2 = p^2$ , from $i_f > 0,\ q_f \neq 0$ , the input also enters mass point 1, but mass point 1 is a node, so the vibration does not increase. If it is a dynamic absorber, it will work as designed.

As described above, the function of a dynamic absorber can be explained from the standpoint of complex vibration intensity.

## 5.6. About Acoustic Active Intensity and Acoustic Reactive Intensity

### 5.6.1. Nature of Acoustic Active Intensity

#### 5.6.1.1. No input or attenuation

The same can be said for acoustic active intensity as for vibration active intensity. In the absence of input and damping, the equations satisfying the acoustic active intensity $\vec{i}_a$ are as follows from equations (40) , (42)

$$\mathrm{div}\vec{i}_a = 0 \tag{88}$$

$$\begin{aligned}\mathrm{rot}\vec{i}_a &= -j\omega\frac{1}{2}\rho\left(\vec{V}_a \times \vec{V}_a^{\,*}\right) \\ &= \frac{\omega}{c_a^2}\frac{\vec{i}_a \times \vec{q}_a}{U}\end{aligned} \tag{89}$$

Then, if there is no input or damping at $\vec{\imath}_a$, it is $\mathrm{div}\vec{\imath}_a = 0$ and there is no upwelling or suction point. It is also generally $\mathrm{rot}\vec{\imath}_a \neq \vec{0}$ and vortices exist. $\vec{\imath}_a$ flows from the antinode of the eigenmode to the node because it flows from the point of higher energy to the point of lower energy. However, since there is no upwelling or suction point, no $\vec{\imath}_a$ is generated or dissipated from the antinode or node. At the antinode position of the mode, $U$ has a value, and at the node position of the mode, $\vec{V}_a \times \vec{V}_a^{\,*}$ has a value. Therefore, it creates a vortex as if it were gushing out at the antinode and sucking in at the node.

If a vortex forms at the antinode position of a mode, the acoustic active intensity will stay there, and this, combined with the eigenmode shape, will increase the vibration. If a vortex forms at the nodal position of a mode and the acoustic active intensity stays there, the sound pressure does not increase because it is at the nodal position, and this is not a problem.

One way to eliminate the vortex is to design it to be $\vec{V}_a \times \vec{V}_a^{\,*} = \vec{0}$ . If this is achieved, $\mathrm{div}\vec{\imath}_a = 0$ and $\mathrm{rot}\vec{\imath}_a = \vec{0}$ , then the vector field created by the acoustic active intensity $\vec{\imath}_a$ is a Laplace field. The Laplace field has the property that the maximum and minimum values exist at the boundary. Therefore, the acoustic active intensity $\vec{\imath}_a$ flows from a place of high energy to a place of low energy without creating vortices.

5.6.1.2. If there is input or attenuation

Considering the case with input and attenuation in the same way, the equation satisfied by the acoustic active intensity $\vec{\imath}$ is as follows

$$\mathrm{div}\vec{\imath}_a = \mathrm{div}\vec{\imath}_f - \mathrm{div}\vec{\imath}_D \tag{90}$$

$$\begin{aligned}\mathrm{rot}\vec{\imath}_a &= -j\omega\frac{1}{2}\rho\left(\vec{V}_a \times \vec{V}_a^{\,*}\right) + \mathrm{rot}\vec{\imath}_f - \mathrm{rot}\vec{\imath}_D \\ &= \frac{\omega}{c_a^2}\frac{\vec{\imath}_a \times \vec{q}_a}{U} + \mathrm{rot}\vec{\imath}_f - \mathrm{rot}\vec{\imath}_D\end{aligned} \tag{91}$$

If we look at the equation (90) of div , we can see that the input position is at the gushing point and the damping position is at the sucking point. If we look at the right side of the equation(91) of rot, the first term means the vortex generated by the eigenmodes, the second term means the vortex generated by the input, and the third term means the vortex generated by the damping.

In order to eliminate the vortex in $\vec{\imath}_a$, $\mathrm{rot}\vec{\imath}_a = \vec{0}$ may be set using the acoustic active intensity $\vec{\imath}_f$ due to input or the acoustic active intensity $\vec{\imath}_D$ due to attenuation. At the same time, $\mathrm{div}\vec{\imath}_a = 0$ can be used for Laplace fields. However, too much attenuation will create a vortex by $\mathrm{rot}\vec{\imath}_D$ . Alternatively, external sound sources can be used to eliminate the divergence and rotation of the acoustic active intensity $\vec{i}_a$.

### 5.6.2. Properties of Acoustic Reactive Intensity

#### 5.6.2.1. No input or attenuation

The same can be said for the acoustic reactive intensity $\vec{q}_a$ in the absence of input and attenuation. From the equations(41) ,(43) , the following equation holds.

$$\mathrm{div}\vec{q}_a = -\omega L \tag{92}$$

$$\mathrm{rot}\vec{q}_a = \vec{0} \tag{93}$$

Then, if there is no input or damping at $\vec{q}_a$, it is $\mathrm{div}\vec{q} \neq 0$ and there is an upwelling or suction point. Also, it is $\mathrm{rot}\vec{q} = \vec{0}$ and there is no vortex.

Here, we consider $L$ in the $x$ direction, partially taken out. However, $\phi$ is the velocity potential.

$$\begin{aligned} L &= T - U \\ &= \frac{1}{2}\rho\dot{u}\dot{u}^* - \frac{1}{2\rho c_a^2}pp^* \\ &= \frac{1}{2}\rho\left(-\frac{\partial\phi}{\partial x}\right)\left(-\frac{\partial\phi^*}{\partial x}\right) - \frac{1}{2\rho c_a^2}(j\omega\phi)(-j\omega\phi) \\ &= \frac{1}{2}\rho\left(\frac{\partial\phi}{\partial x}\right)\left(\frac{\partial\phi^*}{\partial x}\right) - \frac{1}{\rho c_a^2}\omega^2\phi\phi^* \end{aligned} \tag{94}$$

$\phi$ is at its maximum, $\frac{\partial\phi}{\partial x}$ becomes $0$ . In other words, it is the antinode position of the mode, which at this time is $\mathrm{div}\vec{q}_a = -\omega L > 0$ . Conversely, where $\phi$ becomes $0$ , $\frac{\partial\phi}{\partial x}$ has the value. In other words, it is the node position of the mode, which is $\mathrm{div}\vec{q}_a = -\omega L < 0$ . The same is true for the other directions.

$\vec{q}_a$ also flows from the point of high energy to the point of low energy, so it flows from the antinode of the eigenmode to the node. Therefore, an outpouring point is created at the antinode position (sound pressure maximum) of $\mathrm{div}\vec{q}_a > 0$ , and a sucking point is created at the node position (sound pressure 0) of $\mathrm{div}\vec{q}_a < 0$ . In other words, by looking at $\vec{q}_a$ , the antinode and node positions of the mode become clear.

#### 5.6.2.2. If there is input or attenuation

The case with input and damping is considered in the same way as vibration reactive intensity. The equation satisfied by the acoustic reactive intensity $\vec{q}_a$ is as follows

$$\mathrm{div}\vec{q}_a = -\omega L + \mathrm{div}\vec{q}_f \tag{95}$$

$$\mathrm{rot}\vec{q}_a = \mathrm{rot}\vec{q}_f \tag{96}$$

Then, in the case of input and damping in $\vec{q}_a$, the input position is a gushing point and a vortex. Damping has no effect. Therefore, it may be possible to probe the input location by looking at $\vec{q}_a$ .

From the above, $\vec{q}_a$ exists when there is an eigenmode (or standing wave) in the vicinity of the input position or when there is an eigenmode (or standing wave), and it is not attenuated by the structure, only moving from the antinode of the mode to the nodal position. Conversely, an eigenmode exists because the acoustic reactive intensity exists. In particular, where $-L > 0$, that is, where the potential energy $U$ is dominant, if $\vec{V}_a \times \vec{V}_a^{\,*}$ has a value, the acoustic active intensity creates a vortex, causing it to stagnate and the sound pressure to increase, which can be said to become an antinode of the eigenmode.

Also, $\vec{q}_a$ cannot be consumed in damping. In other words, it cannot be $\mathrm{div}\vec{q}_a = 0$ or $\mathrm{rot}\vec{q}_a = \vec{0}$ . Therefore, if it is generated, it will continue to exist without disappearing as long as it is on that frequency. Alternatively, by using an external sound source, the divergence and rotation of the acoustic reactive intensity $\vec{q}_a$ can be eliminated.

## 6. Complex vibrational intensity, complex acoustic intensity, and the coupling of vibration and acoustics

This chapter discusses the coupling of vibration and acoustics using complex vibrational intensity and complex acoustic intensity.

When vibration and acoustics are coupled, the Lagrangian density is given by the following equation.

$$\begin{aligned} L &= T - U \\ &= \rho\left(\frac{1}{2}\dot{u}\dot{u}^* + \frac{1}{2}\dot{v}\dot{v}^* + \frac{1}{2}\dot{w}\dot{w}^*\right) \\ &\qquad - \left(\sigma_{xx}\frac{\partial u^*}{\partial x} + \tau_{xy}\frac{\partial u^*}{\partial y} + \tau_{xz}\frac{\partial u^*}{\partial z} + \tau_{yx}\frac{\partial v^*}{\partial x} + \sigma_{yy}\frac{\partial v^*}{\partial y}\right. \\ &\qquad \left. + \tau_{yz}\frac{\partial v^*}{\partial z} + \tau_{zx}\frac{\partial w^*}{\partial x} + \tau_{zy}\frac{\partial w^*}{\partial y} + \sigma_{zz}\frac{\partial w^*}{\partial z}\right) \\ &\qquad + \frac{1}{2}\rho\left(\frac{\partial \phi}{\partial x}\right)\left(\frac{\partial \phi^*}{\partial x}\right) - \frac{1}{\rho c_a^2}\omega^2\phi\phi^* \end{aligned} \tag{97}$$

This Lagrangian density $L$ appears in the divergence of the vibrational reactive intensity and the acoustic reactive intensity in complex vibrational and acoustic intensities. That is, as follows:

$$\mathrm{div}\vec{q} = \omega L \tag{98}$$

$$\mathrm{div}\vec{q}_a = -\omega L \tag{99}$$

The fact that a vector has a value indicates the existence of upwelling and sucking points. From this result, it can be seen that when vibration and acoustics are coupled, from the perspective of complex vibrational intensity and complex acoustic intensity, the vibrational reactive intensity and acoustic reactive intensity flow to each other through the upwelling and sucking points corresponding to the antinodes and nodes of the modes. As a result, according to equations (46) and (89), the vibrational active intensity and acoustic active intensity create vortices at the antinodes and nodes of the modes.

It should be noted that this relationship does not hold only between complex vibration intensity and complex acoustic intensity. That is, even in the vibration of a structure consisting of multiple structures, the vibrational reactive intensity of each structure couples because it flows to each other through the upwelling and sucking points corresponding to the antinodes and nodes of the modes, and as a result, according to equation (46), the vibrational active intensity creates vortices at the antinodes and nodes of the modes. Naturally, even in acoustics consisting of multiple spaces, the acoustic reactive intensity of each space couples with the upwelling and sucking points corresponding to the antinodes and nodes of the mode, and as a result, according to equation (89), the acoustic active intensity creates vortices at the antinodes and nodes of the mode.

Furthermore, as previously stated, vibrational reactive intensity and acoustic reactive intensity cannot be dissipated by attenuation. Therefore, if a coupled environment exists when vibrational reactive intensity or acoustic reactive intensity occurs, it will continue to exist and coupled without disappearing as long as it remains at that frequency. Alternatively, vibration reactive intensity and acoustic reactive intensity can be eliminated by using external forces or external sound sources.

## 7. Elimination of Complex Vibrational Intensity and Complex Acoustic Intensity Using External Forces and Sound Sources

Previously, we discussed eliminating the divergence and rotation of vibrational active intensity and vibrational reactive intensity by using external forces. Similarly, we discussed eliminating the divergence and rotation of acoustic active intensity and acoustic reactive intensity by using external sound sources.

Of these, it is also possible to consider eliminating only the divergence and rotation of vibrational active intensity and acoustic active intensity. However, vibrational reactive intensity and acoustic reactive intensity remain. That is, vibrational and acoustic standing waves remain. Therefore, these remaining standing waves may adversely affect the structure and space.

Therefore, by simultaneously eliminating the divergence and rotation of vibrational reactive intensity and acoustic reactive intensity using external forces and external sound sources, it is conceivable that the entire complex vibrational intensity and complex acoustic intensity can be eliminated. By doing so, the effects of standing waves can also be eliminated.

## 8. Discussion of previous phenomena and new design guidelines

### 8.1. Considerations from the complex vibration intensity of conventional sound vibration reduction methods

Conventional sound and vibration reduction methods can also be considered from the divergence and rotation of complex vibration intensities.

- When a vortex forms in the antinode of the mode, the vibration active intensity $\vec{\imath}$ stays there, which causes the vibration to increase. The conventional method of applying a dump sheet at the antinode position of the mode was effective because the vibration active intensity $\vec{\imath}$ is sucked in there. If the dump sheet is not effective, it may be because the sucking in by damping is weak and the vortex in the antinode of the mode is not eliminated, or because the damping is too large and the vortex is created by the damping, causing the vibration to increase in the opposite direction.
- Even if a vortex is created at the nodal position of the mode and the vibration active intensity $\vec{\imath}$ stagnates, there is no problem because the vibration does not increase. Therefore, the reason for bringing the input position to the nodal position of the mode is thought to be to prevent the inflow of vibration active intensity $\vec{\imath}$ from the input position by stagnating the vibration intensity that springs at the input position due to the vortex at the nodal position.
- Similarly, it is thought that increasing the point of vibration stiffness can also create a pseudo-node because of creating a minimum of energy by increasing the stiffness. these pseudo-nodes create vortices and form and producing the effect of vibration active intensity stagnation.
- In order to reduce the frequency response function (FRF) peak, the eigenmodes have been properly arranged. This section describes the phenomenon of overlapping eigenfrequencies causing the FRF peak to increase. First, at the location where $\vec{V_e} \times \vec{V_e}^*$ has a value in the mode of the input side (e.g. the suspension or panel) at the frequency in the problem, the vibration active intensity $\vec{\imath}$ creates a vortex, causing the vibration to increase and the input to increase as a result. As a result, the peak was generated in the FRF because the

vibration and sound pressure increased due to the input vibration active intensity $\vec{\imath}$ creating and accumulating vortices at the same frequency at the natural mode's antinode position where $\vec{V_e} \times \vec{V_e}^*$ has a value on the side where a large input was applied (e.g. the vehicle body or an acoustically closed space). Therefore, by shifting the frequencies of the eigenmodes to each other, the vibration active intensity $\vec{\imath}$ does not create a vortex at the problematic frequency because $\vec{V_e} \times \vec{V_e}^* = \vec{0}$ , eliminating the stagnation, and reducing the vibration and sound pressure on the input side or on the input side, the FRF peak can be reduced.

### 8.2. New Design Guidelines

From the nature of vibration active intensity $\vec{\imath}$ , we can also say the following.

- To prevent vibration active intensity $\vec{\imath}$ from flowing into the underbody, or to prevent vibration from increasing in the roof and panels, damping material could be applied to the front section and pillar joints, or to the body frame and panel joints, to absorb power as $\mathrm{div}\vec{\imath} < 0$ , thereby preventing vibration active intensity $\vec{\imath}$ from flowing beyond them. At the same time, it is possible to reduce the flow of vibration active intensity at the joints. At the same time, by increasing the stiffness of the part of the component at the joint where the active intensity is not wanted to flow, a pseudo-node can be created and a vortex can be generated, making it difficult for the active intensity to flow beyond that component by stagnating the active intensity. The amount of flow into the underfloor can also be reduced by increasing the stiffness of a portion of the member at the underfloor entrance, creating a pseudo-node, generating a vortex, and allowing the vibration active intensity to stagnate.
- As in equations (59) and (60), it is conceivable to use $\mathrm{div}\vec{\imath}_D$ and $\mathrm{rot}\vec{\imath}_D$ generated by damping to become $\mathrm{div}\vec{\imath} = 0, \mathrm{rot}\vec{\imath} = \vec{0}$ and create a Laplace field, so that the vibration active intensity $\vec{\imath}$ does not create vortices and consequently does not cause stagnation. Therefore, by first applying damping material at the input position, both vortices, causing by input and damping, can be cancelled together, and the input gushing can be cancelled by the sucking in of the damping at the same time. Furthermore, the excitation point stiffness can be increased to create a vortex. That makes it a pseudo-nodal position, stagnating the vibration active intensity $\vec{\imath}$ and preventing the inflow of $\vec{\imath}$ . Next, if there is a problem with $\vec{V_e} \times \vec{V_e}^* \neq \vec{0}$ , apply damping material to the antinode position of the eigenmode. The vortex at the antinode position with the dumping material can suck in the

stagnant vibration active intensity.

- Considering a closed surface, equations (63) and (64) hold true. Considering the entire car body, including the panels, as a closed surface, it can be seen that if $\vec{\iota}_f = \vec{\iota}_D$, then $\mathrm{div}\vec{\iota} = 0$ and $\mathrm{rot}\vec{\iota} = \vec{0}$. By making the vibration active intensity of the input and damping equal, the entire car body becomes a Laplace field. This is easier to implement than considering each panel individually, as it does not require consideration of the vortices generated by the eigenmodes. As a method, if you want to do it passively, you could indirectly make $\mathrm{div}\vec{\iota} = 0$, $\mathrm{rot}\vec{\iota} = \vec{0}$ by attaching damping material to the input position.

## 9. Conclusion

By considering the divergence and rotation of complex vibration intensity, we were able to show the possibility of explaining the phenomenon of vibration active intensity stagnation. In addition, we used the divergence and rotation concept to explain the phenomena occurring in conventional phenomena and sound and vibration reduction measures. Furthermore, the coupling of vibration and acoustics was discussed using the relationship between complex vibration intensity and complex acoustic intensity. Furthermore, the significance of eliminating complex vibration intensity and complex acoustic intensity by using external forces or external sound sources was also discussed.

In addition, the nature of vibration active intensity suggested the possibility of new design guidelines. Furthermore, from the properties of vibration reactive intensity, we were able to show the possibility of using it to probe the antinode, nodal, and input positions of modes, and the existence of eigenmodes when vibration reactive intensity is present.

## References

Alfredsson, K. S. (1997). *Active and reactive structural energy flow.* Journal of Vibration and Acoustics, Vol. 119.

Garvic, L., & Pavic, G. (1993). *A finite element method for computation of structural intensity by the normal mode approach.* Journal of Sound and Vibration, Vol.164, No.1.

Hidaka, Y., Ankyu, S., & Tachibana, H. (1987). *Sound field analysis using complex acoustic intensity.* The Journal of Acoustical Society of Japan(in Japanese).

Jolly, N., & Pascal, J. C. (2006). *Energy flow relations from quadratic quantities in three-dimensional isotropic medium and exact formulation for one-dimensional* *waves.* Journal of Sound and Vibratioin, Vol. 298.

Liu, Z. S., Lee, H. P., & Lu, C. (2006). *Passive and active interior noise control of box structures using the structural intensity method.* Applied Acoustics, Vol. 67.

Noiseux, D. U. (1970). *Measurement of power flow in uniform beams and plates.* Journal of Acoustical Society of America, Vol.47.

Pavic, G. (1976). *Measurement of structure borne wave intensity, Part I: Formulation of the methods.* Journal of Sound and Vibration, Vol.49, No.2.

Yamazaki, T., & Nakamura, H. (2015). *Passive control of structural intensity on a flat panel by beam attachment.* The 22nd International Congress on Sound and Vibration.