Derivation of Rossiter Formula in Cavity Resonance

May 8, 2026

May 16, 2026 Revised

Takuya Yabu(takuya.yabu@live.jp)

Abstract

Research on automobile whistling and suction sounds has been conducted in the past. For cavity resonance, a type of whistling sound, the cavity resonance frequency has been determined using Rossiter's empirical equation. Therefore, limiting the study to the one-dimensional case, Rossiter formula was derived by solving a system of three differential equations. In addition, equations for the stability conditions of cavity resonance and the sound pressure were derived. Finally, the results were verified through numerical calculations, and reasonable conclusions were obtained.

Nomenclature

L_H: Maximum hole length

ρ, c: Air density and air sound speed

ω: Angular frequency

f, f_c: Frequency and cavity resonance frequency

k: Wavenumber

t: Time

j: Imaginary unit

U, U_{vortex}: Mainstream air velocity and vortex (vorticity) velocity

κ: Ratio of vortex (vorticity) velocity to mainstream air velocity

ϕ: Empirical coefficient for phase lag

M: Mach number

ζ: Vorticity

α: Efficiency of conversion from vorticity to sound pressure determined by edge shape

β: Efficiency of conversion from sound pressure to vorticity

$\delta'(x)$: Differential of Dirac delta function

$H(x)$: Heaviside step function

s: Complex number

1. Introduction

In the past, research has been conducted on the whistling and suction sounds of automobiles (Calvo, Diaz, & San Roman, 2005) (Chien-Hsiung, Lung-Ming , Chang-Hsien , Yen-Loung , & Jik-Chang , 2009) (George, 1990) (Jagtiani, 1972) (Jung & Oh, 1995) (Münder & Carbon, 2022) (Oettle & Sims-Williams, 2017) (Qatu, Abdelhamid, Pang, & Sheng, 2009) (Wang, Chen, & Zhang, 2021) (Zhang, Meng, Li, & Zheng, 2022). Cavity resonance is a type of whistling sound. In cavity resonance, the cavity resonance frequency has traditionally been determined using Rossiter formula, an empirical formula. Therefore, limiting ourselves to the one-dimensional case, we will represent cavity resonance as a physical phenomenon using three differential equations. Rossiter formula will be derived from these three simultaneous differential equations. Furthermore, the equations for the stabilization conditions of cavity resonance and the sound pressure equation will also be derived. Finally, we will verify it with numerical calculations.

These will be discussed below.

2. One-Dimensional Cavity Resonance

2.1. Vorticity Movement Equation

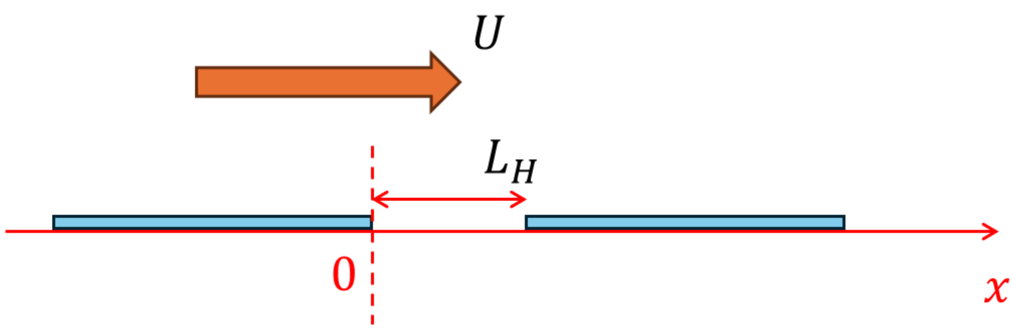

Fig. 1 1-Dimensional Board with Hole

Fig. 1 shows a board with a hole. Here, the conventional definitions of cavity resonance and cavity resonance frequency are given below.

Definition :

When air is flowing at velocity U and there is a hole, a vortex moves from the upstream end to the downstream end of the hole. This vortex collides with the downstream end, generating sound. This generated sound then moves from the downstream end to the upstream end. Sound is a change accompanied by particle velocity, and this particle velocity then collides with the upstream end, generating another vortex. This series of phenomena is defined as "cavity resonance." Furthermore, the reciprocal of the time from when a vortex moves and generates sound until another vortex is generated is defined as the "cavity resonance frequency."

Based on this definition, we derive the differential equation representing cavity resonance, limited to one dimension. First, the movement equation of vorticity $\zeta(x, t)$ is expressed by the following equation. However, the viscosity term is ignored.

$$\frac{\partial \zeta}{\partial t} + U_{vortex} \frac{\partial \zeta}{\partial x} = 0 \tag{1}$$

2.2. Powell's Equation

Next, we will find the relationship between sound pressure and vorticity.

Powell's equation is the relationship between sound pressure and vorticity. Expressed in one dimension, Powell's equation is given by the following:

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \alpha \rho U \zeta(L_H, t) \delta'(x - L_H) \tag{2}$$

2.3. Boundary Conditions

Finally, we find the boundary conditions under which the velocity of the returning particles changes to vorticity. This is given by the following equation.

$$\zeta(0, t) = -\beta \frac{\partial}{\partial x} p(0, t) \tag{3}$$

3. Derivation of Rossiter Fromula

3.1. Solution to the Vorticity Transfer Equation

Rossiter formula is derived by solving the three simultaneous differential equations, equations (1), (2), and (3). First, we solve equation (1). The solution when the boundary condition is $\zeta(0, t)$ is given by the following equation:

$$\zeta(x, t) = \zeta\left(0, t - \frac{x}{U_{vortex}}\right) \tag{4}$$

3.2. Sound Pressure Equation Using Green's Function Derived from Powell's Equation

Next, we solve Powell's equation (2). This can be obtained using Green's functions in the following form First, the Green's function is given by the following formula.

$$G(|x|, t) = -\frac{c}{2} H(ct - |x|) \tag{5}$$

From equation (2), the following equation is obtained.

$$p(x,t) = \int_{-\infty}^{t} \int_{-\infty}^{\infty} G(x,t,\xi,\tau)\alpha\rho U \zeta(\xi,t)\delta'(\xi - L_H)\, d\xi d\tau$$

$$= \int_{-\infty}^{t} \int_{-\infty}^{\infty} -\frac{\alpha\rho U c}{2} \zeta(\xi,t) H(c(t-\tau) - |x-\xi|)\delta'(x - L_H)\, d\xi d\tau \tag{6}$$

Here, $\int f(\xi)\delta'(x - L)d\xi = -f'(L)$ is used.

$$p(x,t) = \int_{-\infty}^{t} \int_{-\infty}^{\infty} -\frac{\alpha\rho U c}{2} \left[-\frac{\partial}{\partial\xi} \zeta(\xi,t) H(c(t-\tau) - |x-\xi|) \right]_{\xi = L_H} d\xi d\tau \tag{7}$$

Continue the calculation. However, $\frac{\partial}{\partial\zeta}\zeta(\xi,\tau)$ is mathematically smaller than $\frac{\partial}{\partial\xi}H(c(t - \tau) - |x - \xi|)$, and physically, the gradual change in vorticity during transport (derivative of ζ) has a small contribution, so it is ignored.

$$p(x,t) = \frac{\alpha\rho U c}{2} \int_{-\infty}^{t} \left[\zeta(L_H,\tau) \frac{\partial}{\partial\xi} H(c(t-\tau) - |x-\xi|) \right]_{\xi = L_H} d\tau \tag{8}$$

Integrate this equation, assuming $x \leq \xi$ and $x \leq L_H$. Also, the derivative of Heaviside step function is the delta function. We also applied the scaling property of the Dirac delta function.

$$p(x,t) = \frac{\alpha\rho U c}{2} \int_{-\infty}^{t} \left[\zeta(\xi,\tau) \frac{\partial}{\partial\xi} H(c(t-\tau) - |x-\xi|) \right]_{\xi = L_H} d\tau$$

$$= \frac{\alpha\rho U c}{2} \int_{-\infty}^{t} \left[\zeta(\xi,\tau) \delta(c(t-\tau) - |x-\xi|) \frac{\partial}{\partial\xi}(-|x-\zeta|) \right]_{\xi = L_H} d\tau$$

$$= \frac{\alpha\rho U c}{2} \int_{-\infty}^{t} [\zeta(\xi,\tau)\delta(c(t-\tau) - |x-\xi|) \cdot (-1)]_{\xi = L_H} d\tau$$

$$= -\frac{\alpha\rho U c}{2} \int_{-\infty}^{t} \zeta(L_H,\tau)\delta(c(t-\tau) - |x - L_H|) d\tau$$

$$= -\frac{\alpha\rho U c}{2} \zeta\left(L_H, t - \frac{L_H - x}{c}\right) \cdot \frac{1}{c}$$

$$\therefore p(x,t) = -\frac{\alpha\rho U}{2} \zeta\left(L_H, t - \frac{L_H - x}{c}\right) \tag{9}$$

Therefore, the following equation is obtained. This equation is the equation for the sound pressure generated by cavity resonance. Also, equation (4) was considered. However, $0 \leq x \leq L_H$.

$$p(x,t) = -\frac{\alpha\rho U}{2} \zeta\left(L_H, t - \frac{L_H - x}{c}\right) = -\frac{\alpha\rho U}{2} \zeta\left(0, t - \frac{L_H}{U_{vortex}} - \frac{L_H - x}{c}\right) \tag{10}$$

3.3. Vorticity Equation Considering Boundary Conditions

Finally, consider equation (3). Substitute equation (10).

$$\zeta(0,t) = -\beta\frac{\partial}{\partial x}\left[-\frac{\alpha\rho U}{2}\zeta\left(L_H, t-\frac{L_H-x}{c}\right)\right]_{x=0}$$

$$=\frac{\alpha\beta\rho U}{2}\frac{\partial}{\partial x}\left[\zeta\left(L_H, t-\frac{L_H-x}{c}\right)\right]_{x=0}$$

$$=\frac{\alpha\beta\rho U}{2}\frac{\partial}{\partial t}\left[\zeta\left(L_H, t-\frac{L_H-x}{c}\right)\cdot\frac{\partial}{\partial x}\left(t-\frac{L_H-x}{c}\right)\right]_{x=0}$$

$$=\frac{\alpha\beta\rho U}{2}\frac{\partial}{\partial t}\left[\zeta\left(L_H, t-\frac{L_H-x}{c}\right)\cdot\frac{1}{c}\right]_{x=0}$$

$$=\frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta\left(L_H, t-\frac{L_H-0}{c}\right)$$

Therefore, the following equation is obtained. Here, equation (4) was considered.

$$\therefore \zeta(0,t) = \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta\left(L_H, t-\frac{L_H}{c}\right) = \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta\left(0, t-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right) \tag{11}$$

3.4. Derivation of the Characteristic Equation

In equation (11), substituting $\zeta(0,t) = \zeta_0 e^{st}$ into both sides, we obtain the following equation:

$$\zeta_0 e^{st} = \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta_0 e^{s\left(t-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}$$

$$=\frac{\alpha\beta\rho U}{2c}s\zeta_0 e^{st}e^{s\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}$$

$$\therefore \zeta_0 e^{st} = \frac{\alpha\beta\rho U}{2c}s\zeta_0 e^{st}e^{s\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)} \tag{12}$$

Eliminating $\zeta_0 e^{st}$ from both sides, we obtain the following equation.

$$1 = \frac{\alpha\beta\rho U}{2c}se^{s\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)} \tag{13}$$

Equation (13) is the characteristic equation.

3.5. Derivation of Rossiter Formula and Cavity Resonance Stabilization Conditions from the Characteristic Equation

By solving the characteristic equation, we derive Rossiter formula and the cavity resonance stabilization conditions.

In the characteristic equation, we consider the properties by setting $s = \sigma + j\omega$. Here, since we want to determine whether "sound is produced or not," we consider the case when $\sigma = 0$. That is, $s = j\omega$. In this case, equation (13) becomes the following equation:

$$1 = \frac{\alpha\beta\rho U}{2c}\omega \cdot j \cdot e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)} \tag{14}$$

In equation (14), we consider the case when the phases of both sides are equal. In this case, sound is produced. Therefore, the following equation must hold.

$$\arg\left(\frac{\alpha\beta\rho U}{2c}\omega\right) + \arg(j) + \arg\left(e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}\right) = -2\pi n \tag{15}$$

In equation (15), $\arg\left(\frac{\alpha\beta\rho U}{2c}\omega\right)$ is a positive value, so it is 0. $\arg(j)$ is $-\frac{\pi}{2}$. $\arg\left(e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}\right)$ is $\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)$. That is, the following equation is obtained.

$$0 - \frac{\pi}{2} + \omega\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right) = -2\pi n$$

$$\omega\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right) = -2\pi n + \frac{\pi}{2}$$

$$2\pi f_c\left(\frac{L_H}{U_{vortex}} + \frac{L_H}{c}\right) = 2\pi n - \frac{\pi}{2}$$

$$f_c = \frac{n - \frac{1}{4}}{\frac{L_H}{U_{vortex}} + \frac{L_H}{c}} \tag{16}$$

Here, we use the following relationship:

$$U_{vortex} = \kappa U \tag{17}$$

$$M = \frac{U}{c} \tag{18}$$

Therefore, equation (16) becomes as follows:

$$f_c = \frac{U}{L_H}\frac{n - \frac{1}{4}}{\left(\frac{1}{\kappa} + M\right)} \tag{19}$$

This is exactly equivalent to Rossiter formula, which is the following equation. Empirically, we have used ϕ as 0.25, but we were able to derive $0.25 = \frac{1}{4}$ from the equation that describes the actual phenomenon.

$$f_c = \frac{U}{L_H}\frac{n - \phi}{\left(\frac{1}{\kappa} + M\right)} \tag{20}$$

Now, when f_c is obtained from equation (16) or equation (19), that is, when $\omega =$

$2\pi f_c$, the gain G on the right side of the characteristic equation (14), which is equation (21), determines whether the self-excited oscillation of the cavity resonance is stable or unstable. That is, if $G > 1$, it is unstable, self-excited oscillation occurs, and the cavity resonance sound gets louder and louder. If $G < 1$, it is stable, and the sound decays until the cavity resonance sound stops. If $G = 1$, it is a critical state, and the cavity resonance sound is emitted at a constant volume.

$$G = \frac{\alpha\beta\rho U}{2c}\omega \tag{21}$$

4. Numerical Example

Based on previous results, a numerical example is shown below.

The physical quantities used in the calculation are shown in Table 1. As an initial value, ζ was given 0.1(1/s) only for the first 0.001(sec).

Table 1 Parameter Name and Value for Numerical Simulation

Parameter Name	Value
L_H	0.5(m)
U_{vortex}	20.4(m/s)
U	34.0(m/s)
α	0.1
β	0.005
c	340.0(m/s)
ρ	1.2(kg/m^3)

The graph below shows the calculated vorticity and sound pressure at $x = 0.0$.

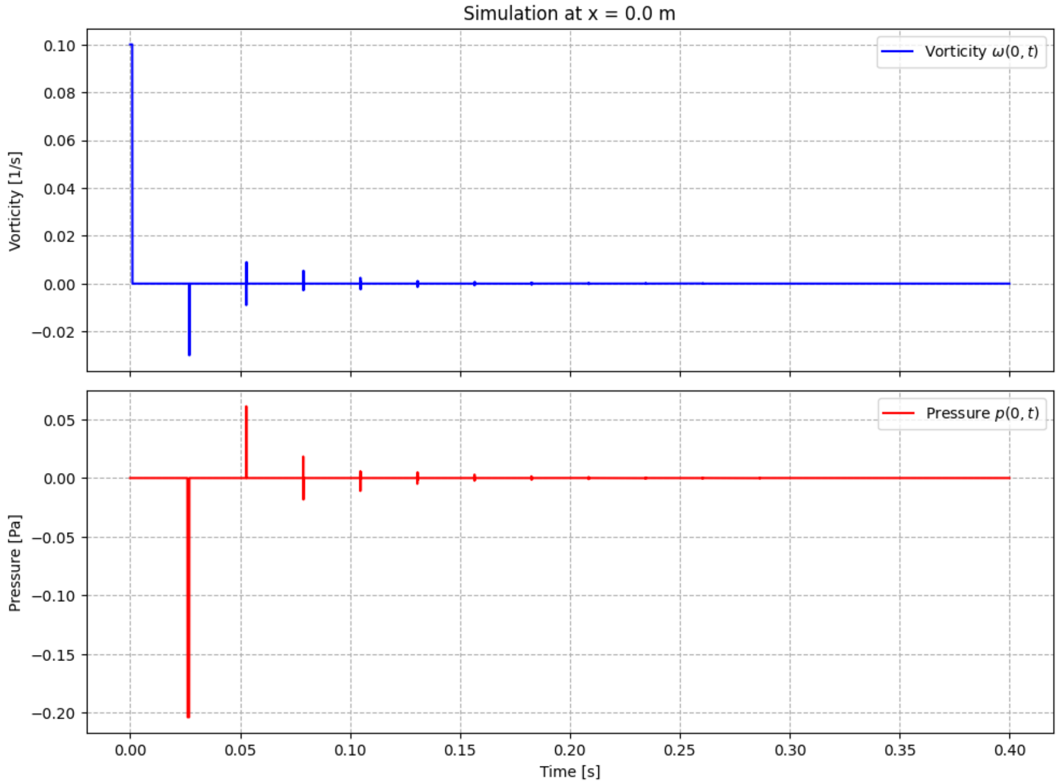

Fig. 2 Simulation Result ($x = 0.0$)

From Fig. 2, it can be seen that sound pressure is generated in the negative direction after a time of $\frac{L_H}{U_{vortex}} + \frac{L_H}{c} = 0.026$(sec), and that vorticity and sound pressure continue to be generated every 0.026(sec) thereafter. In other words, it is a periodic motion. Furthermore, it can be seen that the vorticity and sound pressure gradually decrease and decay until their values disappear. Since $G = \frac{\alpha\beta\rho U}{2c}\omega = 0.0054 < 1$, it can be seen that no self-excited oscillation is occurring.

Next, the graphs of the calculated vorticity and sound pressure at $x = 0.25$ are shown below.

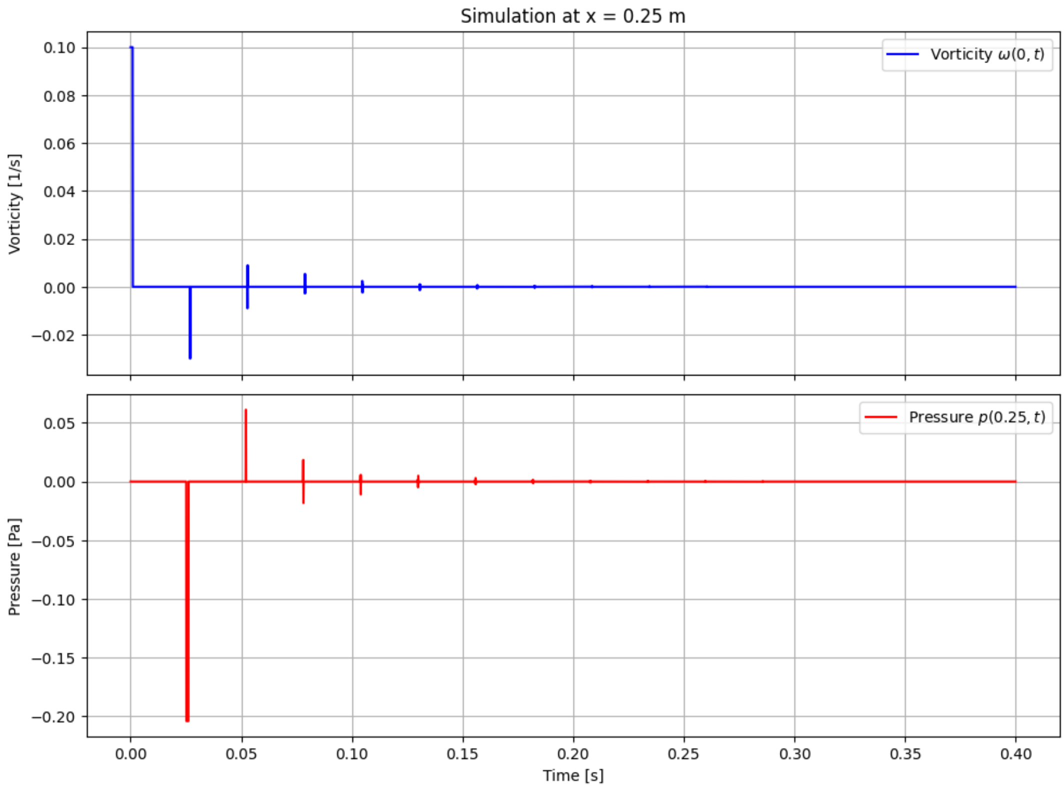

Fig. 3 Simulation Result ($x = 0.25$)

Looking at Fig. 3, since the position is $x = 0.25$, we can see that the time delay from the generation of vorticity to the generation of sound pressure is $\frac{L_H}{U_{vortex}} + \frac{L_H - x}{c} = \frac{L_H}{U_{vortex}} + \frac{L_H - 0.25}{c} = 0.025$(sec), which is shorter than the value at the observation position. Also, the generation interval remains unchanged at 0.026(sec), as in Fig. 2. In other words, the phenomenon is captured well.

Finally, as a forced oscillation, the vorticity at $x = 0$ is given by the following equation, where $f_{x,1}$ is the first resonance frequency (28.87 Hz) according to Rossiter formula.

$$\zeta(0,t) = 0.1 \sin\left(2\pi f_{x,1} t\right) \tag{22}$$

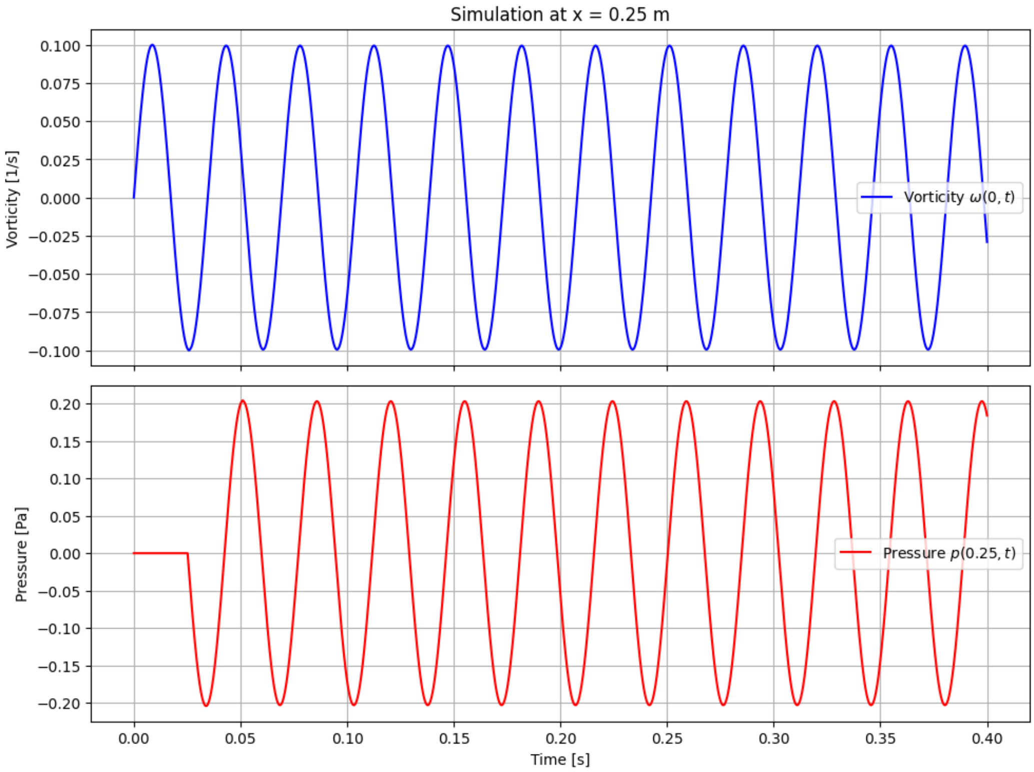

Fig. 4 Simulation Result under Forced Oscillation ($x = 0.25$)

Fig. 4 shows the results for vorticity and sound pressure at $x = 0.25$. It can be seen that even when forced oscilation is applied, sound pressure is generated periodically after a time delay of 0.025(sec).

5. Conclusion

Research on automobile whistling and suction sounds has been conducted in the past. For cavity resonance, a type of whistling sound, the cavity resonance frequency has been determined using Rossiter's empirical equation. Therefore, limiting the study to the one-dimensional case, Rossiter's equation was derived by solving a system of three differential equations. In addition, equations for the stability conditions of cavity resonance and the sound pressure were derived. Finally, the results were verified through numerical calculations, and reasonable conclusions were obtained.

References

Calvo, J. A., Diaz, V., & San Roman, J. L. (2005). *Controlling the turbocharger whistling noise in diesel engines.* International Journal of Vehicle Noise and Vibration, Vol.

2, No. 1. doi:https://doi.org/10.1504/IJVNV.2006.008524

Chien-Hsiung, T., Lung-Ming , F., Chang-Hsien , T., Yen-Loung , H., & Jik-Chang , L. (2009). *Computational aero-acoustic analysis of a passenger car with a rear spoiler.* Applied Mathematical Modelling, Volume 33, Issue 9. doi:https://doi.org/10.1016/j.apm.2008.12.004

George, A. R. (1990). *Automobile Aerodynamic Noise.* SAE Transactions, Vol. 99, Section 6. Retrieved from http://www.jstor.org/stable/44553993

Jagtiani, H. (1972). *The Objective Method of Evaluating Aspiration Wind Noise.* SAE Technical Paper 720506. doi:https://doi.org/10.4271/720506

Jung, W., & Oh, S. (1995). *The Influence of Vehicle Elements to Aspiration Wind Noise.* SAE Technical Paper 950624. doi:https://doi.org/10.4271/950624

Münder, M., & Carbon, C.-C. (2022). *Howl, whirr, and whistle: The perception of electric powertrain noise and its importance for perceived quality in electrified vehicles.* Applied Acoustics, Volume 185. doi:https://doi.org/10.1016/j.apacoust.2021.108412

Oettle, N., & Sims-Williams, D. (2017). *Automotive aeroacoustics: An overview.* Journal of Automobile Engineering, Volume 231, Issue 9. doi:https://doi.org/10.1177/0954407017695147

Qatu, M. S., Abdelhamid, M. K., Pang, J., & Sheng, G. (2009). *Overview of automotive noise and vibration.* International Journal of Vehicle Noise and Vibration, Vol. 5, No. 1-2. doi:https://doi.org/10.1504/IJVNV.2009.029187

Wang, Q., Chen, X., & Zhang, Y. (2021). *An Overview of Automotive Wind Noise and Buffeting Active Control.* SAE International Journal of Vehicle Dynamics, Stability, and NVH, 5(4). doi:https://doi.org/10.4271/10-05-04-0030

Zhang, F., Meng, W., Li, X., & Zheng, C. (2022). *A vehicle whistle database for evaluation of outdoor acoustic source localization and tracking using an intermediate-sized microphone array.* Applied Acoustics, Volume 201. doi:https://doi.org/10.1016/j.apacoust.2022.109113