Derivation of Rossiter Formula in Cavity Resonance

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Abstract

Research on automobile whistling and suction sounds has been conducted in the past. For cavity resonance, a type of whistling sound, the cavity resonance frequency has been determined using Rossiter's empirical equation. Therefore, limiting the study to the one-dimensional case, Rossiter formula was derived by solving a system of three differential equations. In addition, equations for the stability conditions of cavity resonance and the sound pressure were derived. Finally, the results were verified through numerical calculations, and reasonable conclusions were obtained.

Nomenclature

L_H: Maximum hole length

ρ, c: Air density and air sound speed

ω: Angular frequency

f, f_c: Frequency and cavity resonance frequency

k: Wavenumber

t: Time

j: Imaginary unit

U, U_{vortex}: Mainstream air velocity and vortex (vorticity) velocity

κ: Ratio of vortex (vorticity) velocity to mainstream air velocity

ϕ: Empirical coefficient for phase lag

M: Mach number

ζ: Vorticity

α: Efficiency of conversion from vorticity to sound pressure determined by edge shape (dimension: L)

β: Efficiency of conversion from sound pressure to vorticity(dimension: $\mathrm{L^2M^{-1}T}$)

$\delta'(x)$: Differential of Dirac delta function

$H(x)$: Heaviside step function

s: Complex number

L_c: Neck length of the Helmholtz-type cavity

S_c: Opening area of the Helmholtz-type cavity

V_c: Volume of the Helmholtz-type cavity

1. Introduction

In the past, research has been conducted on the whistling and suction sounds of automobiles (Calvo, Diaz, & San Roman, 2005) (Chien-Hsiung, Lung-Ming , Chang-Hsien , Yen-Loung , & Jik-Chang , 2009) (George, 1990) (Jagtiani, 1972) (Jung & Oh, 1995) (Münder & Carbon, 2022) (Oettle & Sims-Williams, 2017) (Qatu, Abdelhamid, Pang, & Sheng, 2009) (Wang, Chen, & Zhang, 2021) (Zhang, Meng, Li, & Zheng, 2022). Cavity resonance is a type of whistling sound. In cavity resonance, the cavity resonance frequency has traditionally been determined using Rossiter formula, an empirical formula. Therefore, limiting ourselves to the one-dimensional case, we will represent cavity resonance as a physical phenomenon using three differential equations. Rossiter formula will be derived from these three simultaneous differential equations. Furthermore, the equations for the stabilization conditions of cavity resonance and the sound pressure equation will also be derived. Finally, we will verify it with numerical calculations.

These will be discussed below.

2. One-Dimensional Cavity Resonance

2.1. Vorticity Movement Equation

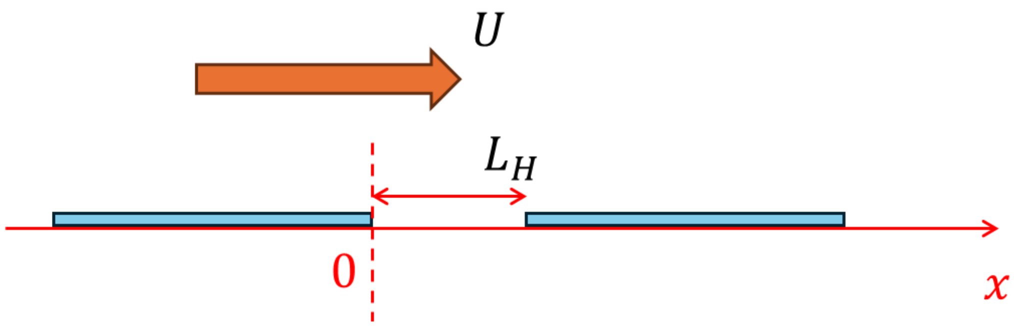

Fig. 1 1-Dimensional Board with Hole

Fig. 1 shows a board with a hole. Here, the conventional definitions of cavity resonance and cavity resonance frequency are given below.

Definition：

When air is flowing at velocity U and there is a hole, a vortex moves from the upstream end to the downstream end of the hole. This vortex collides with the downstream end, generating sound. This generated sound then moves from the downstream end to the upstream end. Sound is a change accompanied by particle velocity, and this particle velocity then collides with the upstream end, generating another vortex. This series of phenomena is defined as "cavity resonance." Furthermore, the reciprocal of the time from when a vortex moves and generates sound until another vortex is generated is defined as the "cavity resonance frequency."

Based on this definition, we derive the differential equation representing cavity resonance, limited to one dimension. First, the movement equation of vorticity $\zeta(x,t)$ is expressed by the following equation. However, the viscosity term is ignored.

$$\frac{\partial \zeta}{\partial t} + U_{vortex}\frac{\partial \zeta}{\partial x} = 0 \tag{1}$$

2.2. Powell's Equation

Next, we will find the relationship between sound pressure and vorticity.

Powell's equation is the relationship between sound pressure and vorticity. Expressed in one dimension, Powell's equation is given by the following:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \alpha\rho U\zeta(L_H,t)\delta'(x-L_H) \tag{2}$$

2.3. Boundary Conditions

Finally, we find the boundary conditions under which the velocity of the returning particles changes to vorticity. This is given by the following equation.

$$\zeta(0,t) = -\beta\frac{\partial}{\partial x}p(0,t) \tag{3}$$

3. Derivation of Rossiter Fromula

3.1. Solution to the Vorticity Transfer Equation

Rossiter formula is derived by solving the three simultaneous differential equations, equations (1), (2), and (3). First, we solve equation (1). The solution when the boundary condition is $\zeta(0,t)$ is given by the following equation:

$$\zeta(x,t) = \zeta\left(0, t - \frac{x}{U_{vortex}}\right) \tag{4}$$

3.2. Sound Pressure Equation Using Green's Function Derived from Powell's Equation

Next, we solve Powell's equation (2). This can be obtained using Green's functions in the following form First, the Green's function is given by the following formula.

$$G(|x|,t) = -\frac{c}{2}H(ct-|x|) \tag{5}$$

From equation (2), the following equation is obtained.

$$p(x,t) = \int_{-\infty}^{t}\int_{-\infty}^{\infty} G(x,t,\xi,\tau)\alpha\rho U\zeta(L_H,\tau)\delta'(\xi-L_H)\,d\xi d\tau$$

$$= \int_{-\infty}^{t}\int_{-\infty}^{\infty} -\frac{\alpha\rho Uc}{2}\zeta(L_H,\tau)H(c(t-\tau)-|x-\xi|)\delta'(x-L_H)\,d\xi d\tau \tag{6}$$

Here, $\int f(\xi)\delta'(x-L)d\xi = -f'(L)$ is used.

$$p(x,t) = \int_{-\infty}^{t}\int_{-\infty}^{\infty} -\frac{\alpha\rho Uc}{2}\left[-\zeta(L_H,\tau)\frac{\partial}{\partial\xi}H(c(t-\tau)-|x-\xi|)\right]_{\xi=L_H} d\xi d\tau \tag{7}$$

Continue the calculation.

$$p(x,t) = \frac{\alpha\rho Uc}{2}\int_{-\infty}^{t}\left[\zeta(L_H,\tau)\frac{\partial}{\partial\xi}H(c(t-\tau)-|x-\xi|)\right]_{\xi=L_H} d\tau \tag{8}$$

Integrate this equation, assuming $x \le \xi$ and $x \le L_H$. Also, the derivative of Heaviside step function is the delta function. We also applied the scaling property of the Dirac delta function.

$$p(x,t) = \frac{\alpha\rho Uc}{2}\int_{-\infty}^{t}\left[\zeta(L_H,\tau)\frac{\partial}{\partial\xi}H(c(t-\tau)-|x-\xi|)\right]_{\xi=L_H} d\tau$$

$$= \frac{\alpha\rho Uc}{2}\int_{-\infty}^{t}\left[\zeta(L_H,\tau)\delta(c(t-\tau)-|x-\xi|)\frac{\partial}{\partial\xi}(-|x-\zeta|)\right]_{\xi=L_H} d\tau$$

$$= \frac{\alpha\rho Uc}{2}\int_{-\infty}^{t}[\zeta(L_H,\tau)\delta(c(t-\tau)-|x-\xi|)\cdot(-1)]_{\xi=L_H} d\tau$$

$$= -\frac{\alpha\rho Uc}{2}\int_{-\infty}^{t}\zeta(L_H,\tau)\delta(c(t-\tau)-|x-L_H|)d\tau$$

$$= -\frac{\alpha\rho Uc}{2}\zeta\left(L_H, t-\frac{L_H-x}{c}\right)\cdot\frac{1}{c}$$

$$\therefore p(x,t) = -\frac{\alpha\rho U}{2}\zeta\left(L_H, t-\frac{L_H-x}{c}\right) \tag{9}$$

Therefore, the following equation is obtained. This equation is the equation for the sound pressure generated by cavity resonance. Also, equation (4) was considered. However, $0 \le x \le L_H$.

$$p(x,t) = -\frac{\alpha\rho U}{2}\zeta\left(L_H, t - \frac{L_H - x}{c}\right) = -\frac{\alpha\rho U}{2}\zeta\left(0, t - \frac{L_H}{U_{vortex}} - \frac{L_H - x}{c}\right) \tag{10}$$

3.3. Vorticity Equation Considering Boundary Conditions

Finally, consider equation (3). Substitute equation (10).

$$\zeta(0,t) = -\beta\frac{\partial}{\partial x}\left[-\frac{\alpha\rho U}{2}\zeta\left(L_H, t - \frac{L_H - x}{c}\right)\right]_{x=0}$$

$$= \frac{\alpha\beta\rho U}{2}\frac{\partial}{\partial x}\left[\zeta\left(L_H, t - \frac{L_H - x}{c}\right)\right]_{x=0}$$

$$= \frac{\alpha\beta\rho U}{2}\frac{\partial}{\partial t}\left[\zeta\left(L_H, t - \frac{L_H - x}{c}\right)\cdot\frac{\partial}{\partial x}\left(t - \frac{L_H - x}{c}\right)\right]_{x=0}$$

$$= \frac{\alpha\beta\rho U}{2}\frac{\partial}{\partial t}\left[\zeta\left(L_H, t - \frac{L_H - x}{c}\right)\cdot\frac{1}{c}\right]_{x=0}$$

$$= \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta\left(L_H, t - \frac{L_H - 0}{c}\right)$$

Therefore, the following equation is obtained. Here, equation (4) was considered.

$$\therefore \zeta(0,t) = \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta\left(L_H, t - \frac{L_H}{c}\right) = \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta\left(0, t - \frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right) \tag{11}$$

3.4. Derivation of the Characteristic Equation

In equation (11), substituting $\zeta(0,t) = \zeta_0 e^{st}$ into both sides, we obtain the following equation:

$$\zeta_0 e^{st} = \frac{\alpha\beta\rho U}{2c}\frac{\partial}{\partial t}\zeta_0 e^{s\left(t - \frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right)}$$

$$= \frac{\alpha\beta\rho U}{2c}s\zeta_0 e^{st}e^{s\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right)}$$

$$\therefore \zeta_0 e^{st} = \frac{\alpha\beta\rho U}{2c}s\zeta_0 e^{st}e^{s\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right)} \tag{12}$$

Eliminating $\zeta_0 e^{st}$ from both sides, we obtain the following equation.

$$1 = \frac{\alpha\beta\rho U}{2c}se^{s\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right)} \tag{13}$$

Equation (13) is the characteristic equation.

3.5. Derivation of Rossiter Formula and Cavity Resonance Stabilization Conditions from the Characteristic Equation

By solving the characteristic equation, we derive Rossiter formula and the cavity resonance stabilization conditions.

In the characteristic equation, we consider the properties by setting $s = \sigma + j\omega$. Here, since we want to determine whether "sound is produced or not," we consider the case when $\sigma = 0$. That is, $s = j\omega$. In this case, equation (13) becomes the following equation:

$$1 = \frac{\alpha\beta\rho U}{2c}\omega \cdot j \cdot e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)} \tag{14}$$

In equation (14), we consider the case when the phases of both sides are equal. In this case, sound is produced. Therefore, the following equation must hold.

$$\arg\left(\frac{\alpha\beta\rho U}{2c}\omega\right) + \arg(j) + \arg\left(e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}\right) = -2\pi n \tag{15}$$

In equation (15), $\arg\left(\frac{\alpha\beta\rho U}{2c}\omega\right)$ is a positive value, so it is 0. $\arg(j)$ is $-\frac{\pi}{2}$. $\arg\left(e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}\right)$ is $\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)$. That is, the following equation is obtained.

$$0 - \frac{\pi}{2} + \omega\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right) = -2\pi n$$

$$\omega\left(-\frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right) = -2\pi n + \frac{\pi}{2}$$

$$2\pi f_c\left(\frac{L_H}{U_{vortex}} + \frac{L_H}{c}\right) = 2\pi n - \frac{\pi}{2}$$

$$f_c = \frac{n - \frac{1}{4}}{\frac{L_H}{U_{vortex}} + \frac{L_H}{c}} \tag{16}$$

Here, we use the following relationship:

$$U_{vortex} = \kappa U \tag{17}$$

$$M = \frac{U}{c} \tag{18}$$

Therefore, equation (16) becomes as follows:

$$f_c = \frac{U}{L_H}\frac{n - \frac{1}{4}}{\left(\frac{1}{\kappa} + M\right)} \tag{19}$$

This is exactly equivalent to Rossiter formula, which is the following equation. Empirically, we have used ϕ as 0.25, but we were able to derive $0.25 = \frac{1}{4}$ from the equation that describes the actual phenomenon.

$$f_c = \frac{U}{L_H} \frac{n - \phi}{\left(\frac{1}{\kappa} + M\right)} \tag{20}$$

Now, when f_c is obtained from equation (16) or equation (19), that is, when $\omega = 2\pi f_c$, the gain G on the right side of the characteristic equation (14), which is equation (21), determines whether the self-excited oscillation of the cavity resonance is stable or unstable. That is, if $G > 1$, it is unstable, self-excited oscillation occurs, and the cavity resonance sound gets louder and louder. If $G < 1$, it is stable, and the sound decays until the cavity resonance sound stops. If $G = 1$, it is a critical state, and the cavity resonance sound is emitted at a constant volume.

$$G = \frac{\alpha\beta\rho U}{2c} \omega \tag{21}$$

3.6. Sound Pressure at a Position Outside the Sound Source ($x \geq L_H$)

We will now discuss the sound pressure at a position outside the sound source ($x \geq L_H$). As before, it can be calculated using the following equation

$$p(x,t) = \frac{\alpha\rho U}{2} \zeta \left(L_H, t - \frac{x - L_H}{c}\right) = \frac{\alpha\rho U}{2} \zeta \left(0, t - \frac{L_H}{U_{vortex}} - \frac{x - L_H}{c}\right) \tag{22}$$

3.7. Derivation of α and β

Equations (2) and (3) are restated below.

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \alpha\rho U \zeta(L_H, t) \delta'(x - L_H) \tag{23}$$

$$\zeta(0,t) = -\beta \frac{\partial}{\partial x} p(0,t) \tag{24}$$

Consider the quasi-static state in Equation (23); that is, neglect the time-dependent terms. This yields the following equation:

$$-\frac{d^2 p}{dx^2} = \alpha\rho U \zeta(L_H) \delta'(x - L_H) \tag{25}$$

Solving this equation using the Green's function yields the following expression

$$p(x) = -\frac{\alpha\rho U \zeta(L_H)}{2} \mathrm{sgn}(x - L_H) + Ax + B \tag{26}$$

Here, the pressure gradient A inside the cavity is closely coupled with the fluid motion at the downstream edge. Considering the steady-state momentum conservation law (the linearized form of the Euler equations) and the spatial balance of the vortex force $\rho U \zeta$ in Howe's acoustic analogy, it is reasonable to assume that the internal pressure gradient is determined by the vorticity characteristics at the downstream end.

Therefore, based on physical consistency, the constant A is given by $A = -\rho U \zeta(L_H)$. This results in the following expression:

$$p(x) = -\frac{\alpha \rho U \zeta(L_H)}{2} \operatorname{sgn}(x - L_H) - \rho U \zeta(L_H) x + B \tag{27}$$

Differentiating both sides with respect to x yields the following expression:

$$\frac{dp}{dx} = -\alpha \rho U \zeta(L_H) \delta(x - L_H) - \rho U \zeta(L_H) \tag{28}$$

Within the cavity ($0 \leq x \leq L_H$), the following expression is obtained:

$$-\frac{dp}{dx} = \rho U \zeta(L_H) \tag{29}$$

Considering the point $x = L_H$, the following equation is obtained:

$$\zeta(L_H) = -\frac{1}{\rho U} \frac{d}{dx} p(L_H) \tag{30}$$

From Equation (24), the following equation is also obtained at $x = 0$:

$$\zeta(0) = -\frac{1}{\rho U} \frac{d}{dx} p(0) \tag{31}$$

Thus, accounting for the time-dependent terms, the following equation is obtained:

$$\zeta(0, t) = -\frac{1}{\rho U} \frac{\partial}{\partial x} p(0, t) \tag{32}$$

Therefore, the following equation is obtained:

$$\beta = \frac{1}{\rho U} \tag{33}$$

Next, consider α. Based on Equation (25) and Howe's vortex sound theory, the following equation holds. Here, $\mathcal{G}_{space}$ is a positive geometric coefficient derived from the sharpness of the edge; the phase lag occurring during the conversion from vortex to sound is assumed to be zero.

$$\alpha = \mathcal{G}_{space} \cdot M \tag{34}$$

In the case of one-dimensional cavity resonance, where the cavity is formed by infinitely thin flat plates, $\mathcal{G}_{space}$ takes the following value:

$$\mathcal{G}_{space} = 1 \tag{35}$$

Therefore, the following equation is obtained:

$$\alpha = M \tag{36}$$

Now, substituting Equation (33) into Equation (21) and Equation (36) yields the following equation:

$$G = \frac{M\omega}{2c} \tag{37}$$

In other words, one can ascertain whether the cavity resonance is stable or unstable based on known values.

Based on the preceding discussion, the solution for vorticity, Equation (11) within the range $0 \le x \le L_H$ are given by the following equations:

$$\zeta(0,t) = \frac{M}{2c}\frac{\partial}{\partial t}\zeta\left(L_H, t - \frac{L_H}{c}\right) = \frac{M}{2c}\frac{\partial}{\partial t}\zeta\left(0, t - \frac{L_H}{U_{vortex}} - \frac{L_H}{c}\right) \tag{38}$$

4. Numerical Example

Based on previous results, a numerical example is shown below.

The physical quantities used in the calculation are shown in Table 1.

Table 1 Parameter Name and Value for Numerical Simulation

Parameter Name	Value
L_H	0.5(m)
κ	0.6(m/s)
U	34.0(m/s)
c	340.0(m/s)
ρ	1.2(kg/m^3)

Fig. 2 shows a graph plotting L_H on the horizontal axis and G on the vertical axis, based on Equation (37) with ω calculated by setting $n = 1$ in Equation (19) and with $\alpha = 1$. The graph reveals that the system is unstable when $L_H < 0.01334$ (m) and stable when $L_H > 0.01334$ (m). This value of L_H is determined by the following equation:

$$L_H = M\frac{\pi(1 - 0.25)}{\left(\frac{c}{\kappa U} + 1\right)} \tag{39}$$

In other words, the maximum hole length L_H serves as the threshold between stability and instability. However, it does not take into account the fact that the phase lag associated with the conversion of vorticity into sound at the downstream end of the cavity (relative to the mainstream air velocity direction) does not ideally become zero. Consequently, this implies that the lower limit for the onset of cavity resonance under idealized conditions is $L_H = 0.01334$ (m).

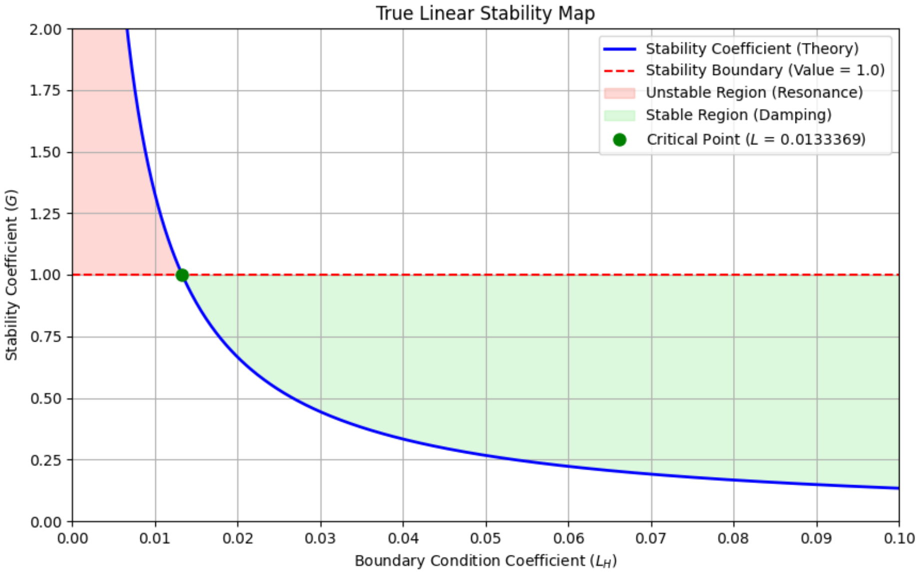

Fig. 2 Numerical Calculation Result (Stability Check)

5. On Helmholtz-Type Cavity Resonance

Consider the case shown in Fig. 3, where a main flow with velocity U passes over a Helmholtz-type cavity. In this scenario, the following equations hold for the vorticity ζ and the sound pressure p:

$$\frac{\partial \zeta}{\partial t} + U_{vortex}\frac{\partial \zeta}{\partial x} = 0 \tag{40}$$

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \alpha \rho U \zeta(L_H, t)\delta'(x - L_H) \tag{41}$$

$$\zeta(0,t) = -\beta\frac{\partial}{\partial x}p(0,t) \tag{42}$$

Furthermore, letting u_c represent the displacement of the air within the neck of the Helmholtz-type cavity, the following equation holds:

$$\rho L_c \frac{\partial u_c}{\partial t^2} = p_{rossiter}(t) - p_c(t) \tag{43}$$

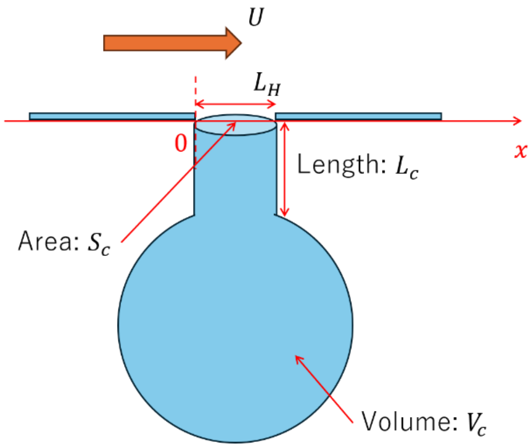

Fig. 3 Helmholtz Type Cavity

Next, consider p_c. Since the neck of the Helmholtz-type cavity oscillates, the rate of volume change is $\frac{V_c+S_c u_c-V_c}{V_c}=\frac{S_c u_c}{V_c}$. Given that the bulk modulus is ρc^2, the following equation holds:

$$p_c=\rho c^2 \frac{S_c}{V_c} u_c \tag{44}$$

Substituting this into Equation (43) yields the following equation:

$$\rho L_c \frac{\partial u_c}{\partial t^2}+\rho c^2 \frac{S_c}{V_c} u_c=p_{rossiter}(t) \tag{45}$$

Regarding Equation (41), the Green's function in the frequency domain for free space is given by the following equation, where $k=\frac{\omega}{c}$:

$$G(x,x';\omega)=-\frac{1}{2jk} e^{-jk|x-x'|} \tag{46}$$

The Green's function taking Equation (45) into account is given by the following equation, where $\gamma_c=\frac{\omega^2}{L_c(\omega_c^2-\omega^2)}$ and $\omega_c=c\sqrt{\frac{S_c}{V_c L_c}}$.

$$G(0,x';\omega)=-\frac{1}{jk+\gamma_c} e^{-jkx'} \tag{47}$$

Applying Equation (47) to Equation (41) yields the following equation.

$$p(0,\omega)=-\frac{jk}{jk+\gamma_c} e^{-jkL_H}\cdot\alpha\rho U\zeta(L_H,\omega) \tag{48}$$

Now, Euler's equation of motion yields the following equation.

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial^2 u_c}{\partial t^2} \tag{49}$$

Considering this in the frequency domain yields the following equation.

$$\frac{\partial}{\partial x} p(0,\omega) = \rho\omega^2 u_c(\omega) \tag{50}$$

Also, considering Equation (45) in the frequency domain yields the following equation, where $p(0,\omega)$ represents the frequency-domain value of $p_{rossiter}(t)$.

$$\rho L_c(\omega_c^2 - \omega^2) u_c(\omega) = p(0,\omega)$$

$$\therefore u_c(\omega) = \frac{1}{\rho L_c(\omega_c^2 - \omega^2)} p(0,\omega) \tag{51}$$

Therefore, substituting this into Equation (50) yields the following equation.

$$\frac{\partial}{\partial x} p(0,\omega) = \frac{\rho\omega^2}{\rho L_c(\omega_c^2 - \omega^2)} p(0,\omega) = \gamma_c p(0,\omega) \tag{52}$$

Equation (42) yields the following equation.

$$\zeta(0,\omega) = -\beta\gamma_c p(0,\omega) \tag{53}$$

From Equation (48), the following equation is obtained.

$$\zeta(0,\omega) = \frac{jk\gamma_c}{jk + \gamma_c} e^{-jkL_H} \cdot \alpha\beta\rho U \zeta(L_H,\omega) \tag{54}$$

Solving Equation (40) yields Equation (4). Considering this solution in the frequency domain results in the following equation:

$$\zeta(0,\omega) = \frac{jk\gamma_c}{jk + \gamma_c} e^{-j\omega\frac{L_H}{c}} \cdot \alpha\beta\rho U \zeta(0,\omega) e^{-j\omega\frac{L_H}{U_{vortex}}}$$

$$\therefore \zeta(0,\omega) = \alpha\beta\rho U \zeta(0,\omega) \cdot \frac{jk}{j\frac{k}{\gamma_c} + 1} e^{-j\omega\left(\frac{L_H}{U_{vortex}} + \frac{L_H}{c}\right)} \tag{55}$$

Thus, the characteristic equation is obtained as follows:

$$1 = \alpha\beta\rho U \frac{j\left(\frac{\omega}{c}\right)}{j\left(\frac{L_H}{c}\right)\frac{\omega_c^2 - \omega^2}{\omega} + 1} e^{-j\omega\left(\frac{L_H}{U_{vortex}} + \frac{L_H}{c}\right)}$$

$$1 = \frac{\alpha\beta\rho U}{c} \omega \cdot j \cdot \frac{1}{j\left(\frac{L_H}{c}\right)\frac{\omega_c^2 - \omega^2}{\omega} + 1} e^{-j\omega\left(\frac{L_H}{U_{vortex}} + \frac{L_H}{c}\right)} \tag{56}$$

Considering the case where ω=ω_c in Equation (56), the following equation is obtained:

$$1 = \frac{\alpha\beta\rho U}{c} \omega \cdot j \cdot e^{-j\omega\left(\frac{L_H}{U_{vortex}} + \frac{L_H}{c}\right)} \tag{57}$$

Consider the condition where the phases on both sides of Equation (57) are equal.

Sound is produced under this condition. Therefore, the following equation must hold:

$$\arg\left(\frac{\alpha\beta\rho U}{c}\omega\right)+\arg(j)+\arg\left(e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}\right)=-2\pi n \tag{58}$$

Since $\arg\left(\frac{\alpha\beta\rho U}{c}\omega\right)$ is a positive value, it is 0. $\arg(j)$ is $-\frac{\pi}{2}$. $\arg\left(e^{j\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)}\right)$ is $\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)$. Thus, the following equation is obtained:

$$0-\frac{\pi}{2}+\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)=-2\pi n$$

$$\omega\left(-\frac{L_H}{U_{vortex}}-\frac{L_H}{c}\right)=-2\pi n+\frac{\pi}{2}$$

$$2\pi f_c'\left(\frac{L_H}{U_{vortex}}+\frac{L_H}{c}\right)=2\pi n-\frac{\pi}{2}$$

$$f_c'=\frac{n-\frac{1}{4}}{\frac{L_H}{U_{vortex}}+\frac{L_H}{c}} \tag{59}$$

Here, the following relations are used:

$$U_{vortex}=\kappa U \tag{60}$$

$$M=\frac{U}{c} \tag{61}$$

Therefore, the Equation (59) becomes:

$$f_c'=\frac{U}{L_H}\frac{n-\frac{1}{4}}{\left(\frac{1}{\kappa}+M\right)} \tag{62}$$

In other words, even for a Helmholtz-type cavity, the same frequencies as those in Rossiter's formula are obtained, and sound is produced at the frequencies given by Equation (59) or Equation (62). The phase lag, which had previously been determined empirically, was also theoretically found to be 0.25. However, regarding f_c', since $\omega=\omega_c$ also holds, the following equation is valid as well:

$$f_c'=\frac{c}{2\pi}\sqrt{\frac{S_c}{V_cL_c}} \tag{63}$$

A very loud sound is produced when Equation (59) and Equation (63) are equal. In other words, a lock-in phenomenon occurs at the frequency where these equations match.

Now, the gain G—represented by Equation (64) below and found on the right-hand side of the characteristic equation (Equation (57)) at the frequency f_c' derived from Equation (59) or (62) (i.e., when $\omega = 2\pi f_c'$)—determines whether the self-excited oscillation known as cavity resonance is stable or unstable. Specifically, if $G > 1$, the system is unstable; self-excited oscillation occurs, and the cavity resonance sound grows increasingly loud. If $G < 1$, the system is stable; the oscillation damps out, and the cavity resonance sound ceases. If $G = 1$, the system is in a critical state, and the cavity resonance sound persists at a constant amplitude.

$$G = \frac{\alpha\beta\rho U}{c}\omega \tag{64}$$

Based on the preceding discussion, the following equations hold:

$$\beta = \frac{1}{\rho U} \tag{65}$$

$$\alpha = \mathcal{G}_{space} \cdot M \tag{66}$$

Here, since the geometry consists of a vertical wall and a horizontal flat plate, $\mathcal{G}_{space}$ is $\frac{2}{3}$. Therefore, Equation (64) yields the following:

$$G = \frac{2M\omega}{3c} \tag{67}$$

In this case, the minimum value of L_H at which $G = 1$ is given by the following equation:

$$L_H = \frac{4}{3}M\frac{\pi(1-0.25)}{\left(\frac{c}{\kappa U}+1\right)} \tag{68}$$

Calculating with the values provided in Table 1 yields $L_H = 0.01778$ (m). The system is unstable if $L_H < 0.01778$ (m) and stable if $L_H > 0.01778$ (m). However, this does not account for the fact that the phase lag—occurring during the conversion of vorticity into sound at the downstream end of the cavity relative to the mainstream air velocity direction—does not ideally become zero. Consequently, this implies that for an idealized Helmholtz-type cavity, the lower limit below which cavity resonance does not occur is $L_H = 0.01778$ (m).

Regarding Helmholtz-type cavities, the oscillation of the cavity neck is treated as a one-dimensional phenomenon in both two-dimensional and three-dimensional cases; therefore, the preceding arguments remain valid for both 2D and 3D configurations.

First, for the two-dimensional case, the following equations hold based on Chapter 5 of the reference (Yabu, About 2D Cavity Resonance, 2026). Here, $\bar{p} = \int_{-L_y}^{\infty} p dy, \bar{u}_c = \int_{-L_y}^{\infty} u_c dy$, where L_y is the length from the opening of the Helmholtz-type cavity to its deepest point.

$$\frac{\partial \zeta}{\partial t} + U_{vortex}\frac{\partial \zeta}{\partial x} = 0 \tag{69}$$

$$\frac{1}{c^2}\frac{\partial^2 \bar{p}}{\partial t^2} - \frac{\partial^2 \bar{p}}{\partial x^2} = \alpha\rho U\zeta(L_H, t)\delta'(x - L_H) \tag{70}$$

$$\zeta(0, t) = -\beta\frac{\partial}{\partial x}\bar{p}(0, t) \tag{71}$$

$$\rho L_c\frac{\partial \bar{u}_c}{\partial t^2} = \bar{p}_{rosster}(t) - \bar{p}_c(t) \tag{72}$$

In these equations, the analysis previously conducted using p and u is now performed using $\bar{p}$ and $\bar{u}_c$. Specifically, for a two-dimensional Helmholtz-type cavity, the phase lag is 0.25—as indicated in the Rossiter equation given by Equation (59) or Equation (62)—and the resonance frequency of the Helmholtz-type cavity is determined by Equation (63). Furthermore, when Equation (59) or Equation (62) equates to Equation (63) and the parameter G given by Equation (64) exceeds 1, a lock-in phenomenon occurs, causing the cavity resonance sound to intensify progressively. This implies that, for an idealized Helmholtz-type cavity, the lower limit value below which cavity resonance does not occur is $L_H = 0.01778$ (m), as derived from Equation (68).

For the three-dimensional case, the following equations hold based on Chapter 5 of the reference (Yabu, About 3D Cavity Resonance, 2026). Here, let $\bar{p} = \int_0^{L_z}\int_{-L_y}^{\infty} p dy\, dz$, $\bar{u}_c = \int_0^{L_z}\int_{-L_y}^{\infty} u_c dy\, dz$, where L_y is the length from the opening of the Helmholtz-type cavity to its deepest point, and L_z is the diameter of the cavity opening.

$$\frac{\partial \zeta}{\partial t} + U_{vortex}\frac{\partial \zeta}{\partial x} = 0 \tag{73}$$

$$\frac{1}{c^2}\frac{\partial^2 \bar{p}}{\partial t^2} - \frac{\partial^2 \bar{p}}{\partial x^2} = \alpha\rho U\left(\frac{2L_z}{\pi}\right)\zeta(L_H, t)\delta'(x - L_H) \tag{74}$$

$$\zeta(0, t) = -\beta\frac{\partial}{\partial x}\bar{p}(0, t) \tag{75}$$

$$\rho L_c\frac{\partial \bar{u}_c}{\partial t^2} = \bar{p}_{rosster}(t) - \bar{p}_c(t) \tag{76}$$

In these equations, the analysis previously conducted using p and u is now performed using $\bar{p}$ and $\bar{u}_c$. Specifically, for a three-dimensional Helmholtz-type cavity, the phase lag is 0.25—as indicated in Rossiter's equation given by Equation (59) or Equation (62)—and the resonance frequency of the Helmholtz-type cavity satisfies Equation (63).

Note, however, that α is treated as $\alpha\left(\frac{2L_z}{\pi}\right)$. Furthermore, when Equation (59) or Equation (62) equates to Equation (63) and the value G given by the following equation exceeds 1, a lock-in phenomenon occurs, causing the cavity resonance sound to intensify progressively.

$$G = \frac{\alpha\beta\rho U}{c}\left(\frac{2L_z}{\pi}\right)\omega \tag{77}$$

Here, based on the preceding discussion, the following equation holds.

$$\beta = \frac{1}{\rho U} \tag{78}$$

$$\alpha = \mathcal{G}_{space} \cdot M \tag{79}$$

Here, regarding $\mathcal{G}_{space}$, since the configuration consists of a vertical wall and a horizontal flat plate, $\mathcal{G}_{space} = \frac{2}{3}$. Therefore, based on Equation (64), the following equation holds:

$$G = \frac{2M}{3c}\left(\frac{2L_z}{\pi}\right)\omega \tag{80}$$

In this case, the lower limit of L_H for which $G = 1$ is given by the following equation for the three-dimensional case:

$$L_H = \frac{2}{3}\frac{M}{c}\left(\frac{2L_z}{\pi}\right)U\frac{2\pi(1-0.25)}{\left(\frac{1}{\kappa}+M\right)} = \frac{8}{3}ML_z\frac{(1-0.25)}{\left(\frac{c}{\kappa U}+1\right)} \tag{81}$$

Setting the opening diameter of the Helmholtz-type cavity to $L_z = 0.02$ (m) and calculating using the values for other variables given in Table 1 yields $L_H = 0.2264 \times 10^{-3}$ (m)=0.2264 (mm). The system is unstable when $L_H < 0.2264$ (mm) and stable when $L_H > 0.2264$ (mm). However, this calculation does not account for the fact that the phase lag during the conversion of vorticity into sound at the downstream edge of the cavity (relative to the main flow direction) is not ideally zero. Therefore, this implies that for an idealized Helmholtz-type cavity, the lower limit below which cavity resonance does not occur is $L_H = 0.2264$ (mm) in the three-dimensional case.

In practice, when blowing air sideways across the opening of a wine bottle, beer bottle, or PET bottle—where the inner diameter is approximately 0.02 (m) and the Mach number is lower than the 0.1 value given in Table 1—the actual lower limit is smaller than $L_H = 0.2264$ (mm). Therefore, since the inner diameter is larger than the lower limit of L_H—where it would become unstable—and remains stable, a sound is produced but quickly fades.

6. Conclusion

Research on automobile whistling and suction sounds has been conducted in the past. For cavity resonance, a type of whistling sound, the cavity resonance frequency has been determined using Rossiter's empirical equation. Therefore, limiting the study to the one-dimensional case, Rossiter's equation was derived by solving a system of three differential equations. In addition, equations for the stability conditions of cavity resonance and the sound pressure were derived. Finally, the results were verified through numerical calculations, and reasonable conclusions were obtained.

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