

# Reference-Dependent Weak Dominance in $2 \times 2$ Coordination Games

By

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## Abstract

In this paper we consider  $2 \times 2$  coordination games with multiple equilibria where pay-offs are measured in an unit of a resource (instead of utilities). The reference point for such games is the pair of max-min payoffs. Each pay-off matrix is associated to two other matrices, one enumerating gains with respect to the max-min pay-off and the other enumerating losses with respect to the max-min pay-off. We prove two propositions about the existence of optimistically weakly dominant action and pessimistically weakly dominant action. The coordination problems are resolved by considering the weakly dominant action for each of the four matrices. For prisoner's dilemma, the optimistically weakly dominant action profile for the payoff matrix incorporating inequality aversion is the action profile that Pareto dominates the unique (Pareto dominated) equilibrium profile of the original game/problem. In the case of chicken, the "wise outcome" can be attained by incorporating inequality aversion.

**1. Introduction:** Two important aspects of interactive decision-making problems (i.e., games) are interpretation of the pay-offs of the decision makers and the attitude of the decision makers participating in the game. In classical game theory, the profile of actions chosen by the decision makers is viewed as a "chosen instrument" and pay-off is interpreted as "utility arising from the chosen instrument". An alternative interpretation of pay-off, that seems to motivate a considerable literature in behavioral game theory and much debate related to solution concepts in game theory, is that of an "earning or reward" of some resource (e.g., money) measurable in an appropriate unit of measurement. What however seems to remain generally unnoticed is that the alternative interpretation of pay-offs, allow the decision makers to redefine their utility functions to which the usual solution (or stability) concepts of classical game

theory can be applied. A notable exception in this respect, as noted in section 8 of this paper, is the work of Fehr and Schmidt (1999).

Among the earliest works on decision-making, that implicitly use the max-min pay-offs of the decision makers, are Brams and Wittman (1981) and Brams (1994).

Section 7 in chapter 4 of Taylor and Pacelli (2008) contains a brief summary of the work reported there. Explicit use of max-min pay-offs in decision-making is available in Schneider and Leland (2015). We use the concept of weak dominance, as in classical game theory, on the matrices obtained by applying the transformations to the pay-offs that are used in Schneider and Leland (2015). By applying the transformations, we get to two different possible outcome (or evaluation) matrices for each decision maker. For each decision maker there is an optimistic scenario in which it considers only gains with respect to its max-min pay-off treating all losses alike and a pessimistic scenario in which it considers only losses with respect to its max-min pay-off treating all gains alike. In different games, the decision makers, perceive each other differently and it is this possibility of differences in perceptions that facilitates solving coordination games using solution concepts in classical game theory.

Unlike the other works that use max-min payoffs as reference point, we use weak dominance in the context of the transformed pay-off matrices. This is possible for the transformed pay-off matrices, although the original pay-off matrices have no weakly dominant actions.

We prove some results about the existence of optimistically weakly dominant action and pessimistically weakly dominant action. It seems that the possibility of existence of pessimistically weakly dominant action is much better than the possibility of existence of optimistically weakly dominant action. Is game theory therefore a “dismal” perspective on human interactions?

The kind of games where both decision makers have the same pay-off matrices (i.e., common pay-off games) are an exception. In such games the best pay-off for the two decision makers can be found by maximizing the expected pay-off to a decision maker.

Apart from common pay-off matrix games, we consider “stag hunt”, “chicken” and “battle of the sexes”. Each of these three games have two equilibria. In stag hunt, there is a better equilibria for both decision makers, that emerges as a optimistically weakly dominant action profile, i.e., both decision makers consider gains with respect to max-min payoff and then both choose a weakly dominant action that emerge from

their respective gain matrix. For “chicken” and “battle of the sexes” it is necessary for one decision maker to choose its optimistically weakly dominant action (i.e., the weakly dominant action for its gain matrix) and the other decision maker to choose its pessimistically weakly dominant action (i.e., the weakly dominant action for its loss matrix).

Contrary to popular misconceptions, in prisoners’ dilemma, there is no coordination problem. Prisoner’s dilemma is an example where a Pareto dominated outcome is stable and the outcome that dominates it is unstable. While the stability of the Pareto superior outcome can be achieved by incorporating inequality aversion in the pay-offs accruing to the decision makers, it is not sufficiently effective in making this outcome uniquely stable. The Pareto dominated outcome continues to remain stable. However, as is well known, a complete resolution of the problem is possible if the decision makers are bound to repeatedly interact with themselves with the same pay-off matrices. Alternatively, as we show here, the Pareto superior outcome is the unique optimistically weakly dominant action profile in prisoners’ dilemma after inequality aversion has been incorporated in the payoff matrices.

If inequality aversion is incorporated in “chicken”, then it could result in a “wise outcome” if and only if the difference between “victory” and a “peace treaty on equal terms” is preferred to the difference between “submission” and “total self-annihilation” by both decision makers. While such inequality aversion would fail to convey the actual purpose of the game, i.e., depicting what happens if a decision-maker is a “rebel without a cause”, such inequality aversion in contexts such as chicken would be the norm in a “*world without greed*”.

**Important observation:** Note that for  $\alpha \in \{1, 2\}$ ,  $\{1, 2\} \setminus \{\alpha\} = \{3 - \alpha\}$ .

**2. Framework of analysis:** There are two decision makers- the row player and the column player- and each has two distinct actions to choose from. Let the row player be denoted by 1, the column player by 2 and the pay-off matrix of player  $h \in \{1, 2\}$  be

$$G_h = \begin{bmatrix} x_h(1,1) & x_h(1,2) \\ x_h(2,1) & x_h(2,2) \end{bmatrix}.$$

The action set of the each player is  $\{1, 2\}$  and a pair  $(i, j) \in \{1, 2\} \times \{1, 2\}$ , with ‘i’ denoting the action (row) chosen by the row player (1) and ‘j’ denoting the action (column) chosen by the column player (2), is referred to as an **action profile**.

The pair  $(G_1, G_2)$  is referred to as a **2×2 bi-matrix game**.

An action profile  $(i^*, j^*)$  is an **equilibrium** for  $(G_1, G_2)$  if for all  $i, j \in \{1, 2\}$ :  $x_1(i^*, j^*) \geq x_1(i, j^*)$  and  $x_2(i^*, j^*) \geq x_2(i^*, j)$ .

$m_1 = \max_{i \in \{1, 2\}} \min_{j \in \{1, 2\}} x_1(i, j)$  is said to be **the reservation pay-off for the row player** and

$m_2 = \max_{j \in \{1, 2\}} \min_{i \in \{1, 2\}} x_2(i, j)$  is said to be **the reservation pay-off for the column player**.

We will refer to the pair  $(m_1, m_2)$  as the **reference pay-off pair**.

For  $h \in \{1, 2\}$  and  $i, j \in \{1, 2\}$ , let  $x_h^*(i, j) = \max \{x_h(i, j) - m_h, 0\}$  and  $\bar{x}_h(i, j) = \max \{m_h - x_h(i, j), 0\}$ .

Clearly,  $x_h^*(i, j)\bar{x}_h(i, j) = 0$  for all  $h \in \{1, 2\}$  and  $\bar{x}_h(i, j)$ .

For  $h \in \{1, 2\}$ , we will refer to the matrix  $G_h^* = \begin{bmatrix} x_h^*(1, 1) & x_h^*(1, 2) \\ x_h^*(2, 1) & x_h^*(2, 2) \end{bmatrix}$  as the **gain matrix**

**for the row player** if  $h = 1$  and the **gain matrix for the column player** if  $h = 2$ .

An action  $i \in \{1, 2\}$  is said to be **optimistically weakly dominant action for the row player** if:  $x_1^*(i, j) \geq x_1^*(3 - i, j)$  for  $j \in \{1, 2\}$ , with strict inequality for at least one  $j \in \{1, 2\}$ .

An action  $j \in \{1, 2\}$  is said to be **optimistically weakly dominant action for the column player** if:  $x_2^*(i, j) \geq x_2^*(i, 3 - j)$  for  $i \in \{1, 2\}$ , with strict inequality for at least one  $i \in \{1, 2\}$ .

If  $i^*$  is an optimistically weakly dominant action for the row player and  $j^*$  is an optimistically weakly dominant action for the column player then we will refer to the pair  $(i^*, j^*)$  as **optimistically weakly dominant action profile**.

For  $h \in \{1, 2\}$ , we will refer to the matrix  $\bar{G}_h = \begin{bmatrix} \bar{x}_h(1, 1) & \bar{x}_h(1, 2) \\ \bar{x}_h(2, 1) & \bar{x}_h(2, 2) \end{bmatrix}$  as the **loss matrix**

**for the row player** if  $h = 1$  and the **loss matrix for the column player** if  $h = 2$ .

An action  $i \in \{1, 2\}$  is said to be **pessimistically weakly dominant for the row player** if:  $\bar{x}_1(i, j) \geq \bar{x}_1(3 - i, j)$  for  $j \in \{1, 2\}$ , with strict inequality for at least one  $j \in \{1, 2\}$ .

An action  $j \in \{1, 2\}$  is said to be **pessimistically weakly dominant for the column player** if:  $\bar{x}_2(i, j) \geq \bar{x}_2(i, 3 - j)$  for  $i \in \{1, 2\}$ , with strict inequality for at least one  $i \in \{1, 2\}$ .

If  $i^*$  is a pessimistically weakly dominant action for the row player and  $j^*$  is a pessimistically weakly dominant action for the column player then we will refer to the pair  $(i^*, j^*)$  as **pessimistically weakly dominant action profile**.

If  $i^*$  is an optimistically weakly dominant action for the row player and  $j^*$  is a pessimistically weakly dominant action for the column player then we will refer to the pair  $(i^*, j^*)$  as **weakly dominant action profile favorable for the row player**.

If  $i^*$  is a pessimistically weakly dominant action for the row player and  $j^*$  is an optimistically weakly dominant action for the column player then we will refer to the pair  $(i^*, j^*)$  as **weakly dominant action profile favorable for the column player**.

**Assumption 1:** If  $G_1 = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$  for 4 real numbers  $a, b, c, d$ , then  $\{x_2(i, j) \mid i \in \{1, 2\} \text{ and } j \in \{1, 2\}\} = \{a, b, c, d\}$ . ■

Suppose  $G_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , i.e., the transpose of  $G_1$ .

Then,  $(G_1, G_2)$  is a **2×2 symmetric bi-matrix game**.

Thus,  $m_1 = m_2 = m \in \{a, b, c, d\}$ .

Let  $a^* = \max\{a-m, 0\}$ ,  $b^* = \max\{b-m, 0\}$ ,  $c^* = \max\{c-m, 0\}$ ,  $d^* = \max\{d-m, 0\}$ ,  $\bar{a} = \max\{m-a, 0\}$ ,  $\bar{b} = \max\{m-b, 0\}$ ,  $\bar{c} = \max\{m-c, 0\}$  and  $\bar{d} = \max\{m-d, 0\}$ .

Thus,  $G_1^* = \begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix}$ ,  $G_2^* = \begin{bmatrix} a^* & b^* \\ c^* & d^* \end{bmatrix}$ ,  $\bar{G}_1 = \begin{bmatrix} -\bar{a} & -\bar{c} \\ -\bar{b} & -\bar{d} \end{bmatrix}$  and  $\bar{G}_2 = \begin{bmatrix} -\bar{a} & -\bar{b} \\ -\bar{c} & -\bar{d} \end{bmatrix}$ .

**3. Weakly dominant action profiles:** The first proposition acts as a “check” on optimism.

**Proposition 1:** (i) If  $c > b > d > a$  then there is no optimistically weakly dominant action for the row player.

(ii) If  $G_1$  is such that the row player *does not* have any optimistically weakly dominant action and  $G_2$  is the transpose of  $G_1$ , then the column player *does not* have any optimistically weakly dominant action.

**Proof:** (i) If  $G_1$  satisfies the requirement in (i), then  $m_1 = d$  and  $G_1^* = \begin{bmatrix} 0 & c-d \\ b-d & 0 \end{bmatrix}$ .

Clearly the row player does not have any optimistically weakly dominant action.

(ii) If  $G_2$  is the transpose of  $G_1$ , then with  $G_1^* = \begin{bmatrix} x_1^*(1, 1) & x_1^*(1, 2) \\ x_1^*(2, 1) & x_1^*(2, 2) \end{bmatrix}$ , it must be the

case that  $G_2^* = \begin{bmatrix} x_1^*(1, 1) & x_1^*(2, 1) \\ x_1^*(1, 2) & x_2^*(2, 2) \end{bmatrix}$ .

First note that  $G_1^* = \begin{bmatrix} x_1^*(1, 1) & x_1^*(1, 2) \\ x_1^*(2, 1) & x_2^*(2, 2) \end{bmatrix}$  must have at least one entry in each row that is equal to zero.

There is no optimistically weakly dominant action for the player, if and only if either  $(x_1^*(1, 1) - x_1^*(2, 1))(x_1^*(1, 2) - x_1^*(2, 2)) < 0$  or  $|x_1^*(1, 1) - x_1^*(2, 1)| = 0$  and  $|x_1^*(1, 2) - x_1^*(2, 2)| = 0$ .

If the minimum in each row of  $G_1$  are in the same column, then the two zero entries of  $G_1^*$  corresponding to them are in the same column. If the absolute value of the difference in the other column is positive, then there is an optimistically weakly dominant action for the row player contrary to assumption in the statement of (ii).

Thus, in this case the entries in the other column must be equal.

Thus, one row of  $G_2^*$  has both entries equal to zero and entries in the other row of  $G_2^*$  are equal. Thus, the column player does not have any optimistically weakly dominant action.

If the minimum in each row of  $G_1$  are in different columns, then the two zero entries of  $G_1^*$  corresponding to them are in different columns. If one the two other entries is zero and the other positive, then  $G_1^*$  has a weakly dominant action contrary to hypothesis. Thus either both of the two other entries are zero or both strictly positive.

If both are zero, then  $G_1^* = G_2^*$  is the “zero matrix” and there does not exist any optimistically weakly dominant action for the column player. In the other case one column of  $G_1^*$  has zero in the first row and a positive entry in the second row and the other column has a positive entry in the first row and zero in the second row. Since  $G_2^*$  is the transpose of  $G_1^*$ , the same must be true for  $G_2^*$ , so that the column player does not have any optimistically weakly dominant action. Q.E.D.

The possibility of existence of pessimistically weakly dominant actions seems to be better than the possibility of existence of optimistically weakly dominant actions.

**Proposition 2:** (i) A pessimistically weakly dominant action for the row player exists if and only if  $\min_{j \in \{1,2\}} x_1(1, j) \neq \min_{j \in \{1,2\}} x_1(2, j)$ .

(ii) A pessimistically weakly dominant action for the column player exists if and only if  $\min_{i \in \{1,2\}} x_2(i, 1) \neq \min_{i \in \{1,2\}} x_2(i, 2)$ .

**Proof:** We prove (i), the proof of (ii) being similar.

If  $\min_{j \in \{1,2\}} x_1(1, j) = \min_{j \in \{1,2\}} x_1(2, j)$ , then it is easy to see that  $\bar{G}_1$  must be the zero matrix and hence the row player has no pessimistically weakly dominant action.

If  $\min_{j \in \{1,2\}} x_1(1, j) \neq \min_{j \in \{1,2\}} x_1(2, j)$ , then without loss of generality suppose

$$\min_{j \in \{1,2\}} x_1(1, j) > \min_{j \in \{1,2\}} x_1(2, j).$$

Thus,  $m_1 = \min_{j \in \{1,2\}} x_1(1, j)$ .

In this case both entries in the first row of  $\bar{G}_1$  are zero. In the second row, both entries are “non-positive” with one entry (corresponding to  $\min_{j \in \{1,2\}} x_1(2, j)$ ) being negative.

Thus, row 1 is a pessimistically weakly dominant action for the row player. Q.E.D.

**4. Stag Hunt:**  $(G_1, G_2)$  is said to be a **stag hunt** if it is a  $2 \times 2$  symmetric bi-matrix game satisfying  $a > b \geq d > c$ .

Thus, there are two equilibria, namely  $(1, 1)$  and  $(2, 2)$ , with  $(1, 1)$  strictly better than  $(2, 2)$  for both decision makers.

In this game  $m = d$ , so that  $G_1^* = \begin{bmatrix} a-d & 0 \\ b-d & 0 \end{bmatrix}$ ,  $G_2^* = \begin{bmatrix} a-d & b-d \\ 0 & 0 \end{bmatrix}$ ,  $\bar{G}_1 = \begin{bmatrix} 0 & -(d-c) \\ 0 & 0 \end{bmatrix}$ ,  $\bar{G}_2 = \begin{bmatrix} 0 & 0 \\ -(d-c) & 0 \end{bmatrix}$ .

Row 1 is an optimistically weakly dominant action for the row player and column 1 is an optimistically weakly dominant action for the column player. Thus,  $(1, 1)$  is an optimistically weakly dominant action profile.

Row 2 is a pessimistically weakly dominant action for the row player and column 2 is a pessimistically weakly dominant action for the column player. Thus,  $(2, 2)$  is an pessimistically weakly dominant action profile.

**5. Chicken:**  $(G_1, G_2)$  is said to be **chicken** if it is a  $2 \times 2$  symmetric bi-matrix game satisfying  $b > a > c > d$ .

This game has two equilibria, namely  $(1, 2)$  and  $(2, 1)$ , with the column player better-off at  $(1, 2)$  than at  $(2, 1)$  and the row player better-off at  $(2, 1)$  than at  $(1, 2)$ .

$m_1 = m_2 = m = c$ .

Thus,  $G_1^* = \begin{bmatrix} a-c & 0 \\ b-c & 0 \end{bmatrix}$ ,  $G_2^* = \begin{bmatrix} a-c & b-c \\ 0 & 0 \end{bmatrix}$ ,  $\bar{G}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -(c-d) \end{bmatrix}$ ,  $\bar{G}_2 = \begin{bmatrix} 0 & 0 \\ 0 & -(c-d) \end{bmatrix}$ .

Row 2 is an optimistically weakly dominant action for the row player and column 2 is an optimistically weakly dominant action for the column player.

Row 1 is a pessimistically weakly dominant action for the row player and column 1 is a pessimistically weakly dominant action for the column player.

Thus,  $(1, 2)$  is a weakly dominant action profile favorable to the column player and  $(2, 1)$  is a weakly dominant action profile favorable to the row player.

**6. Battle of the Sexes:** In this game it is assumed that  $G_2 = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$ , with  $a > b \geq d > c$ .

Thus, there are two equilibria in this game, namely (1, 1) and (2, 2) with (1, 1) better than (2, 2) for the row player and (2, 2) better than (1, 1) for the column player.

Further,  $m_1 = d = m_2$ .

Hence,  $G_1^* = \begin{bmatrix} a-d & 0 \\ b-d & 0 \end{bmatrix}$ ,  $G_2^* = \begin{bmatrix} 0 & 0 \\ b-d & a-d \end{bmatrix}$ ,  $\bar{G}_1 = \begin{bmatrix} 0 & -(d-c) \\ 0 & 0 \end{bmatrix}$  and  $\bar{G}_2 = \begin{bmatrix} -(d-c) & 0 \\ 0 & 0 \end{bmatrix}$ .

Thus, row 1 is an optimistically weakly dominant action for the row player and row 2 is a pessimistically weakly dominant action for the row player.

Further, column 2 is an optimistically weakly dominant action for the column player and column 1 is a pessimistically weakly dominant action for the column player.

Hence, (1, 1) is a weakly dominant action profile favorable for the row player and (2, 2) is a weakly dominant action profile favorable for the column player.

**7. 2×2 common pay-off games:** Suppose  $G_2 = G_1$ . Such games are 2×2 common pay-off games (see Takashi (2009), Emmons, Oesterheld, Critch, Conitzer and Russell (2022), Lahiri (2026)).

The highest payoff to the two decision makers correspond to solutions of the following maximization problem:

Maximize  $\alpha\beta(a+d) + \alpha(1-\beta)c + (1-\alpha)\beta b + (1-\alpha)(1-\beta)d$ , subject to  $\alpha, \beta \in [0, 1]$ .

It is easy to see that if  $x_1(i^*, j^*) \geq x_1(i, j)$  for all  $i, j \in \{1, 2\}$  then  $(i^*, j^*)$  is an equilibrium for  $(G_1, G_2)$  and  $(\alpha^*, \beta^*) = (2-i^*, 2-j^*) \in \{0, 1\} \times \{0, 1\}$  solves the maximization problem.

Conversely, if  $(\alpha^*, \beta^*) \in \{0, 1\} \times \{0, 1\}$  solves the maximization problem, then  $(2-\alpha^*, 2-\beta^*) \in \{1, 2\} \times \{1, 2\}$  is an equilibrium for  $(G_1, G_2)$ .

Suppose suppose  $\min \{b, c\} > \max \{a, d\}$  with  $b \neq c$ . For such games there are two equilibria, namely (1, 2) and (2, 1). If  $b > c$ , then (2, 1) is better for both than (1, 2). If  $c > b$ , then (1, 2) is better for both than (2, 1).

If  $(\alpha^*, \beta^*)$  solves Maximize  $\alpha\beta(a+d) + \alpha(1-\beta)c + (1-\alpha)\beta b + (1-\alpha)(1-\beta)d$ , subject to  $\alpha, \beta \in [0, 1]$ , then it can be easily verified that  $\alpha^* = 1, \beta^* = 0$  if  $c > b$  and  $\alpha^* = 0, \beta^* = 1$  if  $b > c$ .

$\alpha^* = 1$  corresponds to (1, 2) and  $\alpha^* = 0$  corresponds to (2, 1).

Let  $m = \max \{a, d\}$ .

Thus,  $G_1^* = G_2^* = \begin{bmatrix} 0 & c-m \\ b-m & 0 \end{bmatrix}$ ,  $\bar{G}_1 = \bar{G}_2 = \begin{bmatrix} -(m-a) & 0 \\ 0 & -(m-d) \end{bmatrix}$ .

Suppose  $a \neq d$ . Then either  $m-a = 0$  or  $m-d = 0$ , but not both.

If  $m = a$ , then  $(1, 1)$  is a pessimistically weakly dominant action profile and if  $m = d$ , then  $(2, 2)$  is a pessimistically weakly dominant action profile. Neither is an equilibrium for  $(G_1, G_2)$ . There is no weakly dominant action profile favorable to a decision maker in this game.

### 8. Misinterpretation of Prisoner's Dilemma and a Possible Resolution of the

**Problem:**  $(G_1, G_2)$  is a  $2 \times 2$  symmetric bi-matrix game with  $G_1 = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$  satisfying  $c > d > a > b$ . Such a game is said to be a prisoner's dilemma. This game has a unique equilibrium, namely  $(1, 1)$ , though  $(2, 2)$  is better for both decision makers. This game is an example of "instability of a Pareto optimal outcome and stability of a Pareto dominated outcome". It is not a coordination game with more than one equilibrium to choose from. One way to solve this conflict is to adopt the "inequality aversion" approach suggested by Fehr and Schmidt (1999).

Suppose that for  $h \in \{1, 2\}$  there exists real constants  $\alpha_h, \beta_h$  satisfying  $1 > \beta_h \geq \alpha_h > 0$ .

For  $h \in \{1, 2\}$  and  $i, j \in \{1, 2\}$ , let  $\hat{x}_h(i, j) = x_h(i, j) - \alpha_h \max\{x_h(i, j) - x_{3-h}(i, j), 0\} - \beta_h \max\{x_{3-h}(i, j) - x_h(i, j), 0\}$ .

$$\text{Let } \hat{G}_h = \begin{bmatrix} \hat{x}_h(1,1) & \hat{x}_h(1,2) \\ \hat{x}_h(2,1) & \hat{x}_h(2,2) \end{bmatrix}.$$

Thus, in a prisoner's dilemma,  $\hat{G}_1 = \begin{bmatrix} a & c - \alpha_1(c - b) \\ b - \beta_1(c - b) & d \end{bmatrix}$ ,  $\hat{G}_2 =$

$$\begin{bmatrix} a & b - \beta_2(c - b) \\ c - \alpha_2(c - b) & d \end{bmatrix}.$$

While  $(1, 1)$  continues to be an equilibrium for  $(\hat{G}_1, \hat{G}_2)$  (as in Stahl (1999)),  $(2, 2)$  will also be an equilibrium for  $(\hat{G}_1, \hat{G}_2)$  if and only if  $\min\{\alpha_1, \alpha_2\} > \frac{c-d}{c-b} \in (0, 1)$ .

Note that  $\frac{c-a}{c-b} \in (0, 1)$  and  $\frac{c-a}{c-b} > \frac{c-d}{c-b}$ .

Suppose  $\frac{c-a}{c-b} > \max\{\alpha_1, \alpha_2\} \geq \min\{\alpha_1, \alpha_2\} > \frac{c-d}{c-b}$  so that  $a \leq \min\{c - \alpha_1(c-b), c - \alpha_2(c-b)\}$ .

Consider the *gain matrices corresponding to*  $(\hat{G}_1, \hat{G}_2)$ . They must be the pair

$$\left( \begin{bmatrix} 0 & c - \alpha_1(c - b) - a \\ 0 & d - a \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c - \alpha_2(c - b) - a & d - a \end{bmatrix} \right).$$

Since  $d > c - \alpha_h(c-b)$  for  $h \in \{1, 2\}$ , it must be the case that  $d - a > c - \alpha_h(c-b) - a$  for  $h \in \{1, 2\}$ .

Thus, action 2 is an optimistically weakly dominant action for the row player and action 2 is an optimistically weakly dominant action for the column player. Thus, (2, 2) is an optimistically weakly dominant action profile for  $(\widehat{G}_1, \widehat{G}_2)$ .

**9. Chicken with “inequality aversion”:** Chicken is a coordination game, that may benefit the decision makers - although at the expense of its illustrative purpose- if it incorporated inequality inversion as in Fehr and Schmidt (1999).

Recall that in chicken,  $G_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , with  $b > a > c > d$ .

Ideally, the preferred outcome from a completely impartial perspective should be (1, 1). At the same time the outcome should not be (2, 2).

Applying the definitions in the previous section, we get  $\widehat{G}_1 =$

$$\begin{bmatrix} a & c - \beta_1(b - c) \\ b - \alpha_1(b - c) & d \end{bmatrix}, \widehat{G}_2 = \begin{bmatrix} a & b - \alpha_2(b - c) \\ c - \beta_2(c - b) & d \end{bmatrix}, \text{ for some}$$

real constants  $\alpha_h, \beta_h$  satisfying  $1 > \beta_h \geq \alpha_h > 0, h \in \{1, 2\}$ .

$$\frac{b-a}{b-c} \in (0, 1) \text{ and } \frac{c-d}{b-c} > 0.$$

Thus, if  $1 > \alpha_h > \frac{b-a}{b-c} \in (0, 1)$  and  $\beta_h \in [\alpha_h, \min\{\frac{c-d}{b-c}, 1\})$  then (1, 1) will be the unique equilibrium for  $(\widehat{G}_1, \widehat{G}_2)$ .

The existence of such a pair  $(\alpha_h, \beta_h)$  for  $h \in \{1, 2\}$  is possible if and only if  $b - a < c - d$ , which in the context of chicken is quite plausible.

**Note:** Keeping the current geopolitical scenario and the “chicken” game in mind, I humbly request readers not to confuse “standing up to devastating greed for fossil fuels” (to put it mildly) as purposeless rebellion.

## References

1. Brams, S. (1994): Theory of Moves. Cambridge, Cambridge University Press.
2. Brams, S. and Wittman, D. (1981): Nonmyopic equilibria in 2x2 games. Conflict Management and Peace Science, Volume 6, pages 39-62.
3. Emmons, S., Oesterheld, C., Critch, A, Conitzer, V and Russell, S. (2022): For Learning in Symmetric Teams, Local Optima and Global Nash Equilibria. *Proceedings of the International Conference on Machine Learning (ICML)*, Baltimore, Maryland, USA. PMLR 162.
4. Fehr, E. and Schmidt, K. M. (1999) A theory of fairness, competition, and cooperation, *Quarterly Journal of Economics*, 114, 817–68.

5. Lahiri, S. (2026): Common Pay-off Matrix Games and Anti-Diagonal Symmetric Coordination Games. DOI: <https://doi.org/10.31224/6519>.
6. Schneider, M. and Leland, J. W. (2015): Reference dependence, cooperation and coordination in games. *Judgment and Decision Making*, Volume 10, No. 2, pages 123-129.
7. Stahl, S. (1999): A Gentle Introduction to GAME THEORY. Volume 13 in *Mathematical World Series*, American Mathematical Society (Indian Edition).
8. Takashi, U. (2009): Bayesian potentials and information structure: Team decision problems revisited. *Internal Journal of Economic Theory*, Volume 5, Number 3, Pages 271-291.
9. Taylor, A. D. and Pacelli, A. M. (2008): *Mathematics and Politics: Strategy, Voting, Power and Proof* (Second Edition). Springer.