Thin X ray Lens Design And Simulation

Jiaming H. Shen[(1)], Jenny A. Shen[(2)]

[(1)] **Mingshen01@gmail.com, corresponding author,** [(2)] **jiajia.jj,shen@gmail.com**

(US Patent pending. Date 2019-10-28)

Abstract

A proposal of thin sandwich mirror array lens for X-ray telescope Wolter type lens is discussed. We discussed a process of design, fabrication of this thin X-ray lens and conducted a simulation and provided a measurement method for lens focal quality. The lens consists of two connected layers of arrays of rings of mirrors, one is of parabolic mirror and another is of hyperbolic mirrors. In each layer, the mirror coating is deposited on a substrate and next substrate in turn is deposited on this mirror film, leaving no air gap. Repeat this process to form an array of mirrors. Hence form a solid thin disc sandwich mirror lens and each ring of mirror can be curved precisely by polishing process. X ray travels into the transparent substrate and grazes on the surface of the mirror coating facing the substrate. Using geometrical optics ray tracing, we conducted simulation with two artificial distance stars as the source objects, a virtual lens of mirror array and a disc placed at the focal plane as the receiver CCD. Simulation shows that the star located off optical axis forms corn shape patterns by its ray crossing pointa on CCD disc. Those patterns may provide the baseline pattern for further image deconvolution or aberration reducing algorithm. The corn pattern is related to the dimensional structure of the lens and the space location of the star; hence, it contains some information about space location of the star. A simulation of ray tracing video is made as well. The thin sandwich lens we discussed in this paper may provide a way to build a large aperture telescope X ray lens

X-Ray Optics

Total Energy of a particle $\boldsymbol{E}$, its rest energy $\boldsymbol{E_0}$ and its momentum $\boldsymbol{p}$ and light speed $\boldsymbol{c}$ satisfy Einstein mass-energy special relativity equation.

$$\boldsymbol{E = E_0 + pc}$$

Equation 1

And its wavelength satisfies de Broglie wavelength formulism, where $\boldsymbol{h}$ is the Planck constant:

$$\boldsymbol{\lambda = \frac{h}{p} = \frac{hc}{E - E_0}}$$

Equation 2

For any photon, including X-ray photon, $\boldsymbol{E_0} = 0$, its energy range $\boldsymbol{E}$ is 100 eV to 100 KeV which has a wavelength ranging from 10 nm to 0.01 nm. An X ray photon could have wave-particle duality behavior. For X ray source with distributed wavelength, (i.e. not a single wavelength X ray), when X photon go through a structure with dimensional size large than 100 nm, it is safe to assume that the particle property of X ray photon dominates its physics so we can use ray tracing method for simulation without worry about the wave physics such as interference and diffraction. As a comparison, silicon has the diamond cubic crystal structure with a lattice parameter of 0.54 nm. The nearest neighbor distance of two atoms is 0.24 nm so when X ray photon goes through this size structure, we will have X ray diffraction and interference.

The angular resolution of a telescope (resolving power) is limited by diffraction effect of the wave property. For a circular aperture, the first minimum intensity of diffraction pattern and the diffraction angle have relation as Equation 3. Thus the resolving power of a lens depends on its effective aperture diameter D that it satisfies the formula as Rayleigh criterion

$$\vartheta = \mathbf{1.22}\ \frac{\boldsymbol{\lambda}}{\boldsymbol{D}}$$

Equation 3

Here λ is the wavelength. Larger diameter lens and short wavelength are desired for telescopes. Due to the optical index of normal material of high energy photon, such as X-ray photon, is almost a constant value of one, we cannot change the X ray travelling direction with normal transparent materials such as glass. So, we cannot use glass to make X ray lens. Also, X-ray cannot be reflected by a mirror with small reflection angle. Because X ray simply will penetrate the surface or scattered or absorbed by the matter as it passes through. Only small angle grazing (large reflect angle) or diffraction grate could change the X photon travelling direction. (1) (2). So regular parabolic reflection mirror lens does not work for X-ray either.

Today one kind of X-ray telescope lens uses surface grazing to achieve this goal as proposed by Wolter (3). The current design of Wolter type (4) telescope uses an array of co-axis, co-focal mirrors of circular parabolic and hyperbolic surfaces of grazing foil mirrors (5). The space between mirrors are vacuum or with air for photon to travel without any scattering and absorption. Mirror material is coated on the substrate base. Coating material could be metal such as gold or silicon. For those mirrors to be kept in shape, the base of the mirror needs certain mechanical strength and thickness. Currently, the dimension of the lens at z direction (optical axis) is quite thick. Thus, the lens is heavy, and the aperture is not very large (6) (7). The thin sandwich lens we discussed in this paper may provide a way to build a large aperture telescope X ray lens.

Thin X ray Lens

We propose a Wolter type lens with very thin double-layer sandwich mirrors arrays that there is no space gap between mirrors (Figure 1 and Figure 2). The substrate material is almost transparent to X-ray. It provides base substrate and that X ray goes through it, grazing inside the substrate. Its surfaces are polished and coated with the mirror material. Then a new layer of transparent substrate material is deposited on that mirror coating and polished and coated with new layer of mirror. Repeating this process until a desired lens radius is reached. Although this substrate material will have scattering and absorption effect to the X ray photon, since the mirrors layers are very thin in z-direction. This effect could be neglected and can be compensated with lager lens aperture.

The thickness of lens depends on how closer of two mirrors can be made without introducing wave interaction of the photon, and other factors such as the fabrication limitation. If the gap between mirrors are too lager but the lens is too thin, we will not have enough mirror space that some of X ray photons will not be grazed by the mirror. Furthermore, we can break up whole lens disc into sections and build them separately instead of building a large whole lens at once. And assemble those sections together on to a frame such that each section can be adjusted/calibrated with the help of actuator and fine dimensional turning piezo settings. (Figure 3)

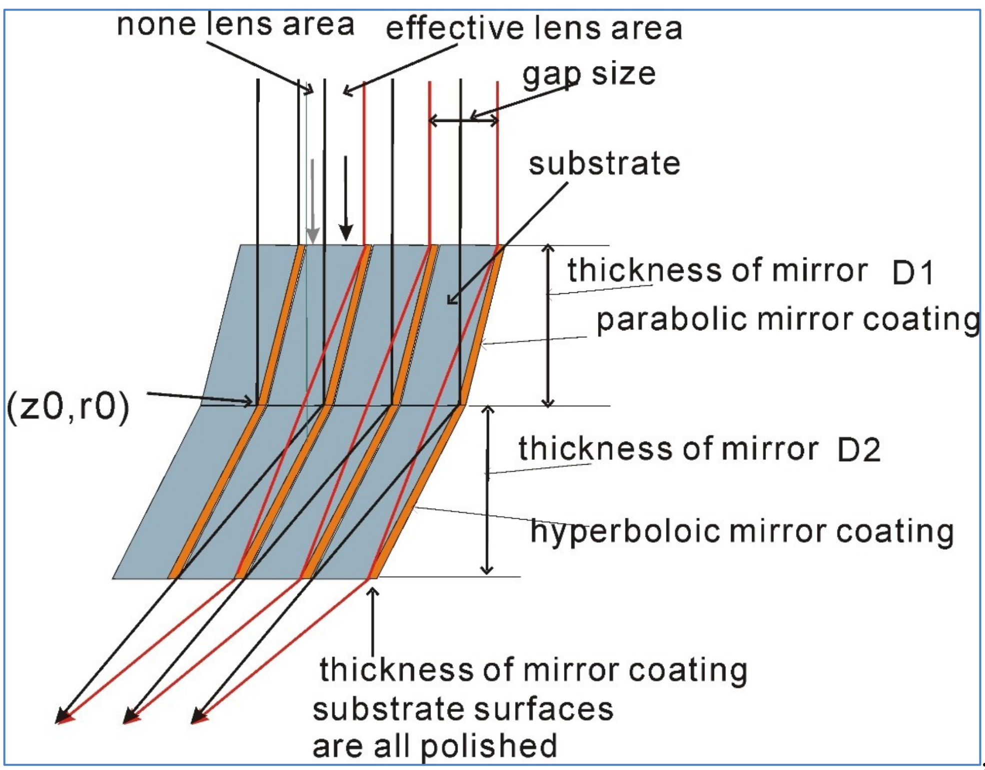
How the X ray photon is grazed inside the substrate needs experimental study. The material used for mirror substrate (or call it base or gap) must be good enough of transparent for X ray and is rigid. Most importantly, the material can be coated with metal such as gold or silicon. And vice versa the substrate material can be coated on to the mirror coating surface. The effect of absorbing, diffusion or scattering of X ray photon should be the less the better. Since the mirror is very thin, the absorbing and scattering of X ray by this gap material is not as important as its coating and rigidity properties. Each coating of the substrate will be polished into required shape before another mirror coat can be applied on. They form sandwich structure and each mirror is coated on the previous shaped and polished substrate. After a section is finished then the top and bottom surface is polished to the design thickness.

Finding a rigid, X-ray transparent material is essential. Starting from a base ring, coating or depositing the substrate material, polish it to parabolic and hyperbolic curves, using laser to calibrate the curves with a monitoring CCD receiver at the focal point during the polishing process. Then coated with grazing film such as gold or silicon as the mirror, then deposit another layer of substrate material, shaping and polishing, then coat another mirror film. Repeat this process to form a thin double-layer mirror array lens with common co-focal points.

The gap material should satisfy those criteria at least:

a) Is hard enough and has certain mechanical and thermal strength and it can be machined and polished to a mirror smoothness that a grazing mirror film can be coated to it. Its surface can be cleaned without weakening the bonding of coatings.
b) At certain environmental condition, such as temperature, it can be deposited or coated to the thin mirror film by a physical or a chemical process without affecting the mirror coat's grazing property.
c) Good transparent property to X-ray, has a low absorbing and scattering effect of X ray.

For X ray, the index of most transparent materials is near a constant one, we can start with those plastic eye glass lens material such as Polycarbonate, Trivex and CR-39 plastic, etc. Mirror coat material can be gold, aluminum or silicon.



,

Figure 1

Side view of sandwich mirror array of thin layer lens. Those rays only reflected once on each mirror surface will be collected by CCD at focal plane. The non-lens areas include the metal mirror coating and those area that the rays directly incidents on to the second layer mirror. The z-thickness and gap size are decided such that there is no wave interference effect. And the vertical rays reflected by parabolic mirror will not be blocked by the mirror in front of it. Also, it is thick enough to have desired mechanical strengthen. When the gap can be made narrower, then the thickness of lens can be reduced but this also increase the number of mirror rings and the difficulty to fabricate mirrors. On the other hand, a very thin mirror can use corn shape curve instead of parabolic or hyperbolic curves. The dimension and the number of mirrors of the array should be a compromised solution of all those design considerations.

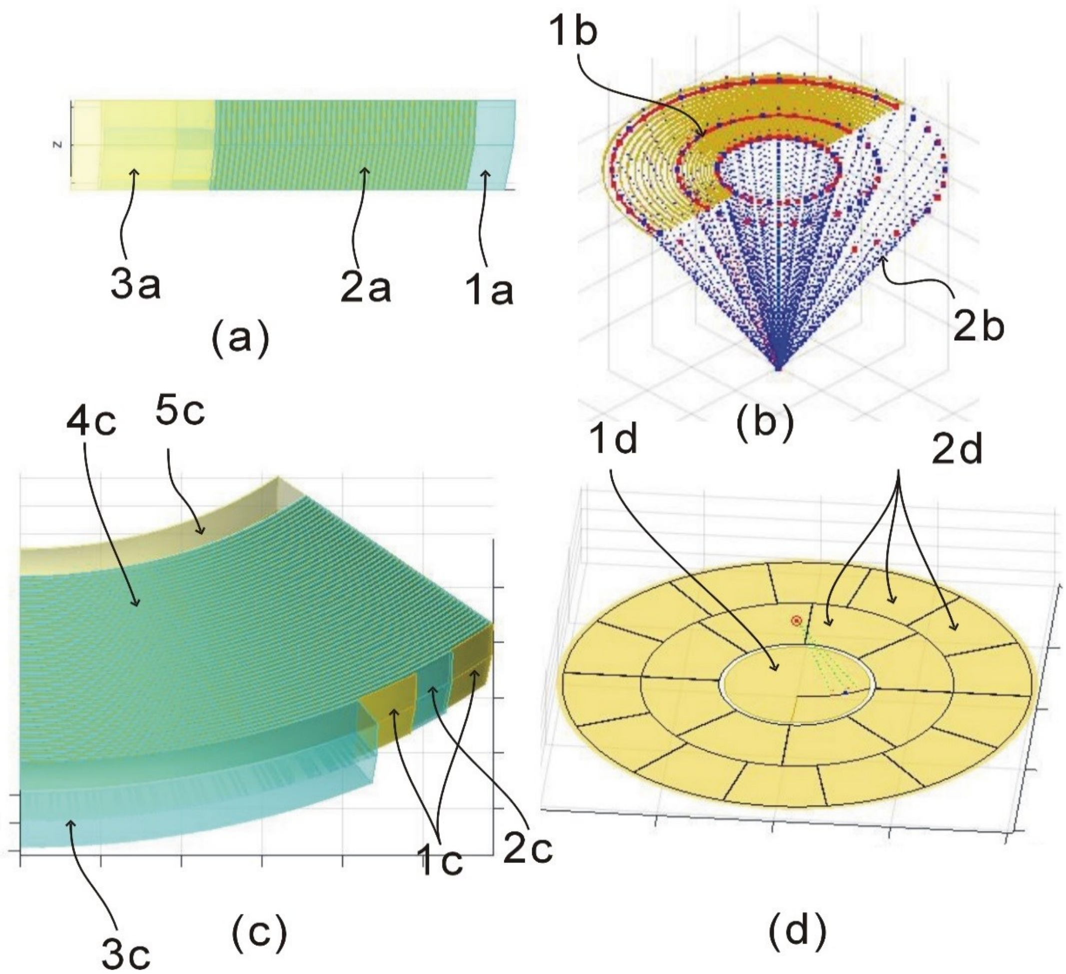


Figure 2

An illustration of thin lens with sandwich mirror array and one of its section (segments). The last mirror coating and substrate are shown partially in (c). None of any supporting fixture is sketched. (a) The side view sandwich of mirror array. (1a) and (3a) are frame with piezo. (2a) is the sandwich mirror array with top layer as parabolic array and bottom layer as hyperbolic array. (b) Illustration of ray tracing: (1b) lens with half shown. (2b) Rays. (c) Illustration one of lens section. (1c) are mirror coats, (2c) transparent substrate. (3c) and (5c) are frames and (4c) is mirror array. (d) Lens comprises of sections. (2d) are lens sections. (1d) Center part can be a regular reflective parabolic mirror telescope or other type telescope for visible light.

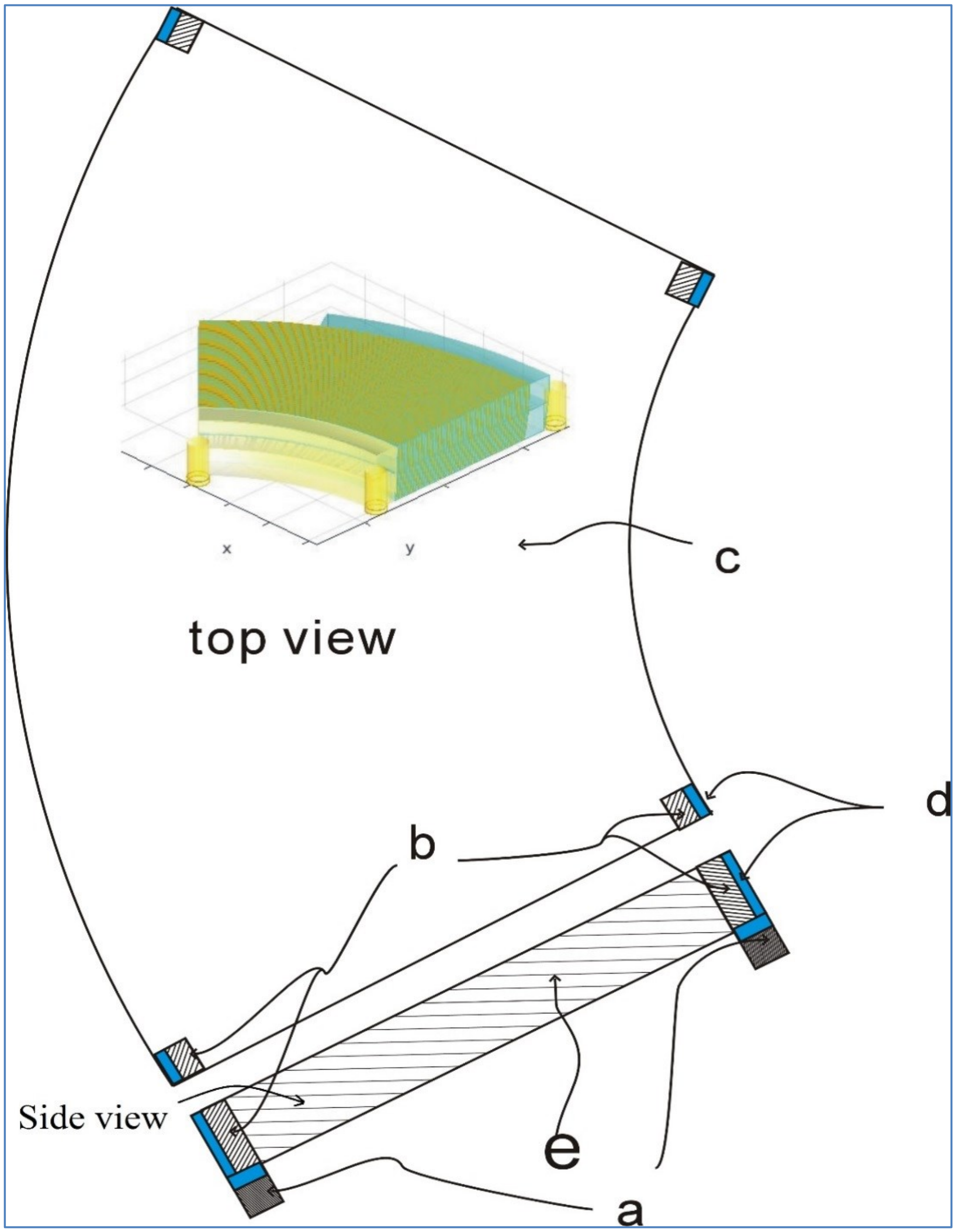


Figure 3

A sketch of one lens section. Illustrated with fixture. A 3D view is inserted for demonstration. The blue bracket is the piezo base and (a) is piezo bloc for z direction adjustment, hence three or four of those piezo sets can be used to tilt the section as well as focusing. (b) They are piezo block sets to adjust along radius direction. The combination of (a) and (b) blocks gives the section 3D dimensional dynamic turning capability with the life feedback from CCD. (c) is the top view of the mirror array sections. Each section can be made separately and mounted to the lens frame. (e) is the side view of mirror array section. (Sandwich mirror is not shown in the picture). Inserted 3D view partially shows the last two layers for viewing purpose. Furthermore, section also can be mounted with actuators for coarse turning. (Not shown in the picture).

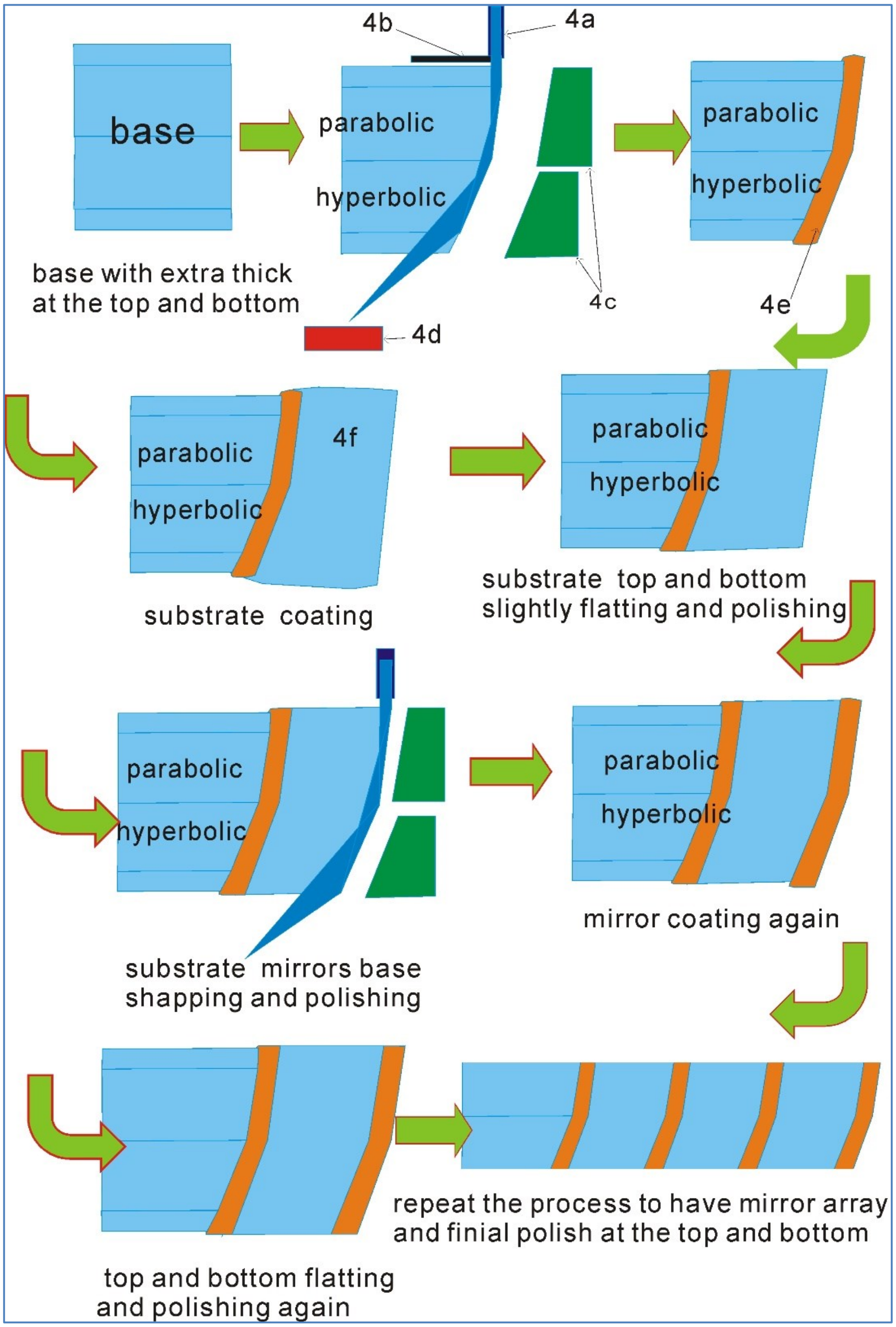


Figure 4

Illustration of mirror ring polishing process. (4a) Monitoring laser for substrate shaping and polishing. Those laser beams not reflected by both parabolic and hyperbolic surface will not focus on the CCD. It also monitors the surface smoothness. (4b) A blocking disk to block unwanted laser beam. (4c) Polishing block. (4d) Monitoring CCD. The pattern of the laser beam image received by CCD at focal plane can be used to monitor the surface of the mirror and its roughness. (4e) Mirror coating.

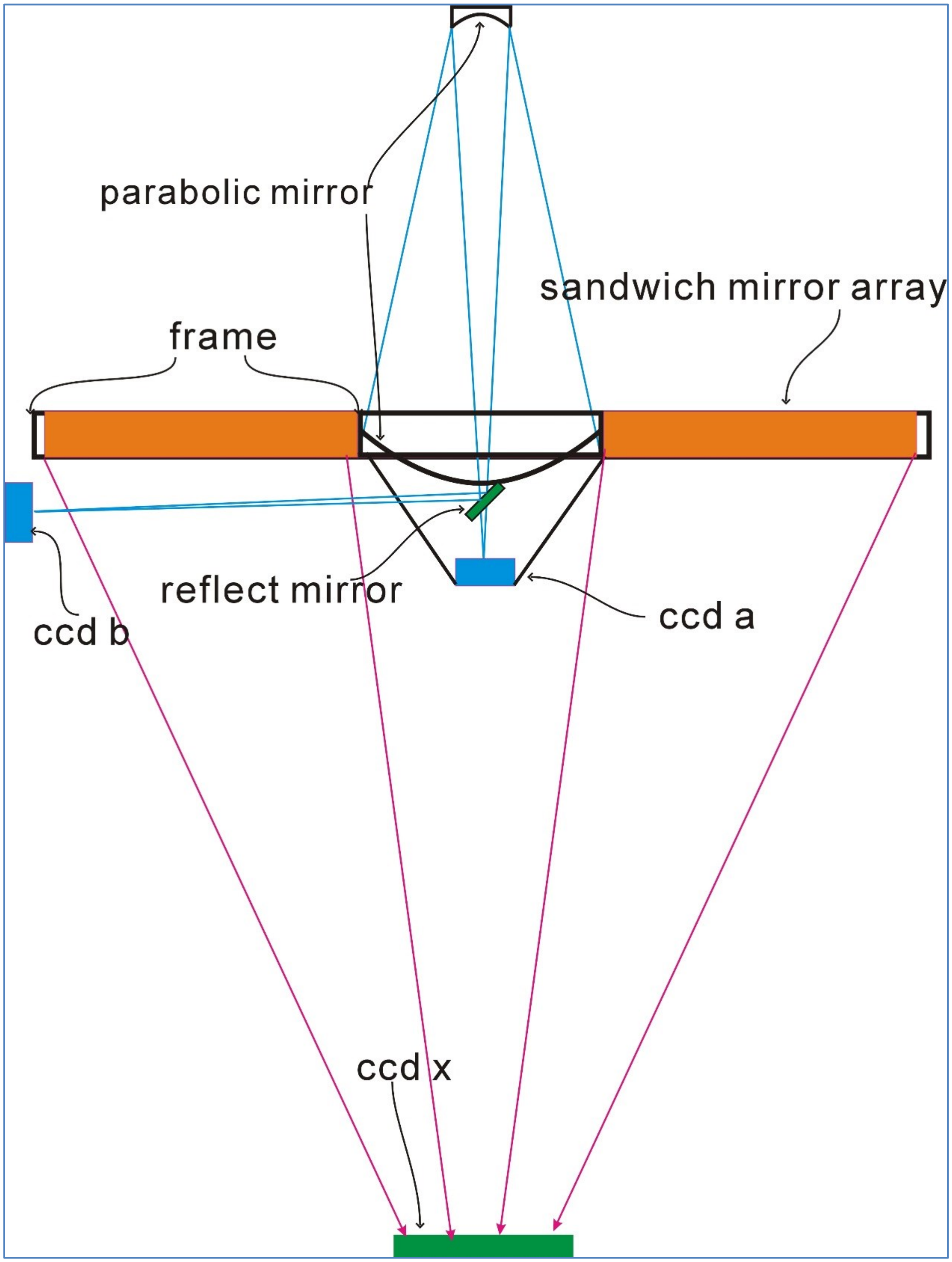


Figure 5

Illustration of thin mirror array lens and a regular reflective telescope at the center. Multiple CCDs for different light source. CCD X for x-ray. Formation (a) has a CCD at the center. Formation (b) has a CCD at side. Those CCD are for regular visible light. Their images can be aligned digitally with CCD x image.

Simulation

MATLAB is used to build two thin layered mirror arrays in a virtual 3-D space. The top one is a parabolic mirror and second one is a hyperbolic mirror.

For the simulation, first form a two-point sources (stars). In the simulation we use data similar to SN185 as the sample star source, so let the stars at distance of 9100 light years of distance. Since the remaining shell of SN185 is about 80 lights in diameter. (Using angular size as 30 arc minutes), so using 1/1000 of its diameter which is 0.01 light year as distance of two stars.

We need initial those parameters: The z value of two co-focal positions. One is parabolic mirror co-focal f_{01} and another is hyperbolic co-focal f_{02} . Hyperbolic mirror has another focal point which is overlap with f_{01} . The bottom of parabolic mirror (z_0 , r_0) . And the thickness of each mirror layer as D_1 and D_2.

To building mirror array in such way that their focal positions are at co-focal. Each of top layer parabolic mirror satisfy this so all the parabolic mirrors have focal points at f_{01}

$$z = \frac{r^2}{4f} + f_{01} - f$$

Equation 4

Where f_{01} is the co-focal position and f is the focal length of each parabolic mirror. Then we build the second layer of the hyperbolic mirror such that its top is connected to the bottom of the first layer parabolic mirror as shown in Figure 1.

From the bottom of the first mirror (z_0 , r_0) , using Equation 4 we can decide f . The top of the mirror has

$$z_3 = z_0 + D_1$$

Equation 5

And from Equation 4 we can get it radius, so we have parameters. (z_3 , r_3).

Hyperbolic mirror has two focal points. Let us put the first focal point at f_{02} and put another focal point at f_{01} . Because of the hyperbolic mirror focal property, all rays which focus at f_{01} will focus at f_{02} when those rays are reflected by the hyperbolic mirror. Thus, this decides the parameters of hyperbolic equation. The distance between f_{01} and f_{02} is 2c, that = $(f_{02} - f_{01})/2$, and (z_0 , r_0) satisfies Equation 6 and Equation 7. So, we have a, b, c and h, where h=a/b. The center of hyperbolic is at $c_0 = f_{01} + c$.

$$\frac{(z - c_0)^2}{a^2} - \frac{r^2}{b^2} = 1$$

Equation 6

$$c^2 = a^2 + b^2$$

Equation 7

Then let the stars emit light rays towards the mirrors. The arrangement of rays is that they evenly incident at the first parabolic mirror ring in a circle. Each mirror takes, say 2N rays per cycle. We can simulate multiple cycle of such rays with different radius. For the rays of each cycle, grouping those rays into N pair that the pair rays separated by 180 degree apart at the two ends of the diameter of the cycle (refer to ray 1 and ray 2 in Figure 6). Then we can find a cross-point on each ray of the pair (refer to points "A" and "B" in Figure 6) that they have the shortest distance from each other. A perfect case is that the distance is 0 which means two rays run across each other that "A" and "B" points are merged into one. For general case in real 3-D space, physically we would find two points instead. N such pair of rays will have 2N cross-points. Those points in 3-D space can be treated as "mass" points. They will have a "mass" center. The average distance of those 2N points to the mass center tells us the focus quality of those rays. The smaller of this value the better. In another way, each ray will also hit a virtual CCD disc placed at the focal plane. The cross points on the disc also have a distribution as in 2-D space (points "a" and "b" in Figure 6 (a)). They form a 2-D "mass" distribution. The average distance of each point to this 2-D "mass" center is a good quantity to measure the focal behavior of the rays. Using one mirror ring simulation to calculate the best CCD position. That is, using one mirror ring do the ray simulation with the star source. Calculate the "mass" center of the cross-point. The center location is the virtual CCD disc center. We can name this process as focal pre-process. This focal pre-process is for virtual flat disc CCD. For curved CCD disc, the mass center may not the best position. By adjusting the position of CCD plane or its surface curvature can be employed to calculate the best results that a minimum average distance to the "mass" center is achieved.

Those points of "A", "B" and "a" and "b" could be used to measure the optical quality of the mirror array lens as well as the CCD position, as well as the curved CCD. A better lens design will have a smaller distribution of "A" and "B" pair and "a" and "b" pair. Their average distances to the common "mass center", i.e. the distribution center is a good value to describe the focus quality. In the simulation, due to limited computational power, only limited mirror rings are simulated with rays. The data is good enough to demonstrate their behavior (Figure 9)

The distribution pattern of point "a" and "b" can be further studied for developing image sharping /deconvolution algorithm. Figure 9 shows those cross points of rays from the blue star, which is away from optical axis, form a corn shape pattern.

Figure 7 shows how to calculate the gap between two mirror rings. The gap ***dr*** between two mirror rings should be that they do not block incident rays reflected from first parabolic mirror. For the compact configuration without consideration of field of view (FOV), the straight ray incident at the bottom of first parabolic mirror decides the value of ***da***, the reflection of the ray at the top of parabolic mirror to the bottom of hyperbolic mirror decides ***db*** and the reflection of the ray at the top of hyperbolic mirror decides ***dc***. The requirement of the gap ***dr*** is that it keeps the points of "a", "b", and "c" not reaching the inner mirror. That is the gap size ***dr***=max(***da,db,dc***) + ***t***. Here ***t*** is the coating thickness of the mirror. The simulation shows that ***da*** is the dominant factor. (Refer to Figure 11 curves)

Figure 10 shows some simulation results. Select four rings along lens radius to have ray traced and simulated. Here is the result of four sets of mirrors ray simulation results. (a) The blue rays from star S2 which is not on optical axis. (b) Red rays from star S1 which is on the optical axis. (c) Rays from stars S1 and S2 crossing over focal plane. (d) Rays from star S2 crossing over focal plane. This is a zoom-in view. A corn shape blurred image is shown. (e) and (f) are side view of those rays from star S1 and S2.

Figure 12 shows the relation of lens radius, number of mirrors and their thickness. Red curve is the thickness as D1=2 cm and D2=1.2 cm. Blue curve is for D1=1 cm and 0.6 cm. The radii of both lenses are 1.5m. Both lenses start from r=0.2m. The gap between two mirrors ***dr*** is decided by the max (***da, db, dc***) in Figure 7. When the thickness is doubled, the number of rings of mirror needed is halved. For this compact configuration and the parameters used, such as the thickness of the mirror coating, the rate of effect idea lenses can be 99% since the only wasted area is the area related to the thickness of coating film. In the simulation, the focal point of parabolic mirror f_{01} is located at origin at z = 0, the first focal point of hyperbolic mirror f_{02} locates at the middle between lens and the origin. For Z0 = 20 (m), f_{02}= (0, 0, 10) in meter. First ring radius is r=0.2 (m). Last ring of mirror is R=1.5 (m). The center of hyperbolic locates at (0, 0, c) that $2c = f_{02}-f_{01}$, which is at (0, 0, 5). Simulations tried two thickness of mirrors, one is (D1, D2) = (1, 0.6) cm and second is (4, 2.4) cm. Another two tries using Z0 = 10 m. The results are in table Table 1 and Table 2

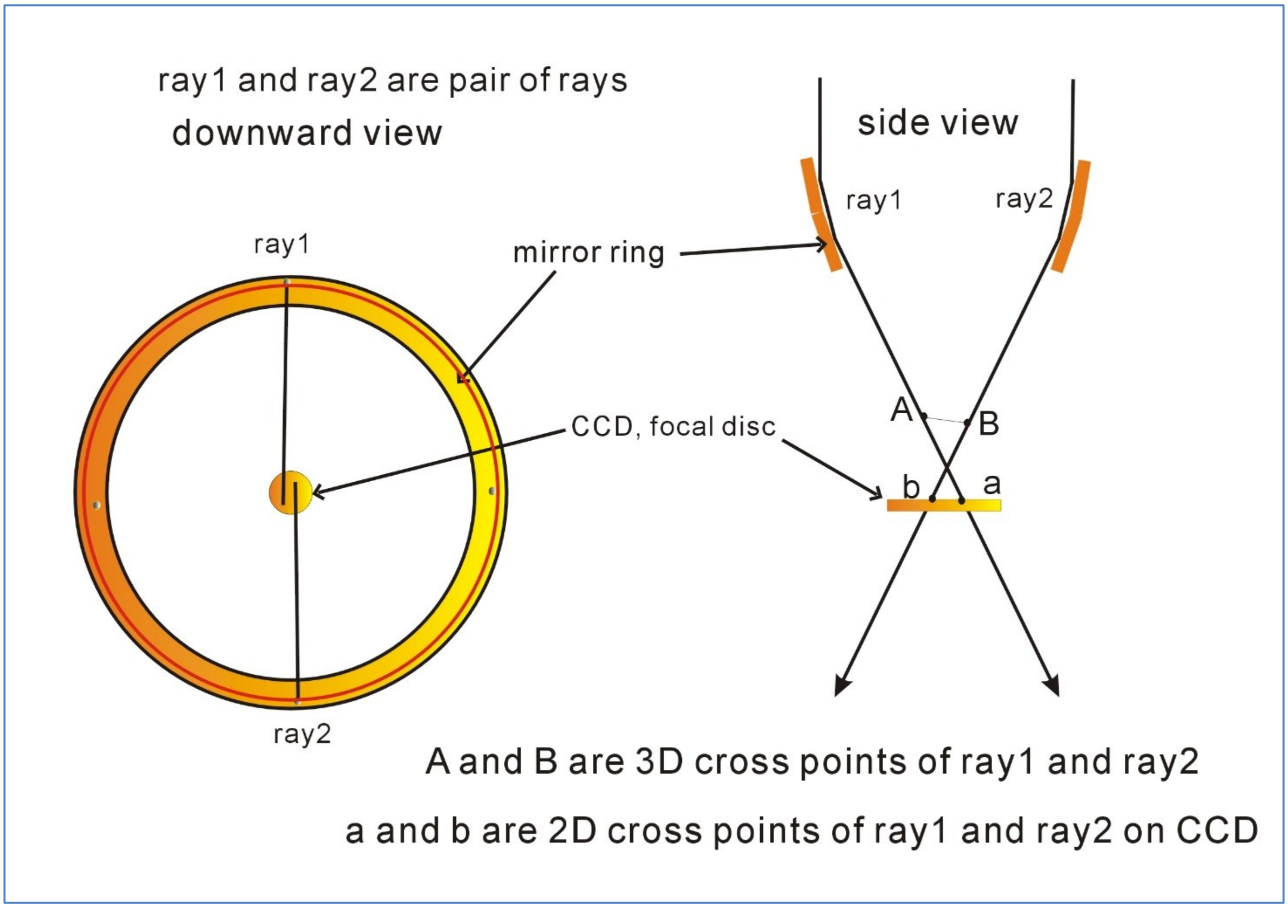


Figure 6

Generating a pair of rays originated from the source to be reflected at the mirror apart of 180 degree (opposite across mirror ring). Using 3-D vector algorithm to calculate their closet cross point pair "A" and "B". These two rays also cross over on the surface of CCD disc at points of "a" and "b". The distribution of those points reflects the focal quality of the mirrors. The average distance of 3D points to it 3D distribution center and 2D points to its 2D distribution center are two good quantities to describe the focal property.

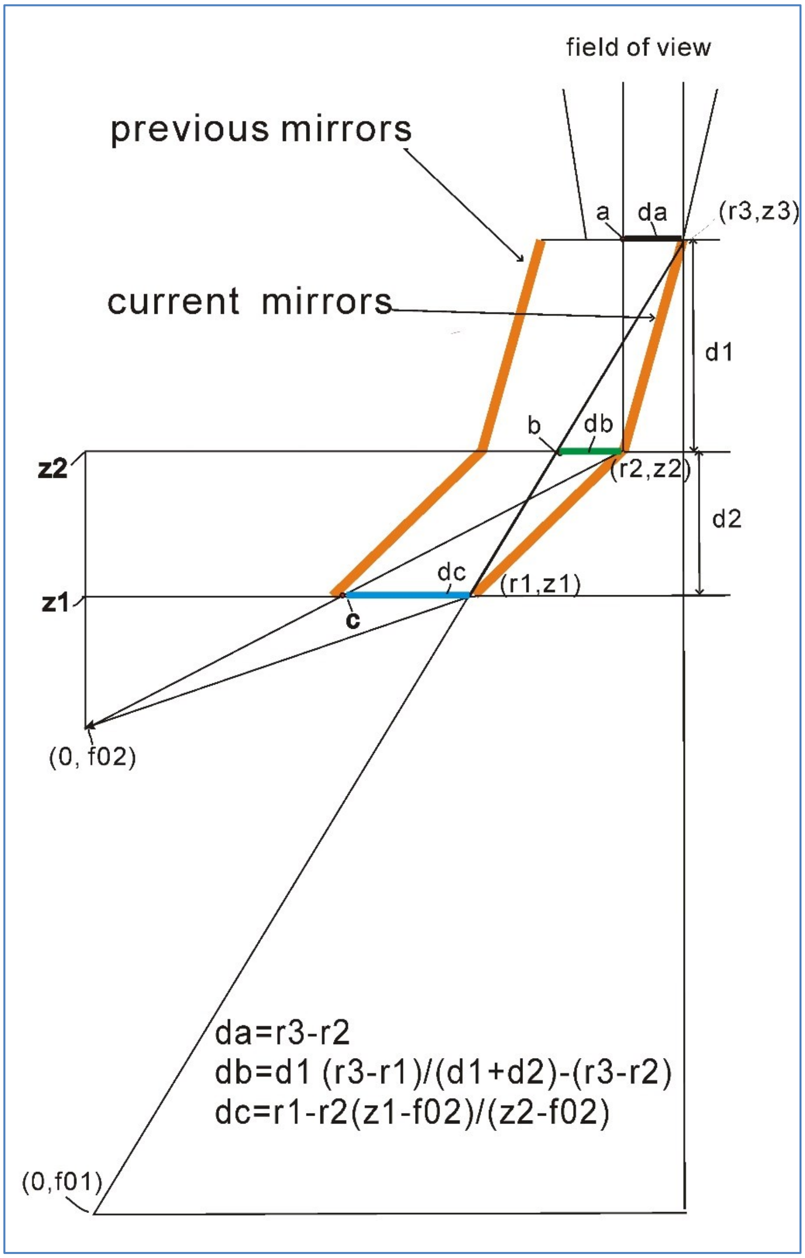


Figure 7

The gap *dr* between two mirror rings should be that it will not block incident rays reflected by first layer parabolic mirror. Three factors decide the gap size. They are *da*, *db*, and *dc* as shown in the picture. Their equations also listed in the chart. The simulation found out that *da* has the largest value. The equations do not consider the field of view (FOV) factor. Otherwise, *dr* must add an extra quantity $d1*tan(\alpha)$ where α is the FOV angle. *d1* is the thickness of first layer parabolic mirror and *d*2 is the thickness of the second hyperbolic mirror. The focal length of first mirror is f01, and f01 and f02 are the focal points of hyperbolic mirror. f01 and f02 are on z axis. The dimension in the picture is not in ratio.

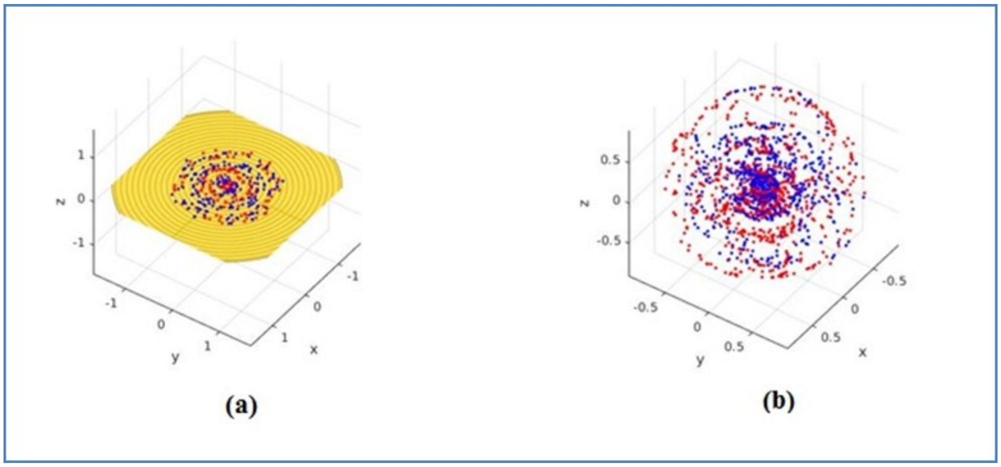


Figure 8

(a)Illustration of cross-point pair "a" and "b" on CCD focal plane and its average distance to the "mass center"

(b)Illustration of cross-point pair "A", "B" distribution and of the average distance to the "mass center". The dimensional size is for demonstration only. Not in real order.

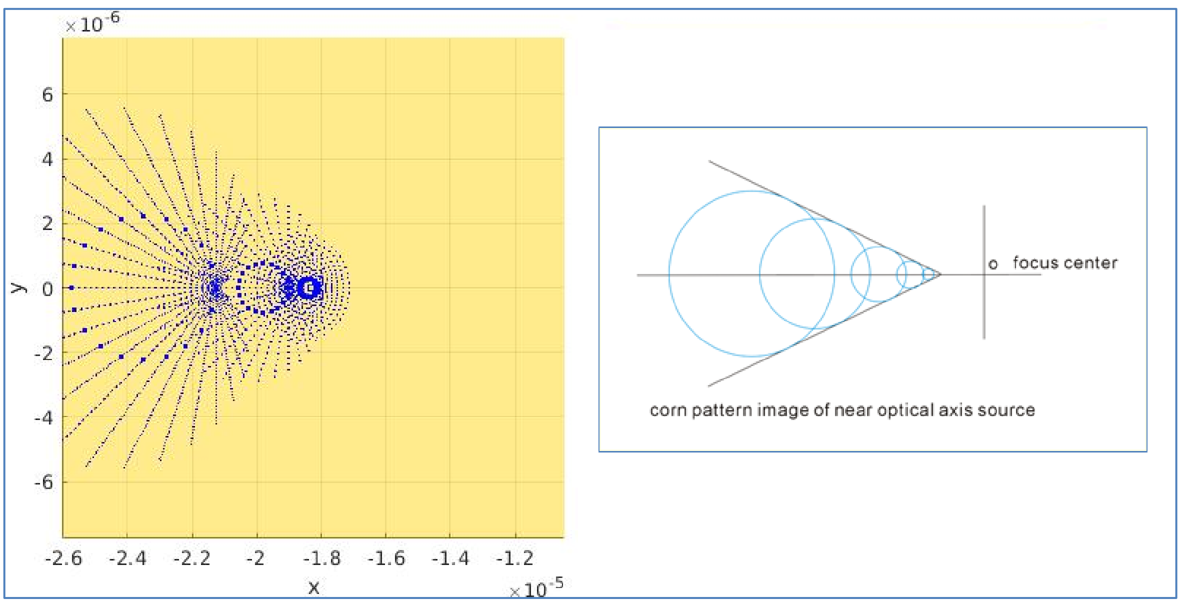


Figure 9

Source star away from optical axis will form a corn shape blurred image. The cross points at CCD focal plane of those rays are reflected from each mirror rings and forms a cycle ring pattern. Those rays from all of those mirror rings form a corn pattern.

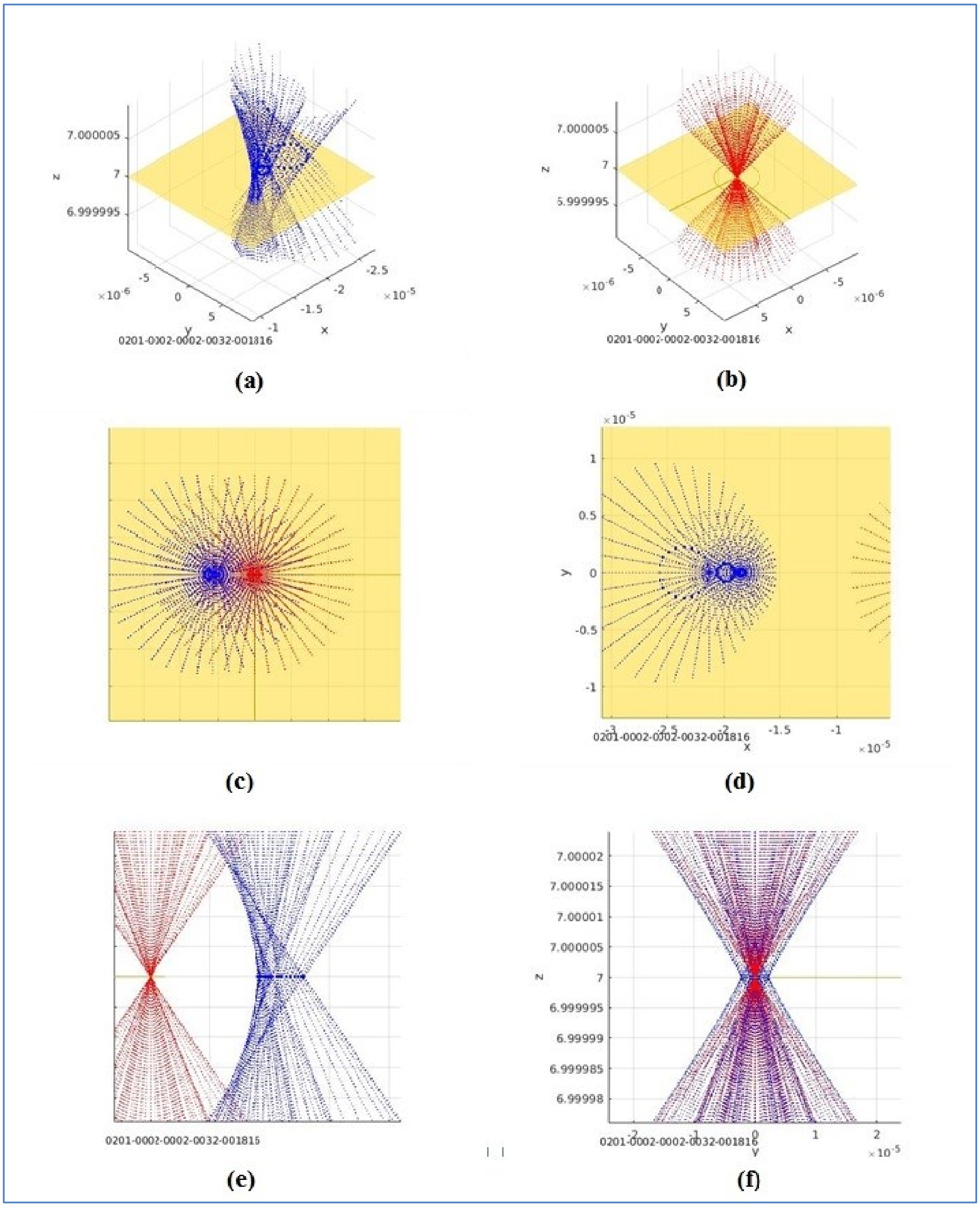


Figure 10

Simulation results. Ray tracing only on four mirrors in this simulation. Ray pairs evenly distributed along the radius of the lens. (a) The blue rays from star S2 which is not on optical axis. (b)The red rays from star S1 which is on the optical axis. (c) Rays from stars S1 and S2 crossing CCD at focal plane. (d) Rays from star S2 crossing CCD over focal plane. This is a zoom-in view. A corn shape blurred image is shown. (e) and (f) are side view of those rays from star S1 and S2.

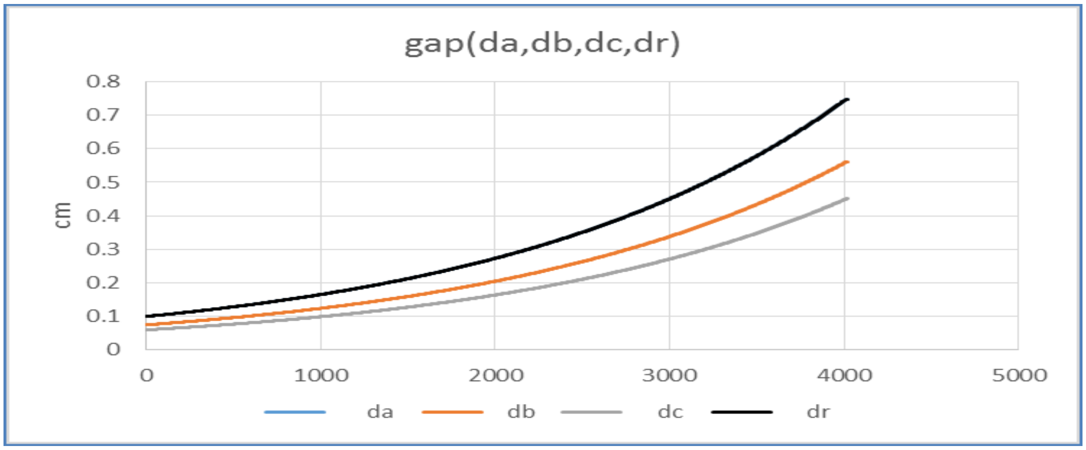


Figure 11

For the simulation, it shows *da* decides the gap size *dr*.

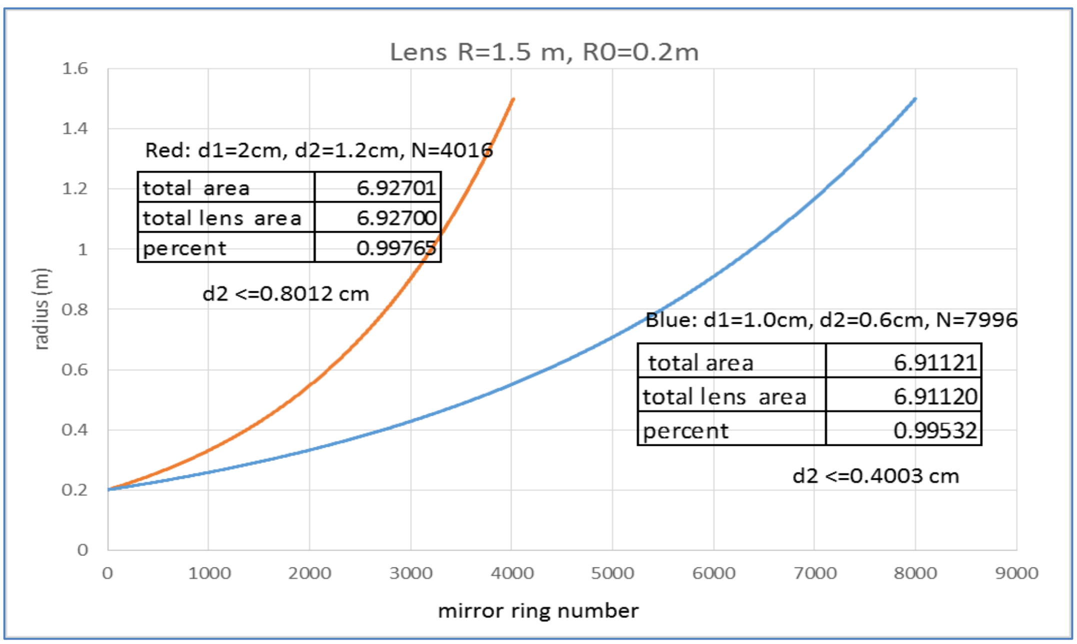


Figure 12

Relationship of lens radius and the number of mirror rings with different lens thickness. Here d2 is the minimum thickness the second mirror needed to reflect all the rays coming from first mirror. The unit of y axis is meter

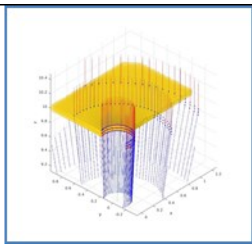
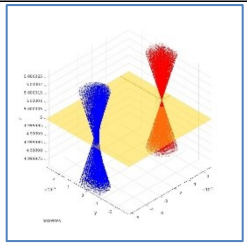
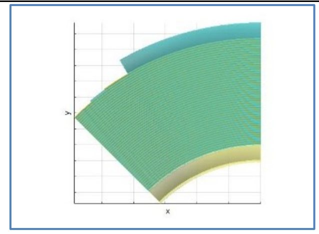
 **Simulation video**	 **Rays near CCD**	 **Demon of mirror array**

Figure 13

Simulation videos of ray tracing on mirror array. Click pictures to play the videos.

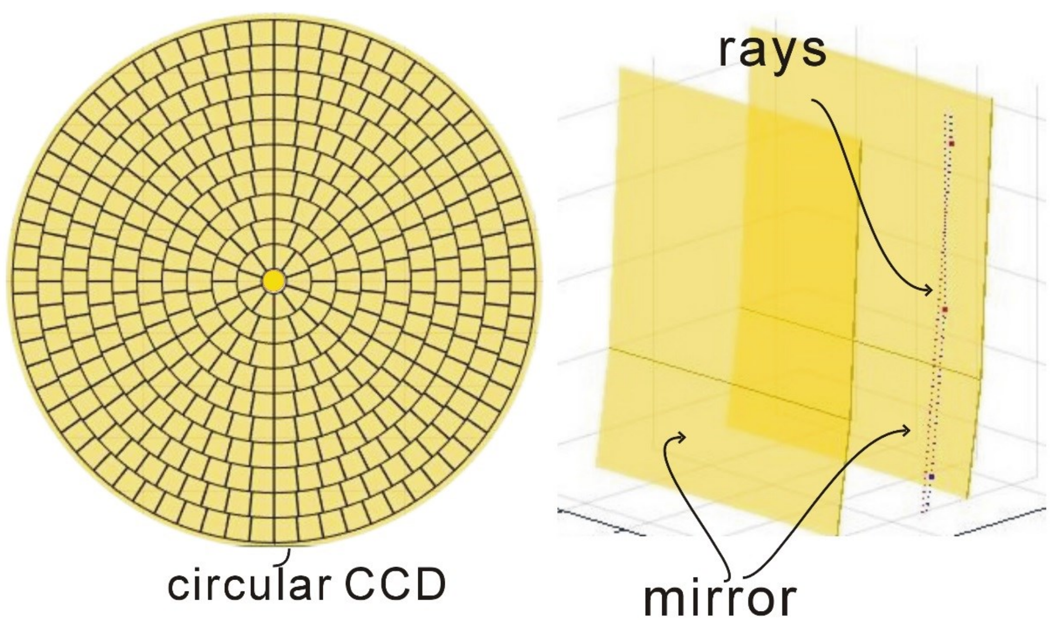


Figure 14

Illustration of circular arranged CCD and rays reflected on mirror array

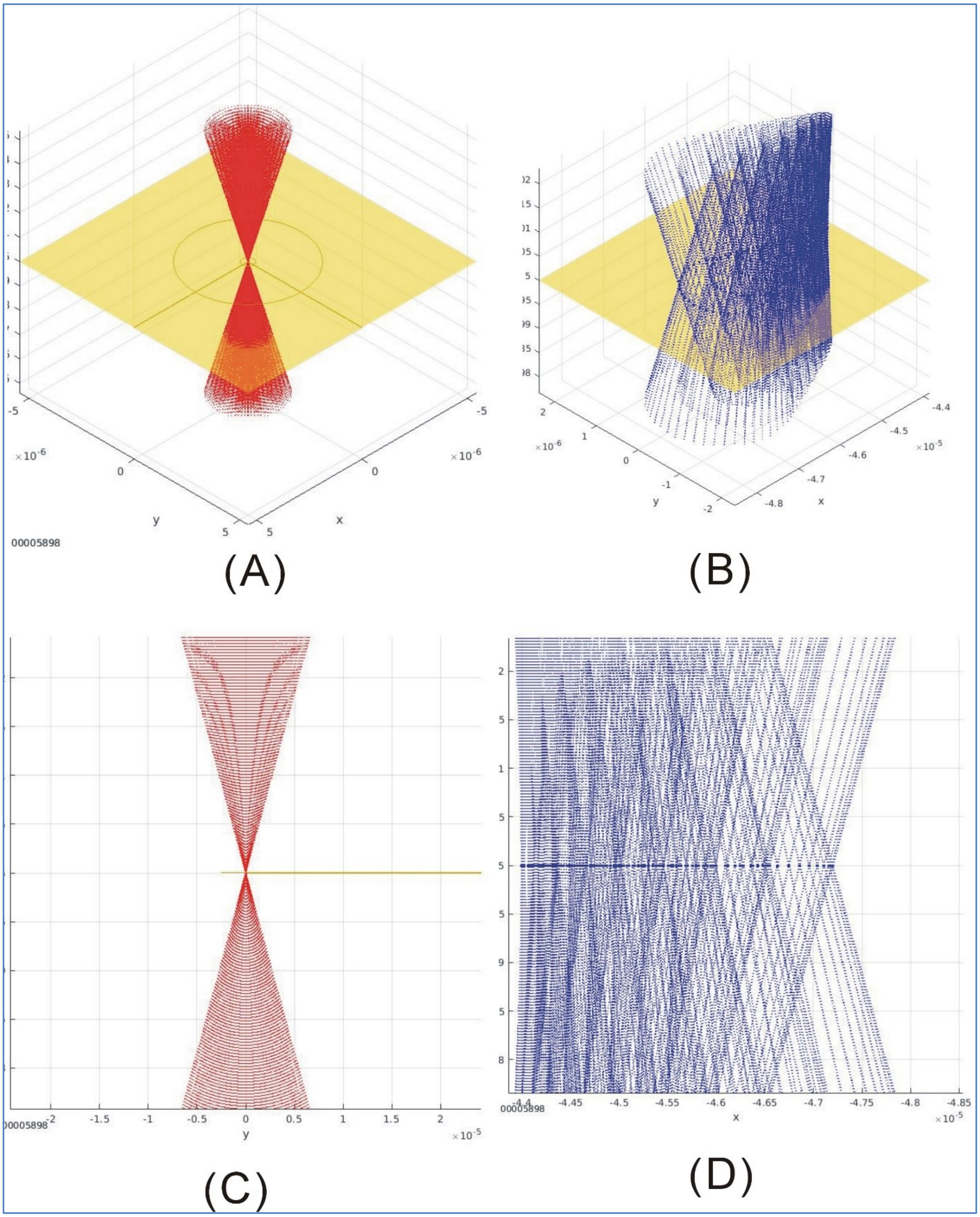


Figure 15

(A)Three-dimensional view of rays from a star on optical axis (Red star). Perfectly focused. (B) Rays from a star off optical axis. Its cross points form a corn pattern. (Blue star). (C)A view at x direction of rays from red star. (D)A view at x direction of rays from blue star. Because if zoom region, only rays from one star are displayed in each figure.

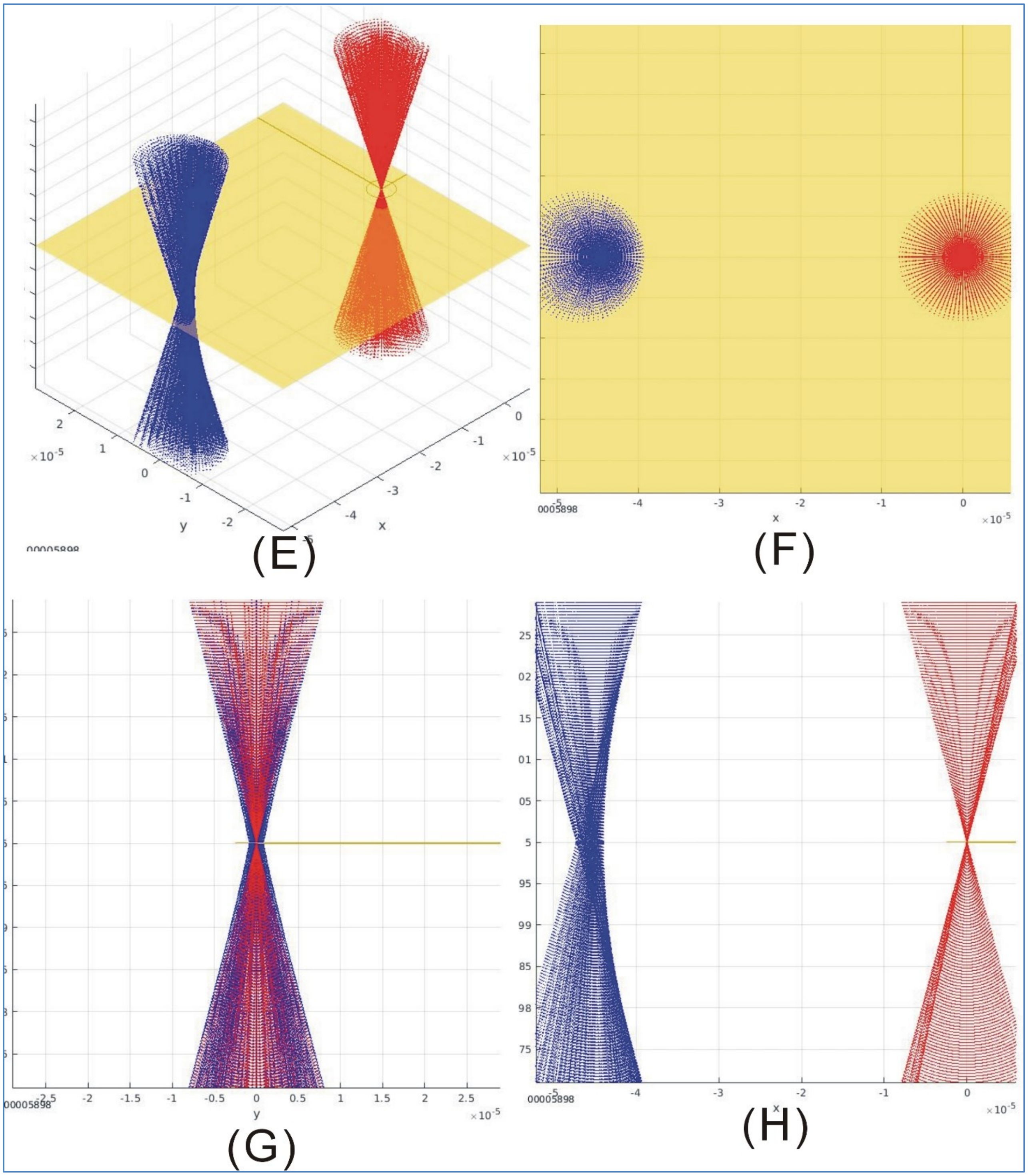


Figure 16

(E)A three-dimensional view of rays from two stars on CCD. Red star is on the optical axis and blue one is off optical axis. (F)A view from –z direction of rays from two stars near CCD. (G)A view of rays at x direction. (H) A view of rays at y direction. Those are zoom out view figures so we can see both stars.

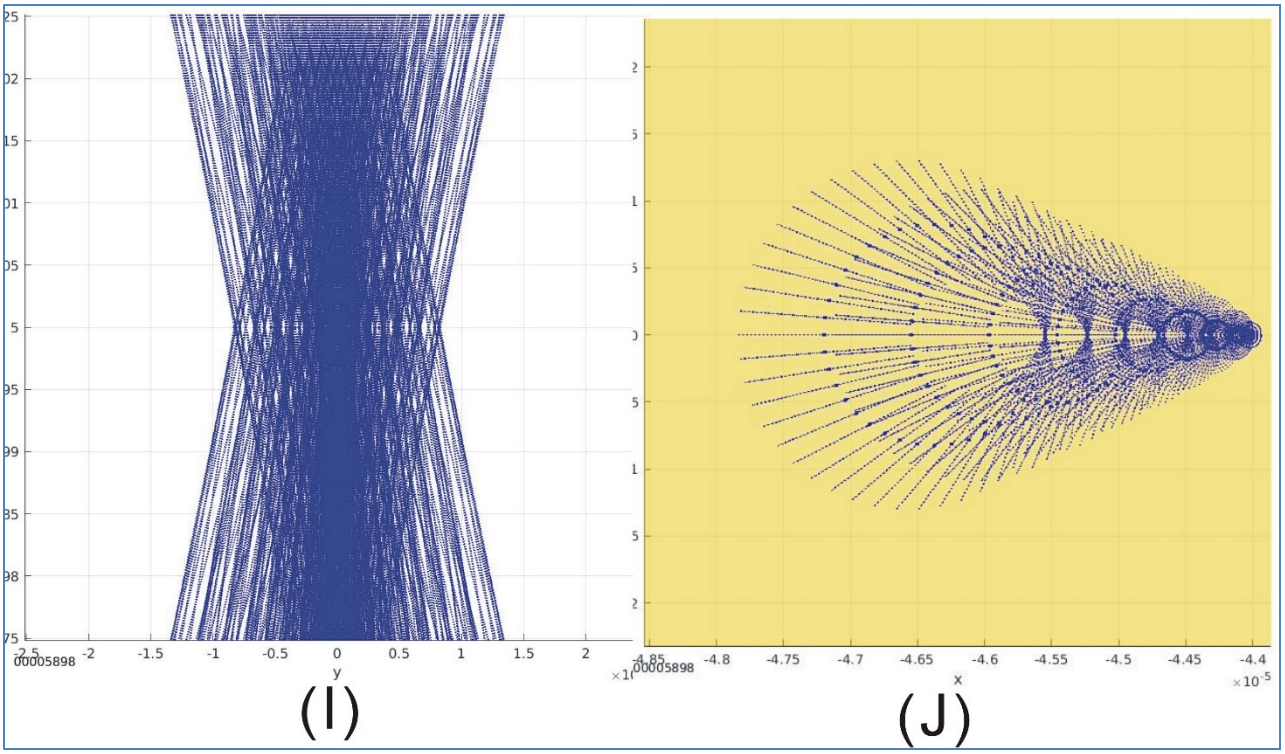


Figure 17

A view of blue star rays at x direction and –z direction near CCD. It shows a corn pattern. The pattern is affected by the dimension of lens mirror as well as the star location. So, it can be used for deconvolution of CCD image to sharpen the image. In turns the image pattern of off optical axis star might give us its distance information as well which worth us to further study. Table 1

r0=0.2 m, r1=1.5 m								
try	ring	Z0	f2	D1	D2	lens area	effect lens area	pcnt
1	1977	20	10	4	2.4	6.98E+00	6.90E+00	9.88E-01
2	7441	20	10	1	0.6	6.95E+00	6.64E+00	9.56E-01
3	1002	10	5	4	2.4	7.02E+00	6.98E+00	9.94E-01
4	3874	10	5	1	0.6	6.96E+00	6.80E+00	9.77E-01

Table 1. All units are in meter except D1 and D2 in cm. For lens starts from radius r0=0.2 to r1=1.5 Z0 is the lens location. The common focal point of first mirror array always at axis origin. The table shows the number of mirror rings needed for different mirror thickness and for different focal length. Mirror one is D1 and mirror two is D2. When thickness D1 increases from 1 cm to 4 cm, Total rings increase by almost 4 times. When focal length reduces by half, the total ring number also reduces by half. Because of gap is calculated using the method in Figure 7, the percentage of effective lens area are very high.

Table 2

3D & 2D Cross-points distribution of rays and distance between two imagery objects											
						star 1 red		star 2 blue		image obj delta distance	
try	f1z	f2z	Z0	D1	D2	3D dr	2D dr	3D dr	2D dr	3D delta	2D delta
1	0	10	20	4	2.4	2.49E-46	3.58E-47	4.99E-07	4.99E-07	8.94E-05	8.94E-05
2	0	10	20	1	0.6	3.55E-47	2.33E-48	4.98E-07	4.98E-07	8.94E-05	8.94E-05
3	0	5	10	4	2.4	1.02E-46	1.18E-47	1.02E-06	1.02E-06	4.69E-05	4.69E-05
4	0	5	10	1	0.6	2.14E-47	2.19E-48	1.01E-06	1.01E-06	4.69E-05	4.69E-05

All units are in m except D1 and D2 are in cm. The cross-point pattern on focal plane from the red star on optical axis, is almost a perfect point. The cross-point from the off optical axis blue star forms a corn shape blur image object. That average distance (3D or 2D dr of blue star) shows that longer focal length produces better results than that of the short one. And this value is not sensitive to the thickness of mirror array. The average distance of two image objects (3D and 2D delta) of the simulation is proportional to their focal length.

Discussion

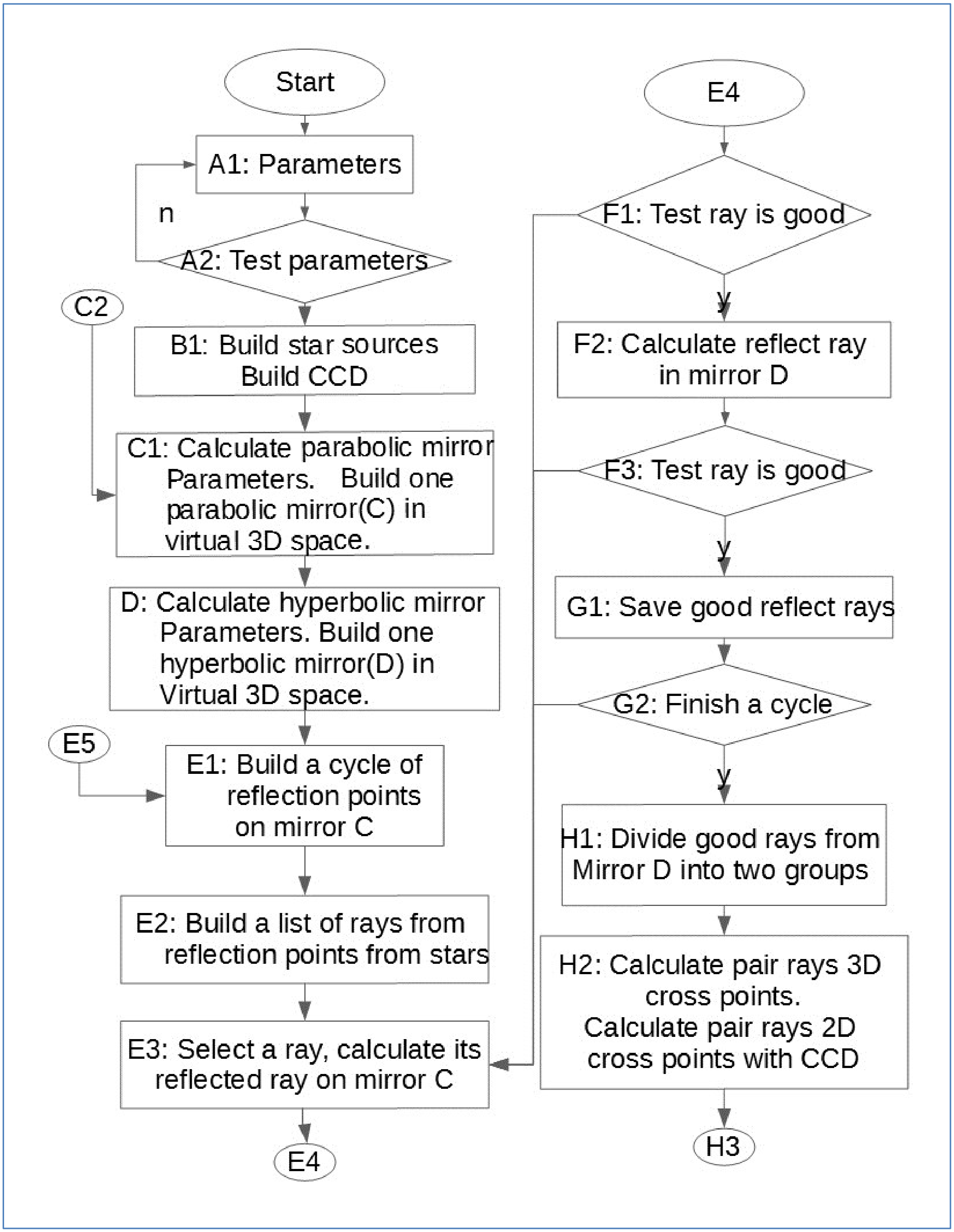
It worth to study the effect to use corn mirror to replace the hyperbolic curve. Simulation is also underway for corn shape mirrors when the thickness of mirror is very thin. Current technology of thin film coating in ready for such mirror array thin lens fabrication. A full simulation of a point source away from optical axis for every mirror of the lens will reveal the image pattern which can be used for image reconstruction to sharp the image. As we seen here the aberration of an off optical axis star forms a corn shape pattern. This pattern is affected by lens and star location. So it is worth to study if we can use this pattern to extra star's location information.

Pseudo Code and flow chart of the simulation

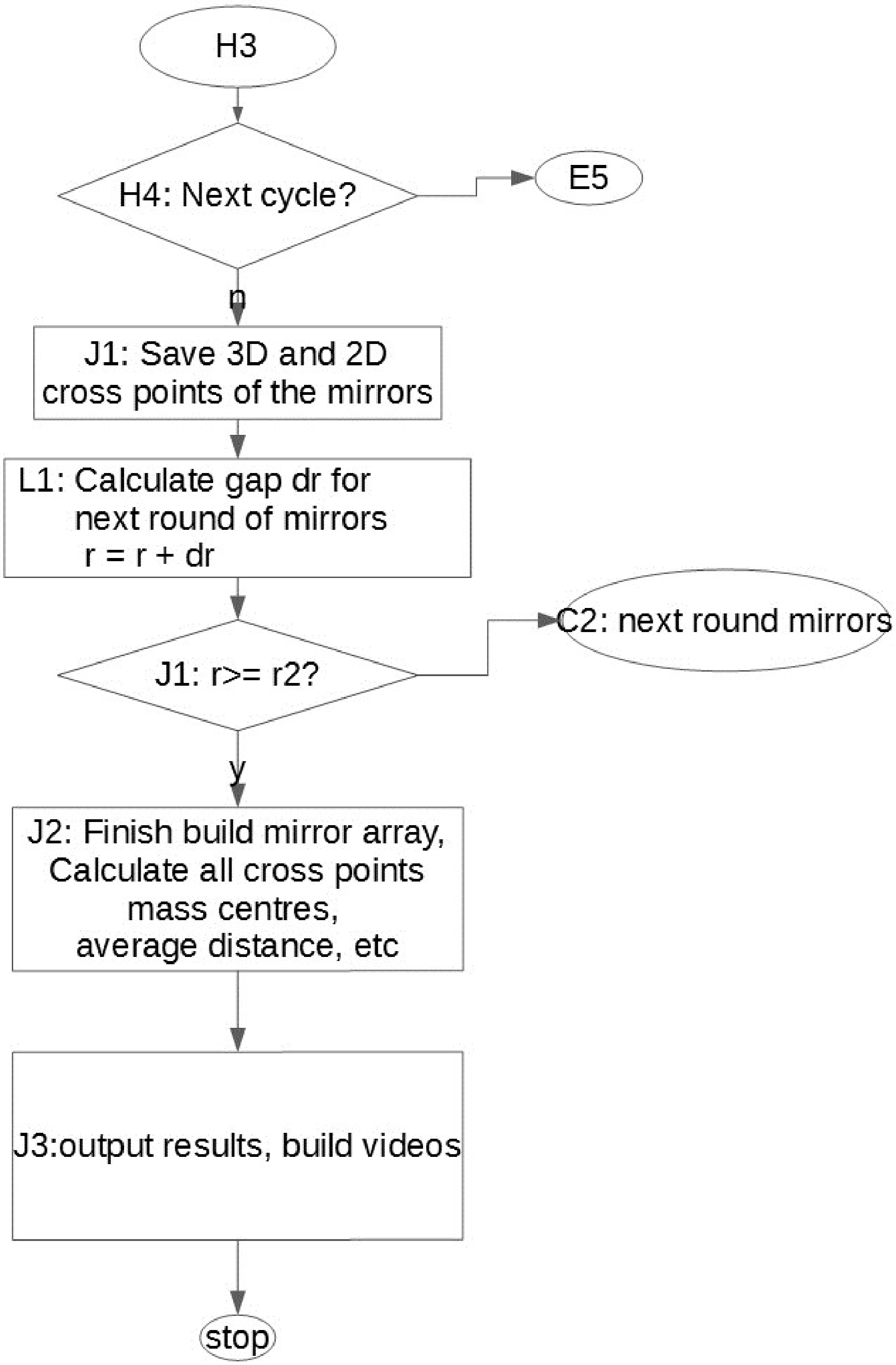
Algorithm start with initial parameters:

The bottom of parabolic mirror is z0, radius is r0. The lens size is R0; Thick of mirror coating is dt. The thickness of parabolic mirror is d1, focal point at f01. The thickness of hyperbolic mirror is d2, focal points are at f01 and f02. CCD is placed at f02. Sources of two stars.

```
K   = 1;
z2 = z0;
r2 = r0;
%build CCD at f02
ccd = set_ssd (f02);
%build two point sources from remote stars.
Sources=set_sources (stars);
set_parameter (z2,r2,f01,f02,d1,d2);
while r2<R0
            [f, z3, r3] = calculate_parapolic_parameter (z2,r2,f01,d1);
           [z1, r1, a, b, c, h]=calculate_hyperpolic_parameter (z2,r2,f01,f02,d2);
           [mirror_array] = build_mirrirs(z1,r1,z2,r2,z3,r3,f,f01,f02,a,b,c,h);
           rays = calculate_ray_parameters (z2,r2,z3,r3,sources);
           rays_out = ray_tracing_grazed_by_mirrors(rays, z1,r1,z2,r2,z3,r3,f,f01,f02,a,b,c,h);
           [3d_cross_points (k), 2d_cross_points (k)]= calculate_cross_points (rays_out, ccd);
           dr=calculate_gap(z1,r1,z2,r2,z3,r3,f,f01,f02,a,b,c,h,dt);
           record_data(z1,r1,z2,r2,z3,r3,f,a,b,c,h,k,dr, 3d_cross_points, 2d_cross_points,rays,
rays_out, k);
           build_moive (rays, rays_out, mirror_array);
           r2=r2+dr+dt;
           k=k+1;
end
lens_focal_quality_results = process_cross_points (3d_cross_points, 2d_cross_points);
```



Start
A1: Parameters
n
A2: Test parameters
C2
B1: Build star sources
Build CCD
C1: Calculate parabolic mirror Parameters. Build one parabolic mirror(C) in virtual 3D space.
D: Calculate hyperbolic mirror Parameters. Build one hyperbolic mirror(D) in Virtual 3D space.
E5
E1: Build a cycle of reflection points on mirror C
E2: Build a list of rays from reflection points from stars
E3: Select a ray, calculate its reflected ray on mirror C
E4
E4
F1: Test ray is good
y
F2: Calculate reflect ray in mirror D
F3: Test ray is good
y
G1: Save good reflect rays
G2: Finish a cycle
y
H1: Divide good rays from Mirror D into two groups
H2: Calculate pair rays 3D cross points.
Calculate pair rays 2D cross points with CCD
H3


H3
H4: Next cycle?
E5
n
J1: Save 3D and 2D cross points of the mirrors
L1: Calculate gap dr for next round of mirrors r = r + dr
J1: r>= r2?
C2: next round mirrors
y
J2: Finish build mirror array, Calculate all cross points mass centres, average distance, etc
J3:output results, build videos
stop

A1:
Decide initial parameters:
Lens size:

(1) r1: start radius,
(2) r2: stop radius,
(3) z0: Lens position.

Lens thickness:

(1) d1: parabolic mirror thickness
(2) d2: hyperbolic mirror thickness

Lens co-focal length:

(1) f1: parabolic co-focal location
(2) f1: hyperbolic first co-focal location
(3) f2: hyperbolic second co-focal location

Coating and array parameter:

(1) dt: mirror coating thickness
(2) dr0: minimum mirror array gap

Grazing and CCD parameters:

(1) a0: minimum grazing angle
(2) rc: CCD radius
(3) zc: CCD Position
(4) Stars color and positions.

A2: Test all A1 parameters that fit for design requirement and satisfy physics law. Such as the minimum grazing angle for all mirrors and optical physics.

References

1. **Modern Magnetic Systems. [Online] September 07, 2018. https://www.is.mpg.de/en/news/novel-polymer-lenses-for-x-ray-microscopes-highly-efficient-and-low-cost.**

2. A radically different type of X-ray space telescope. [Online] https://www.innovationtoronto.com/2019/06/a-radically-different-type-of-x-ray-space-telescope/.

3. How do X-ray telescopes differ from optical telescopes? [Online] http://chandra.harvard.edu/xray_astro/history.html.

4. Wolter telescope. [Online] https://en.wikipedia.org/wiki/Wolter_telescope.

5. xmm-newton. [Online] https://www.cosmos.esa.int/web/xmm-newton/technical-details-mirrors.

6. Overview over many kinds of X-ray optics. [Online] http://www.x-ray-optics.de/index.php/en/types-of-optics/reflecting-optics/curved-mirrors.

7. Caltech. NuStar X-ray Lens. [Online] https://www.nustar.caltech.edu/page/optics.

8. list of x-ray telescope. [Online] https://en.wikipedia.org/wiki/List_of_X-ray_space_telescopes.

9. Spiga, D. [Online] https://arxiv.org/ftp/arxiv/papers/1509/1509.03473.pdf.

10. *Iridium coatings for space based x-ray optics.* Probst, A.-C etc. 2017. 10.1117/12.2296167.

11. Planck constant. [Online] https://en.wikipedia.org/wiki/Planck_constant.

12. *Angular_resolution.* [Online] https://en.wikipedia.org/wiki/Angular_resolution.

13. Stothers, Richard. Is the Supernova of A.D. 185 Recorded in Ancient Roman Literature? Isis. 68 (3): 443–447., 1977.

14. UTK. resolving_power. [Online] http://labman.phys.utk.edu/phys136core/modules/m9/resolving_power.htm.

15. *ASTRO-H, Manufacture of aspherical molding dies for X-ray telescopes.* Namba, Yoshiharu. s.l. : Conference: SPIE, Space Telescopes and Instrumentation, Volume: 9144, July 2014.