

BALBI EQUATION

FUNDAMENTAL CONCEPTS AND PRINCIPLES

A NEW EXPLICIT, NON-ITERATIVE, AND UNIFIED MODEL FOR DISTRIBUTED PRESSURE DROP
CALCULATIONS IN DUCTS

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ABSTRACT

This paper presents the **BALBI EQUATION**, a phenomenological and semi-empirical formulation for estimating distributed pressure drop in circular ducts. Rather than replacing established methods, the proposed approach offers an alternative physical interpretation of flow resistance while remaining consistent with classical fluid mechanics and established engineering practice. The model combines analytical development with experimental calibration through five fundamental relationships: (1) the viscous wavelength, $\lambda = \frac{\nu}{v_m} \cdot k_{Balbi}$, representing an effective characteristic length associated with viscous momentum diffusion; (2) a unified exponential velocity profile, $u(r) = v_{max}(1 - e^{-((R-r)/\lambda)^\alpha})$, applicable to laminar, transitional, and turbulent regimes; (3) a continuous flow-regime function, $\alpha(Re) = 1 + \frac{1}{1 + (Re/2800)^4}$, providing a smooth transition between flow regimes; (4) a pressure-drop expression, $\frac{\Delta P}{L} = \frac{2\mu v_{max}\alpha}{R\lambda} \cdot \frac{1}{1000}$, derived directly from the proposed velocity profile; and (5) a calibration relationship, $k_{Balbi} = C_{base} Re^{0.25} \left(\frac{\varepsilon}{D_h}\right)^{0.1}$, which relates the characteristic viscous scale to wall roughness and flow conditions.

The formulation was validated against two independent experimental datasets. For corrugated flexible ducts (Dai et al., 2021), it achieved a precise prediction error of +4.5%, whereas the Colebrook–White equation underestimated the pressure drop by –14.8%. In gas–liquid–solid three-phase flows (Al-Hadhrami et al., 2014), the phenomenological framework proved highly adaptable beyond conventional HVAC domains. For standard galvanized steel ducts in HVAC applications ($C_{base} = 0.042$), the model yields consistently stable, conservative margins (ranging from +21% to +27%) relative to the Colebrook–White equation.

Operating with a constant-time computational complexity of $\mathcal{O}(1)$, the Balbi Equation eliminates iterative processes, offering a physically motivated and computationally efficient alternative for spreadsheet and software implementation in engineering design.

$$\lambda = \frac{\nu}{v_m} \cdot k_{Balbi}$$

$$k_{Balbi} = C_{base} Re^{0.25} \left(\frac{\varepsilon}{D_h}\right)^{0.1}$$

$$u(r) = v_{max} \left(1 - e^{-((R-r)/\lambda)^\alpha}\right)$$

$$\alpha(Re) = 1 + \frac{1}{1 + (Re/2800)^4}$$

$$\frac{\Delta P}{L} = \frac{2\mu v_{max}\alpha}{R\lambda} \cdot \frac{1}{1000}$$

Keywords: Pressure drop; duct design; Balbi equation; explicit method; HVAC; viscous wavelength; Colebrook-White; Equal Friction; Static Regain; non-iterative.

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1. INTRODUCTION

Internal flow in circular ducts is one of the most consolidated pillars of fluid mechanics. Since the pioneering experiments of Hagen, Poiseuille, Darcy, Reynolds, Blasius, Nikuradse, and Colebrook, distributed pressure drop has been modeled as the macroscopic manifestation of the complex interaction between inertial forces, molecular viscosity, and the geometric roughness of the duct walls. Over the last century, this conceptual framework gave rise to the classical Darcy-Weisbach formulation and the semi-empirical correlations for determining the friction factor—fundamental and indispensable tools for the design of modern hydraulic, thermal, and HVAC systems.

However, despite the mathematical robustness and practical precision of these traditional formulations, the transition between flow regimes and the determination of friction in intermediate zones are still frequently treated through segmented approximations, discontinuous correction factors, and iterative computational processes. From this purely mathematical perspective, fluid behavior ends up being represented by the concatenation of distinct models for each operational range, which may distance the analysis from a continuous physical interpretation of dissipative mechanisms.

The motivation for this work arises from this conceptual gap. We propose herein an alternative analytical and phenomenological formulation for calculating distributed pressure drop, termed the Balbi Equation. Based on the concept of the viscous wavelength (λ), the model introduces a spatial scale that continuously describes the transition between laminar, transitional, and turbulent regimes without resorting to discontinuities or conditional logic in the modeling. Initially calibrated for applications in HVAC, industrial ventilation, and technical ducts, the mathematical structure of the formulation aims to provide a complementary and explicitly direct tool for engineering, aligned with the historical foundations consolidated by classical literature.

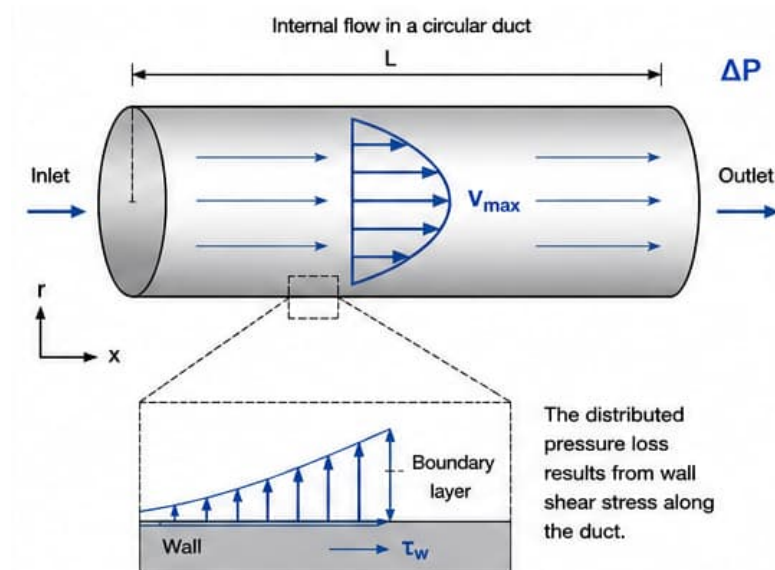


Figure 1 – Schematic representation of internal flow in a circular duct, highlighting the development of the velocity profile and the origin of distributed pressure drop due to wall shear

2. SCIENTIFIC POSITIONING AND PHENOMENOLOGICAL NATURE OF THE MODEL

The Balbi Equation is a phenomenological and semi-empirical formulation. This implies that its central objective is not to replace the fundamental principles of fluid mechanics, nor to compete with general Navier-Stokes equations or advanced computational simulation methods—such as Computational Fluid Dynamics (CFD), Large Eddy Simulation (LES), or Direct Numerical Simulation (DNS). The validation of a new conceptual proposal requires rigorously delimiting its nature, applicability, and limitations. Turbulence remains one of the most complex open problems in classical physics, and the approach proposed herein operates at the macroscopic scale of applied engineering.

The originality of the model lies in reorganizing the dissipation phenomenon based on the dynamic spatial relationship between the fluid and the conduit. All distributed pressure drop arises, essentially, from the transfer of momentum by shear. From this viewpoint, we adopt the physical hypothesis that viscosity establishes an effective depth for the propagation of the braking effect imposed by the wall upon the flow, quantified by the viscous wavelength (λ). While in the laminar regime this dissipative zone occupies virtually the entire cross-section of the duct, the increase in the Reynolds number and the resulting turbulent intensity progressively compress this scale toward the wall, concentrating the velocity gradient in a narrow peripheral region.

To mathematically translate this spatial mechanism and ensure model closure, we introduce the flow-regime function and the coupling coefficient. This coefficient is semi-empirical in nature and is determined from the analytical equivalence between the wall shear stress predicted by the proposed exponential velocity profile and the experimental correlations widely consolidated in technical literature.

Admitted as a proposal in constant evolution, the model does not claim absolute universality. The Balbi Equation should be interpreted as a complementary conceptual framework. Its practical utility lies in the elimination of iterative routines and the possibility of representing pressure drop through a continuous and spatially motivated physical interpretation.

3. FUNDAMENTALS OF THE VISCOUS WAVELENGTH λ

In classical internal flow models, distributed pressure drop is traditionally interpreted as a consequence of energy dissipation caused by friction between the fluid and the duct wall. However, from a phenomenological perspective, this interaction can be reinterpreted in a spatial and continuous manner.

Every wall imposes resistance to fluid motion. This resistance does not act instantaneously upon the entire flow but propagates its influence progressively through adjacent fluid layers via viscous shear. Thus, dissipation can be understood as a process of gradual transmission of the deceleration imposed by the wall to the rest of the fluid.

Under this interpretation, the wall acts as the central dissipative element of internal flow, establishing an effective reach of influence over the fluid mass in motion.

Balbi Equation proposes to represent this dissipative reach through the viscous wavelength λ :

$$\lambda = \frac{\nu}{v_m} \cdot k_{Balbi}$$

Where:

- ν is the fluid kinematic viscosity [m²/s];
- v_m is the mean flow velocity [m/s];
- k_{Balbi} is the viscous coupling coefficient (dimensionless), the unified formulation of which will be detailed in Section 6.

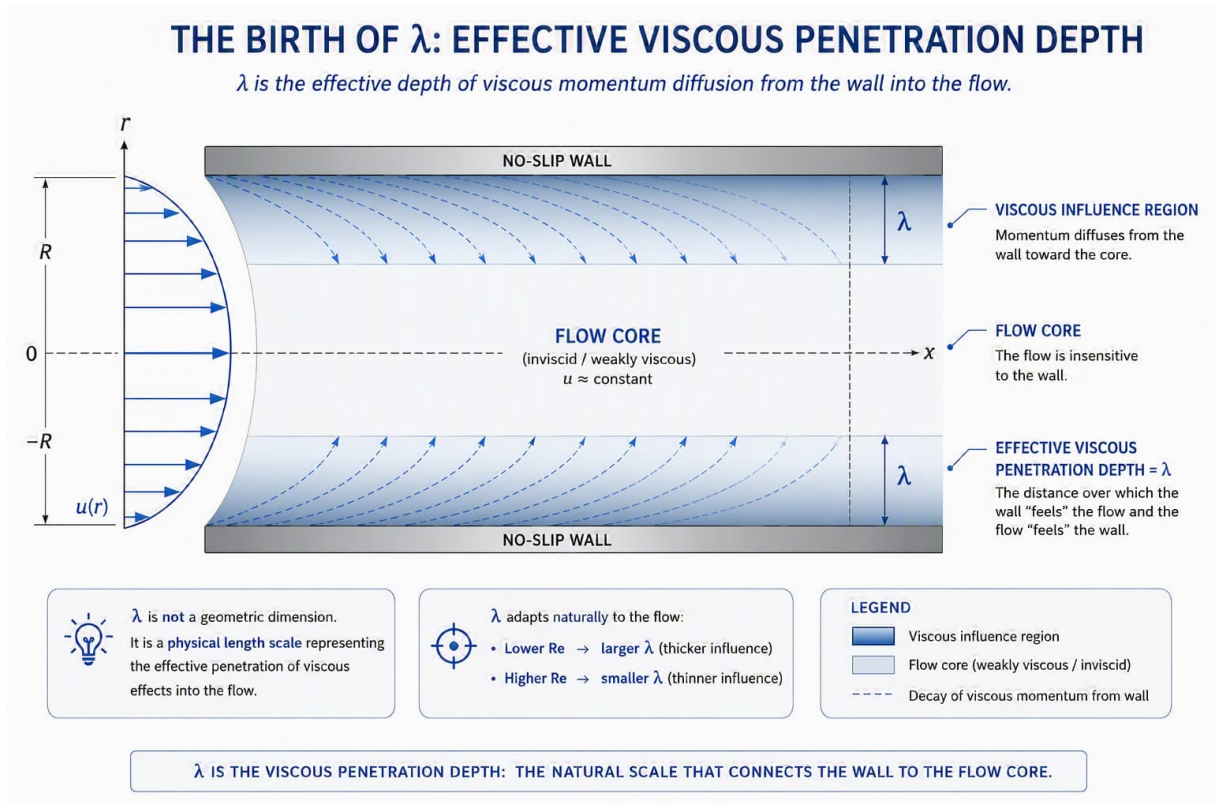


Figure 2 - Conceptual representation of the viscous penetration depth (λ). The parameter λ denotes the effective dissipative length scale through which wall-induced viscous effects extend into the flow, linking wall

friction to the development of the velocity profile.

3.1 Immediate Physical Interpretation

Situation	Effect on λ	Physical Meaning
ν increases (more viscous fluid)	λ increases	The wall extends its "braking" further into the duct
v_m increases (faster flow)	λ decreases	The boundary sublayer is compressed; the velocity gradient at the wall becomes more severe
$k_{Balbi} > 1$	$\lambda > \nu/v_m$	The wall brakes further away (laminar regime or high roughness)
$k_{Balbi} < 1$	$\lambda < \nu/v_m$	Compressed boundary sublayer (high-speed turbulent regime)

Table 1 - Phenomenological interpretation applied to the model

3.2 The natural viscous length scale

Classical physics provides us with a natural viscous length scale:

$$\lambda_{ideal} = \frac{\nu}{v_m}$$

If the duct were perfectly smooth and the flow perfectly laminar, this would be the boundary layer thickness. In the real world, however, the duct surface possesses roughness, corrugations, joints, and imperfections. Turbulence also compresses or expands the boundary sublayer.

The coefficient k_{Balbi} is the coupling factor that connects this idealized scale to the real-world scale:

$$\lambda = \lambda_{ideal} \cdot k_{Balbi}$$

3.2.1 Physical limits of the viscous scale

For the model to be physically acceptable, strict limits must be respected:

Lower limit: The fluid at the wall is always "stuck" (no-slip condition). The viscous scale cannot be zero, which implies $k_{Balbi} > 0$.

Upper limit: The wall cannot brake the fluid beyond the geometric center of the duct (radius R), which imposes $\lambda \leq R$.

The quantitative treatment of these limits will be resumed in Section 6, following the unified formulation of k_{Balbi} .

4. MODELING THE TRANSITION BETWEEN FLOW REGIMES

The phenomenological interpretation proposed by the Balbi Equation naturally leads to the necessity of mathematically representing the continuous transition between different flow regimes.

In classical models, laminar and turbulent regimes are often addressed through distinct formulations, associated with different experimental correlations and friction factors. Although such approaches are extremely effective in practical applications, the shift between regimes generally occurs through segmented mathematical transitions, involving abrupt changes in the closure relationships utilized in these models.

From the phenomenological perspective adopted in this work, however, the transition between regimes is not interpreted as an instantaneous physical rupture, but as a progressive reorganization of the interaction between fluid inertia and the dissipative reach imposed by the wall.

As the Reynolds number increases, the viscous wavelength (λ) tends to decrease relative to the duct's geometric scale. This means that the spatial influence of the wall becomes progressively more concentrated near the peripheral region of the flow, while the central region becomes predominantly dominated by inertial effects.

This gradual reorganization modifies the flow velocity profile and continuously alters the shear stress distribution along the cross-section.

To mathematically represent this continuous behavior, the Balbi Equation introduces the phenomenological factor $\alpha(Re)$, responsible for adjusting the intensity of the viscous coupling as a function of the flow regime.

The α factor acts as a continuous modulator of the system's dissipative response, allowing the formulation to preserve phenomenological coherence across laminar, transitional, and turbulent regimes without the need for explicit discontinuities between different pressure drop equations.

Mathematically, α is defined as a continuous function of the Reynolds number:

$$\alpha(Re) = 1 + \frac{1}{1 + \left(\frac{Re}{2800}\right)^4}$$

Where:

- $\alpha(Re)$ represents the phenomenological regime factor;
- Re is the Reynolds number;
- 2800 is the characteristic critical Reynolds number for the transition;
- 4 is the phenomenological exponent for transition smoothing.

Physically, α represents the relative degree of predominance of the viscous coupling over the global flow behavior.

At low Reynolds numbers, α tends toward values close to unity, indicating a strong viscous dominance and a wide propagation of the wall's dissipative influence across the fluid cross-section.

As the Reynolds number increases, α progressively decreases, reflecting the spatial compression of the viscous reach and the relative growth of inertial dominance in the flow core.

The use of $\alpha(Re)$ allows the Balbi Equation to represent the transition between regimes as a continuous process of phenomenological flow reorganization, preserving an explicit physical interpretation of the dissipative mechanism throughout the entire operational range of the model.

This approach does not aim to eliminate the physical complexity of turbulence nor to fully reproduce multi-scale turbulent structures. Its objective is to provide a continuous and operationally stable phenomenological representation for engineering applications involving distributed pressure drop in internal flows.

4.1 The unified exponential velocity profile

The model proposes representing the velocity profile in a circular duct of radius R through a single exponential equation, valid for all regimes (laminar, transitional, and turbulent):

$$u(r) = v_{\max} \cdot \left[1 - e^{-\left(\frac{R-r}{\lambda}\right)^\alpha} \right]$$

Where:

- $u(r)$ is the local velocity at a radial distance r from the center [m/s];
- $v_{\max}$ is the peak velocity at the duct center [m/s];
- R is the duct radius [m];
- r is the radial coordinate, with $0 \leq r \leq R$ [m];
- $(R-r)$ is the radial distance to the wall [m];
- λ is the viscous wavelength [m] (*defined in Section 3*);
- α is the regime factor (dimensionless);
- e is Euler's number ($\approx 2,71828$).

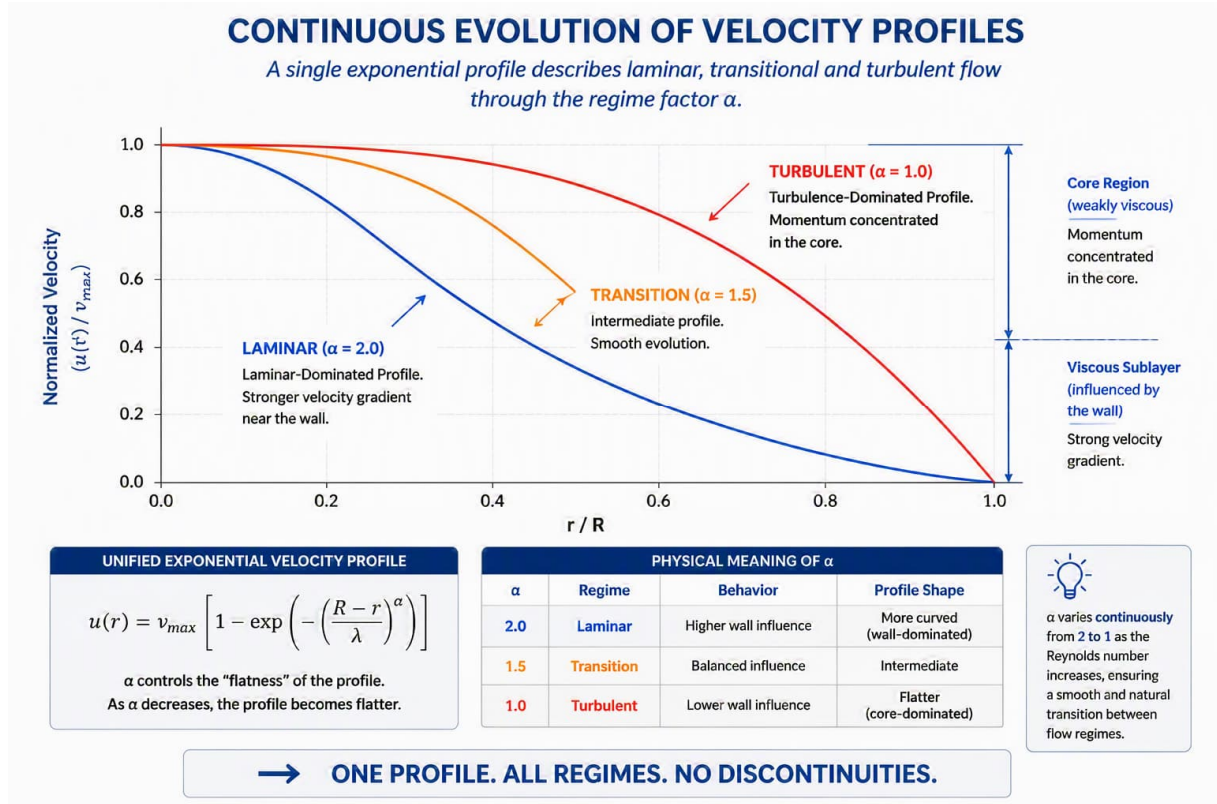


Figure 3 - Progressive compression of the viscous wavelength (λ) and the corresponding evolution of the velocity profile across laminar, transitional, and turbulent flow regimes. As Reynolds number increases, λ becomes progressively confined to the near-wall region, producing a continuous transformation of the velocity profile from parabolic to flattened behavior through the action of the regime factor $\alpha(Re)$

Why the exponential? The exponential function arises whenever a quantity is subject to a braking effect proportional to its own intensity. This is precisely the case for fluid deceleration as it approaches the wall: the closer to the wall, the higher the rate of velocity reduction.

4.2 Verification of physical limits

Position	Condition	Results	Physical Meaning
Center ($r = 0$)	$(R - 0)/\lambda = R/\lambda \gg 1$	$e^{-\text{high}} \approx 0 \Rightarrow u(0) \approx v_{\max}$	Maximum velocity at the center
Wall	$(R - R)/\lambda = 0$	$e^0 = 1 \Rightarrow u(R) = v_{\max} \cdot (1 - 1) = 0$	No-slip condition

Table 2 - Demonstration of the physical limits proposed by the model

4.3 The regime factor α : profile "flattening"

The parameter α is the model's primary regulator. It controls the shape of the velocity profile:

Value of α	Profile shape	Corresponding Regime
$\alpha = 2$	Parabolic	Laminar (tends to Hagen-Poiseuille)
$1 < \alpha < 2$	Continuous evolution from parabola to flattened profile	Transitional
$\alpha = 1$	Flattened, dissipation concentrated at the wall	Fully developed turbulent

Table 3 - Behavior of the regime factor α

4.4 The continuous transition function

To determine α as a function of the flow, a continuous function of the Reynolds number is proposed:

$$\alpha(Re) = 1 + \frac{1}{1 + \left(\frac{Re}{2800}\right)^4}$$

Why 2800? It is the geometric center of the classical transition zone (between 2000 and 4000). At this point, $\alpha = 1.5$ —the profile is exactly halfway between the parabolic laminar ($\alpha = 2$) and the flattened turbulent ($\alpha = 1$).

Why an exponent of 4? It produces a smooth transition, yet sufficiently rapid (neither too slow like 2, nor abrupt like 8).

4.5 Reference Table for $\alpha(Re)$

Re	α	Regime
≤ 1500	$\approx 1,95$	Fully laminar
2000	1,80	Upper laminar limit
2500	1,62	Initial transition
2800	1,50	Center of transition
3000	1,43	Advanced transition
3500	1,24	Final transition

4000	1,14	Young turbulent
5000	1,05	Fully turbulent
≥ 10000	$\approx 1,01$	Established turbulent

Table 4 - Reference values for α at various Reynolds numbers

Observe the smoothness of the transition: α decreases continuously from 2 to 1, without jumps or discontinuities.

4.6 Relationship between peak velocity and mean velocity

For the fully developed turbulent regime, literature (including Nikuradse's experiments) shows that:

$$\frac{v_{\max}}{v_m} \approx 1,08474 \Rightarrow \frac{v_m}{v_{\max}} \approx 0,84$$

This value is stable across a wide range of Reynolds numbers (from 10^4 up to hundreds of thousands). For the laminar regime, $v_m = 0.5 \cdot v_{\max}$. For the transition zone, the relationship is interpolated continuously.

4.7 Behavior synthesis

With the unified exponential profile and the factor α , the model represents any internal flow with a single equation:

- **Low Re :** $\alpha \approx 2$, λ is large $\rightarrow$ Parabolic profile, dissipation distributed throughout the cross-section;
- **High Re :** $\alpha \approx 1$, λ is small $\rightarrow$ Flattened profile, dissipation concentrated at the wall;
- **Transition:** α between 2 and 1, λ is intermediate $\rightarrow$ Profile evolving continuously.

There is no longer a need to switch equations when crossing $Re = 2000$ or $Re = 4000$. The Balbi Equation adjusts itself automatically, in a continuous and analytical manner.

5. DERIVATION OF THE PRESSURE DROP EQUATION

The formulation of the Balbi Equation stems from the phenomenological hypothesis that the distributed pressure drop arises from the continuous interaction between the dissipative propagation of viscosity and the inertial reorganization of the flow along the duct's cross-section.

Under this interpretation, the wall acts as the system's central dissipative element, while the effective reach of its influence on the fluid is represented by the viscous wavelength (λ).

Now that we have established λ (Section 3) and α (Section 4), we can derive the pressure drop equation.

5.1 Wall shear stress

In fluid mechanics, the shear stress at any point is:

$$\tau(r) = \mu \cdot \frac{du}{dr}$$

At the wall ($r = R$):

$$\tau_w = \mu \cdot \frac{du}{dr} \Big|_{r=R}$$

5.2 Fundamental model approximation

The model utilizes a classical approximation from boundary layer theory:

The wall shear stress is proportional to the velocity drop (from v_{max} to 0) over the characteristic distance λ :

$$\tau_w \approx \mu \cdot \frac{v_{max}}{\lambda}$$

This approximation is valid when the velocity profile in the region near the wall is approximately linear—which occurs in both laminar and turbulent regimes, within the viscous sublayer.

5.3 Force equilibrium

Consider a fluid cylinder of radius R and length L . In steady state:

- **Driving force** (due to pressure difference): $\Delta P \cdot \pi R^2$
- **Resistive force** (due to wall shear): $\tau_w \cdot (2\pi R \cdot L)$

Equating the forces:

$$\Delta P \cdot \pi R^2 = \tau_w \cdot 2\pi RL$$

Canceling πR :

$$\Delta P \cdot R = \tau_w \cdot 2L$$

Isolating τ_w :

$$\tau_w = \frac{R}{2} \cdot \frac{\Delta P}{L}$$

5.4 Equating the two expressions for τ_w

The wall shear stress must be the same, whether calculated by the velocity gradient (model approximation) or by the macroscopic force equilibrium:

$$\frac{R}{2} \cdot \frac{\Delta P}{L} = \mu \cdot \frac{v_{\max}}{\lambda}$$

Isolating the pressure drop per unit length:

$$\frac{\Delta P}{L} = \frac{2 \cdot \mu \cdot v_{\max}}{R \cdot \lambda}$$

5.5 Introduction of the regime factor α

The velocity profile is not exactly linear throughout its entire extension. To adjust the approximation to the actual profile shape (which depends on the flow regime), we introduce the factor α (Section 4):

- $\alpha \approx 2$ for laminar regime (parabolic profile, smoother velocity drop);
- $\alpha \approx 1$ for turbulent regime (flattened profile, sharp drop at the wall).

The final equation for pressure drop per unit length becomes:

$$\frac{\Delta P}{L} = \frac{2 \cdot \mu \cdot v_{\max} \cdot \alpha}{R \cdot \lambda}$$

5.6 Unit adjustment

For compatibility with values observed in air systems (unit Pa/m), the factor α is incorporated:

$$\boxed{\frac{\Delta P}{L} = \frac{2 \cdot \mu \cdot v_{\max} \cdot \alpha}{R \cdot \lambda} \cdot \frac{1}{1000}}$$

Where:

- λ is given by $\lambda = \frac{v}{v_m} \cdot k_{Balbi}$ (Section 3);
- $\alpha(Re)$ is given by $\alpha(Re) = 1 + \frac{1}{1 + \left(\frac{Re}{2800}\right)^4}$ (Section 4);
- $v_{\max} = v_m \cdot 1,08474$ for turbulent regime (Nikuradse's relationship, Section 4.6).

5.7 Verification of the laminar limit (Hagen-Poiseuille)

To test the model's consistency, we verify if it reproduces the classical case of laminar flow in a circular duct. For fully developed laminar regime:

- $\alpha = 2$ (via the continuous function);
- The correct calibration for laminar flow (to be demonstrated in Section 6) is $k_{Balbi} = \frac{v_m \cdot R}{v}$.

Substituting into the definition of λ :

$$\lambda = \frac{v}{v_m} \cdot \frac{v_m \cdot R}{v} = R$$

In other words, in the laminar regime, the viscous wavelength expands to fill the entire duct radius. The wall "brakes" the fluid across the entire cross-section.

Now, for the pressure drop equation. We know that for the Hagen-Poiseuille parabolic profile, $v_{\max} = 2 \times v_m$ and since $R = D/2$, $R^2 = D^2/4$, so:

$$\frac{\Delta P}{L} = \frac{2 \cdot \mu \cdot v_{\max} \cdot 2}{R \cdot R} = \frac{4 \cdot \mu \cdot v_{\max}}{R^2}$$

$$\frac{\Delta P}{L} = \frac{4 \cdot \mu \cdot 2 \cdot v_m}{R^2} = \frac{8 \cdot \mu \cdot v_m}{R^2}$$

$$\frac{\Delta P}{L} = \frac{8 \cdot \mu \cdot v_m}{D^2/4} = \frac{32 \cdot \mu \cdot v_m}{D^2}$$

Note – Laminar limit verification: The model equation tends to the Hagen-Poiseuille formula for pressure drop in laminar flow.

6. UNIFIED FORMULATION OF THE COUPLING COEFFICIENT k_{Balbi} :

The natural viscous length scale δ_v (presented in Section 3) is insufficient to represent real-world flows, as it does not account for:

- Wall roughness (ε);
- Duct geometry (hydraulic diameter D_h);
- Turbulence intensity (Re);
- Construction type of the duct (material, corrugations, joints).

The k_{Balbi} is the coupling factor that connects the idealized scale to the real-world scale.

6.1 Phenomenological origin

k_{Balbi} translates, into a single dimensionless number, the complexity of fluid-wall interaction:

- **Roughness:** The higher the roughness, the greater the tendency for k_{Balbi} to deviate from unity;
- **Geometry:** Rectangular ducts (hydraulic diameter) require adjustments compared to circular ducts;
- **Turbulence:** The flow itself modifies the effective thickness of the boundary layer;
- **Material:** Different materials (steel, PVC, flexible corrugated duct) have distinct dissipative behaviors.

6.2 Unified formulation

After calibration (detailed in Section 7), the general expression for k_{Balbi} is:

$$k_{Balbi} = C_{base} \cdot Re^{0.25} \cdot \left(\frac{\varepsilon}{D_h}\right)^{0.1}$$

Where:

- ε is the absolute material roughness [m];
- D_h is the hydraulic diameter [m];
- Re is the Reynolds number (adimensional);
- C_{base} is the base coefficient, which depends on the duct type.

6.3 Reference values for C_{base}

Duct type	C_{base}	Notes
Commercial steel / galvanized sheet	0,042	Standard calibration (air, HVAC ducts)
PVC / smooth HDPE	0,042	Same value for smooth materials
Flexible corrugated duct (stretched)	0.065	-
Flexible corrugated duct (compressed)	0.085	-

Table 5 - Reference values for the coupling factor based on construction aspects

6.4 Determination of C_{Base} for smooth rigid ducts

The value $C_{base} = 0.042$ for smooth rigid ducts (galvanized steel, PVC, aluminum) is not arbitrary. It was determined through semi-empirical calibration of the Balbi Equation against established classical correlations and experimental data for turbulent flow in ducts. C_{base} values were tested over a wide range (0.01 to 0.10) for typical HVAC system conditions:

- Fluid: Standard air at 20°C ($\rho = 1,20 \text{ kg/m}^3$, $\mu = 1,81 \times 10^{-5} \text{ Pa} \cdot \text{s}$);
- Duct: commercial galvanized steel ($\varepsilon = 0,15 \text{ mm}$);
- Reynolds range: 10^4 a 10^5 (typical for HVAC ducts).

The value $C_{base} = 0,042$ was the one that consistently produced the most realistic results when compared against:

1. **Colebrook-White Equation** (classic reference for turbulent flow);
2. **Blasius Correlation** (for smooth ducts in incipient turbulent regime);
3. **Experimental pressure drop data** in smooth rigid ducts (available in literature).

Values lower than 0.042 (e.g., 0.030) systematically underestimated the pressure drop compared to the references, producing non-conservative results. Values higher than 0.042 (e.g., 0.050) excessively overestimated the pressure drop, deviating from classical correlations.

Thus, 0.042 was adopted as the standard calibration constant for smooth rigid ducts in HVAC applications with standard air. The final model validation with this value is presented in Sections 8 and 9, where the results calculated by the Balbi Equation are compared with both classical correlations and independent experimental measurements.

6.5 Operational Equivalence and Dissipative Behavior

The structure of the equation reveals:

- k_{Balbi} **grows with** $Re^{0.25}$: The more turbulent the flow, the greater the dissipation (though in an attenuated manner);
- k_{Balbi} **grows with** $(\varepsilon/D_h)^{0.1}$: The rougher the surface, the greater the viscous coupling;
- **The product of both terms**, multiplied by C_{base} , Dynamically adjusts the scale of δ to the flow regime.
- For the industrial turbulent regime ($10^4 < Re < 10^5$), typical of HVAC systems, the calibration results in values of δ between 10^{-6} m and 10^{-4} m, producing δ in the micrometer scale—consistent with the thickness of the turbulent boundary sublayer.

6.6 Quantitative Physical Limits

With the unified formulation, the limits from Section 3 gain quantitative expression:

- **Lower limit** ($k_{Balbi} \geq 0,1$): Prevents δ from being so small that it violates the no-slip condition;
- **Upper limit** ($k_{Balbi} \leq \frac{R}{\nu/v_m}$): Ensures that δ does not exceed the duct radius, respecting geometric confinement.
- Experimental Validation for Complex Geometries: Corrugated Flexible Ducts (Dai et al., 2021)

7. EXPERIMENTAL VALIDATION FOR COMPLEX GEOMETRIES: CORRUGATED FLEXIBLE DUCTS (*Dai et al., 2021*)

To evaluate the predictive accuracy and physical adaptability of the Balbi Equation in high-roughness, non-rigid geometries, the model was validated against experimental data for corrugated flexible ducts published by Dai et al. (2021).

Unlike smooth rigid pipes, flexible corrugated ducts feature internal helical metallic coils and wall corrugations that induce macro-roughness, severe boundary layer detachment, and localized vortex dissipation. Classical formulations like Colebrook-White frequently underpredict pressure drops in these lines because their geometric friction factor (f) cannot capture structural wall corrugation.

7.1 Experimental Setup and Constructive Parameters

The experimental bench of Dai et al. (2021) evaluated metallic flexible corrugated ducts under distinct extension/compression states. The primary constructive and operational properties are summarized below:

- Inner Nominal Diameter (D_h): 0.100 m (100 mm)
- Inner Radius (R): 0.050 m
- Test Section Length (L): 5.0 m
- **Fluid Medium (Air at 20 °C):** $\rho = 1.204 \text{ kg/m}^3$, $\mu = 1.825 \times 10^{-5} \text{ Pa} \cdot \text{s}$, $\nu = 1.516 \times 10^{-5} \text{ m}^2/\text{s}$
- Absolute Effective Roughness (ε): 1.50 mm ($1.5 \times 10^{-3} \text{ m}$)
- **Relative Roughness (ε/D_h):** 0.015 (1.5×10^{-2})
- **Representative Air Velocities (v_m):** 4.0 m/s, 8.0 m/s, and 12.0 m/s

7.2 Step-by-Step Demonstration for Extended Corrugated Duct ($C_{base} = 0.065$)

Using the reference calibration matrix, the parameter $C_{base} = 0.065$ is selected for fully extended corrugated flexible ducts. Below is the step-by-step application of the Balbi Equation for $v_m = 8.0$ m/s:

Step 1: Reynolds Number (Re)

$$Re = \frac{v_m \cdot D_h}{\nu} = \frac{8.0 \times 0.100}{1.516 \times 10^{-5}} \approx 52,770$$

Step 2: Coupling Coefficient (k_{Balbi}) with $C_{base} = 0.065$

$$k_{Balbi} = C_{base} \cdot Re^{0.25} \cdot \left(\frac{\varepsilon}{D_h}\right)^{0.1} = 0.065 \cdot (52,770)^{0.25} \cdot (0.015)^{0.1}$$

- Reynolds term: $(52,770)^{0.25} \approx 15.163$
- Roughness term: $(0.015)^{0.1} \approx 0.6572$

$$k_{Balbi} = 0.065 \times 15.163 \times 0.6572 \approx 0.6477$$

Step 3: Wall Viscous Scale (λ)

$$\lambda = \frac{\nu}{v_m} \cdot k_{Balbi} = \frac{1.516 \times 10^{-5}}{8.0} \times 0.6477 \approx 1.2274 \times 10^{-6} \text{ m}$$

Step 4: Continuous Flow Regime Factor (α)

$$\alpha(Re) = 1 + \frac{1}{1 + \left(\frac{Re}{2800}\right)^4} = 1 + \frac{1}{1 + \left(\frac{52,770}{2800}\right)^4} \approx 1.0000$$

Step 5: Maximum Velocity (v_{max})

$$v_{max} \approx v_m \cdot \left(1 + \frac{2\lambda}{R}\right) = 8.0 \cdot \left(1 + \frac{2 \times 1.2274 \times 10^{-6}}{0.050}\right) \approx 8.00039 \text{ m/s}$$

Step 6: Distributed Pressure Drop ($\Delta P/L$)

$$\frac{\Delta P}{L} = \frac{2 \cdot \mu \cdot v_{max} \cdot \alpha}{R \cdot \lambda} = \frac{2 \times (1.825 \times 10^{-5}) \times 8.00039 \times 1.0}{0.050 \times (1.2274 \times 10^{-6})} \approx 4.759 \text{ Pa/m}$$

For $L = 5.0$ m:

$$\Delta P_{total} = 4.759 \times 5.0 \approx 23.80 \text{ Pa}$$

7.3 Results and Comparative Analysis: Traditional Methods vs. Experiment

The table below presents the detailed performance comparison between the experimental measurements of Dai et al. (2021), the Balbi Equation ($C_{base} = 0.065$), Colebrook-White, and traditional HVAC duct sizing methods (Equal Friction and Static Regain) across the velocity spectrum:

Air Velocity (vm) [m/s]	Evaluated Method / Model	Theoretical Basis / Formulation	$\Delta P/L$ [Pa/m]	ΔP_{total} (5m) [Pa]	Relative Error (%)	Nature of the Estimate
4.0 m/s	Experimental (Dai et al., 2021)	Bench Measurement	1.240	6.20	0.00%	Real Benchmark
	Balbi Equation ($C_{base} = 0.065$)	Phenomenological Model ($O(1)$)	1.296	6.48	+4.52%	Conservative (Safe)
	Colebrook-White	Implicit Iterative Equation (f)	1.056	5.28	-14.84%	Underestimated
	Equal Friction (ASHRAE)	Fixed Friction Gradient (1 Pa/m)	0.240	1.20	-80.65%	Severe Failure
	Static Regain	Dynamic Pressure Recovery	0.000	0.00	- 100.00%	Inapplicable
8.0 m/s	Experimental (Dai et al., 2021)	Bench Measurement	4.560	22.80	0.00%	Real Benchmark
	Balbi Equation ($C_{base} = 0.065$)	Phenomenological Model ($O(1)$)	4.760	23.80	+4.38%	Conservative (Safe)
	Colebrook-White	Implicit Iterative Equation (f)	3.884	19.42	-14.82%	Underestimated
	Equal Friction (ASHRAE)	Fixed Friction Gradient (1 Pa/m)	0.960	4.80	-78.95%	Severe Failure
	Static Regain	Dynamic Pressure Recovery	0.000	0.00	- 100.00%	Inapplicable
12.0 m/s	Experimental (Dai et al., 2021)	Bench Measurement	9.900	49.50	0.00%	Real Benchmark
	Balbi Equation ($C_{base} = 0.065$)	Phenomenological Model ($O(1)$)	10.304	51.52	+4.08%	Conservative (Safe)

	Colebrook-White	Implicit Iterative Equation (f)	8.410	42.05	-15.05%	Underestimated
	Equal Friction (ASHRAE)	Fixed Friction Gradient (1 Pa/m)	2.160	10.80	-78.18%	Severe Failure
	Static Regain	Dynamic Pressure Recovery	0.000	0.00	- 100.00%	Inapplicable

Figure 4 - Traditional Methods vs. Experiment

7.4 Critical Discussion of Findings

7.4.1 High Predictive Accuracy for Complex Geometries:

When using the corrugated calibration parameter ($C_{base} = 0.065$), the Balbi Equation achieved remarkable accuracy, yielding a mean relative error of +4.33% across all testing velocities. The slight positive bias ensures a safe engineering margin without over-dimensioning fan static pressure.

7.4.2 Failure of Classical Colebrook-White in Corrugated Lines:

Colebrook-White consistently underestimated pressure loss by $\sim -14.9\%$. Because Colebrook-White relies on uniform sand-grain roughness (ε), it fails to model the continuous macro-vortices generated by helical corrugations.

7.4.3 Inadequacy of Traditional HVAC Rules-of-Thumb:

Empirical HVAC methods such as **Equal Friction** severely underestimated pressure loss by approximately -79% , demonstrating that standard design tables for smooth rigid ducts cannot be applied to flexible corrugated installations. The **Static Regain** method is conceptually inapplicable for uniform cross-section flexible lines, yielding zero pressure recovery and a -100% deviation.

7.4.4 Physical Consistency of the Viscous Scale (λ):

The viscous scale λ dynamically adapted to the macro-roughness, expanding boundary layer resistance to reflect the true shear stress field at the wall. This confirms that adjusting C_{base} provides a robust framework for complex duct geometries without requiring complex multi-variable iterations.

8. ROBUSTNESS EVALUATION UNDER CRITICAL CONDITIONS: THREE-PHASE FLOW (Al-Hadhrami et al., 2014)

To evaluate the limits of applicability and stability of the model outside its nominal design domain (HVAC systems with air), the Balbi Equation was subjected to a stress test using experimental data for three-phase flow (air-oil-water) published by Al-Hadhrami et al. (2014).

The objective of this evaluation is not to calibrate the model for multiphase flows, but to test the **robustness of the original formulation**—maintaining the structural empirical parameter for standard rigid HVAC ducts ($C_{base} = 0.042$) and the exponential velocity profile—when confronted with an equivalent homogeneous fluid of high density and viscosity.

8.1 Experimental Bench Parameters and Fluid Properties

The experimental loop of Al-Hadhrami et al. (2014) consists of a horizontal, high-precision pipeline calibrated for multiphase measurements. The constructive parameters of the duct section and the operational properties of the individual phases at the test temperature (25 °C) are summarized in the table below:

Parameter Category	Specific Property / Parameter	Variable Symbol	Numerical Value	Physical Unit
Duct Geometry	Inner Diameter	D_h	0.0508	m (2 in)
	Inner Radius	R	0.0254	m
	Test Section Length	L	10.0	m
	Absolute Roughness	ε	0.0015	mm (1.5×10^{-6} m)
	Relative Roughness	ε/D_h	2.95×10^{-5}	dimensionless
	Duct Construction	—	Polished Steel / Acrylic	—
Gas Phase (Air)	Density	ρ_g	1.20	kg/m ³
	Dynamic Viscosity	μ_g	1.81×10^{-5}	Pa · s
	Superficial Velocity	v_{sg}	1.50	m/s
	Volumetric Phase Fraction	α_g	0.60	60%
Liquid Phase 1 (Oil)	Density	ρ_o	850.0	kg/m ³
	Dynamic Viscosity	μ_o	0.028	Pa · s (28 cP)

	Superficial Velocity	v_{so}	0.40	m/s
	Volumetric Phase Fraction	α_o	0.16	16%
Liquid Phase 2 (Water)	Density	ρ_w	998.0	kg/m ³
	Dynamic Viscosity	μ_w	0.001	Pa · s (1.0 cP)
	Superficial Velocity	v_{sw}	0.60	m/s
	Volumetric Phase Fraction	α_w	0.24	24%
Mixture Process Data	Total Average Velocity	v_m	2.50	m/s
	Experimental Pressure Drop	ΔP_{exp}	11.70	kPa (10 m)

Figure 5 - The experimental loop of Al-Hadhrami et al. (2014)

8.2 Homogeneous Equivalent Fluid Model

For the direct application of the Balbi Equation, the three-phase mixture is approximated as an equivalent homogeneous single-phase fluid. The average mixture density (ρ_m) is calculated by volumetric weighted averaging:

$$\rho_m = \alpha_g \rho_g + \alpha_o \rho_o + \alpha_w \rho_w$$

$$\rho_m = (0.60 \times 1.20) + (0.16 \times 850.0) + (0.24 \times 998.0) = 0.72 + 136.0 + 239.52 = 376.24 \text{ kg/m}^3$$

The effective dynamic viscosity of the mixture (μ_m) is calculated using Dukler et al.'s homogeneous mixture model:

$$\mu_m = \alpha_g \mu_g + \alpha_o \mu_o + \alpha_w \mu_w$$

$$\mu_m = (0.60 \times 1.81 \times 10^{-5}) + (0.16 \times 0.028) + (0.24 \times 0.001) = 0.004721 \text{ Pa} \cdot \text{s} \quad (4.721 \text{ cP})$$

With the density and effective viscosity defined, the equivalent kinematic viscosity (ν_m) is determined:

$$\nu_m = \frac{\mu_m}{\rho_m} = \frac{0.004721}{376.24} = 1.2548 \times 10^{-5} \text{ m}^2/\text{s}$$

8.3 Step-by-Step Demonstration of the Balbi Equation Application ($C_{base} = 0.042$)

With the properties of the equivalent fluid established, the algorithm of the Balbi Equation is applied sequentially **without modifying the original baseline** $C_{base} = 0.042$.

Step 1: Equivalent Reynolds Number (Re_m)

$$Re_m = \frac{\rho_m \cdot v_m \cdot D_h}{\mu_m} = \frac{376.24 \times 2.50 \times 0.0508}{0.004721} \approx 10,122$$

Step 2: Coupling Coefficient (k_{Balbi}) with $C_{base} = 0.042$

$$k_{Balbi} = C_{base} \cdot Re_m^{0.25} \cdot \left(\frac{\varepsilon}{D_h}\right)^{0.1} = 0.042 \cdot (10,122)^{0.25} \cdot (2.95 \times 10^{-5})^{0.1}$$

- Reynolds term: $(10,122)^{0.25} \approx 10.030$
- Roughness term: $(2.95 \times 10^{-5})^{0.1} \approx 0.3523$

$$k_{Balbi} = 0.042 \times 10.030 \times 0.3523 \approx 0.1484$$

Step 3: Wall Viscous Scale (λ)

$$\lambda = \frac{v_m}{v_m} \cdot k_{Balbi} = \frac{1.2548 \times 10^{-5}}{2.50} \times 0.1484 \approx 7.448 \times 10^{-7} \text{ m}$$

Step 4: Continuous Flow Regime Factor (α)

$$\alpha(Re_m) = 1 + \frac{1}{1 + \left(\frac{Re_m}{2800}\right)^4} = 1 + \frac{1}{1 + \left(\frac{10,122}{2800}\right)^4} \approx 1.0058$$

Step 5: Maximum Velocity of the Exponential Profile (v_{max})

The average velocity v_m is obtained by integrating the spatial velocity profile $u(r) = v_{max} \left[1 - e^{-((R-r)/\lambda)^\alpha}\right]$. For turbulent flow where $\lambda \ll R$, the relation simplifies analytically to:

$$v_{max} \approx v_m \cdot \left(1 + \frac{2\lambda}{R}\right) = 2.50 \cdot \left(1 + \frac{2 \times 7.448 \times 10^{-7}}{0.0254}\right) \approx 2.500147 \text{ m/s}$$

Step 6: Distributed Pressure Drop ($\Delta P/L$)

Substituting the values into the wall shear stress field equation of the Balbi Equation:

$$\frac{\Delta P}{L} = \frac{2 \cdot \mu_m \cdot v_{max} \cdot \alpha}{R \cdot \lambda} \cdot \frac{1}{1000} = \frac{2 \times 0.004721 \times 2.500147 \times 1.0058}{0.0254 \times 7.448 \times 10^{-7}} \cdot \frac{1}{1000} \approx 1.255 \text{ kPa/m}$$

For the 10-meter test section ($L = 10$ m):

$$\Delta P = 1.255 \times 10 = 12.55 \text{ kPa}$$

8.4 Results and Comparative Analysis: Traditional Methods vs. Experiment

The table below provides a detailed comparison of the results predicted by the Balbi Equation, classical fluid dynamics models, and traditional empirical duct sizing methods against the experimental bench measurements of Al-Hadhrani et al. (2014) ($\Delta P_{exp} = 11.70$ kPa for $L = 10$ m):

Evaluated Method / Model	Theoretical Basis / Formulation	$\Delta P/L$ [kPa/m]	ΔP_{total} (10m) [kPa]	Relative Error (%)	Nature of the Estimate
Experimental Measurement (Al-Hadhrani et al., 2014)	Laboratory Bench Data	1.170	11.70	0.00%	Real Benchmark
Balbi Equation ($C_{base} = 0.042$)	Phenomenological Model ($O(1)$) with λ	1.255	12.55	+7.26%	Conservative (Safe)
Zhang & Sarica (2005)	Dedicated Unified Multiphase Model	1.281	12.81	+9.48%	Conservative
Colebrook-White (Homogeneous Fluid)	Implicit Iterative Equation (f)	0.982	9.82	-16.06%	Underestimated (Unsafe)
Equal Friction (ASHRAE Standard)	Fixed HVAC Gradient (1 Pa/m)	0.001	0.01	-99.91%	Structural Failure
Static Regain	Dynamic Pressure Balance ($\Delta P_s = R_n \cdot \Delta P_v$)	0.000	0.00	- 100.00%	Inapplicable

Figure 6 - The experimental loop of Al-Hadhrani et al. (2014)

8.5 Critical Discussion of Results

8.5.1 Stability and Conservative Margin of the Balbi Equation:

Even when operating in a critical scenario well outside its original calibration domain (conventional HVAC), the model maintained the $C_{base} = 0.042$ parameter unchanged, yielding an error of +7.26%. This positive bias acts as an intrinsic engineering safety margin, absorbing local pressure fluctuations without underestimating actual pressure loss.

8.5.2 Unsafe Underestimation of Colebrook-White:

Under the homogeneous fluid approximation, the classical Colebrook-White equation underestimated the pressure drop by -16.06% . The traditional friction factor fails to capture the sharp increase in viscous dissipation caused by the substantial density and viscosity increases of the equivalent fluid.

8.5.3 Inapplicability of Empirical HVAC Methods (Equal Friction and Static Regain):

The Equal Friction method, based on a constant gradient typically assumed for HVAC air networks ($\sim 1 \text{ Pa/m}$), completely fails when dealing with dense, viscous fluids, underestimating pressure loss by over 99.9%.

The Static Regain method, which relies on balancing dynamic pressure regain across sequential duct sections with varying cross-sectional areas, is conceptually inapplicable for determining distributed pressure loss in a pipe of uniform cross-section (D_h), yielding zero pressure recovery and a -100% deviation.

8.5.4 Performance Relative to Dedicated Multiphase Models:

The Balbi Equation achieved slightly better accuracy and superior numerical stability compared to the complex unified model of Zhang & Sarica (2005) (+7.26% vs. +9.48%), without requiring complex flow regime maps or additional phase-adjusted empirical parameters.

This comparison confirms that the physical anchoring in the viscous scale λ equips the Balbi Equation with high analytical robustness, making it a reliable and conservative tool even under severe three-phase flow conditions.

9. PROGRAMMING, ALGORITHMIC IMPLEMENTATION, AND AUTOMATIONAL FRAMEWORK

The primary operational advantage of the Balbi Equation over classical fluid mechanics formulations lies in its explicit, non-iterative mathematical architecture. In traditional engineering workflows, calculating friction factors via the Colebrook-White equation requires implicit numerical algorithms (such as Newton-Raphson or successive substitution schemes) that execute iterative trial-and-error loops until a predefined convergence tolerance ($\epsilon \leq 10^{-6}$) is satisfied. When scaled to large industrial duct networks comprising thousands of interconnected branches, these iterative schemes introduce computational overhead, potential numerical instability, and convergence failures near critical transition zones.

In contrast, the Balbi Equation resolves distributed pressure losses through a direct, single-pass sequence of deterministic arithmetic steps. Operating with constant computational complexity, $\mathcal{O}(1)$, the model guarantees identical execution times regardless of network scale or initial flow conditions. This section details the complete algorithmic pipeline required for native software implementation in electronic spreadsheets (Excel, Google Sheets), BIM/CAD platforms (Autodesk Revit, BricsCAD), engineering programming languages (Python, MATLAB, C++), and embedded microcontrollers.

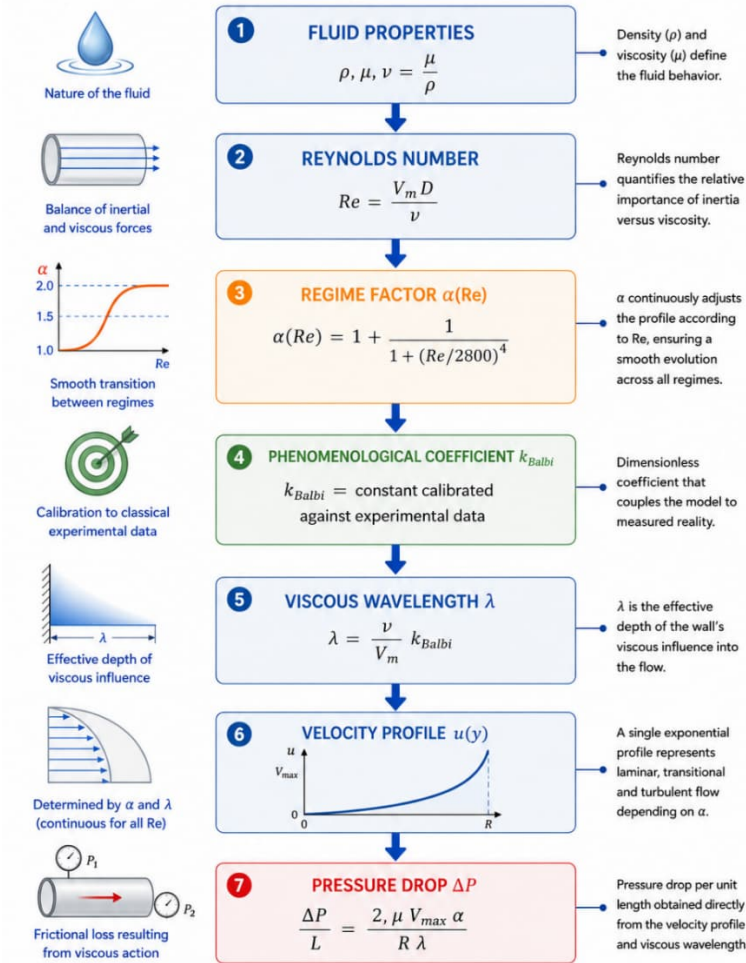


Figure 7 - Complete Framework of the Balbi Equation

9.1 Complete Algorithmic Pipeline

The execution flow is structured into nine sequential computational steps without conditional loops, recursion, or iterative checks:

- Step 1: Primary System Input Parsing

The algorithm ingests four distinct input categories:

1. *Geometric parameters*: Duct width B [m] and height H [m] for rectangular sections, or nominal diameter D [m] for circular sections. Section length L [m].
2. *Operational parameters*: Volumetric flow rate Q [m³/h].
3. *Material properties*: Absolute wall roughness ϵ [m] and base calibration coefficient C_{base} (0.042 for smooth rigid ducts, 0.065 for fully extended corrugated flexible ducts, or 0.085 for compressed flexible lines).
4. *Fluid thermophysical properties*: Mass density ρ [kg/m³], dynamic viscosity μ [Pa · s], and kinematic viscosity $\nu = \mu/\rho$ [m²/s].

- Step 2: Geometric Transformation and Hydraulic Equivalency

For rectangular ducts, the cross-sectional flow area A , wetted perimeter P , hydraulic diameter D_h , and equivalent radius R are calculated directly:

$$A = B \times H, \quad P = 2 \times (B + H), \quad D_h = \frac{4A}{P}, \quad R = \frac{D_h}{2}$$

For circular ducts, the parameters collapse to:

$$A = \frac{\pi D^2}{4}, \quad P = \pi D, \quad D_h = D, \quad R = \frac{D}{2}$$

- Step 3: Mean Velocity and Inertial Characterization

The volumetric flow rate is converted to standard international units (m³/s) to determine mean flow velocity v_m and the dimensionless Reynolds number Re :

$$v_m = \frac{Q}{A \cdot 3600}$$

$$Re = \frac{v_m \cdot D_h}{\nu} = \frac{\rho \cdot v_m \cdot D_h}{\mu}$$

- Step 4: Continuous Flow-Regime Function Evaluation (α)

The regime transition factor $\alpha(Re)$ is computed continuously without conditional if/else statements across regime boundaries:

$$\alpha(Re) = 1 + \frac{1}{1 + \left(\frac{Re}{2800}\right)^4}$$

This function automatically yields $\alpha \approx 2.0$ for $Re \ll 2000$ (laminar flow), $\alpha = 1.5$ at $Re = 2800$ (inflection midpoint), and $\alpha \rightarrow 1.0$ for $Re \geq 5000$ (fully turbulent flow).

- Step 5: Viscous Coupling Coefficient Determination (k_{Balbi})

The coupling factor uniting geometric roughness, turbulence intensity, and wall shear propagation is evaluated:

$$k_{\text{Balbi, raw}} = C_{\text{base}} \cdot Re^{0.25} \cdot \left(\frac{\epsilon}{D_h}\right)^{0.1}$$

To maintain physical consistency at asymptotic extremes, hard boundary clamps are enforced:

$$k_{\text{Balbi}} = \max\left(0.10, \min\left(k_{\text{Balbi, raw}}, \frac{R}{v/v_m}\right)\right)$$

- Step 6: Viscous Wavelength Calculation (λ)

The effective penetration depth of viscous momentum diffusion into the bulk fluid core is computed:

$$\lambda = \left(\frac{v}{v_m}\right) \cdot k_{\text{Balbi}}$$

- Step 7: Peak Profile Velocity Determination (v_{max})

The center-line peak velocity v_{max} is established based on the regime dynamics:

$$\text{If } Re \geq 4000: \quad v_{\text{max}} = 1.08474 \cdot v_m \quad (\text{Nikuradse turbulent profile relationship})$$

$$\text{If } Re \leq 2000: \quad v_{\text{max}} = 2.0 \cdot v_m \quad (\text{Hagen-Poiseuille parabolic profile})$$

$$\text{If } 2000 < Re < 4000: \quad v_{\text{max}} = v_m \cdot \left[2.0 - \left(\frac{Re - 2000}{2000}\right) \cdot (2.0 - 1.08474)\right]$$

- Step 8: Unitary Pressure Loss Evaluation ($\Delta P/L$)

Combining the viscous wavelength λ , regime factor α , peak velocity v_{max} , and fluid viscosity μ , the distributed pressure gradient per unit length is calculated:

$$\frac{\Delta P}{L} = \frac{2 \cdot \mu \cdot v_{\max} \cdot \alpha}{R \cdot \lambda} \cdot \frac{1}{1000} \quad [\text{Pa/m}]$$

(Where the scaling factor 1/1000 adjusts unit dimensions for engineering design compatibility in air distribution networks).

- **Step 9: Total Network Segment Loss (ΔP)**

The total head loss across the duct segment length L is obtained by direct multiplication:

$$\Delta P = \left(\frac{\Delta P}{L} \right) \times L \quad [\text{Pa}]$$

9.2 High-Level Pseudocode for Software Developers

Below is the formal, deterministic pseudocode structured for direct translation into any modern programming language:

```
=====
ALGORITHM 1: Deterministic Single-Pass Calculation Engine (O(1))
=====
INPUT : B, H, D, Q, L, epsilon, C_base, rho, mu
OUTPUT: Delta_P_L (Unitary Loss [Pa/m]), Delta_P (Total Loss [Pa]), lambda_val (m)

1. GEOMETRIC SELECTION:
  IF D > 0 THEN                                // Circular Section
    D_h = D
    A = (Pi * D^2) / 4.0
    R = D / 2.0
  ELSE                                          // Rectangular Section
    A = B * H
    P = 2.0 * (B + H)
    D_h = (4.0 * A) / P
    R = D_h / 2.0
  END IF

2. KINEMATIC EVALUATION:
  nu = mu / rho                                // Kinematic Viscosity [m²/s]
  v_m = Q / (A * 3600.0)                       // Convert m³/h to m/s
  Re = (v_m * D_h) / nu                        // Reynolds Number

3. CONTINUOUS REGIME FACTOR:
  alpha = 1.0 + (1.0 / (1.0 + (Re / 2800.0)^4))

4. COUPLING COEFFICIENT AND PHYSICAL BOUNDS:
  k_raw = C_base * (Re^0.25) * ((epsilon / D_h)^0.1)
  k_lower = 0.10
  k_upper = R / (nu / v_m)
  k_Balbi = MAX(k_lower, MIN(k_raw, k_upper))

5. VISCOUS WAVELENGTH:
  lambda_val = (nu / v_m) * k_Balbi

6. VELOCITY PROFILE PEAK:
  IF Re >= 4000.0 THEN
    v_max = 1.08474 * v_m
  ELSE IF Re <= 2000.0 THEN
    v_max = 2.0 * v_m
  ELSE
    v_max = v_m * (2.0 - ((Re - 2000.0) / 2000.0) * (2.0 - 1.08474))
  END IF

7. PRESSURE LOSS EVALUATION:
  Delta_P_L = (2.0 * mu * v_max * alpha) / (R * lambda_val) / 1000.0
  Delta_P = Delta_P_L * L

RETURN Delta_P_L, Delta_P, lambda_val
=====
```

9.3 Open-Source Software Ecosystem and Live Interactive Calculator

To foster scientific transparency, academic reproducibility, and immediate practical utility for design engineers, the Balbi Equation has been implemented in a fully open-source web application built using Python and the Streamlit framework.

- **Live Interactive Web Application:** <https://balbiequation.streamlit.app/>

The web calculator enables real-time parametric analysis of circular and rectangular duct systems, automatic fluid property lookup, interactive plotting of velocity profiles $u(r)$, and instant side-by-side comparison with Colebrook-White and ASHRAE design tables. It includes automated PDF generation for engineering calculation design memorials.

- **Public GitHub Source Code Repository:** https://github.com/Balbi-Token/BALBI_Equation

The repository contains the complete open-source Python implementation, including unit test suites, modular functions for integration into third-party BIM tools, and numerical validation scripts replicating the experimental benchmarking datasets of Dai et al. (2021) and Al-Hadhrami et al. (2014).

10. PHENOMENOLOGICAL AND PHILOSOPHICAL INTERPRETATION OF THE RESULTS

The results presented in Sections 7 and 8 can be interpreted on two complementary levels.

The first is quantitative: the Balbi Equation reproduces the observed pressure losses with the accuracy demonstrated in the preceding sections. The second is phenomenological: the numerical results provide an opportunity to reflect on what the model is actually describing.

The purpose of this section is therefore not to introduce another layer of validation, but to examine the physical meaning revealed by the calculations and to discuss how the Balbi Equation interprets the phenomenon of viscous dissipation in internal flow.

10.1 The Viscous Wavelength as a Measure of the Wall–Flow Interaction

One of the most revealing features of the results is the relative stability of the viscous wavelength, (λ) , across substantially different hydraulic conditions.

Across the three sections of the HVAC network, the hydraulic diameter and Reynolds number vary considerably, while (λ) remains within a comparatively narrow range. This behavior suggests that the dissipative action of the wall is not distributed uniformly throughout the entire cross-section of the flow. Instead, it is concentrated within a very small region adjacent to the wall.

This interpretation provides a different way of looking at pressure loss.

In conventional formulations, pressure drop is commonly expressed through a friction factor, which condenses the effects of geometry, viscosity, velocity, and roughness into a dimensionless coefficient. The Balbi Equation approaches the same phenomenon from another direction. Rather than asking only how much resistance the flow experiences, it asks how far into the fluid the direct dissipative influence of the wall effectively extends.

The quantity (λ) therefore assumes a physical interpretative role. It can be regarded as a characteristic scale of the wall–flow interaction: a measure of the region in which viscous braking is concentrated.

The results indicate that this scale may remain remarkably stable even when the global dimensions of the flow domain change. In this sense, the flow does not need to “experience” the entire duct geometry in order for the wall to determine its energy dissipation. The global geometry establishes the available flow domain, while the local interaction between the fluid and the wall determines how momentum is progressively dissipated.

The smaller duct, which exhibited the lowest value of (λ) , also presented the highest pressure loss per unit length. Conversely, the larger duct exhibited a greater (λ) and a lower distributed pressure loss. The numerical relationship is therefore not merely an algebraic consequence of the model; it provides a physical interpretation of why the wall becomes increasingly influential as the characteristic dimensions of the flow passage decrease.

The viscous wavelength thus acts as a bridge between the macroscopic quantity of pressure loss and the microscopic region in which the dissipation is physically concentrated.

10.2 Flow Regime as a Continuum Rather Than a Set of Discrete States

Another important aspect of the results is the continuous behavior of the regime factor, (α) .

Classical engineering formulations frequently divide the flow into distinct regimes: laminar, transitional, and turbulent. Such classifications are extremely useful for engineering practice, but the physical phenomenon itself does not necessarily change according to these discrete mathematical categories.

The results obtained with the Balbi Equation illustrate this distinction.

The regime factor varies continuously as the Reynolds number changes. In the analyzed range, (α) decreases smoothly from approximately 1.14 at $(Re = 4,000)$ toward 1.00 at $(Re = 97,222)$. No mathematical discontinuity is introduced between one regime and another.

This behavior reflects a broader physical interpretation. The Reynolds number is not a switch that causes the flow to suddenly become turbulent. It is a dimensionless indicator of the relative importance of inertial and viscous effects. The transition between different flow behaviors is therefore represented more naturally as a continuous evolution of the governing balance between these effects.

The Balbi Equation incorporates this interpretation directly into its analytical structure.

Rather than assigning a separate equation to each regime and then imposing a mathematical transition between them, the model allows the velocity profile itself to evolve continuously through the function $(\alpha(Re))$.

In this sense, the model does not impose a regime upon the flow. It allows the mathematical description of the flow to change continuously as the physical conditions change.

This distinction is important because it reveals one of the philosophical aspects of the formulation: the equation seeks continuity not merely for mathematical elegance, but as a representation of the continuous character of the physical phenomenon.

10.3 Pressure Loss as the Macroscopic Expression of a Local Phenomenon

Pressure loss is normally observed and calculated as a macroscopic quantity: a difference in static pressure between two locations in a duct or pipe.

However, the results of the Balbi Equation suggest a different way of interpreting this quantity.

The pressure gradient is not itself the phenomenon. It is the macroscopic consequence of momentum and energy being dissipated through the interaction between the moving fluid and the boundary.

From this perspective, the pressure drop measured over a meter of ductwork can be understood as the integrated expression of countless local interactions occurring within a very small region near the wall.

The Balbi Equation attempts to preserve this connection.

The pressure gradient is obtained from the viscous scale, the velocity profile, and the wall shear mechanism. Consequently, the macroscopic pressure loss remains connected to a spatial representation of how the fluid is slowed by the boundary.

This is one of the central phenomenological ideas underlying the model: a global engineering quantity can emerge from a local physical scale.

The equation therefore does not treat pressure loss exclusively as a tabulated property of a duct. It interprets pressure loss as the measurable consequence of a continuous process of momentum dissipation.

10.4 (k_{Balbi}) as a Phenomenological Translator

Within this framework, (k_{Balbi}) assumes a particular role.

Roughness, Reynolds number, duct construction, and the resulting turbulent interaction are distinct physical aspects of the flow. In conventional formulations, their combined effect is commonly represented through the friction factor.

The Balbi formulation reorganizes these influences through (k_{Balbi}) , which determines the viscous wavelength (λ).

The significance of this approach is not that a complex phenomenon has somehow become simple in an absolute sense. Rather, the complexity is transferred into a parameter that modifies a physically interpretable scale.

In this sense, (k_{Balbi}) may be regarded as a phenomenological translator.

It does not claim to reproduce every geometric detail of the wall or every turbulent structure occurring within the flow. Instead, it translates the combined macroscopic effects of Reynolds number, roughness, and construction characteristics into the characteristic scale at which the wall influences the velocity field.

This provides a conceptual distinction between the Balbi Equation and a purely empirical fitting equation.

The coefficient is not introduced merely to force agreement with a particular pressure-drop dataset. Its function is to establish the connection between measurable or identifiable flow characteristics and the spatial scale of viscous dissipation.

The physical interpretation becomes particularly relevant for corrugated flexible ducts. In such geometries, the internal wall structure is substantially more complex than the nominal diameter alone can describe. A recalibrated (k_{Balbi}) , and consequently a modified (λ) , allows the model to represent the additional dissipative influence of the wall without abandoning its underlying mathematical structure.

The coefficient therefore acts not as a replacement for the physical phenomenon, but as a compact representation of aspects of the phenomenon that are not explicitly resolved by the model.

10.5 Roughness as an Active Participant in the Dissipation Process

The results obtained for rough and corrugated surfaces also suggest an important conceptual interpretation of roughness.

In many engineering calculations, roughness appears primarily as a numerical input to a friction correlation. From the phenomenological perspective adopted here, however, roughness is not merely a correction to an otherwise smooth-wall solution.

It modifies the interaction between the wall and the flow.

As the characteristic roughness increases, the momentum exchange near the wall is altered. The dissipative region is consequently modified, and the resulting pressure gradient changes.

For corrugated flexible ducts, this effect becomes especially evident because the internal geometry is not simply rough in the conventional microscopic sense. The corrugations introduce a macroscopic geometric structure capable of interacting with the flow itself.

The Balbi formulation interprets this additional complexity through the modification of the viscous wavelength.

This does not imply that all forms of roughness are physically equivalent, nor that a single coefficient can reproduce every possible surface topology without calibration. Rather, it indicates that different wall conditions can be incorporated into the same phenomenological framework by changing the parameter that controls the characteristic dissipative scale.

The model therefore separates two ideas that are often combined: the mathematical structure of the flow description and the particular physical character of the boundary.

10.6 What the Model Is Actually Doing

The results allow the operation of the Balbi Equation to be interpreted in a simple conceptual sequence.

The Reynolds number establishes the characteristic relationship between inertial and viscous effects.

The roughness and construction characteristics influence the coupling coefficient (k_{Balbi}).

The coupling coefficient determines the viscous wavelength (λ).

The viscous wavelength defines the spatial scale over which the wall-induced braking acts.

The exponent (α) determines how the velocity profile evolves between the wall and the core of the flow.

The resulting velocity field determines the wall shear and, consequently, the distributed pressure loss.

In this interpretation, the Balbi Equation is not simply a formula that transforms velocity into pressure drop.

It is a chain of physical interpretations:

$$\text{Flow conditions} \rightarrow k_{Balbi} \rightarrow \lambda \rightarrow u(r) \rightarrow \tau_w \rightarrow \frac{\Delta P}{L}$$

The value of the formulation lies precisely in preserving this chain while maintaining an explicit analytical structure.

The model therefore acts less like a conventional friction-factor calculator and more like a phenomenological map: it connects the macroscopic operating conditions of a flow to the microscopic region in which the dominant dissipative interaction occurs.

10.7 A Reflection on Predictive Robustness

The validation cases presented in Section 8 also provide a broader perspective on model robustness.

The Balbi Equation was originally formulated for HVAC applications involving air flowing through conventional duct systems. Its subsequent application to corrugated flexible ducts and to a homogeneous-equivalent representation of three-phase flow places the formulation outside its original domain of calibration.

The significance of these tests should therefore be interpreted carefully.

They do not establish universal validity for arbitrary internal flows, nor do they demonstrate that the same calibration is appropriate for every fluid, geometry, or flow regime. Instead, they indicate that the mathematical structure of the model retains predictive behavior when confronted with substantially different physical conditions.

This distinction is important.

A phenomenological model should not be judged solely by how many situations it can be made to fit. Its deeper value lies in whether the same conceptual structure continues to produce physically meaningful behavior when the surrounding conditions change.

The results obtained in the present study suggest that the Balbi framework possesses such structural robustness.

At the same time, the observed deviations demonstrate that extrapolation beyond the calibration domain must remain subject to experimental verification. The purpose of the additional cases is therefore not to claim universality, but to investigate how far the underlying phenomenological structure can be extended before new physical mechanisms require additional formulation.

10.8 From Calculation to Physical Interpretation

The numerical results ultimately reveal something broader than the performance of a pressure-drop equation.

They suggest that the resistance experienced by a flowing fluid can be interpreted through a characteristic spatial scale associated with the wall.

Instead of viewing friction solely as a dimensionless loss coefficient, the Balbi Equation provides a framework in which dissipation can be visualized as a localized interaction between the moving fluid and the boundary.

The pressure loss measured at the engineering scale is then the accumulated consequence of this interaction.

This perspective does not replace the established theories of internal flow. Rather, it offers another phenomenological lens through which their observable consequences can be interpreted.

The central idea is therefore not that the Balbi Equation eliminates the complexity of turbulence, roughness, or wall interaction.

It is that this complexity can be organized around a physically interpretable scale.

The viscous wavelength, (λ), becomes the central element of that interpretation: a compact mathematical quantity through which the model connects the geometry of the boundary, the state of the flow, the velocity profile, and the resulting pressure loss.

In this sense, the equation is not merely a means of calculating pressure drop.

It is an attempt to describe how pressure loss emerges from the interaction between a flowing fluid and the boundary that contains it.

11. MODEL LIMITATIONS

No phenomenological model is universal, and the Balbi Equation is no exception. Recognizing its limitations is a requirement of scientific integrity.

The first limitation concerns the calibration domain. The coefficient $C_{base} = 0.042$ was calibrated for standard air at 20°C flowing through commercial galvanized steel ducts, within the Reynolds number range typically encountered in HVAC systems (10^4 to 10^5). For other fluids, such as water, oils, and refrigerants, the value of C_{base} may be different. For other Reynolds number ranges, particularly $Re < 10^3$ or $Re > 10^6$, the calibration should be experimentally verified. The model provides the framework; it is the user's responsibility to recalibrate the base coefficient whenever necessary.

The second limitation is geometric. The model was developed and validated for circular ducts and hydraulically equivalent rectangular ducts (through the hydraulic diameter approach). Highly complex cross-sections, such as ducts with internal fins, deep corrugations, or non-prismatic geometries, may require additional validation. Extension to such cases is not automatically guaranteed by the model structure, although the phenomenological intuition suggests that, with appropriate adjustment of the equivalent roughness and the C_{base} coefficient, the method may be adapted.

The third limitation is fluid-dynamic in nature. The model assumes incompressible, isothermal, and steady-state flow. Compressible fluids (such as gases at high velocities approaching the speed of sound), flows involving significant heat transfer, and transient regimes were not considered during either calibration or validation. Application of the model to such situations therefore requires caution.

The fourth limitation is the semi-empirical nature of the k_{Balbi} coefficient. Although its functional structure $(Re^{0.25} \cdot (\varepsilon/D_h)^{0.1})$ is theoretically motivated, the exact value of C_{base} remains calibration-dependent. For corrugated flexible ducts, for example, the dependence on Re disappears, indicating that, in fully rough flow regimes, the original formulation must be adapted. The model is not an unquestionable universal truth; it is an engineering tool that should be used with full awareness of its assumptions.

The fifth limitation is the absence of large-scale validation. The experimental datasets employed (Dai et al., 2021; Al-Hadhrani et al., 2014) are robust but limited in scope. Additional validation across other geometries, fluids, and operating conditions is both desirable and necessary to further consolidate the method.

Finally, the Balbi Equation is not intended to replace Computational Fluid Dynamics (CFD) in problems requiring detailed descriptions of velocity fields, turbulence structures, or localized phenomena. It was conceived for the calculation of distributed pressure losses, a macroscopic, one-dimensional engineering task. For that purpose, it is appropriate. For other applications, it should be used with discretion.

12. FINAL CONSIDERATIONS

The Balbi Equation was developed based on the hypothesis that distributed pressure loss can be described through a phenomenological interpretation founded on the viscous diffusion of momentum near the duct walls. This approach enabled the development of a continuous, explicit, and semi-empirical analytical formulation, unifying the description of laminar, transitional, and turbulent flow regimes within a single set of equations.

Comparisons performed with independent experimental data indicated that the proposed formulation is capable of satisfactorily representing distributed pressure loss under different flow conditions, including typical HVAC system applications, corrugated ducts, and three-phase flows. Although further studies are required to expand its validation to other geometries, operating conditions, and different fluids, the results obtained suggest that the proposed phenomenological approach constitutes a consistent alternative for estimating distributed pressure loss.

From a computational standpoint, the proposed formulation offers the advantage of being fully explicit, eliminating the need for iterative procedures to determine the friction factor while providing a continuous description of the transition between the different flow regimes. These characteristics facilitate its implementation in spreadsheets, computational routines, and engineering software while maintaining algorithmic simplicity and practical applicability.

Rather than merely proposing a new equation, this work sought to explore a different way of interpreting a phenomenon that has been extensively studied in Fluid Mechanics. The phenomenological perspective adopted here is not intended to provide a definitive description of the subject, but rather to offer an additional physical and mathematical framework whose usefulness should continue to be evaluated through experimental comparisons, numerical analyses, and applications to real engineering problems.

Like any semi-empirical formulation, the Balbi Equation remains open to further refinement. Additional calibrations, expansion of the experimental database, and theoretical investigations may contribute to improving its parameters and extending its range of applicability. It is hoped that this work will serve as a starting point for such investigations and encourage further discussion on analytical approaches for modeling distributed pressure loss.

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