

S1 Velocity Vector Calculation

We averaged the 20 velocity samples before the instillation start time:

Define the subset of the world velocity data:

$$\mathbf{V}^{world} = \begin{bmatrix} v_{x,1} & v_{y,1} & v_{z,1} \\ v_{x,2} & v_{y,2} & v_{z,2} \\ \vdots & \vdots & \vdots \\ v_{x,19} & v_{y,19} & v_{z,19} \end{bmatrix}$$

The mean velocity vector, \mathbf{v} , is calculated by averaging each column:

$$\mathbf{v}_{mean}^{world} = \frac{1}{20} \sum_{i=1}^{20} \mathbf{V}_i^w = \mathbf{v}$$

Where \mathbf{V}_i denotes the i th row vector in \mathbf{V} .

Finally, normalize \mathbf{v} to obtain a unit vector:

$$\mathbf{v}_{unit} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Define the reference unit vector:

$$\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{proj}_{\mathbf{u}} \mathbf{v} = \left(\frac{\mathbf{v}^T \mathbf{u}}{\|\mathbf{v}\|} \right) \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

The \mathbf{X} vector is then:

$$\mathbf{X} = \frac{\mathbf{u} - \text{proj}_{\mathbf{u}} \mathbf{v}}{\|\mathbf{u} - \text{proj}_{\mathbf{u}} \mathbf{v}\|}$$

Calculate the \mathbf{Y} vector:

$$\mathbf{Y} = \frac{\mathbf{v} \times \mathbf{X}}{\|\mathbf{v} \times \mathbf{X}\|}$$

Construct the rotation matrix:

$$\mathbf{R}_{Velocity}^{World} = [\mathbf{X} \ \mathbf{Y} \ \mathbf{v}]$$

This results in a rotation matrix $\mathbf{R}_{Velocity}^{World}$ that reorients the data into the “velocity vector” frame, capturing the direction of bottle motion as the primary axis.