Generalized Risk Dominance with Strong Inequality Aversion Solves The Prisoners’ Dilemma

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Abstract

We show that incorporating strong inequality aversion and generalized risk dominance in prisoners’ dilemma leads to a unique outcome which Pareto dominates the equilibrium action profile of the original problem.

Keywords: 2×2 bi-matrix games, prisoners′ dilemma, inequality aversion, risk dominance

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1. Introduction: As pointed out in Lahiri (2026a), in prisoner’s dilemma, there is no coordination problem. Prisoner’s dilemma is an interactive decision making problem (game) between two participants, where a Pareto dominated outcome is stable and the outcome that dominates it is unstable. While the stability of the Pareto superior outcome can be achieved by incorporating inequality aversion in the pay-offs accruing to the participants (as defined in Fehr and Schmidt (1999)), it is not sufficiently effective in making this outcome “uniquely” stable. Incorporating inequality aversion in the form of a negative externality, requires that the payoffs corresponding to action profiles chosen by the participants/ decision makers/ players *are “instruments” (such as money) that have use-value or utility*, rather than utility itself.

As, pointed out in Rapoport (1965), the pay-offs that are of relevance in game theory, are the utilities derived from the rewards and not reward by itself.

Inequality aversion, as in Fehr and Schmidt (1999), is expressed in terms of two constant marginal “dis-utilities” (i.e., the constant is non-positive) the first constant applicable for the excess of one’s own reward over the reward received by the the other participant, and the second constant applicable for the deficit in one’s own reward compared to that received by the other participant. It is reasonable to assume,

that the “marginal dis-utility” of a deficit in one’s own pay off is at least as strong as the “marginal dis-utility” of an excess in one’s own pay-off. The utility of the payoff of a chosen action profile to a participant, is one’s own reward minus the absolute value of the dis-utility arising from inequality of the rewards received by the participants. The Pareto dominated outcome continues to remain stable. As is well known, a complete resolution of the problem is possible if the decision makers are bound to repeatedly interact with themselves with the same pay-off matrices. In Lahiri (2026a), it is shown that the Pareto optimal action profile is the unique “optimitically weakly dominant action profile” resulting from the coordination game that is obtained after incorporating inequality aversion to the payoff matrices of prisoners’ dilemma.
In this note we show that there is a way to ensure the choice of the Pareto optimal action profile as the unique outcome of prisoner’s dilemma, if in addition to a “strong” inequality aversion assumption that is specific to the context of prisoners’ dilemma, we invoke generalized risk dominance, as defined in Lahiri (2026b), which is a generalization of risk dominance available in Harsanyi and Selten (1988) and Harsanyi (1995). Note that for our present paper, we require to invoke a *stronger version* of inequality assumption than what is required in Lahiri (2026a).
Risk dominance, as in Harsanyi and Selten (1988) and Harsanyi (1995) is based on the assumption that given two different equilibrium action profiles, if the action of a player in one equilibrium profile is different from its action in the other, then the other player “conjectures” the probability with which the former will choose a “randomized strategy” (i.e., the probability with which the first action is chosen) is distributed uniformly over the unit interval. Under such an assumption, an action is said to be risk dominant for a player, if it is a best response for the player with probability “strictly greater than one-half”.
Risk dominance based on uniform probability distributions as conjectures about the “other’s choice” of a randomized strategy has its limitation and therefore needs to be generalized in order to be more meaningful and effective for coordination games in general. If, as in the coordination game arising from prisoners’ dilemma after inequality aversion has been incorporated, one equilibrium action profile is more rewarding for both participants than a second action profile, then each participant will assume that the other player’s probabilistic conjectures lead to the preferred action being a best reply against the preferred action of the former. This kind of thinking resembles the kind of reasoning that takes place in the “ultimatum game” wherein a

player (tacitly?) "conveys" to the other player a preference or a stance, and assumes that the latter's conjectures about the former are informed choices that are in the latter's best interests. This has been discussed in some detail in Lahiri (2026b). The kind of probability distribution function used for generalized risk dominance is referred to as "almost uniform probability distribution function" and there is a some discussion on such probability distribution functions in the first note of the document available at the following link: https://doi.org/10.6084/m9.figshare.32521791

In the context of prisoners' dilemma, application of generalized risk dominance to the coordination game that results after incorporating strong inequality aversion in the original problem, is sufficiently effective in arriving at the Pareto optimal equilibrium action profile as the unique solution of the coordination game.

2. The Framework of Analysis: Let the row player be denoted by 1, the column player by 2 and the pay-off matrix of player $h \in \{1,2\}$ be $G_h = \begin{bmatrix} x_h(1,1) & x_h(1,2) \\ x_h(2,1) & x_h(2,2) \end{bmatrix}$.

The action set of the each player is {1, 2} and a pair $(i, j) \in \{1, 2\} \times \{1, 2\}$,with 'i' denoting the action (row) chosen by the row player (1) and 'j' denoting the action (column) chosen by the column player (2), is referred to as an **action profile**.

The pair (G_1, G_2) is referred to as a **2×2 bi-matrix game**.

Some times it may be convenient to represent (G_1, G_2) as

$$\begin{bmatrix} (x_1(1,1), x_2(1,1)) & (x_1(1,2), x_2(1,2)) \\ (x_1(2,1), x_2(2,1)) & (x_1(2,2), x_2(2,2)) \end{bmatrix}.$$

A **randomized action (or strategy) for the row player** is a real number $p_1 \in [0, 1]$ such p_1 is the probability with which the row player chooses action 1 and 1- p_1 is the probability with which the row player chooses action 2.

Note 1: If $p_1 = 1$, then the strategy p_1 is "action 1" and if $p_1 = 0$, then the strategy "p_1 is action 2".▌

A **randomized action (or strategy) for the column player** is a real number $q_1 \in [0, 1]$ such q_1 is the probability with which the column player chooses action 1 and 1- q_1 is the probability with which the row player chooses action 2.

If $q_1 = 1$, then the strategy q_1 is action 1 and if $q_1 = 0$, then the strategy q_1 is action 2.

Important observation: Note that for $\alpha \in \{1, 2\}$, $\{1, 2\} \backslash \{\alpha\} = \{3- \alpha\}$.▌

An action profile (i^*, j^*) is said to be an **equilibrium action profile** if $x_1(i^*, j^*) \geq x_1(3-i^*, j^*)$ and $x_2(i^*, j^*) \geq x_2(i^*, 3 - j^*)$.

Stahl (1999) contains a detailed discussion on related matters.

An action profile (i, j) is said to be **Pareto dominated** by an action profile (i^+, j^+) if $x_h(i^+, j^+) \geq x_h(i, j)$ for $h \in \{1, 2\}$, *with strict inequality for at least one* $h \in \{1, 2\}$.

An action profile is said to be **Pareto optimal** if it is not Pareto dominated by any other action profile.

Consider the 2×2 bi-matrix game (G_1, G_2) =

$$\begin{bmatrix} (x_1(1,1), x_2(1,1)) & (x_1(1,2), x_2(1,2)) \\ (x_1(2,1), x_2(2,1)) & (x_1(2,2), x_2(2,2)) \end{bmatrix}.$$

For $h \in \{1, 2\}$ and $j \in \{1, 2\}$, let $G_h^{(j)}$ denote the j^{th} column of G_h and $G_{h(j)}$ denote the j^{th} row of G_h.

$\{q_1 \in [0, 1] | G_{1(j)}\begin{pmatrix} q_1 \\ 1-q_1 \end{pmatrix} > G_{1(3\text{-}j)}\begin{pmatrix} q_1 \\ 1-q_1 \end{pmatrix}\}$ is **the set of randomized strategies of the column player against which action j gives the row player a higher expected pay-off than what action 3-j does.**

Similarly, $\{p_1 \in [0, 1] | (p_1 \quad | \quad (1-p_1))G_2^{(j)} > (p_1 \quad | \quad (1-p_1))G_2^{(3-j)}\}$ is **the set of randomized strategies of the row player against which action j gives the column player a higher expected pay-off than what action 3-j does**.

Further, for $j \in \{1, 2\}$, $\{q_1 \in [0, 1] | G_{1(j)}\begin{pmatrix} q_1 \\ 1-q_1 \end{pmatrix} > G_{1(3\text{-}j)}\begin{pmatrix} q_1 \\ 1-q_1 \end{pmatrix}\}$ and $\{p_1 \in [0, 1] | (p_1 \quad | \quad (1-p_1))G_2^{(j)} > (p_1 \quad | \quad (1-p_1))G_2^{(3-j)}\}$ are sub-intervals of [0, 1].

This follows directly from the convexity of each set.

Action $j \in \{1, 2\}$ is said to be *risk dominant for the row player* if the length of the interval $\{q_1 \in [0, 1] | G_{1(j)}\begin{pmatrix} q_1 \\ 1-q_1 \end{pmatrix} > G_{1(3\text{-}j)}\begin{pmatrix} q_1 \\ 1-q_1 \end{pmatrix}\} > \frac{1}{2}$.

Action $j \in \{1, 2\}$ is said to be *risk dominant for the column player* if the length of the interval $\{p_1 \in [0, 1] | (p_1 \quad | \quad (1-p_1))G_2^{(j)} > (p_1 \quad | \quad (1-p_1))G_2^{(3-j)}\} > \frac{1}{2}$.

An action profile (i, j) is said to be a *risk-dominant action profile* if i is risk dominant for the row player and j is risk dominant for the column player.

The concepts of risk dominance introduced above are due to Harsanyi and Selten (1988).

The following concepts are defined in Lahiri (2026b).

For $h \in \{1, 2\}$, let F_h: [0, 1]→[0, 1] be such that for $\alpha \in [0, \frac{1}{2}]$, $F_h(\alpha) = f_h\alpha$ and for $\alpha \in (\frac{1}{2}, 1)$, $F_h(\alpha) = \frac{1}{2}f_h + (2 - f_h)(\alpha - \frac{1}{2})$.

Action i∈{1, 2} is said to be *risk dominant for the row player with respect to* F_1 if the probability of the event $\{q_1 \in [0, 1] | G_{1(j)}\binom{q_1}{1-q_1} > G_{1(3-j)}\binom{q_1}{1-q_1}\}$ determined by F_1 is strictly greater than $\frac{1}{2}$.

Action j∈{1, 2} is said to be *risk dominant for the column player with respect to* F_2 if the probability of the event $\{p_1 \in [0, 1] | (p_1 \quad | \quad (1-p_1))G_2^{(j)} > (p_1 \quad | \quad (1-p_1))G_2^{(3-j)}\}$ determined by F_2 is strictly greater than $\frac{1}{2}$.

An action profile (i, j) is said to be a *risk-dominant action profile with respect to* (F_1, F_2) if i is risk dominant for the row player with respect to F_1 and j is risk dominant for the column player with respect to F_2.

3. Prisoners' Dilemma: Let (G_1, G_2) be a 2×2 (symmetric) bi-matrix game with $G_1 = \begin{bmatrix} a & c \\ 0 & d \end{bmatrix}$ and $G_2 = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$ satisfying c > d > a > 0. Such a game is said to be a prisoner's dilemma. This game has a unique equilibrium action profile, namely (1, 1), though (2, 2) is an action profile that Pareto dominates (1, 1).

As shown in Lahiri (2026a), incorporating the inequality aversion approach suggested by Fehr and Schmidt (1999), in Prisoners' Dilemma, leads to two equilibrium action profiles, i.e., (1, 1) and (2, 2), which is a characteristic feature of coordination games. In the context of Prisoners' Dilemma, the approach suggested by Fehr and Schmidt (1999) would imply the following.

Suppose that for h∈{1, 2} there exists real constants α_h, β_h satisfying $1 > \beta_h \geq \alpha_h > 0$. For h∈{1, 2} and i, j∈{1, 2}, let $\hat{x}_h(i, j) = x_h(i, j) - \alpha_h \max\{x_h(i, j) - x_{3-h}(i, j), 0\} - \beta_h \max\{x_{3-h}(i, j) - x_h(i, j), 0\}$.

Let $\widehat{G}_h = \begin{bmatrix} \hat{x}_h(1,1) & \hat{x}_h(1,2) \\ \hat{x}_h(2,1) & \hat{x}_h(2,2) \end{bmatrix}$.

Thus, in a prisoner's dilemma, $\widehat{G}_1 = \begin{bmatrix} a & c(1-\alpha_1) \\ -\beta_1 c & d \end{bmatrix}$, $\widehat{G}_2 = \begin{bmatrix} a & -\beta_2 c \\ c(1-\alpha_2) & d \end{bmatrix}$.

While (1, 1) continues be an equilibrium for $(\widehat{G}_1, \widehat{G}_2)$, (2, 2) will also be an equilibrium for $(\widehat{G}_1, \widehat{G}_2)$ <u>if and only if</u> $\min\{\alpha_1, \alpha_2\} > \frac{c-d}{c} \in (0, 1)$.

4. Risk dominance in addition to inequality aversion: In $(\widehat{G}_1, \widehat{G}_2)$, action 2 is risk dominant for the row player <u>if and only if</u> the length of the interval $\{q_1 \in [0, 1] | q_1(a+\beta_1 c) < (1-q_1)(d - c(1-\alpha_1))\}$ is greater than $\frac{1}{2}$, i.e., $[0, \frac{1}{2}) \subset\subset$ (strict subset of) $[0, \frac{d - c(1-\alpha_1)}{(d - c(1-\alpha_1))+(a+\beta_1 c)})$.

Thus, in $(\widehat{G}_1, \widehat{G}_2)$, action 2 is risk dominant for the row player if and only if $\frac{d - c(1 - \alpha_1)}{(d - c(1 - \alpha_1)) + (a + \beta_1 c)} > \frac{1}{2}$, which in turn is equivalent to $d > c + a + (\beta_1 - \alpha_1)c$.

However, by hypothesis $c > d > a > 0$ and $\beta_1 \geq \alpha_1$. Thus, $c + a + (\beta_1 - \alpha_1)c > d$ thereby implying action 1 and not action 2 is risk dominant for the row player.

In $(\widehat{G}_1, \widehat{G}_2)$, action 2 is risk dominant for the column player if and only if the length of the interval $\{p_1 \epsilon [0, 1] | p_1(a + \beta_2 c) < (1-p_1)(d - c(1 - \alpha_2))\}$ is greater than $\frac{1}{2}$, i.e., $[0, \frac{1}{2}) \subset\subset [0, \frac{d - c(1 - \alpha_2)}{(d - c(1 - \alpha_2)) + (a + \beta_2 c)})$.

Thus, in $(\widehat{G}_1, \widehat{G}_2)$, action 2 is risk dominant for the column player if and only if $\frac{d - c + \alpha_2 c}{(d - c(1 - \alpha_2)) + (a + \beta_2 c)} > \frac{1}{2}$, which in turn is equivalent to $d > c + a + (\beta_2 - \alpha_2)c$.

However, by hypothesis $c > d > a > 0$ and $\beta_1 \geq \alpha_1$. Thus, $c + a + (\beta_2 - \alpha_2)c > d$ thereby implying action 1 and not action 2 is risk dominant for the column player.

Thus, risk dominance fails to predict the Pareto optimal action profile as the outcome of the game.

5. Generalized Risk dominance in addition to strong inequality aversion: What we know is that for $h \epsilon \{1, 2\}$, $1 > \beta_h \geq \alpha_h > 0$ are such that $\min\{\alpha_1, \alpha_2\} > \frac{c-d}{c} \epsilon (0, 1)$, so that $d - c + \alpha_h c > 0$ for $h \epsilon \{1, 2\}$. Further, $c > a > 0$ and for $h \epsilon \{1, 2\}$, $\beta_h > 0$ so that $a + \beta_h c > 0$.

Thus, $\frac{d - c(1 - \alpha_h)}{(d - c(1 - \alpha_h)) + (a + \beta_h c)} \epsilon (0, \frac{1}{2})$ for $h \epsilon \{1, 2\}$.

Under the assumption that $d > a$, it seems reasonable that in $(\widehat{G}_1, \widehat{G}_2)$ both players will be more inclined towards the payoff pair (d, d) than (a, a). Hence, if both players have been able to convey a strong preference for the payoff pair (d, d) it seems reasonable that each player $h \epsilon \{1, 2\}$ would conjecture a probability distribution function F_h that assigns greater weight to the event that action 2 is a best reply against a strategy (randomization) chosen by player 3-h. This would indeed be the case for a suitable probability distribution function that is *stochastically dominated* by the uniform probability distribution function.

In the context of prisoners' dilemma, the event $\{q_1 \epsilon [0, 1] | \widehat{G}_{1(2)} \begin{pmatrix} q_1 \\ 1 - q_1 \end{pmatrix} > \widehat{G}_{1(1)} \begin{pmatrix} q_1 \\ 1 - q_1 \end{pmatrix}\} = \frac{d - c(1 - \alpha_1)}{(d - c + \alpha_1 c) + (a + \beta_1 c)}$ and the event $\{p_1 \epsilon [0, 1] | (p_1 \quad | \quad (1 - p_1))\widehat{G}_2^{(2)} > (p_1 \quad | \quad (1 - p_1))\widehat{G}_2^{(1)}\} = [0, \frac{d - c(1 - \alpha_2)}{(d - c(1 - \alpha_2)) + (a + \beta_2 c)}]$.

Thus, in $(\widehat{G}_1, \widehat{G}_2)$, (2, 2) is a risk dominant action profile with respect to (F_1, F_2) if and only if for hϵ{1, 2}, the probability of the event that a strategy chosen by player h belongs to the interval $[0, \frac{d - c(1-\alpha_h)}{(d - c(1-\alpha_h))+(a+\beta_h c)}]$ is strictly greater than $\frac{1}{2}$.

In order that such a probability distribution function exists we require the following assumption on the parameters determining $(\widehat{G}_1, \widehat{G}_2)$.

Strong Inequality Aversion Assumption: For hϵ{1, 2}: $3[d - c(1-\alpha_h)] > a + \beta_h c$, i.e., $\frac{a+3(c-d)}{c} < 3\alpha_h - \beta_h$.▌

Strong Inequality Aversion Assumption implies that for hϵ{1, 2}, $4[d - c(1-\alpha_h)] > d - c(1-\alpha_h) + (a + \beta_h c)$, so that $2\times \frac{d - c(1-\alpha_h)}{(d - c(1-\alpha_h))+(a+\beta_h c)} > \frac{1}{2}$. This assumption- as stated- is applicable only in the context of prisoners' dilemma.

Since for hϵ{1, 2}, $\frac{d - c(1-\alpha_h)}{(d - c(1-\alpha_h))+(a+\beta_h c)} \epsilon (0, \frac{1}{2})$, the strong inequality aversion assumption implies that for hϵ{1, 2} there exists a real number $f_h \epsilon (1, 2)$, such that $f_h\frac{d - c(1-\alpha_h)}{(d - c(1-\alpha_h))+(a+\beta_h c)} \epsilon (\frac{1}{2}, 1)$.

For hϵ{1, 2}, let F_h: [0, 1]→[0, 1] be such that for $\alpha\epsilon[0, \frac{1}{2}]$, $F_h(\alpha) = f_h\alpha$ and for $\alpha \epsilon (\frac{1}{2}, 1)$, $F_h(\alpha) = \frac{1}{2}f_h + (2- f_h)(\alpha - \frac{1}{2})$.

Thus, $F_h(\frac{d - c(1-\alpha_h)}{(d - c(1-\alpha_h))+(a+\beta_h c)}) > \frac{1}{2}$ for hϵ{1, 2}.

Hence (2, 2) is a risk dominant action profile for $(\widehat{G}_1, \widehat{G}_2)$.

A numerical example may help in understanding the assumptions we have made so far as well as the conclusion.

Example 1: Let $G_1 = \begin{bmatrix} 5 & 10 \\ 0 & 8 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 5 & 0 \\ 10 & 8 \end{bmatrix}$.

Thus, $\widehat{G}_1 = \begin{bmatrix} 5 & 10(1-\alpha_1) \\ -10\beta_1 & 8 \end{bmatrix}$, $\widehat{G}_2 = \begin{bmatrix} 5 & -10\beta_2 \\ 10(1-\alpha_2) & 8 \end{bmatrix}$.

We require, that for hϵ{1, 2}, there exists $f_h\epsilon(1, 2)$ such that $f_h\frac{8 - 10(1-\alpha_h)}{(8 - 10(1-\alpha_h))+(5+10\beta_h)} > \frac{1}{2}$, i.e., $f_h > \frac{1}{2} \times \frac{8 - 10(1-\alpha_h) +(5+10\beta_h)}{(8 - 10(1-\alpha_h))}$

$\frac{a+3(c-d)}{c} = \frac{11}{10}$ where a = 5, c = 10 and d = 8.

We require $\frac{11}{10} < 2\alpha_h - (\beta_h - \alpha_h)$ along with $0 < \frac{3}{10} < \alpha_h \leq \beta_h < 1$.

For hϵ{1, 2}, let $\alpha_h = \frac{71}{100}$ and $\beta_h = \frac{72}{100}$. Thus, $2\alpha_h - (\beta_h - \alpha_h) = \frac{141}{100} > \frac{11}{10}$.

For h$\in$\{1, 2\}, $\frac{8-10(1-\alpha_h)+(5+10\beta_h)}{(8-10(1-\alpha_h))} = \frac{8-10(1-\frac{71}{100})+(5+10\frac{72}{100})}{(8-10(1-\frac{71}{100}))} = \frac{173}{51}$.

Note that $\frac{1}{2} \times \frac{173}{51} = \frac{173}{102} \in (1, 2)$.

For h$\in$\{1, 2\}, let $f_h \in (\frac{173}{102}, 2)$.

Then, for h$\in$\{1, 2\}, $f_h \frac{8-10(1-\alpha_h)}{(8-10(1-\alpha_h))+(5+10\beta_h)} = f_h \frac{51}{173} > \frac{1}{2}$.

For h$\in$\{1, 2\}, let F_h: [0, 1]$\rightarrow$[0, 1] be such that for $\alpha \in [0, \frac{1}{2}]$, $F_h(\alpha) = f_h\alpha$ and for $\alpha \in (\frac{1}{2}, 1)$, $F_h(\alpha) = \frac{1}{2}f_h + (2 - f_h)(\alpha - \frac{1}{2})$.

Then, for h$\in$\{1, 2\}, $F_h(\frac{51}{173}) > \frac{1}{2}$.

Thus, the action profile (2, 2) is a risk dominant action profile with respect to (F_1, F_2).

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Hence, in the context of prisoners' dilemma, incorporating both strong inequality aversion in addition to generalized risk dominance, leads to the choice of a unique action profile which is Pareto optimal and Pareto dominates the equilibrium action profile in the original problem.

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