

Bitstream Photon Counting Chirped AM Lidar with a Digital Logic Local Oscillator

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Abstract

This paper introduces a new concept for the local oscillator (LO) for the Photon Counting Chirped Amplitude Modulation Lidar (PC-CAML). Rather than using a radio-frequency (RF) analog LO applied electronically either in post-detection mixing or via opto-electronic mixing (OEM) at the detector, or applied via pre-detection mixing using an optical intensity modulator as in previous systems, the new method mixes the single-bit binary counts from the photon counting detector with a single-bit binary LO using an AND binary digital logic gate. This type of LO is called the Digital Logic Local Oscillator (DLLO), and the resulting PC-CAML system is a type of bitstream lidar called bitstream PC-CAML (patent pending).

The key advantage of the DLLO in the bitstream PC-CAML is that it replaces bulky, power-hungry, and expensive wideband RF analog electronics with single-bit digital logic components that can be implemented in inexpensive silicon complementary metal-oxide-semiconductor (CMOS) read-out integrated circuits (ROICs) to make the bitstream PC-CAML with a DLLO more suitable for compact lidar-on-a-chip systems and lidar array receivers than previous PC-CAML systems.

This paper introduces the DLLO for bitstream PC-CAML concept, presents the initial signal-to-noise ratio (SNR) theory with comparisons to Monte Carlo simulation results, and makes suggestions for future work on this concept.

1. Bitstream PC-CAML with a DLLO Concept

The patented photon counting chirped amplitude modulation lidar (PC-CAML) concept has been extensively discussed previously. [ref 1-13] In previous PC-CAML systems, the local oscillator (LO) has been a radio-frequency (RF) analog signal applied using expensive, bulky, and power hungry RF analog electronics either in post-detection mixing or via opto-electronic mixing (OEM) at the photon counting detector, including gating of the detector, or applied via pre-detection mixing using an optical intensity modulator as illustrated in figure 1. [ref 1-13] The reader is referred to references 1-13 for more details on the theory and practice for the traditional PC-CAML system.¹

Alternatively, one can dispense with an LO in the receiver altogether by sending the wideband, high speed single-bit photon count data output from the photon counting detector directly to storage and demodulating the chirp digitally. [ref 14] However, this alternative eliminates a key

¹ Note that in reference 8 from 2013, Zhang, et. al., claim that they were the first to apply the LO signal directly to the gate of the Gm-APD detector in a method that they called smart premixing. However, in reference 3 from 2006, Redman, Ruff, and Giza published results from applying the LO directly to the gate of the Gm-APD detector in a method they called opto-electronic mixing (OEM) to distinguish it from post-detection mixing using an RF LO after the detector, and from pre-detection mixing using an optical modulator prior to the detector. Redman, et. al., used a chirped square wave LO applied to the gate of the Gm-APD, whereas Zhang, et. al., used a pulse width modulated sampling of a chirped sinusoid for their LO applied to the gate of the Gm-APD, which has some advantages over the square wave LO as discussed in references 8 and 11.

advantage of the PC-CAML concept, which is the down conversion of the wide bandwidth ranging signal to the low bandwidth intermediate frequency (IF) regime where low sampling rate analog-to-digital converters (ADCs) can be used to digitize the lidar data which is then sent to storage and digital processing at a low data rate. This down sampling to a lower data rate gains in importance as the number of detectors used increases, such as for larger format array receivers, since it is far easier and less costly to move data from the ROIC to data storage and processing for multiple parallel data streams at low data rates than at high data rates.

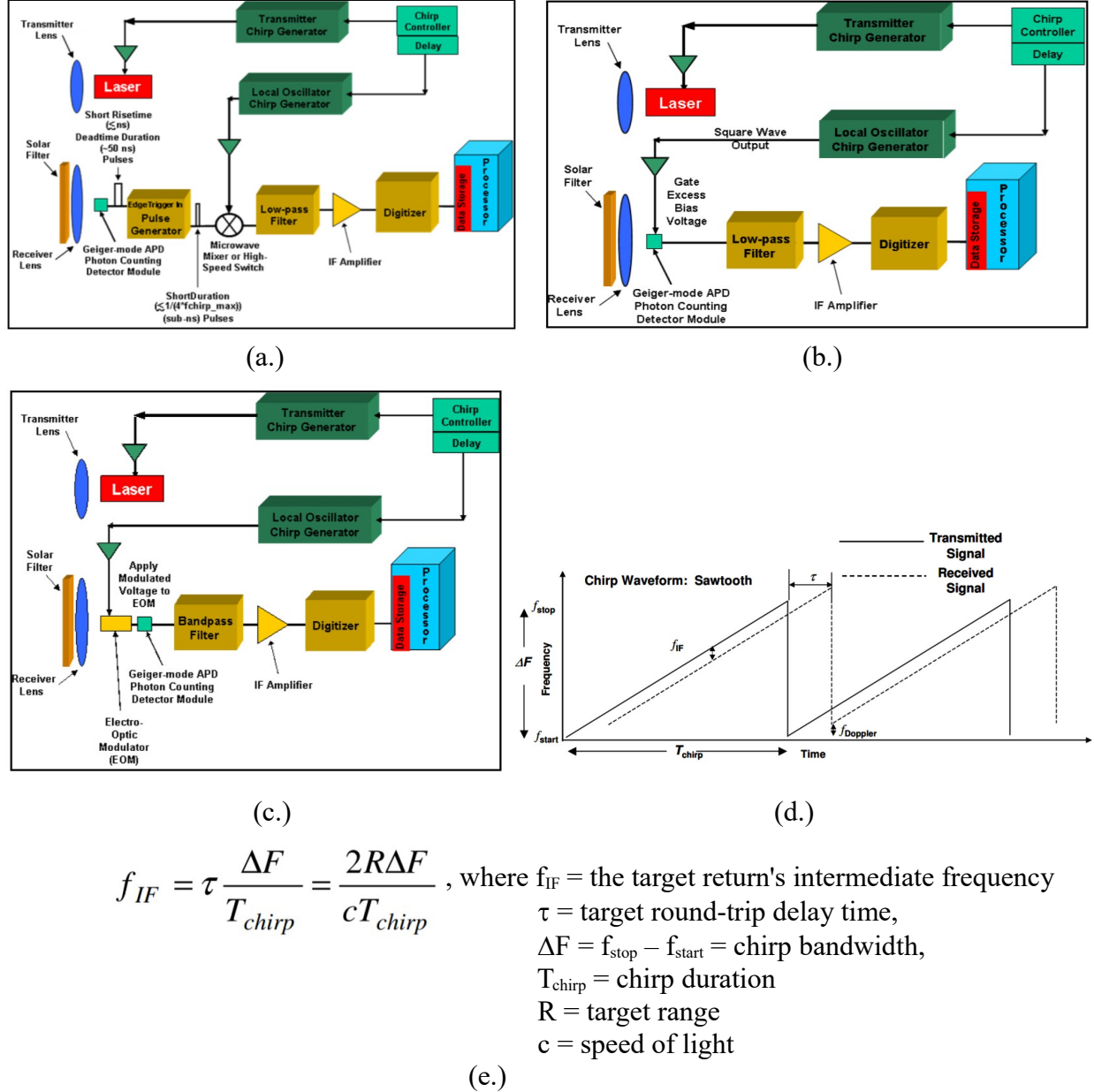


Figure 1. Traditional Photon Counting Chirped Amplitude Modulation Lidar (PC-CAML) Configurations and Waveforms: (a.) Post-detection Mixing Configuration, (b.) Opto-Electronic Mixing (OEM) Configuration, (c.) Predetection Mixing Configuration, (d.) Sawtooth Chirp Waveform applied to the transmitted amplitude modulation, and (e.) Equation relating the stationary target (no Doppler frequency shift) intermediate frequency (IF), f_{IF} , to the target range and chirp waveform parameters. [ref 3 and 5]

The discussion above assumes the use of Geiger-mode Avalanche Photo-Diode (Gm-APD) (also known as Single-Photon Avalanche Diode (SPAD)) photon counting detectors that output a single-bit count for each detection event, but some Gm-APD receivers output multi-bit timestamps for the counts rather than the stream of single-bit counts themselves. For such receivers, at very low count rates, the data rates for the multi-bit timestamp data may be low enough that streaming the timestamp data directly to digital storage for full digital processing of the chirped AM signal may be advantageous even for large format array receivers.

Here I present a new alternative for applying the LO in the PC-CAML concept called the Digital Logic Local Oscillator (DLLO) (patent pending²). For the DLLO, the constant amplitude, single-bit output count of the photon counting detector receiving a chirped AM signal is input to an edge-triggered pulse detector which outputs a very short single-bit digital logic level pulse to one input of an AND binary digital logic gate, and the other input of the AND binary digital logic gate is connected to the single-bit digital logic level binary data stream of a chirped AM LO signal, as shown in figure 2 (a.). The wideband, high speed single-bit digital data output by the AND binary digital logic gate can be either directly sent to storage and fully digitally processed, or sent through an analog or digital low/band pass filter in the IF band, the output of which is either digitized by a low sample rate analog-to-digital converter (ADC), or digitally down sampled to a low sample rate, respectively, as shown in figure 2 (a.). The low sample rate digital data is then sent to storage and digitally processed by the usual methods for chirped AM lidar. In this concept, the AND binary digital logic gate provides the mixing of the single-bit photon count data from the Geiger-mode Avalanche PhotoDiode (Gm-APD) with the single-bit LO data.

An example circuit for the edge-triggered pulse detector is shown in figure 2 (b.). The edge-triggered pulse detector allows the count pulses input to the AND gate to be much shorter than the count pulses output by the Gm-APD, which can be longer than desired due to the Gm-APD dead-time. The edge-triggered pulse detector's output pulse can be as short as a single clock pulse as long as the rising edge of the Gm-APD's count pulse is shorter than a clock pulse, and the delay on the delayed input in the edge-triggered pulse detector circuit is shorter than a clock pulse. Other types of edge-triggered pulse circuits may be used, or the edge-triggered pulse circuit can be eliminated if the Gm-APD count pulses are short enough.

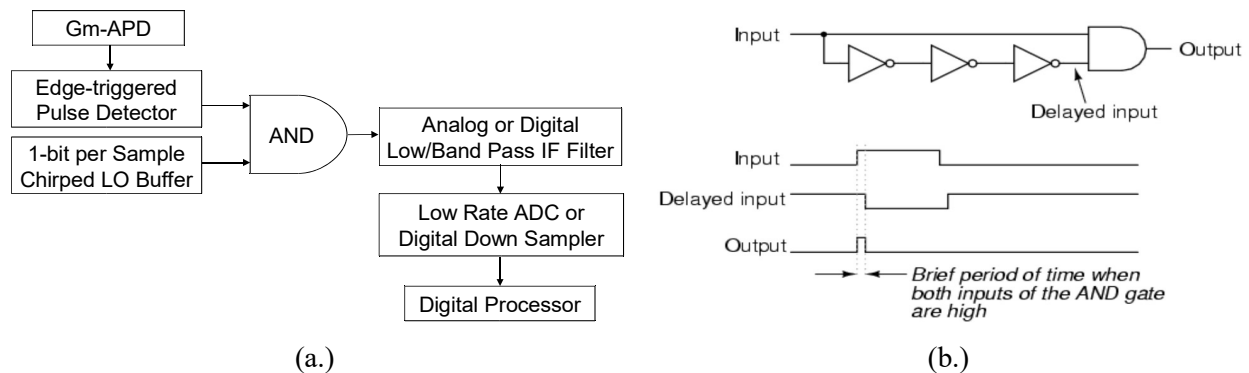


Figure 2. (a.) DLLO for Bitstream PC-CAML Concept Diagram and (b.) Example Edge-triggered Pulse Detector circuit from the online textbook [Lessons in Electric Circuits](https://www.allaboutcircuits.com/textbook/digital/chpt-10/edge-triggered-latches-flip-flops/), at <https://www.allaboutcircuits.com/textbook/digital/chpt-10/edge-triggered-latches-flip-flops/>, by Tony R. Kuphaldt and Dennis Crunkilton, License: <https://www.gnu.org/licenses/dsl.html>.

2 U.S. Provisional Patent Application Number 62931067, "Bitstream Photon Counting Chirped Amplitude Modulation Lidar with a Digital Logic Local Oscillator," Inventor: Brian Carroll Redman, November 5, 2019.

The concept of a DLLO has not been previously applied to lidar, but it was recently introduced for radar and called bitstream or single-bit radar. [ref 15-18] Therefore, I refer to the PC-CAML with a DLLO as a type of bitstream lidar called bitstream PC-CAML (patent pending²).

1.1 Key Advantages of the DLLO in Bitstream PC-CAML

The key advantages of the DLLO in the bitstream PC-CAML are as follows:

1. The DLLO can be implemented in the unit cells of a photon counting lidar receiver's readout integrated circuit (ROIC) by adding AND binary digital logic gates.
2. The signal and LO count data consist of just a stream of single-bit binary data.
3. The DLLO eliminates the need for bulky, power-hungry, and expensive wideband RF analog electronics by replacing them with binary digital logic electronics. Multi-GHz clocks and AND digital logic gates are readily implemented in inexpensive silicon complementary metal-oxide-semiconductor (CMOS) ROICs.
4. The DLLO single-bit binary waveform data can be computed prior to operation and stored in a buffer in each ROIC unit cell, or in a single buffer in the receiver and distributed to each ROIC unit cell in real-time during operation. The buffer can be a circular buffer for continuous repetition of the DLLO waveform.

These advantages make the bitstream PC-CAML with a DLLO more suitable for compact lidar-on-a-chip systems and lidar array receivers than previous PC-CAML systems.

1.2 Differences Between Bitstream PC-CAML and Bitstream Radar

The main differences between the bitstream PC-CAML and the bitstream radar are

1. The bitstream PC-CAML's signal and LO are unipolar, whereas the bitstream radar's signal and LO are bipolar, which makes the appropriate binary digital logic gate an AND gate for the former and an XOR gate for the latter [ref 18, pp. iii, 8, 117].
2. The photon counting detector output in PC-CAML is inherently a single-bit binary data stream even when the transmitted signal is analog, whereas the bitstream radar needs to use thresholds, comparators, and/or limiters to convert the transmitted or received signal to a single-bit binary data stream [ref 18, pp. 6-10, 13, 36-40].
3. The bitstream radar needs dithering on the transmitter or an analog low pass anti-aliasing filter and multi-bit ADC on the receiver to avoid aliasing [ref 18, p. 56], but the bitstream PC-CAML does not need either when the receiver is operating well below saturation due to the alias-free random sampling provided by the received photon counts.

1.3 Unipolar vs. Bipolar Signals

The unipolar nature of the PC-CAML signals makes the AND digital logic gate the appropriate replacement for the analog mixer for the bitstream PC-CAML instead of the XOR (Exclusive OR) digital logic gate used for the bitstream radar [ref 15-18] since the Boolean logic truth table for the AND Boolean operator corresponds to multiplication for 0 and 1, whereas the Boolean logic truth table of the XOR operator corresponds to multiplication for -1 and 1 when mapped to 0 and 1 (with an irrelevant sign inversion on the output)³.

3 The truth table for the Exclusive NOR (XNOR) gate corresponds exactly to multiplication for -1 and 1 when mapped to 0 and 1, but Bjorndal, et. al. [ref 15-18], use an XOR gate which induces a sign inversion in all the

1.4 Inherent Single-bit Binary Received Signal for PC-CAML

The constant amplitude output of the Gm-APD makes its output inherently single-bit binary, i.e., there is either a count, or there is no count. Thus, the bitstream PC-CAML may operate with either a chirped sinusoidal or chirped square wave transmitted waveform without requiring any additional thresholding and/or limiting in the receiver to convert a sinusoid to a square wave, as needed in the bitstream radar when operating with a transmitted chirped sinusoidal signal.

1.5 DLLO Waveforms

The DLLO for the bitstream PC-CAML may be either a single-bit chirped square wave or an analog chirped sinusoid that has been converted to single-bit samples. In the latter case, the single-bit sample pulses may be, for example, pulse width modulated (PWM) or pulse density modulated (PDM) by the amplitude of the analog chirped sinusoid.

The single-bit sampling of the chirped sinusoidal DLLO waveform may be uniform or non-uniform. In the case of uniform sampling, the single-bit sampling rate must be many times the sampling rate of an equivalent multi-bit digitizer to retain the same signal-to-quantization-noise ratio (SQNR) and dynamic range as that of a multi-bit sampled chirped sinusoidal LO waveform. In the case of non-uniform sampling, the average single-bit sampling rate may be much lower than the Nyquist sampling rate if an alias-free sampling scheme, such as a random Poisson point process sampling scheme [ref 19] or an additive random sampling (ARS) scheme [ref 23], is used with enough samples accumulated over the chirp duration to provide sufficient SQNR, and if the clock rate is at least at the Nyquist rate of twice the highest chirp frequency.[ref 24]

Note that the single-bit chirped sinusoidal LO samples will be computed before operation of the bitstream PC-CAML and stored in a buffer for streaming in real-time during operation. Therefore, the complexity of computing the single-bit chirped sinusoidal LO samples is dealt with once and the results stored prior to operation. Thus, there is no additional complexity in streaming the resulting single-bit LO samples whether they are for a chirped sinusoidal LO or a chirped square wave LO, other than possibly streaming the single-bit chirped sinusoidal LO samples at a much higher rate if uniform sampling were used to generate the single-bit samples.

The advantage of having a single-bit sampled chirped sinusoidal LO instead of a single-bit chirped square wave LO is that whether the signal is a chirped square wave or a chirped sinusoid, mixing the signal and the LO will not produce harmonics in the IF spectrum.

Due to the increased complexity of generating the single-bit sampled chirped sinusoidal LO data, however, the remainder of this introductory paper confines the discussion to a DLLO consisting of a single-bit chirped square wave. Simulation of the bitstream PC-CAML with a single-bit sampled chirped sinusoidal DLLO is suggested for future work to be reported in another paper.

1.6 Harmonics and their Mixing Products

At signal levels well below receiver saturation with a chirped sinusoidal signal, the bitstream PC-

results. The sign inversion corresponds to a 180 degree phase shift, which is irrelevant, and the XOR has one less logic element (the NOT logic element), so the XOR gate is preferred.

CAML will be mixing a randomly sampled chirped sinusoidal modulation of the received signal count rate with a chirped square wave LO. Just as for the linear response receiver based CAML with a chirped sinusoidal signal and chirped square wave LO, this does not produce harmonics in the mixed signal's intermediate frequency (IF) power spectrum.

However, when the transmitted waveform is a chirped square wave or when the receiver starts to saturate so that it “squares off” the received waveform, the bitstream lidar will be mixing a chirped square wave signal and a chirped square wave LO which will produce odd-order harmonics and their mixing products.

These odd-order harmonics and their mixing products also occur in the bitstream radar, for which both the signal and LO are always chirped square waves. Bjorndal, et. al., have shown three ways of dealing with these odd-order harmonics to mitigate their effects in bitstream radar by suppressing them or even by utilizing them for improved range resolution [ref 15-18], and which can also be used in the bitstream PC-CAML:

1. Known pseudorandom or random dithering of the chirp signal's phase with or without noise shaping to spread out the harmonics over the spectrum
2. Transmitted waveform additional delay to shift the harmonics beyond the intermediate frequency (IF) corresponding to the desired unambiguous range
3. Detection at a harmonic frequency for improved range resolution.

As Bjorndal points out, dithering the chirp signal's phase can spread out the energy of the harmonics over the power spectrum, but this increases the noise floor. [ref 18, pp. 12, 56, 62-65]

1.7 Aliasing

Dithering can also be used to address the issue of aliasing. As Bjorndal states “If we sample, either by transmitting a sampled bitstream, or sample on the receiver without an anti-aliasing filter then anything above the sample rate is going to fold down...Avoiding aliasing on the receiver side is best done with an analog low-pass filter and a multi-bit ADC ..., while aliasing on the transmitter can be mitigated with intentional dithering...” [ref 18, p.56] Use of an anti-aliasing filter with a multi-bit ADC in the wideband signal path on the receiver, however, complicates the receiver, and dithering on the transmitter complicates the transmitter.

Fortunately, at signal levels well below saturation, the quantum randomness of the photon stream and noise counts in the PC-CAML provides nonuniform random sampling of the transmitted waveform, which can eliminate the aliasing inherent in uniform sampling. The Nyquist sampling theorem applies only to uniform sampling. For example, Beutler showed that any spectral distribution is alias-free if randomly sampled by a Poisson point process. [ref 19]

However, for the bitstream PC-CAML, as the signal level increases into receiver saturation so that all the available clock time interval samples start filling up with ones, the sampling becomes more like uniform sampling and aliasing can occur.

Gatt, et. al., show that the lidar signal counts from a photon counting Gm-APD have a negative binomial distribution rather than a Poisson distribution due to the speckle diversity. [ref 20, pp. 3263-3265] (Note that the form of the negative binomial distribution used in Gatt, et. al., is also

called the Gamma Poisson (mixture) distribution. [ref 21-22]). They also model the noise counts with a Poisson distribution. The resulting signal plus noise counts for the Gm-APD based lidar have a Negative Binomial Plus Poisson (NBPP) distribution resulting from the convolution of the negative binomial and Poisson distribution given in Gatt, et. al.. [ref 20, p. 3264] When the counts per matched filter impulse response time are much less than one, the counts distribution is approximately Poisson [ref 20], so the alias-free property of Poisson sampling holds in this limit.

The author is not aware of any prior work showing that an NBPP point process has the alias-free property in general. Determining whether or not the NBPP point process provides alias-free random sampling in general is beyond the scope of this paper and is suggested for future work.

However, the counts from the Gm-APD can be viewed as the result of an additive random sampling (ARS) process. An ARS process is defined by having sample times given by $t_n = t_{n-1} + \tau_n$, where n is an integer, and $\{\tau_n\}$ is a family of independent identically distributed positive random variables. This says that the current count sample time is the previous count sample time plus a random delay time, which is the case for the Gm-APD counts for PC-CAML. An important result for our purposes from the thesis of King Chuen Lo is that an ARS process is alias-free for any distribution of $\{\tau_n\}$ for a sampling duration T larger than T_a , where T_a depends on the probability density function of τ . [ref 23, p. 52]

Luo points out in his thesis that the foregoing tacitly assumes that the time intervals $\{\tau_n\}$ are continuously distributed. [ref 24, p. 99] However, in a real sampled system, the sample time intervals are discrete at the clock time interval, Δ . Luo shows that for discrete clock time intervals of Δ , additive random sampling is alias-free only if the signal is band-limited in $[-1/2\Delta, 1/2\Delta]$. [ref 24, p. 100] Luo also shows that for exponentially distributed $\{\tau_n\}$, which is the case for a Poisson point process, for such a band-limited signal, the sampling is alias-free for any number of counts accumulated. [ref. 24, pp. 80 and 84-86]

Thus, the average count rate for an ARS process can be much lower than the Nyquist sampling rate for the signal bandwidth if a sufficient number of additive random counts are accumulated, but the clock rate must be at least twice the signal bandwidth for alias-free sampling.

1.8 Dead-Time and Count Pulse Rise Time Limitations on Signal Bandwidth

One might think that the chirp signal's maximum frequency would be limited to no more than $1/2t_d$, where t_d = the Gm-APD's dead-time. As Redman, et. al., demonstrated, however, it is the rise-time of the count pulse, not the dead-time, that sets the upper frequency limit for PC-CAML with pre- or post-detection mixing. [ref 2-4].

On the other hand, for OEM mixing by modulating the bias voltage of the Gm-APD directly with a square-wave LO signal, the highest useful chirp frequency is limited by the Gm-APD dead-time. [ref 2-4, 8-9, 13] If, however, the LO is applied by modulating the gate open duration of the Gm-APD such that the gate open duration is proportional to the amplitude of a sinusoidal LO, then the dead-time limit on the highest chirp frequency can be eliminated by setting the longest gate open duration, Δt_g , and the shortest time sample interval between starts of opening the gate, t_{gs} , such that $t_{gs} - \Delta t_g > t_d$, where t_d is the dead-time. [ref 11, p. 11802] In this case, the highest

chirp frequency is limited by the smallest gate duration that can be applied to the Gm-APD. Luo's analysis does not take into account the finite rise-time of the edges of the count pulses. If the clock time interval, Δ , is much larger than the count pulse rise-time, then the signal must be band-limited within $[-1/2\Delta, 1/2\Delta]$ as stated by Luo. If, however, Δ is less than or equal to one-half the count pulse rise-time, then the signal bandwidth must be less than or equal to half the bandwidth of the count pulse's rising edge. When the count pulse rise-time equals 2Δ , so that the count pulse rising edge is sampled at the Nyquist rate, then the signal must be band-limited within $[-1/2t_r, 1/2t_r] = [-1/4\Delta, 1/4\Delta]$, where t_r is the count pulse rise-time.

Gatt, et. al., state "...a signal with intrinsic diversity M behaves much like a Poissonian signal when $m_s \ll M$. That is to say, quantum noise dominates over speckle noise" where m_s is the average number of signal photoelectrons per matched filter impulse response time and M is the speckle diversity. [ref 20, p. 3264] For well designed photon counting lidars, in order to prevent receiver saturation, transmitter power and/or receiver throughput control will be used to enforce $m_s \ll M$ since the minimum of M is 1, and if m_s were not much smaller than 1, the receiver would be nearly saturated. Also for well designed photon counting lidars, measures such as using narrow band optical filters and low dark count rate Gm-APDs, will be taken to insure that $m_n \ll 1$, where m_n is the average number of noise photoelectrons per matched filter impulse response time, otherwise, the arm probability will be too low for practical operation. (See reference 20 for a more detailed discussion of m_n and the arm probability.)

Therefore, for a well designed bitstream PC-CAML with transmitter power and/or receiver throughput control, and low noise count rate, the signal plus noise will be well approximated by Poisson distributed signal plus noise, and the signal spectrum will be alias-free for any accumulation duration if the signal's bandwidth is within the band limits set by the clock time interval or the count pulses' rise-time as discussed above. In addition, since the counts form an ARS process, even for high count rates, but count rates still well below receiver saturation, the spectrum will be alias-free for a sufficient number of accumulated counts if the signal bandwidth, count pulse rise-time, and clock time interval meet the requirements discussed above.

1.9 Detection at a Harmonic Frequency for Improved Range Resolution

When using a chirped square wave signal, detection at a harmonic frequency for improved range resolution can be done for processing bitstream PC-CAML data in the same manner as described by Bjorndal, et. al., for bitstream radar [ref 16-18], but further investigation into that technique is beyond the scope of this paper and is suggested for future work. If the harmonics are to be used for improved range resolution, the additional bandwidth of the harmonics to be used for this purpose must be included in the signal bandwidth limits for alias-free sampling discussed above.

1.10 Transmitted Waveform Delay for Shifting Harmonics Beyond the Unambiguous Range

Since dithering increases the noise floor and is not needed for anti-aliasing for a well designed bitstream PC-CAML operating at signal levels well below saturation, and since detection at a harmonic frequency is beyond the scope of this paper, in this paper, just the effectiveness of the transmitted waveform delay technique to shift the frequencies of the harmonics beyond the frequency corresponding to the desired unambiguous range for the bitstream PC-CAML with a DLLO is demonstrated with simulation results.

The additional delay on the transmitted waveform is equivalent to an advance of the LO waveform. Therefore, the additional transmitted waveform delay can be implemented by starting the LO waveform earlier than the start of the transmitted waveform and extending the LO's chirp duration and frequency along the chirp's temporal slope for complete overlap with the round-trip delayed received chirp signal's duration.

2. Bitstream PC-CAML with a DLLO SNR Theory

The purpose of this paper is to introduce the new bitstream PC-CAML with a DLLO concept and to show through simulation results how it works. The purpose of this paper is not to develop a comprehensive theory of operation for the new concept, so the initial SNR theory presented herein has a limited range of applicability to the new concept, and an improved theory needs to be developed in future work.

The initial electrical power signal-to-noise ratio (SNR) theory used herein for the bitstream PC-CAML with a DLLO concept is derived from the SNR theory for photon counting Gm-APDs developed by Gatt, Johnson, and Nichols. [ref 20, pp. 3268-3269] The theory developed by Gatt, et. al., is for detection of pulsed lidar returns. This theory, however, can be applied to the single-bit chirped AM waveform and DLLO as simulated in Mathcad^{®4} by making the following adjustments:

1. Reducing the SNR by a factor of 16 to account for losses due to the one-sided (aka unipolar) signal and LO waveforms of the bitstream PC-CAML as discussed in the papers presenting the original PC-CAML concept by Redman, Ruff, and Giza [ref 2-4].
2. Adding quantization noise due to the single-bit quantization
3. Setting the arm probability, PA, in the SNR equations of Gatt, et. al., [ref 20, pp. 3268-3269] to 1 since in the theory as used herein, the counts per matched filter impulse response time are output counts after having been subjected to the arm probability.
4. Multiplying the theoretical SNR by $10^{-0.176}$ to account for the scalloping loss due to the Hann window applied to the data in the simulations (see section 3) to reduce sidelobes.

Equation (41) from Gatt, et. al., [ref 20, p. 3269] modified as described above becomes

$$SNR_{theory} = \frac{10^{-0.176} N_s \left[1 - \left(\frac{M}{M + m_{scounts}} \right)^M \right]^2}{16 \left[1 - e^{-(m_{ncounts} + m_{qncounts})} \left(\frac{M}{M + m_{scounts}} \right)^M \right] \left[1 - \left[1 - e^{-(m_{ncounts} + m_{qncounts})} \cdot \left(\frac{M}{M + m_{scounts}} \right)^M \right] \right]} \quad (1.)$$

where N_s = Number of clock time interval samples accumulated over the chirp duration

M = speckle diversity

$m_{scounts}$ = average number of signal counts output by the Gm-APD per clock time interval

$m_{ncounts}$ = average number of noise counts output by the Gm-APD per clock time interval
(includes all of the additive noise sources except for quantization noise)

$m_{qncounts}$ = the average number of quantization noise counts per clock time interval.

4 Mathcad[®] is a registered trademark of PTC Inc. or its subsidiaries in the U.S. and in other countries.

Note that the clock time interval equals the matched filter impulse response time and the dead-time in this theory and in the simulations discussed in section 3.

Also note that formulating the SNR theory in terms of the signal and noise counts output by the Gm-APD per matched filter impulse response time after having been subjected to the dead-time and arm probability constraints as done in equation (1.), rather than in terms of the signal and noise photoelectrons per matched filter impulse response time prior to the dead-time and arm probability constraints as done by Gatt et. al., has the practical advantage for comparison to experimental results of using the noise only count rates and signal plus noise count rates that would be measured in an experiment without having to have an auxiliary measurement of the incident photon rates and the Gm-APD's photon detection efficiency. (Note that the average signal only count rate in an experiment is easily calculated by subtracting the measured average noise only count rate from the measured average signal plus noise count rate.) However, for use in system performance modeling, the original formulation of the SNR theory by Gatt et. al. is probably more useful since the photon rates will be calculated by the mathematical models and the Gm-APD's photon detection efficiency will be specified in the system design parameters.

This SNR theory does not include the effects of energy loss from the fundamental IF frequency to higher order odd harmonics and their mixing products for the chirped square wave modulation waveforms in the bitstream PC-CAML. This SNR theory also assumes 100% modulation depth for the PC-CAML signal.

2.1 Sinusoidal Modulation Depth Loss Near Saturation

As the average number of signal counts output by the Gm-APD per available clock time interval increases towards 1, the apparent modulation depth of a sinusoidal chirped AM waveform decreases due to the Gm-APD being able to output at most only a single count per dead-time which is set equal to the clock time interval in the simulations (see section 3). For the sinusoidal signal, when the average count rate is so high that a count is output for every clock time interval, the single-bit count data stream looks like that of a constant, unmodulated signal corresponding to a modulation depth of zero and therefore, the SNR goes to zero.

2.2 Saturation Effects on Chirped Square Wave Signals

It must be noted that the number of available clock time intervals for a signal count (i.e., for a 1) for a fully modulated chirped square wave signal is half that of a chirped sinusoidal signal. This is because even with the negative binomial signal fluctuation statistics, there are no signal counts in the low areas of the square wave since the signal is exactly zero for those areas. (There are noise counts in those areas, but here we are discussing just the clock time intervals available for signal counts.) Thus, for the fully modulated chirped square wave, half of the clock time intervals will be 0 and only half of the clock time intervals will be available for a signal count.

Therefore, for a fully modulated chirped square wave signal, as the mean number of signal counts output by the Gm-APD per clock time interval available for a signal count increases towards 1, the signal becomes more like a deterministic chirped square wave signal that is uniformly sampled at the clock rate, and the floor of the power spectrum of the mixer output becomes dominated by sidelobes and perhaps aliasing, depending on the sampling rate and signal

bandwidth, rather than dominated by noise for the low noise rates of a well designed PC-CAML. These effects of Gm-APD receiver saturation on the chirped sinusoidal and chirped square wave forms are illustrated in figures 6 and 7 below with results from the Monte Carlo simulations. These effects, however, are not included in the initial SNR theory represented by equation (1.).

2.3 Different Effects of Speckle Diversity on Sinusoidal and Square Wave Signals

As shown by the Monte Carlo simulation results in section 3, figure 6 (a.), the value of the speckle diversity, M , affects the SNR for the chirped sinusoidal signal as predicted by the SNR theory. However, the simulation results in figure 6 (b.) show that the value of M makes no difference in the SNR for the chirped square wave signal in contrast to the theory's predictions. The following is an untested conjecture to explain these results, but more work on the SNR theory is clearly needed.

For a fully modulated chirped square wave where the signal is high, if any signal is detected, the output level of the Gm-APD is the same constant high level regardless of amplitude fluctuations on the input optical signal. Where the signal is zero, there are no fluctuations since there are no signal counts in those areas, just noise counts. Although the photon arrival rates will vary with the speckle amplitude fluctuations in the high areas of the chirped square wave signal, this may just look like additive random sampling of the high areas of the chirped square wave with a different distribution of random samples depending on M , which does not change the SNR. Therefore, for the chirped square wave signal this causes the effect of the speckle induced signal amplitude fluctuations on the SNR to be nearly eliminated.

If this is true, then if the chirped square wave were not fully modulated so that the small, but non-zero, signal level in the low areas of the square wave would be subject to amplitude fluctuations, then the amplitude fluctuations over the whole signal would be detected as photon arrival rate fluctuations, which would reduce the SNR with increasing amplitude fluctuations corresponding to decreasing M .

For the chirped sinusoidal signal there is some signal everywhere except at the exact nulls of the fully modulated chirped sinusoidal waveform so that the speckle induced amplitude fluctuations cause photon arrival rate fluctuations at almost all points on the signal, and these fluctuations are higher for lower M causing the SNR to be lower for lower M just as seen in the Monte Carlo results and theory predictions for the chirped sinusoidal signal case.

2.4 Single-bit Quantization Noise

Lastly, noise due to single-bit quantization must be calculated to use in the SNR theory of equation (1.). The quantization noise is computed from the Signal-to-Quantization-Noise Ratio in dB (SQNR_{dB}) given by equation 2.38 of Bjorndal [ref. 18, p. 40]:

$$\text{SQNR}_{\text{dB}} = 6.02 \cdot N_{\text{bits}} + 1.76, \text{ where } N_{\text{bits}} = \text{the number of bits of digitization. (2.)}$$

For a single-bit, $N_{\text{bits}} = 1$, the quantization noise per 1-bit count is then $1/10^{0.778} = 1/6$. Therefore, the average quantization noise per clock time interval is the average number of signal plus noise counts per clock time interval times $1/6$, which is given by the following in the simulations:

$$m_{q_n_counts_avg} = m_{spn_counts_avg}/6 \quad (3.)$$

where $m_{q_n_counts_avg}$ = the average number of quantization noise counts per clock time interval in the simulations

$m_{spn_counts_avg}$ = the average number of signal plus noise counts per clock time interval in the simulations.

In the initial SNR theory, equation (1.), $m_{qncounts}$ is set equal to the value of $m_{q_n_counts_avg}$ for the simulations for which the SNR theory and simulation results are being compared.

Development of an SNR theory that includes the effects of quantization noise, aliasing, the harmonics and their mixing products, less than 100% signal modulation depth, and Gm-APD saturation for the bitstream PC-CAML that is applicable for both chirped sinusoidal and chirped square wave signals and LOs is beyond the scope of this paper and is suggested for future work.

3. Monte Carlo Simulation Results Compared to Initial SNR Theory

The Monte Carlo simulations use the `rnbinom` and `rpois` functions in Mathcad® to generate the negative binomial distributed signal counts and Poisson distributed noise counts, respectively, in accordance with the theory of Gatt, et. al.. [ref 20] The signal, LO, and noise models with the `rnbinom` and `rpois` functions with their arguments are shown below.

$$signal_i := 0.5 \cdot \left[1 + \sin \left[2 \cdot \pi \cdot \left[f_0 + k_f \cdot \left(t_i - \frac{2 \cdot R_{tgt}}{c} - \tau_{txdelay} \right) \right] \cdot t_i \right] \right] \quad \text{for a sinusoidal waveform (4. a.)}$$

$$signal_i := 0.5 \cdot \left[1 + \text{sign} \left[\sin \left[2 \cdot \pi \cdot \left[f_0 + k_f \cdot \left(t_i - \frac{2 \cdot R_{tgt}}{c} - \tau_{txdelay} \right) \right] \cdot t_i \right] \right] \right] \quad \text{for a square waveform (4. b.)}$$

$$LO_i := 0.5 \cdot \left[1 + \text{sign} \left[\sin \left[2 \cdot \pi \cdot (f_0 + k_f \cdot t_i) \cdot t_i \right] \right] \right] \quad \text{for a square wave LO (5.)}$$

where f_0 = the start frequency of the chirp

k_f = the temporal slope of the chirp = $(f_s - f_0)/T_{chirp}$, where f_s = the stop frequency of the chirp, and T_{chirp} = the duration of the chirp

t_i = the i^{th} clock time interval from the start of the chirp

R_{tgt} = the target range for the simulations

$c = 3 \cdot 10^8$ in m/s, the speed of light in vacuum.

$\tau_{txdelay}$ = an additional delay added to the transmitted waveform to move the harmonics beyond the IF of the desired unambiguous range.

$\text{sign}[]$ = the sign of the argument function, which returns +1 for a non-negative number for the argument and -1 for a negative number for the argument.

Note that the value of the target range used in the simulations is chosen so that the resulting round-trip delay time makes the intermediate frequency (IF) for the target return signal exactly equal to some frequency sample in the power spectrum computed in the simulations to avoid complications in computing the SNR for comparison to the theory due to the target range peak straddling two frequency samples.

The random Poisson distributed noise counts and negative binomial distributed signal counts per clock time interval are given by equations (6.) and (7.), respectively:

$$m_{n_counts_rnd_{i,j}} := \text{if}\left(\text{rpois}\left(N_{\text{trials}} \cdot P_{n_counts_i}\right)_j > 0, 1, 0\right) \quad (6.)$$

$$m_{s_counts_rnd_{i,j}} := \text{if}\left[\left(\text{rnbino}\left(N_{\text{trials}}, M, p_i\right)_j > 0\right) \wedge \left(m_{n_counts_rnd_{i,j}} = 0\right), 1, 0\right] \quad (7.)$$

where In Mathcad®, the function if(logical expression, A, B) means that if the logical expression is true, then the result is A, otherwise the result is B

In Mathcad®, the symbol ^ represents the Boolean logical AND

i = row index over clock time intervals

j = column index over trials

Note: The “if” statements in equations (6.) and (7.) enforce the restriction of having at most one count output by the Gm-APD per dead-time, which equals the clock time interval in the simulations.

In Mathcad®, rnbino(m,n,p) returns a vector of m random numbers having the negative binomial distribution:

Negative binomial

$$\binom{n+k-1}{k} \cdot p^n \cdot (1-p)^k$$

in which $0 < p \leq 1$ and n and k are integers, $n > 0$ and $k \geq 0$.

(8.)

where for the Gamma Poisson (mixture) parameterization used by Gatt, et. al., we have

$m = N_{\text{trials}}$ = the number of trials in the simulations

$n = M$ = the speckle diversity

$$p := \frac{M}{M + n_{pe_avg}}$$

where n_{pe_avg} = the mean number of signal photoelectrons in a clock time interval in the simulations prior to applying the limit of 1 count output by the Gm-APD per dead-time, which equals the clock time interval in the simulations.

$$n_{pe_avg_i} := 5.1 \cdot 10^{-2} \cdot \text{signal}_i$$

(where the numerical factor multiplying the signal is varied to vary the number of signal counts in the simulations.)

In Mathcad®, rpois(m,λ) returns a vector of m random numbers having the Poisson distribution:

Poisson

$$\frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

in which $\lambda > 0$ and k is a non-negative integer.

(9.)

where $m = N_{\text{trials}}$ = the number of trials in the simulations

$\lambda = p_{\text{n_counts}}$ = the mean number of noise counts in each clock time interval in the simulations prior to applying the limit of 1 count output by the Gm-APD per dead-time, which equals the clock time interval in the simulations.

The following expression in Mathcad® performs the logical AND on the single-bit signal plus noise counts and the single-bit LO data streams to produce the single-bit mixer output:

$$\text{Mixed}^{<j>} := \overrightarrow{(m_{\text{spn_counts_rnd}}^{<j>} \wedge \text{LO})} \quad (11.)$$

where $m_{\text{spn_counts_rnd}}^{<j>} := \overrightarrow{(m_{\text{s_counts_rnd}}^{<j>} + m_{\text{n_counts_rnd}}^{<j>})}$ = the signal plus noise counts
 j = column index over trials, and the superscript $<j>$ indicates the j^{th} column of a 2D array.

The single-bit mixer output is Hann windowed to reduce sidelobes, and zero padded to eight times its original length. The Hann window is given by

$$w_{\text{hann}_i} := \sin\left(\frac{\pi \cdot i}{N_s - 1}\right)^2 \quad (12.)$$

The Hann windowed and zero padded single-bit mixer output is digitally band pass filtered by a super Gaussian filter that filters out the DC peak and high frequencies prior to down sampling to prevent aliasing. The digital super Gaussian band pass filter for the zero padded signal is defined in the frequency domain by the following in the Mathcad® simulations:

$$\text{SGBPF}_{ii} := \text{if} \left[ii \leq \text{ceil}\left(\frac{N_s \cdot 8}{2}\right), \exp\left[\frac{-ii^{50}}{(121 \cdot 8)^{50}}\right], \exp\left[\frac{-(ii - N_s + 1)^{50}}{(121 \cdot 8)^{50}}\right] \right] \quad (13.)$$

$$\text{SGBPF}_0 := 0$$

$$\text{SGBPF}_1 := 0$$

$$\text{SGBPF}_{8 \cdot N_s - 1} := 0$$

where $\text{ceil}()$ = the round up to the next highest integer function

ii = index over the frequency bins

Figure 3 is a graph of the frequency response of the super Gaussian band pass filter used in the simulations as defined by equation (13.). The filter's upper cut-off frequency is set just below one-half the simulation's 2 GHz clock rate divided by 32, which equals 31.25 MHz, for anti-aliasing when down sampling by a factor of 32.

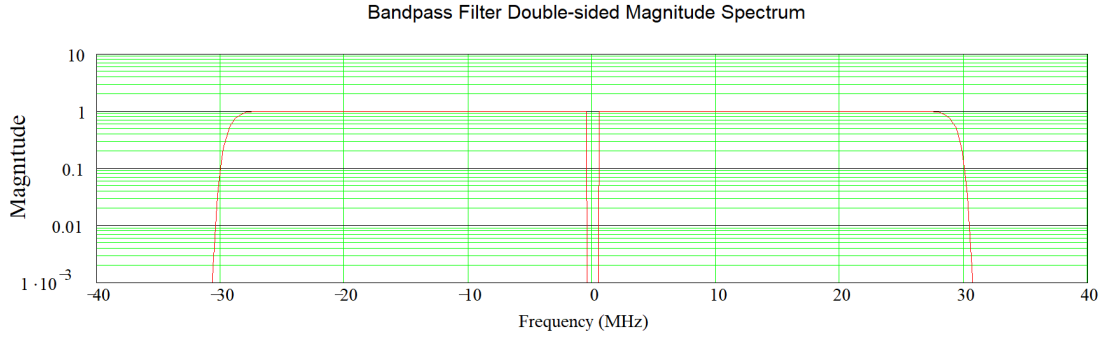


Figure 3. Super Gaussian band pass filter frequency response.

The super Gaussian band pass filter function is multiplied by the complex fast Fourier transform (cfft) of the Hann windowed and zero padded single-bit mixer output. The real part of the inverse cfft (icfft) of the resulting product is the filtered mixer output which is then down sampled by a factor of 32 as follows:

$$iv := 0 \dots 8 \cdot \frac{N_s}{32} - 1 \quad (14.)$$

$$\text{Mixed_filt_ds}_{iv,j} := \sum_{n=0}^{32-1} \text{Mixed_filt}_{32 \cdot iv + n,j} \quad (15.)$$

where Mixed_filt_ds = the filtered and down sampled Hann windowed and zero padded mixer output

Mixed_filt = the filtered Hann windowed and zero padded mixer output prior to down sampling

The power spectrum of the filtered and down sampled Hann windowed and zero padded single-bit mixer output is computed in Mathcad® for each trial. The resulting power spectra are averaged together to produce the mean power spectrum for all the trials.

The mean noise floor of this mean power spectrum is computed over the portion of the spectrum between the target signal's fundamental IF and the third harmonic of that frequency. The value of this mean noise floor is the denominator in computing a simulation's mean SNR.

The peak value of the mean power spectrum at the fundamental IF minus the value of the mean noise floor is used as the numerator in computing a simulation's mean SNR.

Note that since the simulated data are quantized to one bit, the single-bit quantization noise is inherently included in the simulated power spectra.

The Monte Carlo simulations generate 64 realizations of signal plus noise for an up-chirp AM signal waveform. The simulations were run for the following parameter values:

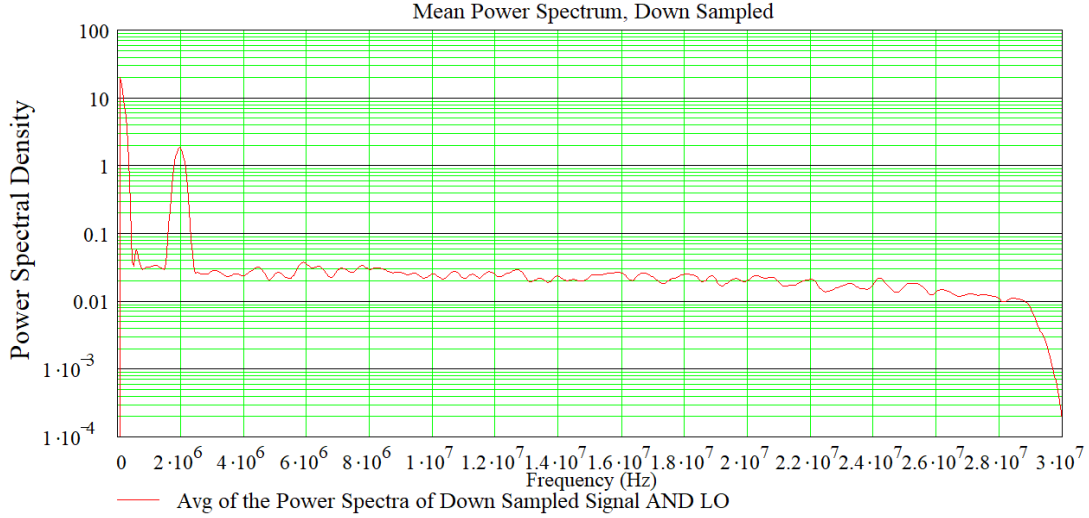
N_{trials} = Number of simulation trials = 64	N_s = Number of clock time intervals = 2^{13}
Δt = Clock time interval = 0.5 ns	t_{chirp} = chirp duration = 4.096 μs
f_0 = 100 MHz = chirp start frequency	f_s = 500 MHz = chirp stop frequency
Target Range = 2.99963379 m	Target IF = 1.953125 MHz

$M = \text{speckle diversity} = 1 \text{ or } 1\text{E}+06$

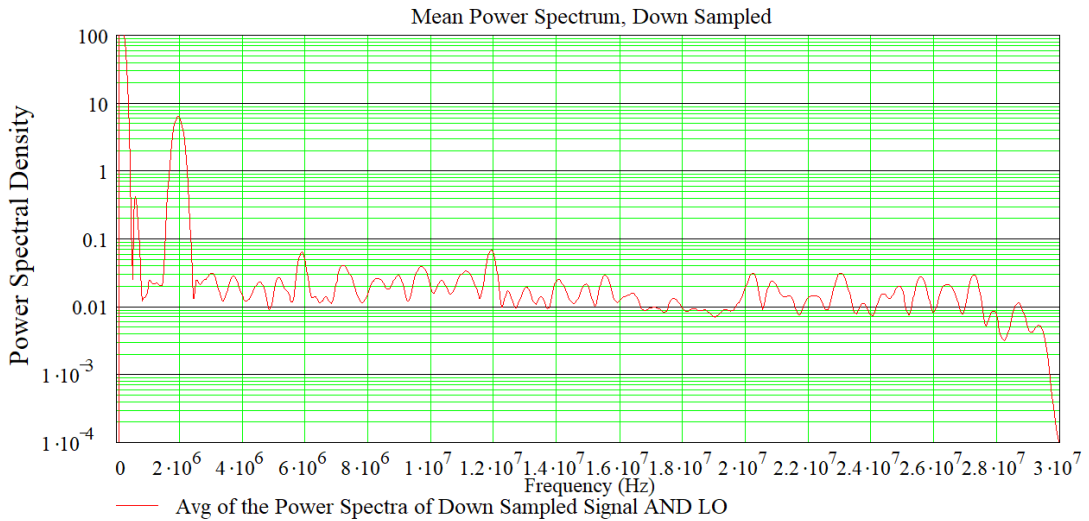
$m_{n_counts_avg} = \text{average number of additive noise counts per clock time interval} = 1\text{E}-04$

$m_{s_counts_avg} = \text{average number of signal counts per clock time interval} = \text{from } 0.05 \text{ to } 1$

Figure 4 shows the mean power spectrum of the Hann windowed and zero padded single-bit mixer output after band pass filtering and down sampling by a factor of 32, for each of low and high signal levels from the simulations for a chirped sinusoidal signal.



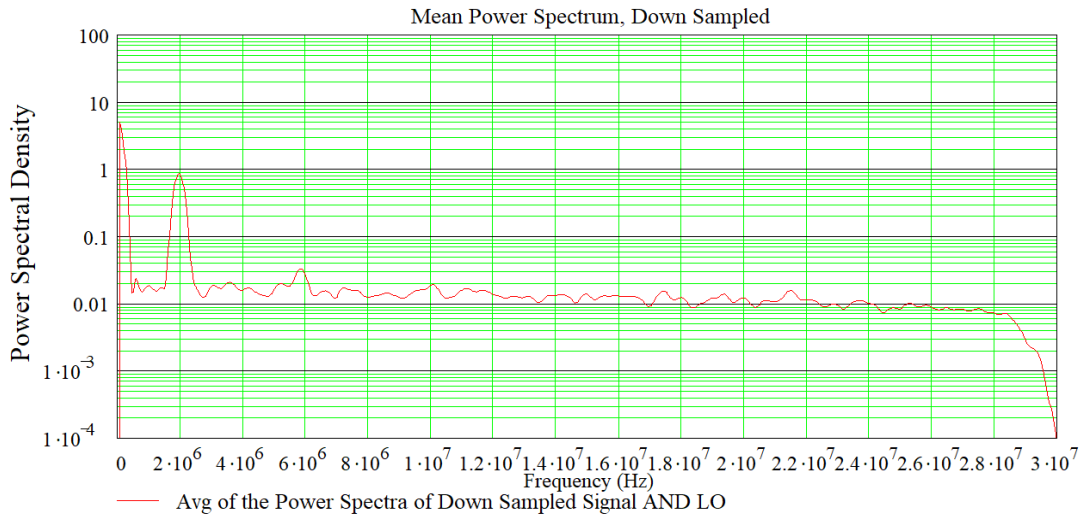
(a.) Low signal counts per clock time interval ($m_{s_counts_avg} = 0.2$) - chirped sinusoidal signal



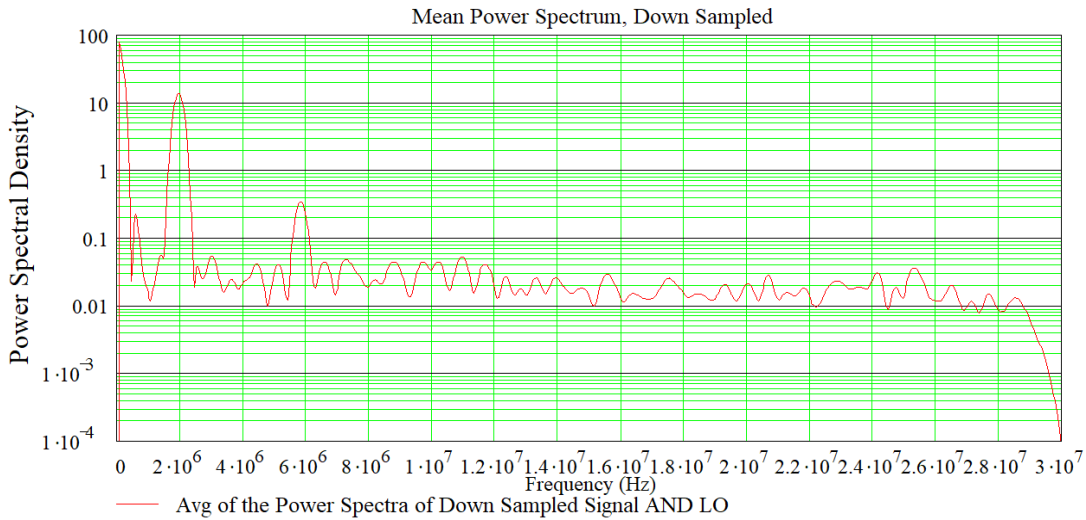
(b.) High signal counts per clock time interval ($m_{s_counts_avg} = 0.8$) - chirped sinusoidal signal

Figure 4. Mean filtered and down sampled signal AND LO plus noise power spectrum from the Mathcad® Monte Carlo simulations for a chirped sinusoidal signal with $m_{n_counts_avg} = 1\text{E}-04$ and $M=1\text{E}+06$ for each of low and high signal levels (a.) Low signal counts per clock time interval ($m_{s_counts_avg} = 0.2$) and (b.) High signal counts per clock time interval ($m_{s_counts_avg} = 0.8$).

Figure 5 shows the mean power spectrum of the Hann windowed and zero padded single-bit mixer output after band pass filtering and down sampling by a factor of 32, for each of low and high signal levels from the simulations for a chirped square wave signal.



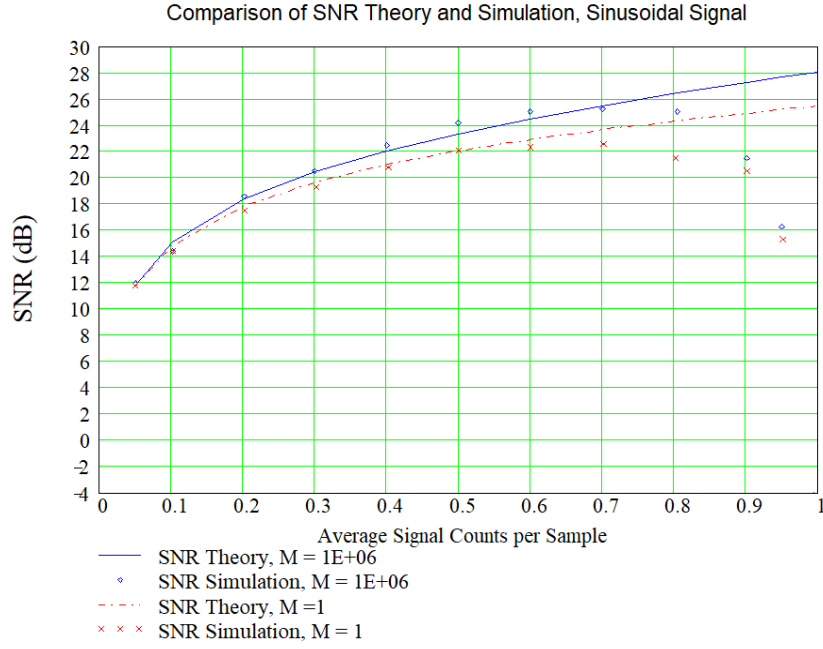
(a.) Low signal counts per clock time interval ($m_{s_counts_avg} = 0.2$) - chirped square wave signal



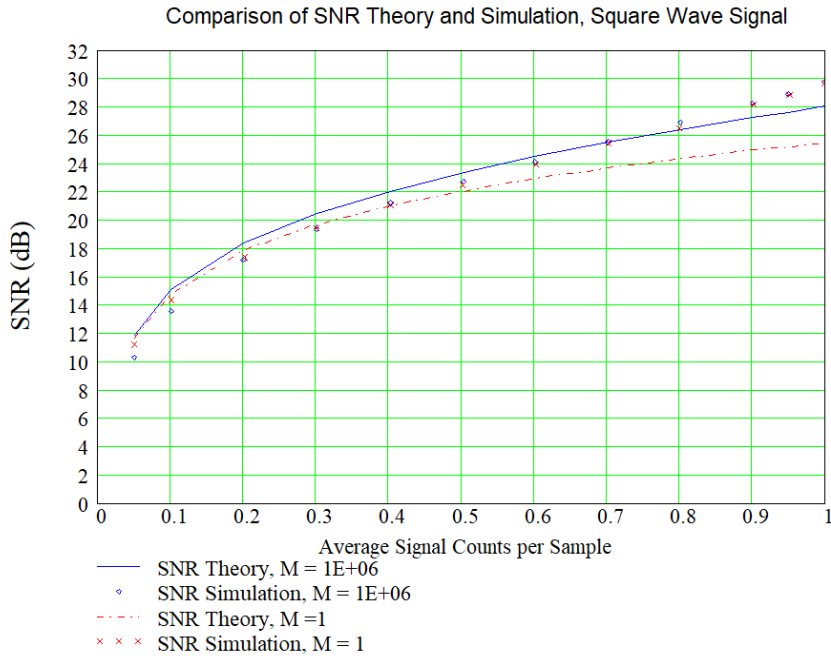
(b.) High signal counts per clock time interval ($m_{s_counts_avg} = 0.8$) - chirped square wave signal

Figure 5. Mean filtered and down sampled signal AND LO plus noise power spectrum from the Mathcad® Monte Carlo simulations for a chirped square wave signal with $m_{n_counts_avg} = 1E-04$ and $M=1E+06$ for each of low and high signal levels (a.) Low signal counts per clock time interval ($m_{s_counts_avg} = 0.2$) and (b.) High signal counts per clock time interval ($m_{s_counts_avg} = 0.8$).

Figure 6 shows plots of the mean SNR vs. mean signal counts per clock time interval ($m_{s_counts_avg}$) from the simulations and the SNR theory for the chirped sinusoidal signal (top) and the chirped square wave signal (bottom) as a function of $m_{s_counts_avg}$ for $M = 1$ and $M = 1E+06$, with $m_{n_counts_avg} = 1E-04$.



(a.) Chirped Sinusoidal Signal



(b.) Chirped Square Wave Signal

Figure 6. Comparison of SNR results for the Monte Carlo simulations for (a.) a chirped sinusoidal signal and (b.) a chirped square wave signal, and the SNR theory as a function of average signal counts per clock time interval, $m_{s_counts_avg}$, with the average number of noise counts per clock time interval, $m_{n_counts_avg}$, set equal to $1E-04$.

The simulation results for the chirped sinusoidal signal and the SNR theory are in good agreement up to an average signal counts per clock time interval, $m_{s_counts_avg}$, of about 0.6 for $M=1$ and 0.7 for $M=1E+06$. As discussed in section 2.1, above these levels the saturation effects

of power loss to the harmonics and their mixing products, and of modulation depth loss make the simulations' SNR results rollover with increasing $m_{s_counts_avg}$. The SNR theory over estimates the SNR for these high signal levels since the effects of power loss to the harmonics and their mixing products, and of modulation depth loss are not included in the SNR theory.

For the chirped square wave signal, the simulation results and the theory are qualitatively different in that the simulation results for $M = 1$ and $M = 1E+06$ do not differ significantly even for higher signal levels as they do for the SNR Theory results. See the earlier discussion of a tentative conjecture to explain this in subsection 2.3. Even so, for the chirped square wave signal, the SNR values for the simulation results for both low and high M are within about 2 dB of the initial SNR theory values for high M over the range of $m_{s_counts_avg}$ plotted.

At $m_{s_counts_avg}$ above about 0.8, the SNR values for the chirped square wave signal simulation results exceed those of the SNR theory, whereas the SNR values for the chirped sinusoidal signal simulation results roll over to approach zero as $m_{s_counts_avg}$ approaches 1.

As explained in section 2.1, the latter behavior is due to the clock time interval samples all being filled with 1's as $m_{s_counts_avg}$ approaches 1 for the chirped sinusoidal signal. This looks like a loss in modulation depth for the sinusoidal signal until at $m_{s_counts_avg} = 1$, the modulation depth goes to 0 and the signal looks like a constant level signal with no modulation.

As explained in section 2.2, for the fully modulated chirped square wave signal, however, the clock time interval samples where the zero's of the chirped square wave are located are never filled by 1's for any signal level, and those clock time interval samples can only have an occasional 1 due to noise at the low noise levels in the simulations and in a well designed PC-CAML. Therefore, as $m_{s_counts_avg}$ approaches 1, only the clock time interval samples corresponding to the high levels of the chirped square wave become filled with 1's, and this causes the waveform to approach the uniformly sampled deterministic chirped square wave form as $m_{s_counts_avg}$ approaches 1. This results in the floor of the power spectrum of the chirped square wave signal plus noise becoming dominated by sidelobes and perhaps aliasing, depending on the signal bandwidth and clock rate, as $m_{s_counts_avg}$ approaches 1.

This is illustrated by the graph in figure 7 of the raw power spectra prior to filtering and down sampling for the simulated PC-CAML's stochastic chirped square wave signal for $m_{s_counts_avg} = 1$, $m_{n_counts_avg} = 1E-4$, and $M = 1E+06$ ANDed with a deterministic chirped square wave LO (red trace), and a noiseless, deterministic chirped square wave signal multiplied by a deterministic chirped square wave LO (blue trace). Clearly, the power spectrum, including the floor between the spectral peaks, for the mixer outputs with the stochastic chirped square wave signal for $m_{s_counts_avg} = 1$ ANDed with the chirped square wave LO is nearly identical to that of the noiseless, deterministic chirped square wave signal multiplied by the chirped square wave LO.

Note that a well designed PC-CAML system using transmitter power and/or receiver throughput control to prevent saturation would operate at the lower signal levels where the initial SNR theory agrees with the simulation results, and where the spectral floor is dominated by noise. Also note that for the average signal counts per clock time interval regime of 0.05 to 0.7, the theory and simulation results predict SNRs of about 10-25 dB for the additive noise level of $m_{n_counts_avg} = 1E-4$ used in this study. This range of SNRs is sufficient for many lidar applications.

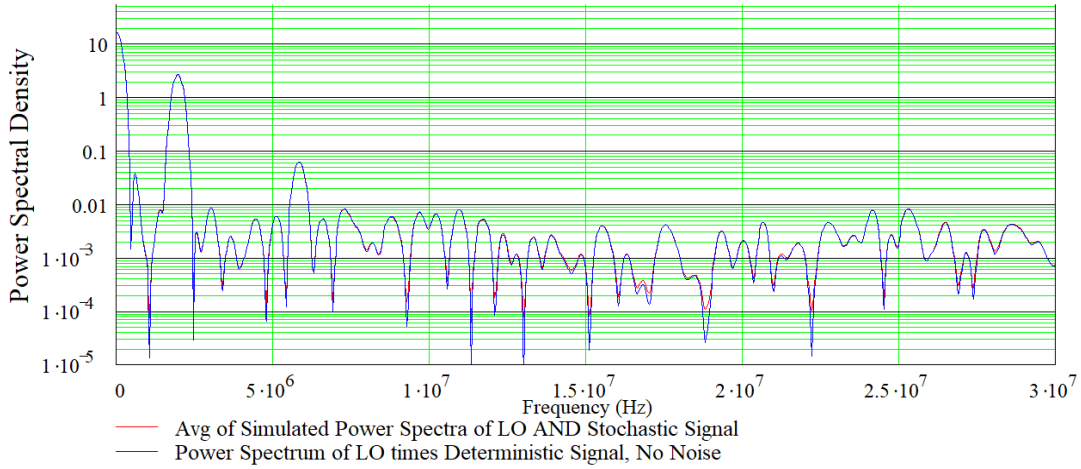


Figure 7. Comparison of simulated raw power spectra prior to filtering and down sampling of the chirped square wave LO ANDed with the stochastic chirped square wave signal for $m_{s_counts_avg} = 1$, $m_{n_counts_avg} = 1E-4$, and $M = 1E+06$ (red trace) versus the chirped square wave LO multiplied by the noiseless, deterministic chirped square wave signal (blue trace).

These SNR results will vary with the mean number of additive noise counts per clock time interval. For the number of clock time intervals accumulated of 2^{13} used in the simulations, the mean total number of noise counts accumulated over the chirp duration of $4.096 \mu s$ in the simulations is about 0.82, corresponding to an average noise count rate of about 200 kHz for $m_{n_counts_avg} = 1E-4$. This mean noise count rate is on the high side compared to the dark noise count rates of many Gm-APD receivers, but can be comparable to the dark noise plus solar background noise count rates in some scenarios.

As a spot check to verify that the SNR theory and simulation results agree at higher noise levels for a signal level below saturation, the simulation was run for a chirped sinusoidal signal at two much higher noise levels for $m_{s_counts_avg} = 0.5$ with the results shown in table 1.

Table 1. Comparison of SNR from Theory and Simulation Results for Higher Noise Levels

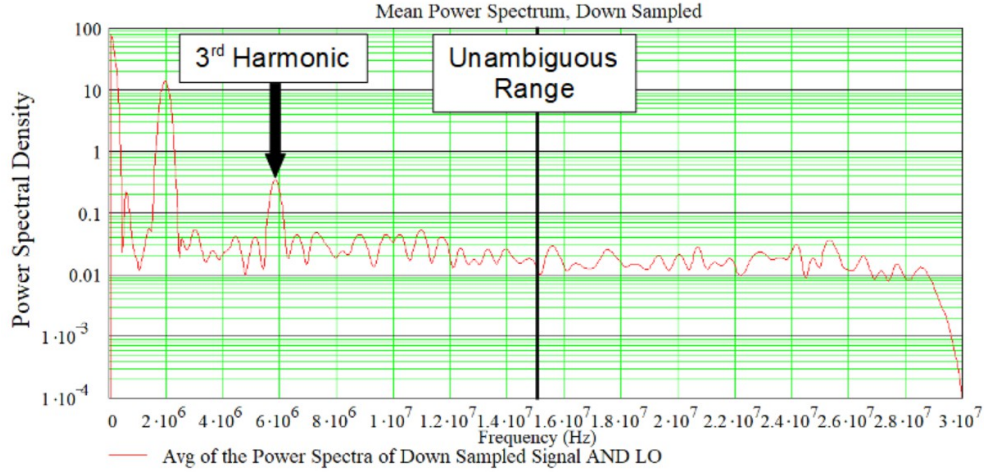
$m_{n_counts_avg}$	Mean Noise Count Rate	M	SNR Theory	SNR Simulation	% Difference	Ratio in dB
5E-03	10 MHz	1E+06	216.15	226.09	4.4%	0.2 dB
1E-01	200 MHz	1E+06	215.08	190.29	-13%	-0.5 dB
1E-01	200 MHz	1	151.77	123.39	-23%	-0.9 dB

The results shown in table 1 indicate that the initial SNR theory probably provides reasonable SNR estimates for a chirped sinusoidal signal at a level well below saturation even with high noise levels that are well above the noise count rates expected for a well designed PC-CAML.

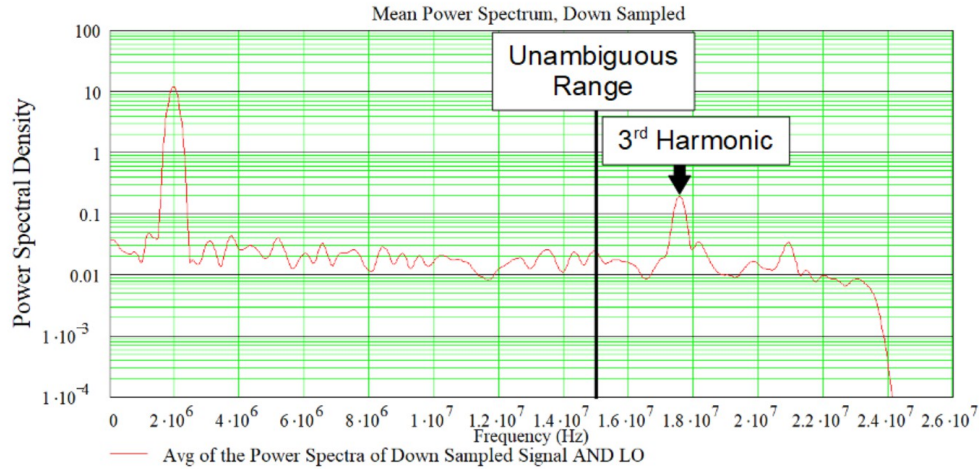
Note that prior to adding the quantization noise term, the SNR theory over estimated the SNR compared to the simulation results for low $m_{s_counts_avg}$ (less than about 0.2), but after addition of the quantization noise terms, the SNR theory is in good agreement with the simulation results for

these low values of $m_{s_counts_avg}$. Therefore, it is important to include the quantization noise caused by the single-bit quantization inherent in the bitstream PC-CAML with DLLO concept.

4. Simulation Results for the Transmitted Waveform Delay to Shift the Harmonics



(a.) No additional Delay on the Transmitted Waveform



(b.) With an additional Delay of 59.99 ns on the Transmitted Waveform

Figure 8. Power Spectrum without (a.) and with (b.) an additional delay of 59.99 ns on the transmitted chirped square wave signal, with $m_{s_counts_avg} = 0.8$, $m_{n_counts_avg} = 1E-04$, and $M = 1E+06$.

Figure 8 shows the mean power spectrum without (a.) and with (b.) an additional 59.99 ns delay on the transmitted chirped square wave signal. In the latter case, the frequencies plotted have all been digitally shifted by subtracting the IF corresponding to the additional transmitted waveform delay to return the fundamental IF peak corresponding to the target range to its original position without the additional delay. These simulation results demonstrate the ability of the additional delay on the transmitted waveform to move the harmonics beyond the desired unambiguous range.

This technique works because adding a delay to the transmitted waveform spreads the frequency difference between the fundamental IF and its n^{th} harmonic since the n^{th} harmonic has n times the chirp bandwidth making its frequency slope over the same waveform duration n times steeper. (See ref 18, pp. 11-12, and ref 17, section 3.1 for more details on this technique.) Bjorndal points out that in addition to the harmonics, there will also be a number of cross-products, so care must be taken such that the cross terms are also easy to filter out. [ref 18, p. 12.] As Bjorndal, et. al., note “If the desired unambiguous range is large, the above approach may necessitate an impractically long delay.” [ref 17]. In this case, Bjorndal, et. al., provide alternative methods for dealing with the harmonics as detailed in references 17 and 18.

5. Conclusion

The concept and initial performance modeling and simulation results for the new bitstream PC-CAML with a DLLO system were presented in this paper. The results of the initial SNR theory and Monte Carlo simulations presented herein indicate that the bitstream PC-CAML with a DLLO performs as expected for low signal levels.

At high signal levels nearing saturation, the SNR for the simulation results for a chirped sinusoidal signal rolls off due to energy losses to higher order harmonics and to reductions in the apparent modulation depth of the received AM waveform, neither of which are included in the initial SNR theory presented herein. For a fully modulated chirped square wave signal, at high levels near saturation, however, the SNR continues to increase as the mixer output approaches that for a deterministic chirped square wave signal, and the floor of the power spectrum of the mixer output becomes dominated by sidelobes and perhaps aliasing, depending on the signal bandwidth and sample rate. Methods for removing or reducing aliasing developed for bitstream radars can be applied to bitstream PC-CAML if necessary.

The issue of the higher order harmonics generated by the chirped square wave modulation can be mitigated by using the same methods for this used in bitstream radar. The effectiveness for bitstream PC-CAML with a DLLO of one of these methods, that of applying an additional delay on the transmitted waveform to move the harmonics to frequencies higher than the intermediate frequency of the desired unambiguous range, was demonstrated herein with simulation results.

The key advantages of the bitstream PC-CAML with a DLLO are that it can be implemented in the unit cells of a photon counting lidar receiver's ROIC by adding AND binary digital logic gates, and the received signal and LO just consist of streams of single-bit binary data, eliminating the need for bulky, power-hungry, and expensive wideband RF analog electronics. The DLLO single-bit binary waveform data can be computed prior to operation and stored in a buffer in each ROIC unit cell, or in a single buffer and distributed to each ROIC unit cell in real-time during operation. The DLLO data buffer can be a circular buffer for continuous repetition of the DLLO single-bit binary waveform data. Multi-GHz clocks and AND digital logic gates are readily implemented in inexpensive silicon CMOS ROICs, and Multi-tens-of-GHz can be attained with more expensive technologies. These advantages make the bitstream PC-CAML with a DLLO more suitable for compact lidar-on-a-chip systems and lidar array receivers than previous PC-CAML systems.

6. Suggestions for Future Work

Suggestions for future work include:

1. Develop an SNR theory to include the effects of energy loss from the fundamental IF to the higher order odd harmonics and their mixing products in the power spectrum.
2. Develop an SNR theory to include the effects of less than 100% modulation depth for both chirped sinusoidal and chirped square wave signals.
3. Develop an SNR theory that works for both chirped sinusoidal and chirped square wave signals and LOs over the whole range of signal levels, including saturation levels, and for any dead-times, matched filter impulse response times, and clock time intervals.
4. Perform modeling and simulations for the bitstream PC-CAML with a single-bit sampled chirped sinusoidal DLLO for uniform sampling and for non-uniform alias-free sampling.
5. Determine whether or not negative binomial plus Poisson (NBPP) distributed random sampling is alias-free for all spectra for any number of count samples accumulated.
6. For bitstream PC-CAML with a DLLO, investigate detection at a harmonic frequency as used for improved range resolution in bitstream radar.
7. Investigate other techniques for suppressing harmonics and sidelobes for application to the bitstream PC-CAML with a DLLO system.
8. Perform lab experiments to verify the SNR theory and Monte Carlo simulation results.
9. Build and test breadboard and brassboard bitstream PC-CAML with a DLLO prototypes.

7. Acknowledgments

The author thanks Philip Gatt of Lockheed Martin Coherent Technologies (LMCT) and Barry Stann of the United States Army Research Laboratory (ARL) for their helpful discussions and suggestions. The work reported herein was funded and executed solely by the author.

8. References:

1. Patent US7675610B2, "Photon counting, chirped AM LADAR system and related methods," Inventors: Brian Redman, William C. Ruff, and Barry L. Stann, Assignee: US Secretary of Army. Available at <https://patents.google.com/patent/US7675610B2/en>.
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