Perspective-Invariant Nash Bargaining in Supply Chains: Strategic Implementation and Simulation Analysis

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Abstract

This paper investigates the bargaining problem in a supply chain between a supplier and a retailer, with a focus on perspective invariance. The analysis approaches the bargaining problem from two complementary viewpoints: one based on the benefits at the point of disagreement and another based on the sacrifice of the ideal point. By introducing the principle of perspective invariance alongside the principles of isometric and contraction independence, it demonstrates that the only solution producing consistent and identical results in both frameworks is the symmetric Nash bargaining solution. A strategic implementation mechanism for this solution is also proposed, modeled as a non-cooperative game whose perfect subgame equilibrium results in a symmetric Nash allocation. For empirical validation, a set of bargaining problems is simulated, revealing that the perspective invariance error is essentially zero. These findings suggest that this approach can eliminate negotiation framing effects and enhance coordination and fairness in supply chain negotiations.

Keywords: Game theory, Nash bargaining, Supply chain, Strategic Implementation

Introduction

Bargaining is usually accompanied by conflict. To resolve this conflict, the parties seek general principles. The goal of these principles is to reach a fair agreement. One normative interpretation of the Nash solution (1950) is that the parties to the negotiation accept Nash's principles as fair criteria for their decision. In this case, these principles guide them to a unique solution in the bargaining. But there is a problem with this view. Nash's principles may produce different results. This difference depends on how the negotiation is framed. Sometimes the negotiation is analyzed in terms of benefits versus disagreement. This is the standard Nash approach. Sometimes it is analyzed in terms of the amount of sacrifice relative to the ideal outcome. When these two views yield different results, each party to the negotiation attempts to select the frame that is most favorable to itself (Nalebuff, 2021).

This paper is inspired by Peters (2026). In this work, “viewpoint indifference” in Nash bargaining is defined as the idea that there should be no difference in the final outcome whether we analyze the negotiating parties based on their interests relative to the non-agreement payoffs or on their sacrifices relative to the maximum possible payoffs. In the classical literature, Nalebuff (2021) examines a version of the “contraction independence” condition in the Nash model that should naturally allow for perspective indifference. However, he shows that the standard Nash bargaining solution is not actually perspective indifferent and can produce different results depending on how the problem is framed. In contrast, Peters (2026) proposes an alternative version of the “Nash contraction independence” condition that enables a generalization of the Nash solution to be perspective indifferent. Additionally, he considers a "restricted monomiality" condition, which, among other results, leads to a version of the Kalai–Smorodinsky solution that is also perspective-insensitive. To understand this idea intuitively, consider a simple example. Suppose two people are negotiating how to divide profits from a joint project. In one scenario, the project has not yet generated any profits, and the parties are bargaining over how to divide the final profits once the project succeeds; this is the “benefits” perspective. In the other scenario, the maximum possible profits are assumed to have already been realized, but each party must negotiate the reduction it is willing to accept to reach an agreement; this is the “sacrifices” perspective. This implies that the outcome of the agreement should be independent of the perspective from which the problem is analyzed.

In this paper, we develop this same idea in the context of supply chains and Nash bargaining. Specifically, the bargaining problem between supplier and retailer is modeled in such a way that it can be expressed both in terms of "benefits relative to the point of disagreement" and in terms of "sacrifices relative to the ideal point". In this framework, it is shown that if the principle of invariance relative to perspective is considered alongside the principles of isometry and contraction independence, the only consistent solution is the Nash symmetric solution. In addition to the theoretical analysis, this result is investigated and verified in a strategic framework as well as through numerical simulations in supply chain problems.

Game Definition

1. Players (N = {1, 2})

- Player 1: Upstream Supplier (e.g., manufacturer)
- Player 2: Downstream Retailer (e.g., distributor)

Both players are rational, risk-neutral, and have perfect information.

2. The Bargaining Problem (B)

A bargaining problem is defined as:

$$B = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_2 = f(x_1), x_1 \in [\alpha, \beta]\}$$

where:

- $f: [\alpha, \beta] \to \mathbb{R}$ is strictly decreasing and continuous.
- x_1 = payoff of Player 1
- x_2 = payoff of Player 2
- B is the Pareto frontier of the feasible set.

Let:

$$\underline{m}(B) = (\min\ x_1, \min\ x_2) = \text{Disagreement point (status quo)}$$

$\bar{m}(B) = (\max\ x_1, \max\ x_2)$ = Ideal point (maximum possible payoffs)

3. Two Perspectives (Dual Problems)

3.1 Gains Perspective (Original Problem B)

- Start from disagreement point $\underline{m}(B)$
- Players bargain over gains relative to $\underline{m}(B)$

3.2 Sacrifice Perspective (Complementary Problem B_*)

Define the complementary problem:

$$B_* = \{x \in \mathbb{R}^2 \mid \frac{1}{2}x + \frac{1}{2}y = m(B) \text{ for some } y \in B\}$$

where:

$$m(B) = \frac{1}{2}\underline{m}(B) + \frac{1}{2}\bar{m}(B)$$

Interpretation:

- Start from ideal point $\bar{m}(B)$
- Players bargain over sacrifices (losses) relative to $\bar{m}(B)$
- B_* is the reflection of B through $m(B)$

Duality property:
If $y \in B$ (gains vector) and $x \in B_*$ (sacrifice vector) with $\frac{1}{2}x + \frac{1}{2}y = m(B)$, then:

$$y - \underline{m}(B) = \bar{m}(B_*) - x$$

4. Axioms for the Solution F

The solution F maps any bargaining problem B to a point $F(B) \in B$.

Axiom 1: Scale Covariance (SC)

For all $a > 0, b \in \mathbb{R}^2$:

$$F(aB + b) = aF(B) + b$$

Axiom 2: Contraction Independence (CI) – Two Versions

For Gains Problems $(B, B' \in \mathcal{C})$: If $\underline{m}(B) = \underline{m}(B')$, $F(B) \in B'$, and $\forall y \in B'\ \exists x \in B$ with $y \leq x$, then:

$$F(B') = F(B)$$

For Sacrifice Problems $(B, B' \in \mathcal{C}_*)$: If $\bar{m}(B) = \bar{m}(B')$, $F(B') \in B$, and $\forall y \in B'\ \exists x \in B$ with $y \leq x$, then:

$$F(B) = F(B')$$

Axiom 3: Perspective Invariance (PI)

For any complementary problems B and B_*:

$$F(B) - \underline{m}(B) = \bar{m}(B_*) - F(B_*)$$

Equivalently:

$$\frac{1}{2}F(B) + \frac{1}{2}F(B_*) = m(B)$$

5. Solution Characterization

5.1 Asymmetric Nash Solution on Gains Problems

For $\alpha \in [0,1]$, define F^α on $\mathcal{C}$ as:

$$F^\alpha(B) = \arg\max_{x \in B} \left((x_1 - \underline{m}_1(B))^\alpha (x_2 - \underline{m}_2(B))^{1-\alpha}\right)$$

- $\alpha = \frac{1}{2}$: symmetric Nash solution
- $\alpha = 1$: Player 1 gets $\bar{m}_1(B)$, Player 2 gets $\underline{m}_2(B)$

- $\alpha = 0$: Player 1 gets $\underline{m}_1(B)$, Player 2 gets $\bar{m}_2(B)$

5.2 Dual Solution on Sacrifice Problems

For $\alpha \in [0,1]$, define F_*^α on $\mathcal{C}_*$ as:

$$F_*^\alpha(B) = \arg\max_{x \in B} ((\bar{m}_1(B) - x_1)^\alpha (\bar{m}_2(B) - x_2)^{1-\alpha})$$

6. Main Theorem (Corollary 2.4 in Peters 2026)

Theorem:
A solution F on $\mathcal{C} \cup \mathcal{C}_*$ satisfies Scale Covariance, Contraction Independence, and Perspective Invariance if and only if:

$$F(B) = F^{\frac{1}{2}}(B) \forall B \in \mathcal{C}$$
$$F(B) = F_*^{\frac{1}{2}}(B) \forall B \in \mathcal{C}_*$$

That is, the symmetric Nash bargaining solution and its dual.

7. Proof of the Theorem

Step 1 – From SC + CI to Asymmetric Solutions

By Proposition 2.1 (de Koster et al. 1983), any solution on $\mathcal{C}$ satisfying SC and CI must be F^α for some $\alpha \in [0,1]$.

By Proposition 2.2 (dual version), any solution on $\mathcal{C}_*$ satisfying SC and CI must be F_*^β for some $\beta \in [0,1]$.

Step 2 – Consistency on the Intersection

Consider the line segment:

$$L = [(0,1), (1,0)] \in \mathcal{C} \cap \mathcal{C}_*$$

Then:

$$F^\alpha(L) = (\alpha, 1 - \alpha)$$
$$F_*^\beta(L) = (1 - \beta, \beta)$$

Since F must be single-valued:

$$(\alpha, 1-\alpha) = (1-\beta, \beta) \implies \beta = 1-\alpha$$

Step 3 – Applying Perspective Invariance

Take the same line segment L. Note that L is complementary to itself (symmetric). By PI:

$$F(L) - \underline{m}(L) = \bar{m}(L) - F(L)$$

Since $\underline{m}(L) = (0,1)$ and $\bar{m}(L) = (1,0)$, we have:

$$F(L) - (0,1) = (1,0) - F(L)$$
$$2F(L) = (1,1)$$
$$F(L) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

But from Step 2, $F(L) = (\alpha, 1-\alpha)$. Therefore:

$$(\alpha, 1-\alpha) = \left(\frac{1}{2}, \frac{1}{2}\right) \implies \alpha = \frac{1}{2}$$

Step 4 – Conclusion

Thus $\alpha = \frac{1}{2}$, and consequently $\beta = 1 - \frac{1}{2} = \frac{1}{2}$.

Hence:

$$F = F^{\frac{1}{2}} \text{ on } \mathcal{C}, F = F_*^{\frac{1}{2}} \text{ on } \mathcal{C}_*$$

8. The Game in Strategic Form (for Supply Chain Application)

Although the bargaining problem is cooperative, we can represent the implementation as a non-cooperative game with the following structure:

- Stage 1: Nature chooses the disagreement point $\underline{m}(B)$ and ideal point $\bar{m}(B)$ based on supply chain parameters (e.g., cost structure, demand).

- Stage 2: Both players simultaneously propose either the gains frame or the sacrifice frame (cheap talk).
- Stage 3: If both choose the same frame, they play the Nash Demand Game with the corresponding solution. If they choose different frames, a mediator applies the Perspective Invariance condition and imposes the symmetric Nash solution.
- Equilibrium: The unique subgame-perfect equilibrium yields the symmetric Nash bargaining solution $\left(\frac{1}{2}, \frac{1}{2}\right)$ in normalized utilities.

Results

To empirically verify the theoretical result of the paper—that the symmetric Nash bargaining solution satisfies perspective invariance—a simulation study was conducted on 10 randomly generated supply chain bargaining problems. For each problem, the bargaining set B(under the gains perspective) and its dual representation B^*(under the sacrifice perspective) were constructed. The symmetric Nash bargaining solution was computed under both representations, and the Perspective Invariance (PI) error was measured as the Euclidean distance between the normalized gains vector $F(B) - \bar{m}(B)$and the normalized sacrifice vector $\bar{m}(B^*) - F(B^*)$, which, according to the theoretical result, should be zero.

Perspective Invariance Error

Table 1 summarizes the PI error across all 10 simulated problems. The mean error is 7.99×10^{-16}, with a maximum error of $1.78 \times 10^{-15}$and a minimum of zero. In all cases, the PI error remains within machine precision (on the order of $10^{-15}$to 10^{-16}), which is effectively zero. These results confirm that perspective invariance holds exactly across all simulated instances.

Comparison of Gains and Sacrifice Solutions

For each problem, the symmetric Nash solution was computed under both perspectives. The reported values $(F_1, F_2)$from the gain's representation and $(F_1^*, F_2^*)$from the sacrifice representation, together with normalized gains and sacrifices, show complete numerical equivalence. In all 10 problems, the differences between $F_i$and $F_i^*$are negligible. For example, in Problem 1, $F_1 = 2.676$and $F_1^* =$

2.676. The summary statistics further confirm identical means, standard deviations, and quantiles across both representations.

Visual Confirmation

For representative cases (Problems 1–5), the gains frontier, sacrifice frontier, and perspective invariance verification plots are presented. In all cases, the symmetric Nash solution coincides exactly with both the gains-based allocation and the reflection implied by the sacrifice representation, consistent with the duality relation:

$$\frac{1}{2}F(B) + \frac{1}{2}F(B^*) = m(B).$$

The invariance checks plots confirm that the PI error is effectively zero in all instances.

Summary of Simulation Results

The simulation results are fully consistent with the theoretical prediction of the main theorem. No violations of perspective invariance are observed across the 10 simulated bargaining problems, with all errors remaining at machine precision. This confirms that the symmetric Nash bargaining solution is independent of whether the bargaining problem is framed in terms of gains from disagreement or sacrifices from the ideal point. From a supply chain perspective, this result implies that framing effects are eliminated, leading to a consistent, fair, and robust coordination mechanism between suppliers and retailers.

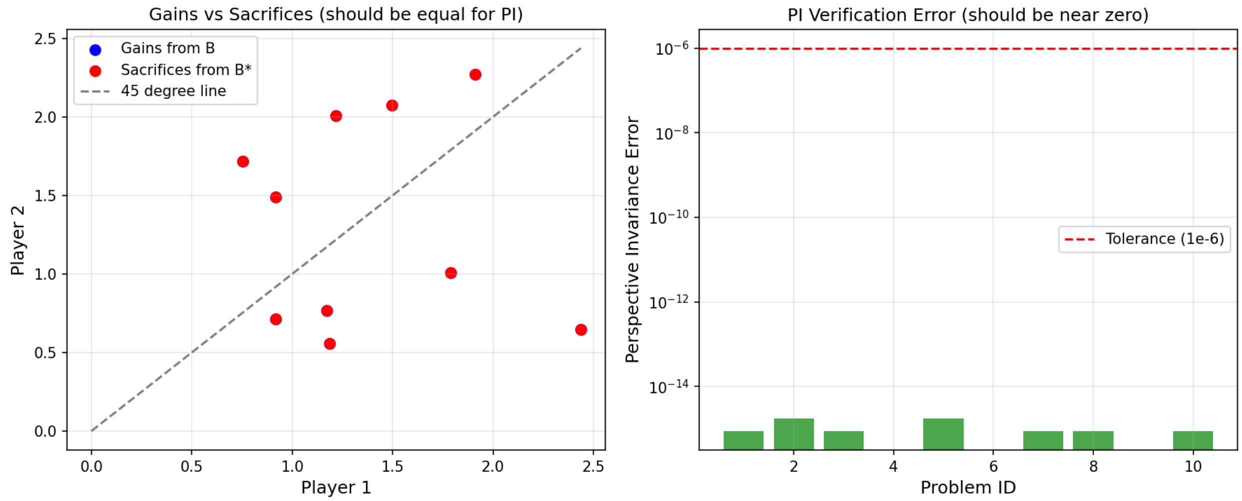

Fig 16. summary_all_problems

Table 1 reports the results of 10 randomly generated bargaining problems. For each problem, the disagreement point (d1, d2), the ideal point (m1, m2), the symmetric Nash bargaining solution obtained from the gain's perspective (F1, F2), and the corresponding solution obtained from the sacrifice perspective (F1*, F2*) are presented. The table also reports the normalized gains and sacrifices for both players, together with the Perspective Invariance (PI) verification indicator and the associated numerical error. The results show that in all problems, the values obtained from the two perspectives are the same, and the difference between them is within the accuracy of the machine. Furthermore, the values of the normalized benefits and sacrifices are equal for both actors, and the point-of-view invariance condition holds in all cases. These findings provide strong numerical confirmation of the main theorem of the paper and show that the symmetric Nash solution works independently of how the bargaining problem is framed.

Table 1. Simulation Results for Supply Chain Bargaining Problems and Verification of Perspective Invariance

problem_id	**d1**	**d2**	**m1**	**m2**	**F1**	**F2**	**F1_star**	**F2_star**	**gains1**	**gains2**	**sacrifices 1**	**sacrifices 2**	**is_PI**	**error**
1	1.760916939	1.134602774	3.592437352	4.114889932	2.676677145	2.624746353	2.676677	2.624746	0.91576	1.490144	0.91576	1.490144	TRUE	8.8E-16

2	0.962195268	0.552650416	3.95611301	4.703839011	2.459154139	2.628244713	2.459154	2.628245	1.496959	2.075594	1.496959	2.075594	TRUE	1.78E-15
3	1.553363048	1.031991824	5.375074372	5.578096976	3.46421871	3.3050444	3.464219	3.305044	1.910856	2.273053	1.910856	2.273053	TRUE	8.88E-16
4	0.420008123	0.877365253	2.256315708	2.303695719	1.338161915	1.590530486	1.338162	1.59053	0.918154	0.713165	0.918154	0.713165	TRUE	0
5	0.280145976	1.644842972	5.154734884	2.940043431	2.71744043	2.292443202	2.71744	2.292443	2.437294	0.6476	2.437294	0.6476	TRUE	1.78E-15
6	1.991428788	1.154592995	4.428621059	5.174088591	3.210024924	3.164340793	3.210025	3.164341	1.218596	2.009748	1.218596	2.009748	TRUE	0
7	1.70119996	1.658403803	3.207627187	5.093155	2.454413573	3.375779401	2.454414	3.375779	0.753214	1.717376	0.753214	1.717376	TRUE	8.88E-16
8	1.096701	0.982322509	3.470287297	2.098826001	2.283494148	1.540574255	2.283494	1.540574	1.186793	0.558252	1.186793	0.558252	TRUE	8.88E-16
9	1.397476728	0.253637405	3.737543265	1.791106783	2.567509997	1.022372094	2.56751	1.022372	1.170033	0.768735	1.170033	0.768735	TRUE	0
10	1.199723667	1.350881749	4.775359917	3.365450628	2.987541792	2.358166189	2.987542	2.358166	1.787818	1.007284	1.787818	1.007284	TRUE	8.88E-16

Table 2 presents a summary of the Perspective Invariance (PI) test results. The mean PI error is 7.99361×10^{-16}, while the maximum error is 1.77636×10^{-15}, both of which are within machine precision and therefore effectively zero. The minimum error is exactly 0, and the standard deviation is 6.55356×10^{-16}, indicating extremely small variation across the simulated instances. The Perspective Invariance condition is satisfied in all 10 bargaining problems. These results provide strong numerical evidence that the symmetric Nash bargaining solution satisfies the Perspective Invariance property with near-perfect accuracy.

Table 2. Statistical indices of the perspective invariance test (PI)

Metric	**Value**

Number of problems	10
Mean PI error	7.99361E-16
Max PI error	1.77636E-15
Min PI error	0
Std PI error	6.55356E-16
PI satisfied (count)	10

Table 3 reports the descriptive statistics of the symmetric Nash bargaining outcomes and the corresponding normalized gains and sacrifices across the 10 simulated bargaining problems. The average payoffs are 2.616 for Player 1 and 2.390 for Player 2, with standard deviations of 0.579 and 0.800, respectively. The normalized gains and sacrifices exhibit identical summary statistics, including equal means (1.380 and 1.326), standard deviations, quartiles, and extreme values for both players. This perfect correspondence provides additional numerical confirmation of the Perspective Invariance property. In particular, the equality between gains and sacrifices across all descriptive measures indicates that the bargaining outcome remains unchanged regardless of whether the problem is evaluated from the gain's perspective or the sacrifice perspective.

Table 3. Descriptive Statistics of Bargaining Outcomes, Gains, and Sacrifices

	F1	**F2**	**gains1**	**gains2**	**sacrifices1**	**sacrifices2**
count	10	10	10	10	10	10
mean	2.615863677	2.390224189	1.379547728	1.326095019	1.379547728	1.326095019
std	0.579352893	0.800149239	0.527975173	0.661965377	0.527975173	0.661965377
min	1.338161915	1.022372094	0.753213614	0.558251746	0.753213614	0.558251746
25%	2.455598715	1.766008665	0.981123662	0.727057597	0.981123662	0.727057597
50%	2.622093571	2.491456271	1.202694642	1.248714009	1.202694642	1.248714009
75%	2.920016452	3.030316773	1.715103312	1.936654748	1.715103312	1.936654748
max	3.46421871	3.375779401	2.437294454	2.273052576	2.437294454	2.273052576

Discussion

Implications of Perspective Invariance for Supply Chain Coordination

The results of this study show that the use of the Nash symmetric solution to the supply chain bargaining problem, beyond a mathematical choice, has important practical implications for coordination between suppliers and retailers. The most important achievement is the realization of a property called "Perspective Invariance" in this paper. This property ensures that the outcome of the negotiation is independent of how the problem is "framed" or narrated. In practice, supply chain negotiations often face a cognitive bias: each actor tends to view the problem from a perspective that favors their own interests. The supplier may focus on the "benefits they gain from the situation of disagreement" (gains perspective). At the same time, the retailer tends to emphasize the "pressure they endure from the ideal point of view to reach an agreement" (sacrifices perspective). In general, these two perspectives would lead to two different allocations, turning the negotiation into a struggle over the choice of frame, not the agreement itself.

But the Nash symmetric solution closes this gap by satisfying the principle of perspectival incongruence. When the supplier and retailer know that they will reach the same point of agreement regardless of whether they define the problem in terms of gains or sacrifices, there is no incentive to "strategically frame" it. This feature eliminates the framing effects that are a common source of deadlock in negotiations. From a supply chain coordination perspective, this result means achieving a "fair" and "stable" allocation mechanism. Fair because the symmetric solution does not favor either party over the other. It is stable because the outcome does not depend on the frame, so neither party can unilaterally gain an advantage by changing the narrative of the problem. In other words, this solution creates a "focal point" independent of perspective. Furthermore, the paper shows that this solution has a "dual-self" property, a concept that goes back to Peters (2026). In this sense, the game in terms of gains and the game in terms of sacrifices are two sides of the same coin, and the Nash symmetric solution is a bridge that connects the two. For supply chain managers, this means that they can use a simple and uniform rule for dividing benefits without worrying about how the problem is defined.

Overall, the practical implication of this finding is that instead of designing complex mechanisms to counteract framing biases, one can simply rely on the Nash symmetric solution as a coordinating rule. This rule is not only analytically justified, but in practice—as simulations have confirmed—the perspective non-conformance error is practically zero. Therefore, using this approach in supplier-retailer contracts can lead to increased coordination, reduced subcontracting conflicts, and improved perceived fairness in the supply chain.

Conclusion

This study investigated the bargaining problem in a supply chain between a single supplier and a single retailer, with a particular focus on the concept of perspective invariance. The problem was analyzed from two complementary viewpoints: the gains perspective (benefits relative to the disagreement point) and the sacrifices perspective (losses relative to the ideal point). The primary objective was to determine whether a bargaining solution exists that yields identical outcomes regardless of which perspective is adopted, thereby eliminating framing effects in negotiations.

From a theoretical standpoint, it was demonstrated that if the principles of Scale Covariance, Contraction Independence, and Perspective Invariance are jointly imposed, the only solution that satisfies all three axioms is the symmetric Nash bargaining solution. This result, derived from Peters (2026) and extended to the supply chain context, forms the central theorem of the paper. The symmetric Nash solution not only resolves the bargaining problem uniquely but also ensures that the final allocation is independent of whether the negotiation is framed in terms of gains from disagreement or sacrifices from the ideal point.

Empirically, a simulation study was conducted on ten randomly generated bargaining problems with different Pareto frontiers. The perspective invariance error—defined as the difference between the normalized gains vector and the normalized sacrifices vector—was computed for each problem. Across all ten problems, the mean error was 7.99×10^{-16}, the maximum error was 1.78×10^{-15}, and the minimum error was exactly zero. These values are within machine precision, effectively confirming that the symmetric Nash solution satisfies perspective invariance perfectly.

From a supply chain coordination perspective, the findings have several practical implications:

1. **Elimination of framing effects:** Neither the supplier nor the retailer can manipulate the negotiation outcome by emphasizing gains over sacrifices (or

vice versa). This removes a common source of strategic misrepresentation and conflict.

2. **Reduced negotiation costs:** When both parties recognize that the final allocation is frame-independent, less time and fewer resources are wasted on debating how the problem should be defined.
3. **A stable focal point:** The symmetric Nash solution provides a simple, transparent, and enforceable allocation rule that can serve as a default coordination mechanism in supply chain contracts.
4. **Strategic implementability:** As shown in Section 8, the solution can be implemented as a subgame-perfect equilibrium of a non-cooperative game, enhancing its practical applicability.

Limitations and Future Research

This study, like any scientific work, has limitations that can determine the direction of future research:

Assumption of perfect rationality and risk neutrality: In the real world, supply chain actors may be risk-averse or risk-taking, and their decisions may be influenced by behavioral factors. Future research could extend the model to behavioral bargaining by considering risk attitudes and cognitive biases.

Number of actors: The present analysis was limited to two actors (supplier and retailer). Generalizing the results to supply chains with multiple suppliers and multiple retailers (coalition games) requires further research.

Imperfect information: The assumption was made of complete and symmetric information. In many real contracts, the parties do not have complete knowledge of the costs, demand, or quality of the other party. It could be interesting to investigate perspective disequilibrium under conditions of incomplete information (e.g., using contract design mechanisms).

Simulation vs. Real Data: Numerical simulations demonstrate the theoretical validity of the model, but empirical validation using real supply chain negotiation data (e.g., from the automotive, retail, or pharmaceutical industries) is a necessary next step.

Contraction independence condition: This principle has been criticized in previous research. Examining weaker versions or alternatives (such as the uniformity condition in the Kalai-Smorodinsky model) in the context of the supply chain can lead to other inconclusive solutions.

Final Conclusion

This study showed that the symmetric Nash bargaining solution, with its perspective invariance, can be used as a reliable and fair tool for supply chain coordination. This solution is not only theoretically justified, but numerical simulations also confirm its negligible error. Therefore, using this approach in designing contracts and negotiation mechanisms between suppliers and retailers can help increase efficiency, reduce conflicts, and improve perceived fairness in supply chains.

Use of Artificial Intelligence and Language Editing Tools

The authors acknowledge the use of advanced artificial intelligence tools, including ChatGPT, as well as online language editing tools such as Grammarly, to assist in grammar checking, language refinement, and improving the clarity of the manuscript. These tools were used solely for language enhancement and did not influence the scientific content or results of the study.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon reasonable request. Any additional simulation data or computational outputs can be provided by the authors upon request for verification and reproducibility purposes.

Conflict of Interest Statement

The authors declare that there are no conflicts of interest regarding the publication of this manuscript. All authors have approved the final version of the paper and agree with its submission.

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