

# ***Hexa-Wankel:***

A Geometric and Thermodynamic Hypothesis  
for a Six-Apex Rotary Engine with Two-Phase Combustion

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Conceptual paper. This document is not a prototype claim, not a final patent claim, and not a manufacturing specification. Performance claims are presented as design-threshold conditions that must be verified through CAD, CFD, FEA, seal-tribology testing, and dynamometer testing.

## Abstract

This paper formulates the *Hexa-Wankel* as a hypothesis for a six-apex rotary engine intended to combine the power density of the triangular *Wankel* engine with the thermal-sealing predictability of the piston engine. The design definition used here is a convex six-apex rotor operating within a Wankel-like five-lobe baseline housing, not a flat hexagonal housing. Two combustion zones are fixed in housing coordinates and operate in phase-separated sequence; the total fuel dose is not doubled, but divided into two pulses,  $Q/2+Q/2$ . The design objective is not to reduce the mean engine temperature, but to render the thermal and pressure maps more stable, periodic, and predictable enough for apex-seal force to be specified more precisely. The paper derives an initial kinematic closure for an  $N$ -apex rotor, the envelope regularity condition, swept-volume scaling, friction thresholds, apex-only leakage thresholds, Fourier thermal-harmonic analysis, the relationship between phase angle and the torque kernel  $K_i(\theta) = dV_i/d\theta$ , and the gas-exchange constraints of a six-chamber architecture. As an initial validation step, the paper also includes a two-dimensional kinematic CAD model and numerical chamber-volume calculation for the six-apex/five-lobe baseline. This validation establishes basic geometric feasibility and chamber-volume periodicity, but it does not establish performance superiority; the torque kernel, phased-combustion compatibility, port timing, CFD, FEA, and three-dimensional leakage remain subsequent tests. The principal result is therefore not a final efficiency prediction, but a set of threshold conditions: to avoid being defeated by seal friction, the six-apex candidate must satisfy  $\lambda_F \lambda_v < 0.5$ ; to avoid being defeated by leakage, it must satisfy  $2\lambda_A \sqrt{\lambda_p \lambda_t} < 1$ ; and to approach a modern petrol piston engine at approximately 40% brake thermal efficiency at a target  $r = 10.5$ , the total realisation factor must satisfy  $\Xi_H > 0.713$ . The *Hexa-Wankel* is consequently a coherent first-order design hypothesis, but it remains falsifiable: it fails if the six-apex/five-lobe housing cannot produce sealed chambers, an adequate compression ratio, a two-phase torque kernel, sufficient port timing, and measurable reductions in thermal variance and seal loss.

**Keywords:** Wankel engine; rotary engine; apex seal; phased combustion; thermal harmonics; five-lobe housing; effective displacement; threshold theory.

## 1 Introduction

The classical Wankel engine possesses mechanical advantages that are difficult for piston engines to reproduce: compactness, low vibration, few reciprocating components, and high power density. In a conventional Wankel engine, a three-apex rotor forms three working chambers inside an epitrochoidal housing; the eccentric shaft rotates three times for one material revolution of the rotor, so that one rotor produces one power event per shaft revolution in an effective four-stroke cycle [5]. This compactness makes the Wankel engine attractive for small vehicles, range extenders, drones, and other applications in which power-to-weight ratio is important.

The weaknesses of the Wankel engine, however, are equally fundamental. Its long and thin combustion chamber does not burn as ideally as the cylinder of a piston engine. The apex seal must seal a moving chamber while sweeping through housing regions with non-uniform thermal maps. The literature on apex seals identifies gas leakage, wear, vibration, and possible seal lift-off as central problems in rotary engines [6, 7, 8]. Compared with a piston engine, the Wankel engine faces a more difficult sealing problem because its seals do not operate in a simple cylindrical geometry, but along a continuously varying housing trajectory.

The advantage of the conventional piston engine is not merely its historical efficiency, but the predictability of the ring-cylinder system. In a piston engine, ring tension, gas pressure behind the ring, oil film, clearance, and thermal expansion can be calculated within a stable

cylindrical path. Modern piston engines can therefore reach high thermal efficiency; Toyota, for example, states that its Dynamic Force Engine achieves 40% thermal efficiency for the conventional petrol variant and 41% for the hybrid variant [4]. The main problem for a new rotary architecture is therefore not simply to increase the number of chambers, but to make the seal and thermal environment sufficiently predictable, in a manner analogous to the piston system.

The *Hexa-Wankel* is proposed as a hypothesis addressing this problem. Its core is not a naive “six-sided Wankel”, but a six-apex rotary architecture that uses two phased combustion pulses to stabilise the distribution of heat and pressure. A single fuel dose is divided into two pulses, so the target is not additional energy, but a cleaner conversion of the same chemical energy in terms of torque angle, peak pressure, thermal gradient, and seal load.

## 2 Design Definition and Hypothesis Statement

### 2.1 Working definition

In this paper, the term *Hexa-Wankel* denotes the following conceptual design:

1. the rotor has six apexes and six convex sides;
2. the kinematic baseline is Wankel-like: a six-apex rotor moves inside a five-lobe housing obtained from the envelope of apex trajectories;
3. the term “hexa” refers to the number of rotor apexes/chambers, not to the number of housing lobes;
4. two combustion zones are placed at distinct housing-coordinate positions and operate in a phased sequence;
5. the total fuel per cycle is not doubled, but divided into two pulses:

$$Q_{tot} = Q_1 + Q_2, \quad Q_1 = Q_2 = \frac{1}{2}Q_{tot}. \quad (1)$$

Under this definition, the *Hexa-Wankel* creates no additional energy. The claim is that the same chemical energy may be converted with lower peak stress, lower thermal variance, and lower leakage.

### 2.2 Main hypothesis

If a six-apex rotor in a five-lobe housing can produce sealed chambers with a competitive compression ratio, two positive expansion windows, sufficient gas exchange, and a more stable thermal map, then two combustion pulses  $Q/2 + Q/2$  may allow a rotary engine to approach piston-like seal predictability while retaining Wankel-like power density.

The operative word is “if”. This paper does not assert that the *Hexa-Wankel* is necessarily more efficient. It derives the threshold conditions under which the claim is not physically empty.

### 3 Kinematic Closure: Six-Apex Rotor in a Five-Lobe Housing

#### 3.1 Reference case: the conventional Wankel

In the conventional Wankel engine, a three-apex rotor moves inside a two-lobe housing. The internal gearing constrains the eccentric shaft to rotate three times for one material revolution of the rotor [5, 1]. In other words, the rotor apex count  $N = 3$  is associated with a two-lobe housing and a shaft-to-rotor speed ratio of 3 : 1.

To remove geometric ambiguity, this paper adopts the following Wankel-like generalisation as its baseline:

$$N = 6, \quad L = N - 1 = 5, \quad (2)$$

where  $N$  is the number of rotor apexes and  $L$  is the number of housing lobes. The baseline *Hexa-Wankel* is therefore:

$$\boxed{\text{six-apex rotor in a five-lobe Wankel-like housing.}} \quad (3)$$

This choice is essential. Without kinematic closure, chamber volume, phase angle, port timing, and thermal balancing would not refer to the same mechanical object.

#### 3.2 Baseline motion law

Let  $\theta_s$  be the eccentric-shaft angle relative to the housing. The rotor centre travels around a circle of eccentricity  $e$ :

$$\mathbf{C}(\theta_s) = e \begin{pmatrix} \cos \theta_s \\ \sin \theta_s \end{pmatrix}. \quad (4)$$

For a Wankel-like  $N$ -apex baseline, the material orientation of the rotor relative to the housing is written as:

$$\psi(\theta_s) = -\frac{\theta_s}{N} + \psi_0. \quad (5)$$

For the *Hexa-Wankel*,  $N = 6$ , hence:

$$\psi(\theta_s) = -\frac{\theta_s}{6} + \psi_0. \quad (6)$$

The shaft therefore performs six revolutions for one material revolution of the rotor. This is the baseline kinematic assumption. Mechanically, the law may be represented by a fixed gear and an internal rotor gear with a 5 : 6 tooth ratio in the Wankel-like sense: the fixed sun gear has  $5q$  teeth and the internal rotor gear has  $6q$  teeth for some manufacturing-suitable integer  $q$ . This ratio is not a detailed tooth-profile specification, but the minimal kinematic closure required for a six-apex rotor to operate against a five-lobe housing. Other conjugate-pair designs may employ different motion laws, but they are not treated as the main design in this paper.

The six local rotor apexes are defined as:

$$\mathbf{a}_k = \rho_k \begin{pmatrix} \cos \alpha_k \\ \sin \alpha_k \end{pmatrix}, \quad \alpha_k = \frac{2\pi k}{6}, \quad k = 0, \dots, 5. \quad (7)$$

For a first-order symmetric rotor,  $\rho_k = \rho$ . The global position of apex  $k$  is:

$$\mathbf{A}_k(\theta_s) = \mathbf{C}(\theta_s) + \mathbf{R}(\psi(\theta_s))\mathbf{a}_k, \quad (8)$$

where  $\mathbf{R}$  is the two-dimensional rotation matrix. The ideal housing is the envelope of the trajectories of all apexes:

$$\mathcal{H} = \partial \left( \bigcup_{\theta_s \in [0, 2\pi N)} \bigcup_{k=0}^5 \mathbf{A}_k(\theta_s) \right). \quad (9)$$

Equation (9) is the geometric definition that must be realised in CAD. The paper does not claim that the conceptual drawing is already a manufacturing profile.

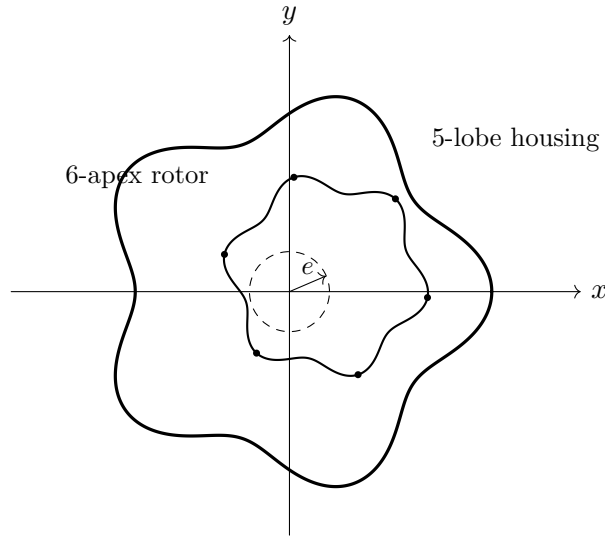


Figure 1: Initial *Hexa-Wankel* schematic as a kinematic baseline: six rotor apexes/chambers inside a five-lobe Wankel-like housing.

### 3.3 Consistency between housing and rotor coordinates

Two different coordinates must be separated:

1. housing coordinates, where ports and combustion zones are fixed;
2. rotor/shaft coordinates, where the chamber work phase angle is calculated.

Because  $\psi(\theta_s) = -\theta_s/N$ , the optimum phase angle for mechanical work is not automatically the same as a  $180^\circ$  separation in the housing. The two opposed combustion zones in housing coordinates should therefore be read as a thermal-balancing target, whereas their compatibility with  $\Delta\phi^*$  must be calculated from  $V_i(\theta_s)$  and  $K_i(\theta_s)$ .

### 3.4 Baseline firing sequence

To remove operational ambiguity, this paper uses the following baseline firing sequence. The six chambers are indexed  $i = 0, \dots, 5$  following the rotor-apex order. The right combustion zone  $Z_R$  serves the even-indexed chambers (0, 2, 4), whereas the left combustion zone  $Z_L$  serves the odd-indexed chambers (1, 3, 5). Consequently, the  $Q/2 + Q/2$  fuel split is not defined for a single chamber, but for an equivalent event pair, for example one even event at  $Z_R$  and the following odd event at  $Z_L$ .

$$Q_{\text{pair}} = Q_R + Q_L, \quad Q_R \simeq Q_L \simeq \frac{1}{2}Q_{\text{pair}}, \quad (10)$$

with a candidate mechanical offset  $\Delta\phi$  to be selected so that both pulses fall within the positive part of the torque kernel. At the chamber-individual level, even chambers receive their pulse from  $Z_R$  and odd chambers receive their pulse from  $Z_L$ ; what is compared with one triangular Wankel pulse is the effective event pair  $Q_{\text{pair}}$ , not twice the fuel. This scheme is not intended to be the only possible firing order, but an analytical baseline: even and odd chambers are separated so that the two housing zones can provide thermal balancing, while mechanical optimisation remains determined by  $K_i(\theta_s) = dV_i/d\theta_s$ .

## 4 Envelope Geometry and Regularity

### 4.1 Initial curve family

For first-order analysis, the following trochoidal/peritrochoidal curve family is used:

$$\mathbf{H}_n(\theta) = R \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} + e \begin{pmatrix} \cos n\theta \\ \sin n\theta \end{pmatrix}, \quad 0 \leq \theta < 2\pi. \quad (11)$$

Drogosz emphasises that Wankel geometry must be formalised mathematically because the housing curve, rotor, and eccentricity parameters determine the chamber geometry [1]. Equation (11) is not the final profile, but a scaling law for regularity and initial displacement. The final housing profile must still be calculated from the kinematic envelope in Equation (9); in other words,  $\mathbf{H}_n(\theta)$  is a first-order approximation for studying regularity limits and displacement scale, not a substitute for the actual chamber boundary. Since the baseline *Hexa-Wankel* uses a six-apex rotor in a five-lobe housing, every final volume figure must be derived from the rotor-housing pair in Equations (8)-(9), not from  $\mathbf{H}_n$  alone.

The derivative is:

$$\mathbf{H}'_n(\theta) = R \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix} + ne \begin{pmatrix} -\sin n\theta \\ \cos n\theta \end{pmatrix}. \quad (12)$$

Its norm is:

$$\|\mathbf{H}'_n(\theta)\|^2 = R^2 + n^2e^2 + 2nRe \cos((n-1)\theta). \quad (13)$$

The minimum occurs when  $\cos((n-1)\theta) = -1$ , hence:

$$\min_{\theta} \|\mathbf{H}'_n(\theta)\| = |R - ne|. \quad (14)$$

The no-cusp condition is therefore:

$$\boxed{R > ne.} \quad (15)$$

For the six-apex candidate:

$$R > 6e. \quad (16)$$

If  $R = 105$  mm, then:

$$e < 17.5 \text{ mm}. \quad (17)$$

With a conservative eccentricity  $e_H = 7.5\text{--}10$  mm, the regularity margin is greater than with  $e = 15$  mm.

## 4.2 Status of the displacement formula

The first-order swept-volume formula used here is:

$$V_{d,n} \simeq 2nbRe \sin\left(\frac{2\pi}{n}\right), \quad (18)$$

where  $b$  is the rotor width/depth. This formula must be read as a *scaling law*, not as a substitute for chamber-volume computation. The final displacement must be calculated from the actual chamber area:

$$V_i(\theta_s) = bA_i(\theta_s). \quad (19)$$

For  $n = 3$ :

$$V_{d,3} = 6bRe \sin\left(\frac{2\pi}{3}\right) = 3\sqrt{3}bRe, \quad (20)$$

which is consistent with the usual form of the Wankel displacement formula. For  $n = 6$ :

$$V_{d,6} = 12bRe \sin\left(\frac{2\pi}{6}\right) = 6\sqrt{3}bRe. \quad (21)$$

Thus, at the same  $R$ ,  $e$ , and  $b$ :

$$\frac{V_{d,6}}{V_{d,3}} = 2. \quad (22)$$

This result must not be read as proof of doubled efficiency. It means only that, for the same effective displacement, the six-apex candidate may reduce either  $e$  or  $b$ .

### 4.3 Scaling for equal effective displacement

To give the six-apex candidate an effective displacement equivalent to the three-apex reference, three scaling strategies are available:

Strategy	First-order condition	Design consequence
width reduction	$b_H = \frac{1}{2}b_W$	axially shorter engine
eccentricity reduction	$e_H = \frac{1}{2}e_W$	improved orbit, sliding severity, and regularity margin
hybrid scaling	$b_H e_H = \frac{1}{2}b_W e_W$	compromise between packaging and tribology

Table 1: Scaling strategies for equalising the effective displacement of the six-apex candidate and the three-apex reference.

Take, for example:

$$R = 105 \text{ mm}, \quad e_W = 15 \text{ mm}, \quad b = 80 \text{ mm}. \quad (23)$$

Then:

$$V_{d,3} = 3\sqrt{3}(0.080)(0.105)(0.015) \quad (24)$$

$$\approx 6.55 \times 10^{-4} \text{ m}^3 \quad (25)$$

$$\approx 655 \text{ cc}. \quad (26)$$

For  $n = 6$  with  $e_H = 7.5 \text{ mm}$ :

$$V_{d,6} = 6\sqrt{3}(0.080)(0.105)(0.0075) \quad (27)$$

$$\approx 655 \text{ cc}. \quad (28)$$

At equal effective displacement, the six-apex candidate can therefore use half the eccentricity of the three-apex reference. This provides a physical basis for the hypothesis of reduced sliding severity, but it does not yet prove leakage reduction or efficiency improvement.

## 5 Compression Ratio from Chamber Volume

Compression ratio must not be assumed from a target value. It must be calculated from the actual chamber volume:

$$r_i = \frac{V_{i,\max}}{V_{i,\min}} = \frac{V_{c,i} + V_{d,i}}{V_{c,i}}. \quad (29)$$

With  $V_d = 655 \text{ cc}$ , a target  $r = 10.5$  requires:

$$V_c = \frac{V_d}{r - 1} = \frac{655}{9.5} = 68.9 \text{ cc}. \quad (30)$$

This clearance corresponds to:

$$\frac{V_c}{V_d} = 0.105. \quad (31)$$

A target  $r = 10.5$  therefore means that the minimum volume must be approximately 10.5% of the swept volume. This is plausible as an order-of-magnitude target, but it does not prove that the six-apex/five-lobe housing can achieve it. The geometric criterion to be satisfied is:

$$r_{H,i} = \frac{\max_{\theta_s} A_i(\theta_s)}{\min_{\theta_s} A_i(\theta_s)} \geq 10. \quad (32)$$

If CAD gives  $r_{H,i} < 9$ , then the high-efficiency claim fails or requires modification through a rotor pocket, flank curvature, or variable-compression geometry.

## 6 Initial Kinematic CAD Validation and Actual Chamber Volume

To test whether the kinematic definition in Section 3 can generate actual computable chambers, a first-order two-dimensional kinematic CAD model was produced. This model is not a final manufacturing CAD, but an initial geometric validation of two features: the housing-rotor form and the chamber-volume curves  $V_i(\theta)$ .

The model parameters are:

$$N = 6, \quad L = 5, \quad R = 60 \text{ mm}, \quad e = 8 \text{ mm}, \quad b = 80 \text{ mm}. \quad (33)$$

The regularity condition is satisfied because:

$$R > Ne, \quad 60 > 48. \quad (34)$$

The housing is calculated from the five-lobe envelope trajectory, whereas the rotor is represented as six convex flanks with six apex contact points. Chamber area  $i$  is calculated from the closed polygon between the corresponding housing segment and rotor flank. The volume is obtained as:

$$V_i(\theta) = bA_i(\theta), \quad (35)$$

where  $A_i(\theta)$  is the numerical chamber area in  $\text{mm}^2$  and  $b$  is the rotor width in mm. Conversion to cc uses:

$$V_{i,cc}(\theta) = \frac{bA_i(\theta)}{1000}. \quad (36)$$

The raw numerical result indicates that all chambers are symmetric, with the following values:

The important finding is that the ideal geometry without a pocket gives a near-zero  $V_{min}$ . Hence the raw compression ratio  $r_{raw} \approx 672$  is unrealistic and must not be used as a performance claim. The design-relevant result is the required additional clearance:

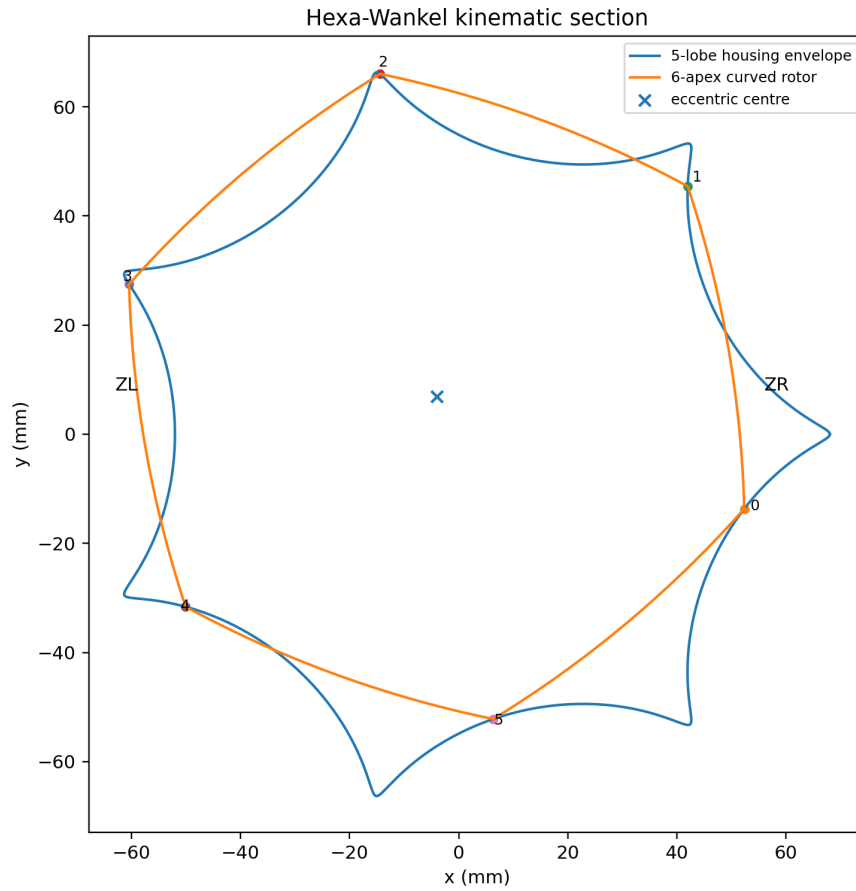


Figure 2: Two-dimensional kinematic CAD validation: a six-apex rotor within the baseline five-lobe housing. ZR and ZL denote two fixed combustion zones in housing coordinates.

Model	$V_{min}$	$V_{max}$	$V_d$	$r_{raw}$
Raw geometric chamber	0.053 cc	35.839 cc	35.786 cc	672.1
Clearance-corrected $r = 10.5$	3.767 cc	39.552 cc	35.786 cc	10.5

Table 2: Actual chamber-volume validation for the baseline  $R = 60$  mm,  $e = 8$  mm, and  $b = 80$  mm. The raw value must not be read as a real engine compression ratio because it does not include a combustion pocket or clearance volume.

$$V_{c,add} = \frac{V_d}{r_t - 1} - V_{min,raw}, \quad (37)$$

where  $r_t = 10.5$ . With  $V_d = 35.786$  cc and  $V_{min,raw} = 0.053$  cc, one obtains:

$$V_{c,add} = 3.714 \text{ cc}. \quad (38)$$

The corrected minimum volume becomes:

$$V_{min,corr} = 3.767 \text{ cc}, \quad (39)$$

and the corrected maximum volume becomes:

$$V_{max,corr} = 39.552 \text{ cc}. \quad (40)$$

The initial kinematic CAD model therefore supports two limited conclusions. First, actual chambers can be defined and numerically calculated in the six-apex/five-lobe baseline. Second, the target  $r = 10.5$  does not arise automatically from the ideal envelope, but requires an explicit combustion-pocket or clearance-volume design. This strengthens the paper's position: compression ratio is a geometric requirement to be designed, not a performance assumption.

A combustion pocket is not merely a mathematical volume addition. In a real engine, the pocket must be located and shaped so that it does not create a large dead volume, excessive flame travel, strong near-wall quenching, or increased hydrocarbon loss. The clearance-corrected result in Table 2 must therefore be read as a combustion-chamber geometry target, not as proof that combustion is already efficient. Pocket design is one of the decisive issues for the *Hexa-Wankel* before CFD analysis can be undertaken.

## 6.1 Limits of interpretation of the 2D model

The results in Section 6 must be read strictly as two-dimensional geometric validation, not as engine-performance validation. The model shows that chambers can be formed and actual volume  $V_i(\theta)$  can be computed, but it does not show that two-phase combustion, three-dimensional sealing, gas exchange, or thermal deformation will work better than in a Wankel or piston engine.

There are five main limits. First, the model remains two-dimensional: it does not include side-housing thickness, corner seals, side seals, oil film, axial thermal expansion, or housing deformation. Second, the additional clearance of 3.714 cc per chamber, approximately 10.4% of  $V_d$ , is not a defect in itself, but it is a combustion-pocket design burden. The pocket shape must preserve compression ratio without excessive surface-to-volume ratio, quenching, or hydrocarbon loss. Third, phased combustion has not yet been verified against the actual torque kernel. A 180° separation in housing coordinates provides only a thermal-balancing target; it must be tested against  $K_i(\theta) = dV_i/d\theta$  so that both pulses occur during positive expansion windows. Fourth, port timing is still represented by a worst-case estimate; with six chambers, intake and exhaust duration per chamber may become shorter and may reduce volumetric efficiency. Fifth, the model scale remains small, with  $V_d \approx 35.8$  cc per chamber. Scaling up to a total engine of 650–1300 cc must be recalculated because surface-to-volume ratio, seal force, sliding velocity, thermal distortion, and cooling requirements do not scale linearly.

The present validation status is therefore: basic 2D geometry passes; performance does not yet pass. Efficiency claims still depend on the next stages, especially numerical torque kernel analysis, phase-angle optimisation, combustion-pocket design, port timing, CFD, FEA, and apex-corner-side seal leakage modelling.

Validated at this stage	Not yet validated / next stage
Six-apex/five-lobe kinematic closure and baseline gear law	Torque, power, BSFC, or BTE advantage over Wankel/piston engines
Periodic chamber formation in the 2D model	Numerical torque kernel and two-pulse combustion compatibility
Actual chamber-volume computation $V_i(\theta)$	Actual port timing and volumetric efficiency
Required clearance volume for target $r = 10.5$	Combustion-pocket design, flame travel, quenching, and HC loss
Initial envelope regularity and 2D CAD visualisation	Apex-corner-side seal leakage, oil film, FEA, CFD, and dynamometer testing

Table 3: Validation status of the present paper. The table separates results shown by the 2D kinematic model from performance claims that still require testing.

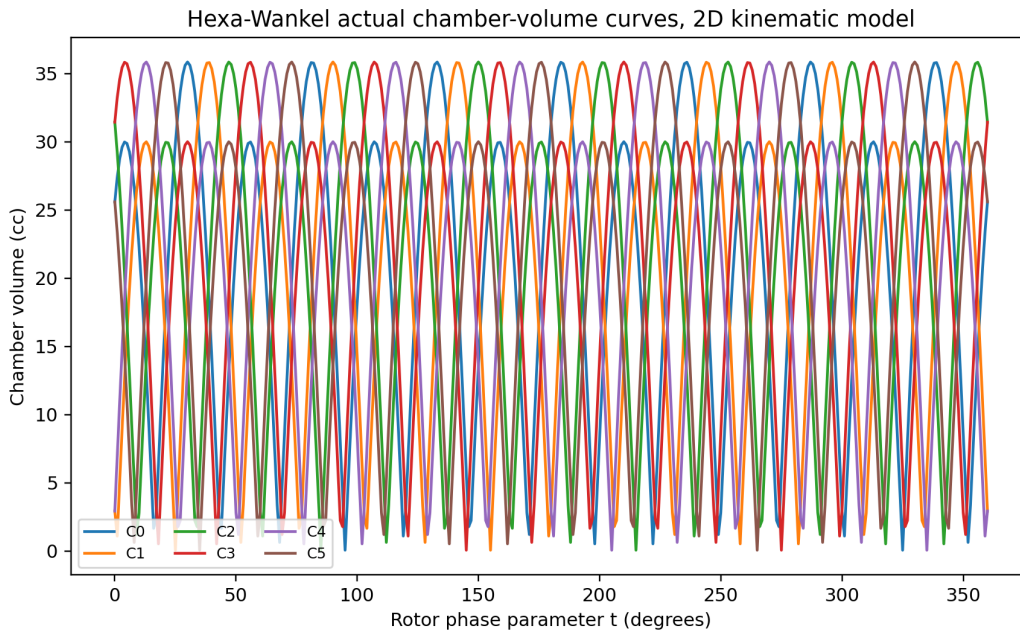


Figure 3: Raw chamber-volume curves from the 2D kinematic model. The near-zero minimum volume shows why a real design must include a combustion pocket or clearance volume.

## 7 Indicative Work and Phase Angle

For any rotary chamber, the indicated work is:

$$W_i = \oint p_i(\theta) dV_i(\theta). \quad (41)$$

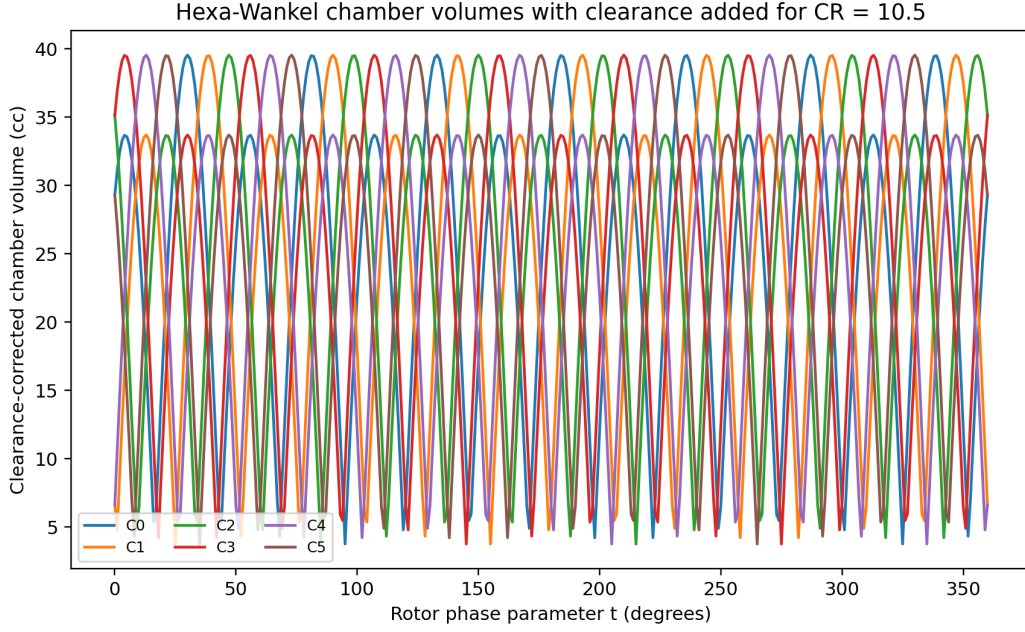


Figure 4: Clearance-corrected chamber-volume curves for the target compression ratio  $r = 10.5$ . These curves are suitable for subsequent torque-kernel extraction, but not yet for performance prediction.

The torque contribution can be written as:

$$\tau_i(\theta) = p_i(\theta) \frac{dV_i}{d\theta}. \quad (42)$$

Define the chamber torque kernel:

$$K_i(\theta) = \frac{dV_i}{d\theta}. \quad (43)$$

Only intervals for which  $K_i(\theta) > 0$  can produce positive expansion work. A two-pulse combustion strategy is therefore mechanically meaningful only if both pulses are placed on positive and sufficiently large sections of  $K_i$ .

For the triangular *Wankel* reference, a simplified single-pulse work model is:

$$W_W \approx \int_{\theta_a}^{\theta_b} p_W(\theta) K_W(\theta) d\theta. \quad (44)$$

For the *Hexa-Wankel* two-pulse candidate:

$$W_H(\Delta\phi) \approx \frac{1}{2} \int p_R(\theta) K_R(\theta) d\theta + \frac{1}{2} \int p_L(\theta + \Delta\phi) K_L(\theta + \Delta\phi) d\theta. \quad (45)$$

The optimum mechanical phase angle is therefore:

$$\Delta\phi^* = \arg \max_{\Delta\phi} W_H(\Delta\phi). \quad (46)$$

This is not merely the statement that the best phase is the best phase, because  $K_i(\theta)$  must be obtained from the actual chamber geometry. Once  $V_i(\theta)$  is computed by CAD, Equation (46) becomes a numerical test. If the optimum  $\Delta\phi^*$  conflicts with the 180° thermal-balancing separation in housing coordinates, the design must be revised.

## 8 Thermal Stabilisation Through Fourier Harmonics

### 8.1 Not a colder engine

The *Hexa-Wankel* hypothesis is not that the engine is simply colder. A combustion engine must retain sufficient gas temperature during expansion. The relevant target is not lower mean temperature, but lower thermal non-uniformity in the housing:

$$\Delta T_h = \max_{x \in \mathcal{H}} T(x) - \min_{x \in \mathcal{H}} T(x), \quad (47)$$

or, more generally, lower thermal variance:

$$\sigma_T^2 = \frac{1}{|\mathcal{H}|} \int_{\mathcal{H}} (T(x) - \bar{T})^2 dx. \quad (48)$$

Let the housing heat-flux distribution from a combustion zone at angular position  $\theta_R$  be:

$$\dot{q}_R(\theta) = q_0 F(\theta - \theta_R). \quad (49)$$

Expand  $F$  as a Fourier series:

$$F(\theta) = \sum_{m=-\infty}^{\infty} F_m e^{im\theta}. \quad (50)$$

If the second combustion zone is placed at:

$$\theta_L = \theta_R + \pi, \quad (51)$$

then the total housing heat flux is:

$$\dot{q}_H(\theta) = q_0 F(\theta - \theta_R) + q_0 F(\theta - \theta_L) \quad (52)$$

$$= q_0 \sum_m F_m e^{im(\theta - \theta_R)} [1 + e^{-im\pi}] \quad (53)$$

$$= q_0 \sum_m F_m e^{im(\theta - \theta_R)} [1 + (-1)^m]. \quad (54)$$

For odd  $m$ :

$$1 + (-1)^m = 0. \quad (55)$$

Thus all odd harmonics vanish. The physical interpretation is that two opposed combustion zones can suppress the first-order thermal asymmetry of the housing. This is the strongest mathematical argument for the thermal-stabilisation claim.

## 8.2 Limitation of the Fourier analysis

The cancellation in Equation (54) is a housing-coordinate result. It does not automatically prove mechanical optimality. Mechanical work depends on the rotor/shaft phase angle through  $K_i(\theta)$ , whereas thermal balancing depends on fixed housing positions. Therefore, the following compatibility condition is necessary:

$$|\Delta\phi^* - \Delta\phi_{thermal}| < \epsilon_\phi, \quad (56)$$

where  $\Delta\phi_{thermal}$  is the phase separation implied by the two opposed housing zones after being mapped into the chamber work coordinate. If Equation (56) fails, the thermal and mechanical optima are not aligned.

## 9 Surface-to-Volume Hazard

Dividing one chamber into two smaller events can reduce peak heat flux, but it can also increase the surface-to-volume ratio. If one chamber of volume  $V$  is divided into two geometrically similar chambers of volume  $V/2$ , then:

$$A_{new} = 2A \left(\frac{1}{2}\right)^{2/3} = 2^{1/3}A \approx 1.26A. \quad (57)$$

This is a 26% increase in total wall area for the same volume. The effect is beneficial only if the reduction in thermal non-uniformity and seal load exceeds the additional wall heat loss. Thus the design condition is:

$$\Delta W_{thermal\ stability} > \Delta Q_{wall\ loss}. \quad (58)$$

This is why the *Hexa-Wankel* must be treated as a threshold theory rather than a claim of automatic efficiency.

## 10 Seal, Friction, Leakage, and Wear

### 10.1 Apex-seal force

A first-order apex-seal load model is:

$$F_s = F_{spring} + F_{gas} + F_{inertia}. \quad (59)$$

The contact pressure is:

$$p_c = \frac{F_s}{A_c}. \quad (60)$$

A seal must satisfy two conflicting conditions:

$$F_s > F_{lift-off}, \quad (61)$$

but:

$$F_s \text{ must not be so large that friction and wear dominate.} \quad (62)$$

The *Hexa-Wankel* hypothesis is that a more periodic pressure and thermal map may reduce the required safety margin in  $F_{spring}$ .

## 10.2 Friction threshold

For the triangular Wankel reference, write the total apex friction loss as:

$$P_{f,W} = 3\mu F_W v_W. \quad (63)$$

For the six-apex candidate:

$$P_{f,H} = 6\mu F_H v_H. \quad (64)$$

Define:

$$\lambda_F = \frac{F_H}{F_W}, \quad \lambda_v = \frac{v_H}{v_W}. \quad (65)$$

For the six-apex candidate not to lose against the triangular Wankel reference in apex friction:

$$P_{f,H} < P_{f,W}. \quad (66)$$

Substitution gives:

$$6\mu F_H v_H < 3\mu F_W v_W, \quad (67)$$

or:

$$\boxed{\lambda_F \lambda_v < 0.5.} \quad (68)$$

This is a severe condition. If  $e_H = e_W/2$  reduces sliding severity, a first estimate might suggest a reduced  $\lambda_v$ . However, a value such as  $\lambda_v = 0.8$  is a conservative threshold value rather than an optimistic kinematic prediction, because equal output power may require a different shaft speed and dynamic contact condition. If  $\lambda_v = 0.8$ , then:

$$\lambda_F < 0.625. \quad (69)$$

Therefore the six-apex architecture must reduce effective seal force by more than 37.5% merely to break even in apex-friction loss.

### 10.3 Leakage threshold and exposure time

For simplified compressible leakage through a small effective gap:

$$\dot{m} = C_d A_g \sqrt{2\rho\Delta p}. \quad (70)$$

Let the six-apex candidate change gap area, pressure difference, and exposure time by:

$$A_{g,H} = \lambda_A A_{g,W}, \quad \Delta p_H = \lambda_p \Delta p_W, \quad (71)$$

$$t_{open,H} = \lambda_t t_{open,W}. \quad (72)$$

The total leakage exposure scales approximately as:

$$M_{leak,H} \propto 6\lambda_A \sqrt{\lambda_p \lambda_t}. \quad (73)$$

For the triangular Wankel:

$$M_{leak,W} \propto 3. \quad (74)$$

For leakage not to worsen:

$$6\lambda_A \sqrt{\lambda_p \lambda_t} < 3, \quad (75)$$

hence:

$$\boxed{2\lambda_A \sqrt{\lambda_p \lambda_t} < 1.} \quad (76)$$

The parameter  $\lambda_t$  connects the leakage threshold with port and chamber timing. A shorter high-pressure exposure window can help satisfy Equation (76), but it may also reduce intake/exhaust duration and volumetric efficiency. The present model is apex-seal-only. Corner seals and side seals are not modelled here. For a six-apex rotor the number of corner interfaces also increases, and the full leakage model must treat apex, corner, and side seals separately, in the spirit of Picard et al. [8].

### 10.4 Wear threshold

Using Archard's wear relation:

$$V_{wear} = k \frac{F_s L}{H}, \quad (77)$$

where  $L$  is sliding distance and  $H$  is hardness. With six apexes, the total wear source count doubles. The six-apex design is advantageous only if the reduction in  $F_s$  and thermal instability is large enough to offset the increased number of sliding interfaces.

## 11 Six-Chamber Gas Exchange

Gas exchange must be evaluated because the six-chamber geometry may shorten the intake and exhaust windows. Define:

$$\Delta\theta_{int,i} = \theta_{IC,i} - \theta_{IO,i}, \quad (78)$$

$$\Delta\theta_{exh,i} = \theta_{EC,i} - \theta_{EO,i}. \quad (79)$$

The volumetric efficiency is:

$$\eta_v = \frac{m_{air,actual}}{\rho_{air}V_d}. \quad (80)$$

As a conservative first-order estimate, if six chamber events replace three chamber events over an equivalent rotor sequence and the port arc is not enlarged, then:

$$\Delta\theta_{int,H} \sim \frac{1}{2}\Delta\theta_{int,W}. \quad (81)$$

This is only a worst-case estimate, not final port timing. It assumes linear scaling with chamber count; the actual port duration depends on the five-lobe envelope, chamber trajectory, port geometry, and shaft-speed mapping. Equation (81) is important because it can work in opposite directions:

$$\lambda_t < 1 \quad \text{helps reduce leakage exposure,} \quad (82)$$

$$\eta_{v,H} < \eta_{v,W} \quad \text{hurts breathing and power.} \quad (83)$$

Therefore the design constraint is:

$$\eta_{v,H} \geq \eta_{v,W} - \epsilon_v, \quad (84)$$

with  $\epsilon_v$  small enough that the torque and efficiency gains are not lost through poor breathing.

## 12 Efficiency as a Threshold Condition

For a spark-ignition engine, the ideal Otto efficiency is commonly written as:

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}, \quad (85)$$

where  $r$  is compression ratio and  $\gamma$  is the heat-capacity ratio. This is only an upper-bound thermodynamic model; real brake thermal efficiency depends on combustion, heat transfer, leakage, friction, and pumping losses [3].

Let:

$$\eta_b = \eta_{otto}\Xi, \quad (86)$$

where:

$$\Xi = M_c M_h M_l M_f M_p. \quad (87)$$

Here  $M_c$  represents combustion completeness,  $M_h$  wall heat-transfer loss,  $M_l$  leakage,  $M_f$  mechanical friction, and  $M_p$  pumping loss. For  $\gamma = 1.35$  and  $r = 9$ :

$$\eta_{otto,9} = 1 - 9^{-0.35} \approx 0.5365. \quad (88)$$

If a triangular Wankel reference has  $\eta_b \approx 0.339$ , then:

$$\Xi_W = \frac{0.339}{0.5365} = 0.632. \quad (89)$$

For a *Hexa-Wankel* candidate at  $r = 10.5$ :

$$\eta_{otto,10.5} = 1 - 10.5^{-0.35} \approx 0.561. \quad (90)$$

To exceed the triangular Wankel reference:

$$0.561\Xi_H > 0.339, \quad (91)$$

therefore:

$$\boxed{\Xi_H > 0.604.} \quad (92)$$

To reach a modern piston benchmark of approximately 40% BTE:

$$0.561\Xi_H > 0.40, \quad (93)$$

hence:

$$\boxed{\Xi_H > 0.713.} \quad (94)$$

Thus the strongest defensible claim is: the *Hexa-Wankel* can outperform a modern piston engine only if thermal-pressure stabilisation and loss reduction raise  $\Xi_H$  above 0.713 at  $r_H = 10.5$ .

### 13 Comparison with Piston Engines at Equal Effective Displacement

Comparison with piston engines must not use raw nominal cc. In a rotary engine, displacement depends on chamber swept volume, the number of events per shaft revolution, and effective air throughput. In this paper, *effective displacement* means the air-fuel volume actually compressed and expanded to produce work, normalised against comparable work events. All comparisons

with piston engines must be read in this sense, not as an administrative equivalence between rotary and piston cc.

At equal effective displacement, the *Hexa-Wankel* may have a power-density advantage because:

1. it lacks a large reciprocating piston mass;
2. torque may become more continuous if the two phased pulses succeed;
3. eccentricity can be reduced for the same displacement;
4. packaging may be more compact than a multi-cylinder piston engine.

Efficiency, however, does not automatically improve. Modern piston engines have optimised combustion, cooling, friction, porting, and ring-pack design for more than a century. A fair comparison must therefore satisfy:

$$\eta_{b,H} > \eta_{b,P}. \quad (95)$$

For a modern piston engine with  $\eta_{b,P} \approx 0.40$ , the  $r_H = 10.5$  candidate requires  $\Xi_H > 0.713$ . This is a demanding target. The strongest thesis is therefore not that the *Hexa-Wankel* is necessarily more efficient, but that it has a theoretical path for turning rotary power density into efficiency, provided that sealing and thermal mapping truly become as predictable as those of the piston system.

## 14 Prior Art and Location of Novelty

This paper does not claim that all its components are new. Wankel geometry, trochoidal housing, apex-seal studies, and multiple-apex sealing have been discussed in the literature [1, 2, 9, 10]. Rose and Yang, for example, show that wide/multiple apex-seal designs can incorporate seal profiles into the housing design process and may improve sealing [9]. Warren also discusses a multi-apex sealing grid as an approach that can provide multiple leakage barriers and reduce the likelihood of seal separation [10].

The conceptual novelty of the *Hexa-Wankel* lies in the following combination:

1. definition of a six-apex rotor in a five-lobe Wankel-like baseline housing;
2. effective-displacement scaling that permits reduced eccentricity at equal displacement;
3. two phased combustion zones with unchanged total fuel,  $Q/2 + Q/2$ ;
4. use of Fourier harmonic cancellation to formulate thermal stabilisation;
5. formulation of seal friction and leakage as thresholds, not as optimistic assumed values;
6. an explicit aim: to bring the rotary engine closer to piston-like thermal-sealing predictability.

The claimed originality is therefore not “six-sided rotary” in a general sense, but a threshold-based architecture combining kinematic scaling, phased combustion, thermal-harmonic reduction, and seal predictability.

## 15 Validation Status and Further Programme

Steps 1-2 of the validation programme have been carried out at the first-order two-dimensional model level. The kinematic CAD model constructs the six-apex rotor, the gear law (6), and the five-lobe housing as an envelope. The actual chamber-volume calculation produces  $V_i(\theta)$ ,  $V_d = 35.786$  cc per chamber in the  $R = 60$  mm,  $e = 8$  mm,  $b = 80$  mm model, and an additional clearance requirement of 3.714 cc for the target  $r = 10.5$ .

The technical status is: *basic 2D geometry passes; performance does not yet pass*. The model has shown periodic chamber formation and chamber-volume computability, but it has not shown torque advantage, combustion advantage, sealing advantage, port-timing feasibility, or brake thermal efficiency advantage. For the hypothesis to become a robust engineering theory, the further programme must proceed in the following order:

1. **Numerical torque kernel:** calculate  $K_i(\theta_s) = dV_i/d\theta_s$  from the corrected actual volume curves. This determines whether two sufficiently positive expansion windows exist for the two combustion pulses.
2. **Phase optimisation:** determine  $\Delta\phi^*$  from Equation (46) using the numerical  $K_i(\theta_s)$ , not an abstract torque illustration.
3. **Thermal-mechanical compatibility:** test whether the  $180^\circ$  separation in housing coordinates ( $\theta_L = \theta_R + \pi$ ) is compatible with the mechanical  $\Delta\phi^*$ . If it is not, the design must choose between thermal balancing and torque optimisation, or alter the firing order.
4. **3D CAD refinement:** optimise rotor flank, combustion pocket, clearance, side housing, and axial volume so that  $r_i \geq 10$  is obtained without cusp, self-intersection, large dead volume, or a pocket with excessive surface-to-volume ratio.
5. **Port timing and gas exchange:** compute  $\Delta\theta_{int}$ ,  $\Delta\theta_{exh}$ , overlap,  $\eta_v$ , mass-flow capability, and pumping work  $W_p$  in the actual five-lobe housing.
6. **CFD combustion:** test flame travel, wall heat transfer, incomplete combustion, pressure trace, and the effect of the combustion pocket on quenching and hydrocarbon loss.
7. **FEA housing:** calculate thermal distortion, thermal stress, and  $\sigma_T^2$  to ensure that thermal stabilisation is not merely a feature of the ideal Fourier model.
8. **Seal tribology and leakage:** model apex, corner, and side seals separately; measure  $F_s$ , lift-off, leakage area, friction, oil film, and wear.
9. **Scaling and dynamometer test:** repeat the calculation at 650–1300 cc effective displacement, then measure BSFC and BTE on a physical prototype or experimental rig.

With this status, the paper has passed initial geometric validation, but not performance validation. Efficiency claims must remain threshold-theoretic until the torque kernel, gas exchange, CFD, FEA, tribology, and dynamometer tests are completed.

## 16 Falsification Conditions

The *Hexa-Wankel* hypothesis fails, or must be substantially revised, if any of the following conditions occurs:

1. the six-apex/five-lobe housing forms a cusp, self-intersection, or unsealable chamber;
2. the actual chamber volume does not permit  $r \geq 10$  without a large dead volume;
3. the torque kernel does not provide two adequate positive expansion windows;
4. the two-zone combustion separation  $\theta_L = \theta_R + \pi$  is incompatible with  $\Delta\phi^*$ ;
5. port timing causes a significant loss of volumetric efficiency;
6.  $\lambda_F\lambda_v \geq 0.5$ , so seal friction does not improve;
7.  $2\lambda_A\sqrt{\lambda_p}\lambda_t \geq 1$ , so leakage does not improve;
8. thermal variance  $\sigma_{T,H}$  is not lower than that of the triangular Wankel;
9. scaling to 650–1300 cc effective displacement produces a surface-to-volume ratio, seal force, or thermal deformation that removes the design advantage;
10. simulated and dynamometer BTE do not exceed the Wankel comparator.

## 17 Conclusion

The *Hexa-Wankel* is a six-apex rotary-engine design hypothesis that attempts to combine Wankel power density with piston-like thermal-sealing predictability. The baseline design is a six-apex rotor in a five-lobe Wankel-like housing, with gear law  $\psi = -\theta_s/6$ . Two phased combustion points divide one fuel dose into two pulses,  $Q/2 + Q/2$ , rather than doubling the fuel.

The geometric analysis shows that the first-order envelope family has the regularity condition  $R > ne$ . Displacement scaling shows that the six-apex candidate has  $V_{d,6}/V_{d,3} = 2$  at the same  $R$ ,  $e$ , and  $b$ ; therefore, at equal effective displacement, the six-apex design can reduce eccentricity or width. This provides a basis for the reduced-sliding-severity hypothesis, but it is not proof of efficiency.

The initial geometric contribution is no longer merely symbolic, but includes a two-dimensional kinematic CAD validation. At the baseline  $R = 60$  mm,  $e = 8$  mm, and  $b = 80$  mm, the actual chamber can be calculated with  $V_d = 35.786$  cc per chamber. The raw model gives a near-zero  $V_{min}$ , showing that a real design must include a combustion pocket or clearance volume. With an additional clearance of 3.714 cc, the target  $r = 10.5$  can be represented in the initial model. This result, however, proves only basic 2D geometric feasibility; it does not prove torque advantage, phased-combustion compatibility, port-timing feasibility, leakage reduction, or BTE advantage.

The principal thermal contribution is the Fourier harmonic analysis: two opposed combustion zones in housing coordinates cancel the odd harmonics of heat flux, so the thermal map may become more stable. Compatibility between thermal balancing and mechanical phase angle must nevertheless be calculated from the torque kernel  $K_i = dV_i/d\theta_s$ , which can now be extracted from the actual chamber-volume curves.

Tribologically, six apexes are advantageous only if  $\lambda_F\lambda_v < 0.5$  for friction and  $2\lambda_A\sqrt{\lambda_p}\lambda_t < 1$  for leakage. In efficiency terms, a candidate with target  $r = 10.5$  must satisfy  $\Xi_H > 0.604$  to outperform the 33.9% Wankel reference and  $\Xi_H > 0.713$  to approach a modern piston engine at approximately 40% BTE. The *Hexa-Wankel* is therefore a coherent and falsifiable concept, but it is not yet a final performance theory until housing geometry, chamber volume, phase angle, gas exchange, and seal data are verified through numerical and experimental validation.

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