

**Shared lexicographic equilibrium strategy profile for a finite number of pair-valued bi-matrix games**

**By**

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**Abstract**

We show that a strategy profile is a lexicographic equilibrium strategy profile for an arbitrary finite set of pair-valued bi-matrix games if and only if it solves a bi-linear programming problem and the *optimal value pair* of the bi-linear programming problem is zero. An immediate consequence of this result, is that if the outcome matrix for the row player is a convex combination of its outcome matrices in the finite collection and the outcome matrix for the column player is a (possibly different) convex combination of its outcome matrices in the finite collection, then a strategy profile is a lexicographic equilibrium strategy profile for the game associated with pair of pair-valued matrices if and only if it satisfies the same two conditions.

**Keywords:** lexicographic equilibrium strategy profile, finite set of pair-valued bi-matrix games, bi-linear programming problem, optimal value pair, convex hull

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**1. Introduction:** In the seminal work of Mangasarian and Stone (1964), the main result states that a strategy profile of a bi-matrix game (i.e., a randomization for each of the two players- a row player and a column player- over the non-empty finite set of actions available to each) is an equilibrium strategy profile (i.e., no one benefits from unilateral deviations) if and only if it solves a certain bi-linear programming problem. In Lahiri (2026), we introduce the concept of a pair-valued bi-matrix game, which extends the two-person zero sum version of the same that is discussed in Radhakrishnan and Saikeerthana (2020). As in Radhakrishnan and Saikeerthana (2020), we assume in Lahiri (2026) that the outcomes of the game are lexicographically ordered and introduce the concept of a lexicographic equilibrium strategy profile for pair-valued bi-matrix game. This solution concept, generalizes the

related solution concept in Radhakrishnan and Saikethana (2020) for zero-sum pair-valued bi-matrix games. Interesting examples of lexicographic preferences (see Varian (1978)) are available in Goswami, Mitra and Sen (2018).

In this note we show that a strategy profile is a lexicographic equilibrium strategy profile for an arbitrary non-empty finite set of pair-valued bi-matrix games if and only if it solves a generalized form of the bi-linear programming problem in Mangasarian and Stone (1964) and the *optimal value pair* of the bi-linear programming problem is zero. Since we maximize with respect to lexicographic preferences and since lexicographic preferences are linear orders, the concept of an “optimal value pair” is meaningful in our context. This result generalizes proposition 3 in Lahiri (2026) which is concerned with lexicographic equilibrium strategy profile for a single pair-valued bi-matrix game. As in the case of proposition 3 in Lahiri (2026), we need, this additional condition about the optimal value pair being zero, since- as shown in Lahiri (2026)- unlike in the case of a single bi-matrix game, there is no guarantee that a pair-valued bi-matrix game has a lexicographic equilibrium strategy profile. The existence of an equilibrium for a bi-matrix game implies that the optimal value of the objective function in the bi-linear programming problem in Mangasarian and Stone (1964) is zero. An immediate consequence of our first major result, is that if the outcome matrix for the row player is a convex combination of its outcome matrices in the finite collection and the outcome matrix for the column player is a (possibly different) convex combination of its payoff matrices in the finite collection, then a strategy profile is a lexicographic equilibrium strategy profile for the game associated with the pair of pair-valued matrices if and only if it satisfies the same two conditions.

Bhurjee (2016), attempts to extend the scope of the main result in Mangasarian and Stone (1964) to the case of two bi-matrix games, where each entry in the payoff matrix of each player in the second bi-matrix game is “greater than or equal to” the corresponding entry in the payoff matrix of the same player in the first bi-matrix game. The possibility of non-existence of a shared equilibrium profile for the kind of bi-matrix games discussed in Bhurjee (2016) is shown in example 2 in (the appendix section of) Lahiri (2026).

Since, a bi-matrix game is a special case of a pair-valued bi-matrix game in which all entries in the outcome matrix of a player have the same value of the second coordinate, the relevant results in Bhurjee (2016) are implied by corresponding results in this note.

**2. The Model:** For positive integers  $m, n$  consider an interactive decision making context (game) with two players/participants/decision-makers who we denote by 'a' and 'b' respectively, and where player 'a' has  $m$  actions to choose from and 'b' has 'n' actions to choose from.

A **strategy for player 'a'** is a point in  $\Delta^{m-1} = \{x \in \mathbb{R}_+^m \mid \sum_{i=1}^m x_i = 1\}$  and a **strategy for player 'b'** is a point in  $\Delta^{n-1} = \{x \in \mathbb{R}_+^n \mid \sum_{j=1}^n y_j = 1\}$ .

A strategy for the player 'a' is a probability distribution on  $\{1, \dots, m\}$ , with each coordinate of the strategy denoting the probability with which the player 'a' chooses the corresponding action. A strategy for player 'b' is a probability distribution on  $\{1, \dots, n\}$ , with each coordinate of the strategy denoting the probability with which player 'b' chooses the corresponding action.

**Notation 1:** For  $i \in \{1, \dots, m\}$ , let  $E^{(m,i)}$  be the **m-dimensional i<sup>th</sup> unit coordinate column vector** (i.e., 1 in its  $i^{\text{th}}$  coordinate and 0 in all other coordinates) and let  $E^{(m)} = \sum_{h=1}^m E^{(m,h)}$  be the **m-dimensional sum vector**. For  $j \in \{1, \dots, n\}$ , let  $E^{(n,j)}$  be the **n-dimensional j<sup>th</sup> unit coordinate column vector** (i.e., 1 in its  $j^{\text{th}}$  coordinate and 0 in all other coordinates) and let  $E^{(n)} = \sum_{k=1}^n E^{(n,k)}$  be the **n-dimensional sum vector**. ■

If a strategy  $x$  for player 'a' is such that  $x = E^{(m,i)}$  for some  $i \in \{1, \dots, m\}$ , then  $x$  is said to be the **i<sup>th</sup> pure strategy for the player 'a'**.

If a strategy  $y$  for player 'b' is such that  $y = E^{(n,j)}$  for some  $j \in \{1, \dots, n\}$ , then  $y$  is said to be the **j<sup>th</sup> pure strategy for the player 'b'**.

A pair  $(x, y)$  where  $x$  is a strategy for player 'a' and  $y$  is a strategy for player 'b' is said to be a **strategy profile**.

If both  $x$  and  $y$  are pure strategies then the strategy profile  $(x, y)$  is said to be a **pure strategy profile**.

**Note 1:** If  $(x, y)$  is a pure strategy profile, then we may represent it as  $(i, j)$  if  $x$  is the  $i^{\text{th}}$  pure strategy for player 'a' and  $j$  is the  $j^{\text{th}}$  pure strategy for player 'b'. ■

Let  $\mathbb{R}^{m \times n}$  be the set of all  $m \times n$  matrices.

Given  $C \in \mathbb{R}^{m \times n}$  and  $(i, j) \in \{1, \dots, m\} \times \{1, \dots, n\}$ , let  $c_{ij}$  denote the  $(i, j)^{\text{th}}$  entry of  $C$ ,  $C_i$  denote the  $i^{\text{th}}$  row of  $C$  and  $C^j$  denote the  $j^{\text{th}}$  column of  $C$ .

Let  $\mathbb{R}^{(2, m \times n)}$  denote the set of all  $m \times n$  matrices whose entries are pairs of real number.

If  $V \in \mathbb{R}^{(2, m \times n)}$ , then for all  $(i, j) \in \{1, \dots, m\} \times \{1, \dots, n\}$ , the typical (though not invariable) notations are as follows:  $(i, j)^{\text{th}}$  entry of  $V$  will be denoted by  $(v_{ij}^{(1)}, v_{ij}^{(2)})$ , the  $i^{\text{th}}$  row of  $V$  will be denoted by  $V_i$  and the  $j^{\text{th}}$  column of  $V$  will be denoted by  $V^j$ . For our purposes here, a member of  $\mathbb{R}^{(2, m \times n)} \times \mathbb{R}^{(2, m \times n)}$  will be referred to as a **pair-valued bi-matrix game** and  $\mathbb{R}^{(2, m \times n)} \times \mathbb{R}^{(2, m \times n)}$  as the **set of all pair-valued bi-matrix games**.

Let  $\succcurlyeq$  be a binary relation on  $\mathbb{R}^2$  such that for all  $(\gamma_1, \eta_1), (\gamma_2, \eta_2) \in \mathbb{R}^2$ :  $(\gamma_1, \eta_1) \succcurlyeq (\gamma_2, \eta_2)$  if and only if either  $\gamma_1 > \gamma_2$  or  $\gamma_1 = \gamma_2$  and  $\eta_1 \geq \eta_2$ .

Clearly  $\succcurlyeq$  is a linear order on  $\mathbb{R}^2$ . It is generally known as the **lexicographic preference ordering on  $\mathbb{R}^2$** .

**Notation 2:** Given a pair-valued bi-matrix game  $(V(a), V(b))$ , for  $r \in \{1, 2\}$  and  $\alpha \in \{a, b\}$ , let  $V^{(r)}(\alpha)$  be the  $m \times n$  real-valued matrix whose  $(i, j)^{\text{th}}$  term for  $(i, j) \in \{1, \dots, m\} \times \{1, \dots, n\}$  is  $v_{ij}^{(r)}(\alpha)$ . The  $i^{\text{th}}$  row of  $V^{(r)}(\alpha)$  for  $i \in \{1, \dots, m\}$  is denoted by  $V_i^{(r)}(\alpha)$  and the  $j^{\text{th}}$  column of  $V^{(r)}(\alpha)$  for  $j \in \{1, \dots, n\}$  is denoted by  $V^{(r)j}(\alpha)$ . For  $(x, y) \in \Delta^{m-1} \times \Delta^{n-1}$  and  $\alpha \in \{a, b\}$ ,  $x^T V(\alpha) y = \sum_{i=1}^m \sum_{j=1}^n x_i y_j (v_{ij}^{(1)}(\alpha), v_{ij}^{(2)}(\alpha)) = \sum_{i=1}^m \sum_{j=1}^n (x_i y_j v_{ij}^{(1)}(\alpha), x_i y_j v_{ij}^{(2)}(\alpha))$ . ■

Given a pair-valued bi-matrix game  $(V(a), V(b))$ , a strategy profile  $(x, y)$  is said to be a **lexicographic equilibrium strategy profile** for  $(V(a), V(b))$  if  $x^T V(a) y \succcurlyeq V_i(a) y$  for all  $i \in \{1, \dots, m\}$  and  $x^T V(b) y \succcurlyeq x^T V^j(b)$  for all  $j \in \{1, \dots, n\}$ .

**Note 2:** It ought to be emphasized that the entries of  $V(a)$  and  $V(b)$  are *not* (necessarily) *payoffs*. They are “*outcomes*” that can be linearly ordered using lexicographic preferences. ■

**3. Some Preliminary Lemmas:** For the sake of completeness we reproduce two lemmas in Lahiri (2026) along with their proofs

**Lemma 1:** (1) Suppose  $u^{(1)}, u^{(2)}, v^{(1)}, v^{(2)} \in \mathbb{R}^2$  satisfy  $u^{(1)} \succcurlyeq v^{(1)}$  and  $u^{(2)} \succcurlyeq v^{(2)}$ . Then,  $u^{(1)} + u^{(2)} \succcurlyeq v^{(1)} + v^{(2)}$ .

(2) For all  $u, v \in \mathbb{R}^2$  and  $\theta \in \mathbb{R}_{++}$ :  $u \succcurlyeq v$  if and only if  $\theta u \succcurlyeq \theta v$ .

(3) For all  $u, v \in \mathbb{R}^2$ :  $u \succcurlyeq v$  if and only if  $-u \succcurlyeq -v$ .

**Proof:** (1) For  $k \in \{1, 2\}$ ,  $u^{(k)} \succcurlyeq v^{(k)}$  if and only if either  $u_1^{(k)} > v_1^{(k)}$  or  $u_1^{(k)} = v_1^{(k)}$  &  $u_2^{(k)} \geq v_2^{(k)}$ .

Thus,  $\{k \mid u_1^{(k)} > v_1^{(k)}\} \neq \emptyset$  implies  $u_1^{(1)} + u_1^{(2)} > v_1^{(1)} + v_1^{(2)}$  which implies  $u^{(1)} + u^{(2)} \succcurlyeq v^{(1)} + v^{(2)}$ .

If  $\{k \mid u_1^{(k)} > v_1^{(k)}\} = \emptyset$ , then  $u_1^{(1)} + u_1^{(2)} = v_1^{(1)} + v_1^{(2)}$ . Further, in this case, for  $k \in \{1, 2\}$  it is the case that  $u_2^{(k)} \geq v_2^{(k)}$  so that  $u_2^{(1)} + u_2^{(2)} \geq v_2^{(1)} + v_2^{(2)}$ .

Thus,  $u^{(1)} + u^{(2)} \succcurlyeq v^{(1)} + v^{(2)}$ .

(2) For  $u, v \in \mathbb{R}^2$  and  $\theta \in \mathbb{R}_{++}$ :  $[u_1 > v_1 \text{ if and only if } \theta u_1 > \theta v_1]$  &  $[u_1 = v_1 \text{ along with } u_2 \geq v_2 \text{ if and only if } \theta u_1 = \theta v_1 \text{ along with } \theta u_2 \geq \theta v_2]$ .

Thus,  $u \succcurlyeq v$  if and only if  $\theta u \succcurlyeq \theta v$ .

(3) Let  $u, v \in \mathbb{R}^2$  and  $u \succcurlyeq v$ . Thus, either  $u_1 > v_1$  or  $u_1 = v_1$  &  $u_2 \geq v_2$ .

Thus, either  $-v_1 > -u_1$  or  $-v_1 = -u_1$  &  $-v_2 \geq -u_2$ .

Thus,  $-v \succcurlyeq -u$ .

The converse follows from  $-(-v) = v$  and  $-(-u) = u$ . Q.E.D.

The following lemma is obtained by applying a standard induction argument to parts 1 and 2 of lemma 1 and the fact that if for some positive integer  $N \geq 2$ ,  $\langle u^{(k)} \mid k = 1, \dots, N \rangle$  is an array in  $\mathbb{R}^2$  and  $\langle \theta^{(k)} \mid k = 1, \dots, N \rangle$  is an array in the closed interval  $[0, 1]$

satisfying  $\sum_{k=1}^N \theta^{(k)} = 1$ ,  $\theta^{(N)} > 0$ , then  $\sum_{k=1}^N \theta^{(k)} u^{(k)} = (1 - \theta^{(N)}) \sum_{k=1}^{N-1} \frac{\theta^{(k)}}{1 - \theta^{(N)}} u^{(k)} + \theta^{(N)} u^{(N)}$ .

**Lemma 2:** Suppose that for some positive integer  $N \geq 2$ ,  $\langle u^{(k)} \mid k = 1, \dots, N \rangle$  and  $\langle v^{(k)} \mid k = 1, \dots, N \rangle$  are arrays in  $\mathbb{R}^2$  satisfying  $u^{(k)} \succcurlyeq v^{(k)}$  for all  $k \in \{1, \dots, N\}$ . Let  $\langle \theta^{(k)} \mid k = 1, \dots, N \rangle$ ,  $\langle \lambda^{(k)} \mid k = 1, \dots, N \rangle$  be arrays in the closed interval  $[0, 1]$  satisfying  $\sum_{k=1}^N \theta^{(k)} = \sum_{k=1}^N \lambda^{(k)} = 1$ . Then,  $\sum_{k=1}^N \theta^{(k)} u^{(k)} \succcurlyeq \sum_{k=1}^N \lambda^{(k)} v^{(k)}$ .

Lemmas 1 and 2 are used to prove the main result in the next section.

#### 4. Shared lexicographic Equilibrium Strategy Profile For a Finite Number of

**Pair-valued Bi-matrix Games:** For a positive integer  $K$ , let  $\{(V(a, k), V(b, k)) \mid k = 1, \dots, K\}$  be a non-empty finite set of pair-valued bi-matrix games.

**Proposition 1:** A strategy profile  $(x, y)$  is a lexicographic equilibrium strategy profile for every pair-valued bi-matrix game in the set  $\{(V(a, k), V(b, k)) \mid k = 1, \dots, K\}$  if and only if there exists arrays  $\langle \alpha^*(k) \mid k = 1, \dots, K \rangle$ ,  $\langle \beta^*(k) \mid k = 1, \dots, K \rangle$  in  $\mathbb{R}^2$  that along with  $x, y$  satisfies the following “two” conditions:

(1)  $x, y, \langle \alpha^*(k) \mid k = 1, \dots, K \rangle, \langle \beta^*(k) \mid k = 1, \dots, K \rangle$  solve the bi-linear programming problem BLP defined thus: Maximize (with respect to  $\succcurlyeq$ )  $\sum_{k=1}^K [w^T (V(a, k) +$

$V(b, k)z - \alpha(k) - \beta(k)$ ], subject to  $\alpha(k) \geq V_i(a, k)z$ ,  $i = 1, \dots, m$ ,  $k \in \{1, \dots, K\}$ ,  $\beta(k) \geq w^T V(b, k)^j$ ,  $j = 1, \dots, n$ ,  $k \in \{1, \dots, K\}$ ,  $w \in \Delta^{m-1}$ ,  $z \in \Delta^{n-1}$ ,  $\alpha(k), \beta(k) \in \mathbb{R}^2$ ,  $k \in \{1, \dots, K\}$ .

$$(2) \sum_{k=1}^K [x^T (V(a, k) + V(b, k))y - \alpha^*(k) - \beta^*(k)] = (0, 0).$$

**Proof: Observation:** Note that if  $z, w, \langle \alpha(k) | k = 1, \dots, K \rangle, \langle \beta(k) | k = 1, \dots, K \rangle$  satisfies the constraints of BLP, then then by lemma 2,  $[\alpha(k) \geq V_i(a, k)z, i = 1, \dots, m, k \in \{1, \dots, K\}, \beta(k) \geq w^T V(b, k)^j, j = 1, \dots, n, k \in \{1, \dots, K\}, w \in \Delta^{m-1}, z \in \Delta^{n-1}]$  implies  $[\alpha(k) \geq w^T V(a, k)z$  and  $\beta(k) \geq w^T V(b, k)z$ , so that by repeated application of part 1 of lemma 1, we get  $\sum_{k=1}^K (\alpha(k) + \beta(k)) \geq \sum_{k=1}^K w^T (V(a, k) + V(b, k))z$ .

Adding  $-\sum_{k=1}^K (\alpha(k) + \beta(k))$  to both sides of we get  $(0, 0) \geq \sum_{k=1}^K [w^T (V(a, k) + V(b, k))z - \alpha(k) - \beta(k)]$ . ■

Suppose  $(x, y)$  is a lexicographic equilibrium strategy profile for every pair-valued bi-matrix game in the set  $\{(V(a, k), V(b, k)) | k = 1, \dots, K\}$ .

For  $k \in \{1, \dots, K\}$ , let  $\alpha^*(k) = x^T V(a, k)y$  and  $\beta^*(k) = x^T V(b, k)y$ .

Then,  $x, y, \langle \alpha^*(k) | k = 1, \dots, K \rangle, \langle \beta^*(k) | k = 1, \dots, K \rangle$  satisfy all the constraints of BLP and  $\sum_{k=1}^K [x^T (V(a, k) + V(b, k))y - \alpha^*(k) - \beta^*(k)] = (0, 0)$ .

Thus,  $x, y, \langle \alpha^*(k) | k = 1, \dots, K \rangle, \langle \beta^*(k) | k = 1, \dots, K \rangle$  solve BLP and satisfy (2).

Now suppose  $x, y, \langle \alpha^*(k) | k = 1, \dots, K \rangle, \langle \beta^*(k) | k = 1, \dots, K \rangle$  solve BLP and satisfy (2).

Since,  $x, y, \langle \alpha^*(k) | k = 1, \dots, K \rangle, \langle \beta^*(k) | k = 1, \dots, K \rangle$  satisfy the constraints of BLP, it must be the case that for all  $k \in \{1, \dots, K\}$ ,  $\alpha^*(k) \geq V_i(a, k)y$ ,  $i = 1, \dots, m$  and  $\beta^*(k) \geq x^T V^j(b, k)$ ,  $j = 1, \dots, n$ .

For  $k \in \{1, \dots, K\}$ , let  $\alpha^*(k) = (\alpha_1^*(k), \alpha_2^*(k))$  and  $\beta^*(k) = (\beta_1^*(k), \beta_2^*(k))$ .

Thus, for all  $k \in \{1, \dots, K\}$ :

$$\alpha_1^*(k) > V_i^{(1)}(a, k)y \text{ or } \alpha_1^*(k) = V_i^{(1)}y \text{ and } \alpha_2^*(k) \geq V_i^{(2)}(a, k)y \text{ for } i \in \{1, \dots, m\}$$

&

$$\beta_1^*(k) > x^T V^{(1)j}(b, k) \text{ or } \beta_1^*(k) = x^T V^{(1)j}(b, k) \text{ and } \beta_2^*(k) = x^T V^{(2)j}(b, k) \text{ for } j \in \{1, \dots, n\}.$$

Since,  $(x, y) \in \Delta^{m-1} \times \Delta^{n-1}$  and  $\sum_{k=1}^K [x^T (V^{(1)}(a, k) + V^{(1)}(b, k))y - \alpha_1^*(k) - \beta_1^*(k)] = 0$ , it must be the case that  $x_i > 0$  implies  $\alpha_1^*(k) = V_i^{(1)}(a, k)y$  and  $y_j > 0$  implies  $\beta_1^*(k) = x^T V^{(1)j}(b, k)$ .

Thus, for all  $k \in \{1, \dots, K\}$ :  $x^T V^{(1)}(a, k)y = \alpha_1^*(k)$  and  $x^T V^{(1)}(b, k)y = \beta_1^*(k)$ .

Since  $(x, y) \in \Delta^{m-1} \times \Delta^{n-1}$  and  $\sum_{k=1}^K [x^T(V^{(2)}(a, k) + V^{(2)}(b, k))y - \alpha_2^*(k) - \beta_2^*(k)] = 0$ , it must be the case that for all  $k \in \{1, \dots, K\}$ :  $x_i > 0$  implies  $\alpha_2^*(k) = V_i^{(2)}(a, k)y$  and  $y_j > 0$  implies  $\beta_2^*(k) = x^T V^{(2)j}(b, k)$ .

Thus, for all  $k \in \{1, \dots, K\}$ :  $\alpha_2^*(k) = x^T V^{(2)}(a, k)y$  and  $\beta_2^*(k) = x^T V^{(2)}(b, k)y$ .

Thus, for  $k \in \{1, \dots, K\}$ :  $\alpha^*(k) = x^T V(a, k)y$  and  $\beta^*(k) = x^T V(b, k)y$ .

Since, for all  $k \in \{1, \dots, K\}$ ,  $\alpha^*(k) \geq V_i(a, k)y$ ,  $i = 1, \dots, m$  and  $\beta^*(k) \geq x^T V^j(b, k)$ ,  $j = 1, \dots, n$ , it follows that  $(x, y)$  is a lexicographic equilibrium strategy profile for  $(V(a), V(b))$ .

**5. Shared Equilibrium For Convex Hull of Outcome Matrices:** Given  $\{(V(a, k), V(b, k)) \mid k = 1, \dots, K\}$  as above, let  $\text{conv}\{V(a, k) \mid k = 1, \dots, K\}$  denote the convex hull of the  $m \times n$  pair-valued matrices in  $\{V(a, k) \mid k = 1, \dots, K\}$  and let  $\text{conv}\{V(b, k) \mid k = 1, \dots, K\}$  denote the convex hull of the  $m \times n$  pair-valued matrices in  $\{V(b, k) \mid k = 1, \dots, K\}$ .

**Proposition 2:**  $(x, y)$  is a lexicographic equilibrium strategy profile for all  $(V(a), V(b)) \in \text{conv}\{V(a, k) \mid k = 1, \dots, K\} \times \text{conv}\{V(b, k) \mid k = 1, \dots, K\}$  if and only if  $(x, y)$  is a lexicographic equilibrium strategy profile for all bimatrix games in  $\{(V(a, k), V(b, k)) \mid k = 1, \dots, K\}$ .

**Proof:** If  $(x, y)$  is a lexicographic equilibrium strategy profile for all  $(V(a), V(b)) \in \text{conv}\{V(a, k) \mid k = 1, \dots, K\} \times \text{conv}\{V(b, k) \mid k = 1, \dots, K\}$ , then  $(x, y)$  must be a lexicographic equilibrium strategy profile for all  $(V(a), V(b)) \in \{(V(a, k), V(b, k)) \mid k = 1, \dots, K\}$ .

Hence suppose  $(x, y)$  is a lexicographic equilibrium strategy profile for all  $(V(a), V(b)) \in \{(V(a, k), V(b, k)) \mid k = 1, \dots, K\}$ .

Thus, for all  $k \in \{1, \dots, K\}$ ,  $x^T V(a, k)y \geq V_i(a, k)y$  for all  $i \in \{1, \dots, m\}$  and  $x^T V(b, k)y \geq x^T V^j(b, k)$  for all  $j \in \{1, \dots, n\}$ .

For  $\theta, \eta \in \Delta^{K-1} = \{\xi \in \mathbb{R}_+^K \mid \sum_{k=1}^K \xi_k = 1\}$ , consider the pair-valued bi-matrix game

$$(\sum_{k=1}^K \theta_k V(a, k), \sum_{k=1}^K \eta_k V(b, k)).$$

Thus, by lemma 2,  $x^T(\sum_{k=1}^K \theta_k V(a, k))y = \sum_{k=1}^K \theta_k x^T V(a, k)y \geq \sum_{k=1}^K \theta_k V_i(a, k)y =$

$$(\sum_{k=1}^K \theta_k V_i(a, k))y \text{ for all } i \in \{1, \dots, m\} \text{ and } x^T(\sum_{k=1}^K \eta_k V(b, k))y =$$

$$\sum_{k=1}^K \eta_k x^T V(b, k)y \geq \sum_{k=1}^K \eta_k x^T V^j(b, k) = x^T(\sum_{k=1}^K \eta_k V^j(b, k)) \text{ for all } j \in \{1, \dots, n\}.$$

Thus,  $(x, y)$  is a lexicographic equilibrium strategy profile for

$$(\sum_{k=1}^K \theta_k V(a, k), \sum_{k=1}^K \eta_k V(b, k)).$$

An immediate consequence of propositions 1 and 2 is the following result.

**Theorem 1:**  $(x, y)$  is a lexicographic equilibrium strategy for all  $(V(a), V(b)) \in \text{conv}[\{V(a, k) \mid k = 1, \dots, K\}] \times \text{conv}[\{V(b, k) \mid k = 1, \dots, K\}]$  if and only if there exists arrays  $\langle \alpha^*(k) \mid k = 1, \dots, K \rangle, \langle \beta^*(k) \mid k = 1, \dots, K \rangle$  in  $\mathbb{R}^2$  that along with  $x, y$  satisfies the following “two” conditions:

- (1)  $x, y, \langle \alpha^*(k) \mid k = 1, \dots, K \rangle, \langle \beta^*(k) \mid k = 1, \dots, K \rangle$  solve the bi-linear programming problem BLP defined thus: Maximize (with respect to  $\succcurlyeq$ )  $\sum_{k=1}^K [w^T(V(a, k) + V(b, k))z - \alpha(k) - \beta(k)]$ , subject to  $\alpha(k) \succcurlyeq V_i(a, k)z, i = 1, \dots, m, k \in \{1, \dots, K\}, \beta(k) \succcurlyeq w^T V(b, k)j, j = 1, \dots, n, k \in \{1, \dots, K\}, w \in \Delta^{m-1}, z \in \Delta^{n-1}, \alpha(k), \beta(k) \in \mathbb{R}^2, k \in \{1, \dots, K\}$ .
- (2)  $\sum_{k=1}^K [x^T(V(a, k) + V(b, k))y - \alpha^*(k) - \beta^*(k)] = (0, 0)$ .

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