Constructal design of glider wings: the intersection of aerodynamic and structural asymptotes

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Abstract

When designing an aircraft, aerodynamic efficiency must be taken into account as well as structural integrity. While high-aspect-ratio wings minimize induced drag, they exponentially increase the bending moment and the required structural weight at the root of the wings. In this paper, “Constructal Theory” is applied to theoretically predict the optimal wing span for a glider. Instead of using complex and high-cost approaches such as “Fluid-Structure Interaction” to predict the optimal wing span, in this paper aerodynamic drag and structural weight are treated as two competing flow resistances that restrict the movement of the flying system. By formulating the total resistance function and applying the principle of the intersection of asymptotes, we analytically derive the geometric configuration that minimizes the global resistance. This mathematical optimization process shows that the geometry of an optimal wing is not a random occurrence, but it is a deterministic physical phenomenon driven by the equipartition of imperfection.

Keywords: Constructal Law, Wing Optimization, Intersection of Asymptotes, Fluid-Structure Interaction, Aspect Ratio.

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1. Introduction

Maximizing aerodynamic efficiency has always challenged aircraft designers throughout the history. To achieve this, designers tried extreme geometric configurations, such as in high-performance gliders like Daedalus, where wingspans are pushed to their limits for incomparable lift-to-drag ratios. However, stretching the wing to reduce induced drag exponentially increases the root bending moment. This requires heavier load-bearing structures, negating aerodynamic gains. Resolving this conflict typically relies on costly multidisciplinary computer simulations. This paper argues the resulting optimal architecture is a deterministic physical phenomenon governed by a fundamental law of design in nature. The Constructal Law states that for a finite-size flow system to persist, it must evolve to provide easier access to its imposed currents. A glider wing hosts two competing "currents": airflow seeking minimum aerodynamic resistance, and mechanical stresses seeking minimum structural weight. Treating the wing as a macroscopic flow architecture, this study analytically demonstrates that the optimum wingspan arises at the "intersection of asymptotes"—where aerodynamic and structural drags balance. We prove this search is the natural evolution of a system distributing its imperfections evenly.

2. The Aerodynamic Asymptote

In Constructal Theory, the evolution of a flow architecture is driven by the minimization of global flow resistance [1]. For flying machines such as gliders, the first fundamental "current" is the air passing over the wing. The purpose of this macroscopic flow is to minimize aerodynamic drag, which acts as the primary resistance to the system's forward motion.

In aircrafts, the total aerodynamic drag is the combination of parasitic and induced drag, but in high-aspect-ratio gliders this aerodynamic drag is mostly dominated by the induced drag since such aircrafts operates with high lifting coefficients. The induced drag (D_i) is a direct physical consequence of the lift force resulting from the pressure difference between the upper and lower surfaces of the wing, creating tail vortices. In classical aerodynamics, induced drag is expressed as follows [2]:

$$D_i = \frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2 \tag{1}$$

where, "W" is the weight of the aircraft (assuming steady-level flight where Lift = Weight), "e" is the Oswald span efficiency number, "q_∞" is the freestream dynamic pressure ($q_\infty = \frac{1}{2}\rho V^2$), and "$b$" is the wingspan.

To isolate the geometric variable that determines the macroscopic architecture of the wing (b), we can group operational and environmental parameters (W, q_∞, e) under a single aerodynamic constant (C_1).

$$R_{aero} = \frac{C_1}{b^2} \tag{2}$$

Equation (2) defines our aerodynamic asymptote which is the first extreme of our design space. This shows that the historical trend that we discussed in glider design is true. In order to allow easier access to airflow and reduce aerodynamic drag to near zero, the wingspan must approach infinity.

3. The Structural Asymptote

While aerodynamics dictates an infinite wing span to reduce drag, solid mechanics imposes a contradictory physical penalty. In Constructal Theory, the wing is a macroscopic flow architecture transferring mechanical stresses. The "current" here is the flow of bending stresses from aerodynamic lift, which must safely reach the wing root. Modeled as a cantilever beam with a simplified uniform lift distribution ($\omega = \frac{W}{b}$), the bending moment (M_{root}) occurring at the wing root for each half-span is calculated as [3]:

$$M_{root} = \int_{0}^{\frac{b}{2}} \omega \cdot y \, dy = \frac{Wb}{8} \tag{3}$$

Equation (3) shows that as the wingspan (b) increases, the bending moment at the root increases linearly. However, the structural cost is not just the moment itself, but the physical material needed to withstand that moment. Based on the Constructal principle of "equipartition of imperfection" (where the maximum allowable yield stress, σ_{max}, is distributed uniformly along the structure to avoid weak points) [1], the required structural volume (V_{struct}) which can be seen in Equation (4) is a function of both the bending moment and the span.

$$V_{struct} = \left(\frac{k \cdot \rho_{mat} \cdot g \cdot W}{8 \cdot \sigma_{max} \cdot h} \right) b^2 \tag{4}$$

As a result, the structural weight (W_{struct}) that needs to support this bending moment without failing is not linearly, but exponentially (second power) increases with the wingspan. This weight represents the second macroscopic resistance to the flying system's movement. By grouping the material properties (density, yield stress) into a single structural constant (C_2), the structural flow resistance (R_{struct}) can be defined as follows:

$$R_{struct} = C_2 b^2 \tag{5}$$

Equation (5) defines our structural asymptote which is the second extreme of our design space. It shows an opposite characteristic to the aerodynamic asymptote and as wingspan approaches to zero, structural weight penalty disappears.

4. Constructal Optimization and Conclusion

The total macroscopic resistance (R_{total}) is the sum of aerodynamic and structural resistances. Superimposing the asymptotes yields:

$$R_{total} = R_{aero} + R_{struct} = \frac{C_1}{b^2} + C_2 b^2 \tag{6}$$

In Equation (6), we can clearly see our fundamental conflict in wing design. To find the optimal design for our glider, the total resistance must be minimized. According to Constructal Theory, this optimization occurs at the geometric "intersection of asymptotes" [1]. Mathematically, the minimum total resistance is found by taking the derivative of R_{total} with respect to the wingspan (b) and equating it to zero:

$$\frac{dR_{total}}{db} = -\frac{2C_1}{b^3} + 2C_2 b = 0 \tag{7}$$

Solving Equation (7) for the span yields the optimal wingspan (b_{opt}) as:

$$b_{opt} = \left(\frac{C_1}{C_2}\right)^{\frac{1}{4}} \tag{8}$$

The optimal span is analytically determined by the ratio of aerodynamic conditions to structural constraints. In conclusion, the optimal design of high-performance glider wings can be explained by the Constructal Law, beyond complex computer simulations. The architecture evolves to provide the easiest access to both airflow and internal mechanical stresses. This specific wingspan, emerging from the equipartition of imperfection, proves that engineered flying machines mimic the evolution of design in nature.

References

[1] A. Bejan and S. Lorente, *Design with Constructal Theory*. Hoboken, NJ, USA: John Wiley & Sons, 2008.
[2] J. D. Anderson Jr., *Fundamentals of Aerodynamics*, 6th ed. New York, NY, USA: McGraw-Hill Education, 2016.
[3] T. H. G. Megson, *Aircraft Structures for Engineering Students*, 6th ed. Oxford, UK: Butterworth-Heinemann, 2016.