A 2-D Subthreshold Analytical model for Short Channel Effects in Nanowire MOSFETs (Si, Ge)

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***Abstract*-The present analysis proposes a 2D analytical model of potential and current for Nanowire MOSFETs in the subthreshold region for a moderately doped channel. The analytical expression for subthreshold current has been used to investigate the short channel parameters such as threshold voltage roll off and subthreshold slope. The model has been validated for Silicon and Germanium Nanowire MOSFETs by verifying the analytical results obtained with simulated results using Device3D.**

***Index Terms*— MOSFETs, Nanowire, Short-Channel Effect (SCE), Channel material**

I.INTRODUCTION

In order to exploit the ultimate potential of the CMOS technology and to continue the scaling of MOSFET according to the ITRS provisions [1], new device architectures are being investigated. Various nonplanar device structures [2-3] have been proposed in recent years to improve the electrostatic control of the channel potential. Nanowire MOSFETs(NW-MOS) have been recognized as one of the possible choices to continue the scaling of CMOS beyond conventional scaling limits [4-6]. 1-D Surface Potential based model for Nanowire MOSFET [7]and 2-D model of undoped Nanowire MOSFET [8] along with extraction of Short channel effects have already been derived. Recently a two dimensional model for surrounding gate MOSFET considering channel doping have been proposed by Wu and Su [9] , however a two dimensional model for Nanowire MOSFET(NW-MOS) with channel doping for extraction of short channel parameters has not been reported.

The channel doping is crucial factor to be considered to evaluate the performance of NW-MOS. In this paper we propose the model of NW-MOS with moderate channel doping extendable to undoped channel NW-MOS. The further higher doping requires the addition of quantum mechanical and band to band tunneling effects which require more rigorous computations. However to keep the model simple and compact these effects have not been included in the present model. This model has been validated for Silicon. The model also been extended for Ge based NW-MOS which is another important alternative [10-14] for Nanowire CMOS applications due to higher carrier mobility.

In section II we first derive a complete two dimensional model for the potential and current for Nanowire MOSFET in subthreshold region. The important parameters for SCEs such as threshold voltage roll off and subthreshold slope have been extracted in Section III and IV and results have been verified using simulations with Device3D. The model has been verified for both Silicon and Germanium, and various parameters as required for calculation and simulation have been taken from [15].

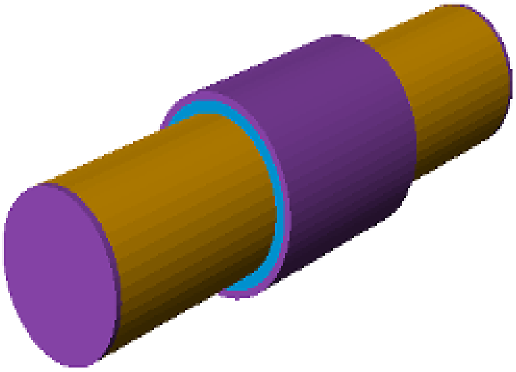

Fig.1 3-D view of Nanowire MOSFET

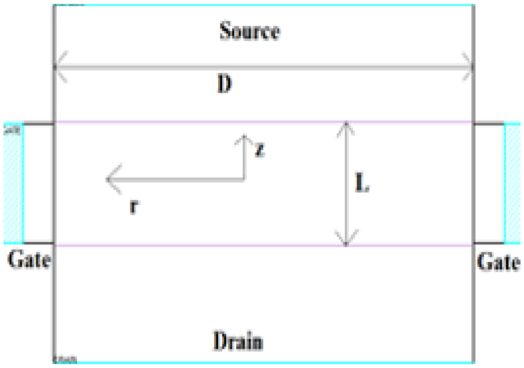

Fig.2 2-D Cross sectional view of Nanowire MOSFET

II. FULL TWO DIMENSIONAL POTENTIAL AND SUBTHRESHOLD CURRENT DERIVATION

Figure1 shows the cylindrical Nanowire MOSFET and Figure2 shows the two dimensional view of the Nanowire MOSFET along with the coordinate system used in this model. If the doping concentration is assumed to be uniform, the two dimensional poisson equation in cylindrical coordinates based on the depletion approximation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\psi\left(r,z\right)\right)+\frac{\partial^2}{\partial z^2}\psi\left(r,z\right)=-\frac{qN_a}{\in_{Si}} \qquad (1)$$

where N_a is the doping concentration in the substrate of a n-channel MOSFET; $\in_{Si}$ is the dielectric permittivity of the substrate and $\psi\left(r,z\right)$ is the electrostatic potential in the substrate. The source and drain junctions in this model are assumed to be abrupt. The boundary conditions are as follows:

$$\frac{\partial\psi\left(r,z\right)}{\partial r}\Big|_{r=0}=0 \qquad (2)$$

$$\in_{Si}\frac{\partial}{\partial r}\psi\left(r,z\right)\Big|_{r=\frac{D}{2}}=C_i\left[V_{GS}-V_{fb}-\psi\left(r=\frac{D}{2},z\right)\right] \qquad (3)$$

$$\psi\left(r,z=0\right)=V_{bi} \qquad (4)$$

$$\psi\left(r,z=L\right)=V_{DS}+V_{bi} \qquad (5)$$

where L and D are effective channel length and channel diameter respectively. t_i is the thickness of gate insulator. V_{GS} and V_{DS} are the voltage bias of the gate and drain terminal respectively, V_{fb} is the flat band voltage and V_{bi} is the built-in voltage of the source/drain to the channel.

By applying the superposition principle, the electrostatic potential the nanowire MOSFET can written as

$$\psi\left(r,z\right)=\psi_1\left(r\right)+\psi_2\left(r,z\right) \qquad (6)$$

where $\psi_1\left(r\right)$ is the solution of 1-D poisson equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\psi\left(r,z\right)\right)=-\frac{qN_a}{\in_{Si}} \qquad (7)$$

And can be expressed as

$$\psi_1\left(r\right)=\alpha_1 r^2+\alpha_2 \qquad (8)$$

$\psi_2\left(r,z\right)$ is the solution of 2-D laplace equation and can be expressed as

$$\psi_2\left(r,z\right)=\sum_n\left[\beta_n\sinh(k_n y)+\beta_n^{/}\sinh(k_n(L-y))\right]J_0(k_n r) \qquad (9)$$

Here $J_0(x)$ denotes the zeroth-order Bessel function of the first kind. The boundary condition at the silicon/insulator interface requires the electric field and potential to be continuous at $r = D/2$, thus k_n can be determined by

$$J_0\left(k_n\frac{D}{2}\right)=\frac{\in_{Si}}{C_i}k_n J_1\left(k_n\frac{D}{2}\right) \qquad (10)$$

where $C_i=2\in_{Si}/\left(D.\ln\left(1+\frac{2t_i}{D}\right)\right)$ (11)

is the gate oxide capacitance[16].

The coefficients β_n and $\beta_n^{/}$ can be expressed as

$$\beta_n=\frac{\int_0^{D/2}\left[\psi_2\left(\rho,0\right)-\alpha_1 r^2-\alpha_2\right]J_0(k_n r)\sinh(k_n L)\in_{Si}rdr}{\int_0^{D/2}\left(J_0(k_n r)\sinh(k_n L)\right)^2\in_{Si}rdr} \qquad (12)$$

$$\beta_n^{/}=\frac{\int_0^{D/2}\left[\psi_2\left(\rho,L\right)-\alpha_1 r^2-\alpha_2\right]J_0(k_n r)\sinh(k_n L)\in_{Si}rdr}{\int_0^{D/2}\left(J_0(k_n r)\sinh(k_n L)\right)^2\in_{Si}rdr} \qquad (13)$$

The aforementioned integrals can be carried out using basic characteristics of Bessel functions [17] to get the coefficients for the equations (8),(9) which can be expressed as

$$\alpha_1=-\frac{qN_a}{4\in_{Si}} \qquad (14)$$

$$\alpha_2=V_{GS}-V_{fb}+\frac{qN_a}{4\in_{Si}}\frac{D}{2}\left(\frac{D}{2}+2\frac{\in_{Si}}{C_i}\right) \qquad (15)$$

$$\beta_n=\frac{\left\{\begin{array}{c}-\alpha_1\left[\frac{1}{k_n}\left(\frac{D}{2}\right)^3J_1\left(k_n\frac{D}{2}\right)-2\left(\frac{1}{k_n}\right)^2\left(\frac{D}{2}\right)^2J_2\left(k_n\frac{D}{2}\right)\right]\\+\left(V_{DS}+V_{bi}-\alpha_2\right)\frac{1}{k_n}\frac{D}{2}J_1\left(k_n\frac{D}{2}\right)\end{array}\right\}}{\left(\frac{D}{2}\right)^2\left(\left(\frac{C_i}{k_n\in_{Si}}\right)^2+1\right)\left(J_0\left(k_n\frac{D}{2}\right)\right)^2\sinh\left(k_n L\right)} \qquad (16)$$

$$\beta_n^{/}=\frac{\left\{\begin{array}{c}-\alpha_1\left[\frac{1}{k_n}\left(\frac{D}{2}\right)^3 J_1\left(k_n\frac{D}{2}\right)-2\left(\frac{1}{k_n}\right)^2\left(\frac{D}{2}\right)^2 J_2\left(k_n\frac{D}{2}\right)\right]\\+\left(V_{bi}-\alpha_2\right)\frac{1}{k_n}\frac{D}{2}J_1\left(k_n\frac{D}{2}\right)\end{array}\right\}}{\left(\frac{D}{2}\right)^2\left(\left(\frac{C_i}{k_n\in_{Si}}\right)^2+1\right)\left(J_0\left(k_n\frac{D}{2}\right)\right)^2\sinh(k_nL)}\tag{17}$$

In the following derivation we assume that only the first term of the series expansion of the potential is dominant [18-19]. The results presented in the analysis are consistent with the approximation since the Fourier-Bessel series coefficients decay rapidly. Thus the final 2-D potential for the cylindrical nanowire MOSFET can be expressed as

$$\psi\left(r,z\right)=\alpha_1 r^2+\alpha_2+\left[\beta_1\sinh(k_1 z)+\beta_1^{/}\sinh(k_1(L-z))\right]J_0(k_1 r)\tag{18}$$

Once the analytical potential is obtained, the subthreshold current can be derived based on the current continuity equation

$$I_{ds}(z)=2\pi\mu q\frac{n^2{}_i}{N_a}\frac{\int_0^{V_{DS}} e^{-qV(z)/kT}dV\left(z\right)}{\int_0^{L}\frac{dy}{\int_0^{D/2} e^{q\psi(r,z)/kT}rdr}}\tag{19}$$

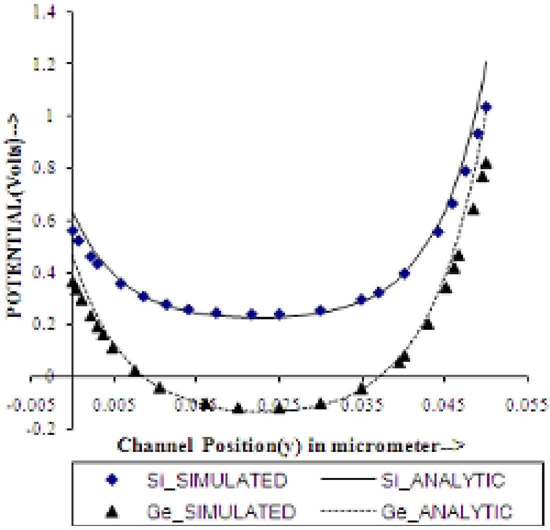

Fig.3 Analytical and simulated results for channel potential of Si and Ge Nanowire MOSFETs for D=20nm, L=50nm, V_G=0V, V_D=0.5V, t_i=1.5nm, $N_a=10^{15}$ cm^{-3}

$$=2\pi\mu kTq\frac{n^2{}_i}{N_a}\frac{\left(1-e^{-qV_{DS}/kT}\right)}{\int_0^{L}\frac{dy}{\int_0^{D/2} e^{q\psi(r,z)/kT}rdr}}\tag{20}$$

The results for the subthreshold current have been plotted in the Figure 4 for Si and Ge respectively with gate voltage being ramped till the threshold voltage of the respective device.

III. THRESHOLD VOLTAGE ROLLOFF MODEL

The threshold voltage roll off ΔV_t can be obtained from the parallel shift of the $I_D V_{GS}$ curves (in log scale) of a short-channel device with respect to the long-channel device, i.e.

$$I_{DS}(\text{short channel})\ L=I_{DS}(\text{long channel})\ e^{-q\Delta V_t/kT}\tag{21}$$

The position of minimum surface potential (0,y_c) obtained y differentiating equation (18) and setting the derivative equal to zero is given as

$$y_c=\frac{L}{2}-\frac{1}{2k_1}\ln\frac{c_1e^{k_1L/2}-b_1e^{-k_1L/2}}{b_1e^{k_1L/2}-c_1e^{-k_1L/2}}\simeq\frac{L}{2}-\frac{1}{2k_1}\ln\frac{c_1}{b_1}\tag{22}$$

The minimum surface potential is obtained approximately as

$$\psi_{\min}=\psi\left(0,z_c\right)\doteq V_g-\Delta\phi+2\sqrt{b_1c_1}\,e^{-\frac{k_1L}{2}}\tag{23}$$

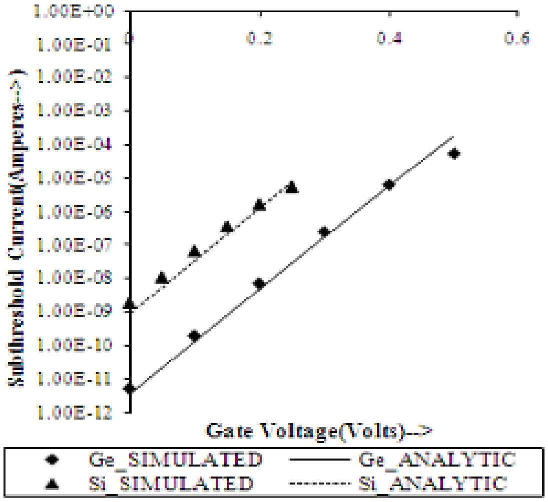

Fig. 4 Subthreshold Current Vs. Gate Voltage for Si and Ge Nanowire MOSFETs for D=20nm, L=50nm, V_D=0.5V, t_i=1.5nm, $N_a=10^{15}$ cm^{-3}

Using Taylor Series expansion we expand the expression of $\psi(r,z)$ at $\psi(0,z_c)$ and keep the expression upto second order terms. The simplified expression of the potential in the nanowire MOSFET can be written as

$$\psi(r,z)=\psi_{\min}+(y-y_c)^2 k_1^2 e^{-\frac{k_1L}{2}}\sqrt{\beta_1\beta_1^{/}}\sinh(k_1L) + \frac{r^2}{2}\left[2\alpha_1-k_1^2 e^{-\frac{k_1L}{2}}\sqrt{\beta_1\beta_1^{/}}\sinh(k_1L)\right] \tag{24}$$

This simplified form of the potential can be used to get the final analytical expression of the subthreshold current from (20).Substituting (23) into the extraction equation (21), one can derive

$$\Delta V_t=\Delta V_{t1}+\Delta V_{t2}+\Delta V_{t3} \tag{25}$$

Where ΔV_{t1} corresponds to the minimum surface potential and is given by

$$\Delta V_{t1}=-2e^{-\frac{k_1L}{2}}\sqrt{\beta_1\beta_1^{/}}\sinh(k_1L) \tag{26}$$

ΔV_{t2} represents the modification induced by the second order terms of the potential in the channel direction and ΔV_{t3} corresponds to the modification induced by the second order term of the potential in the radial direction. These are given by following expressions:

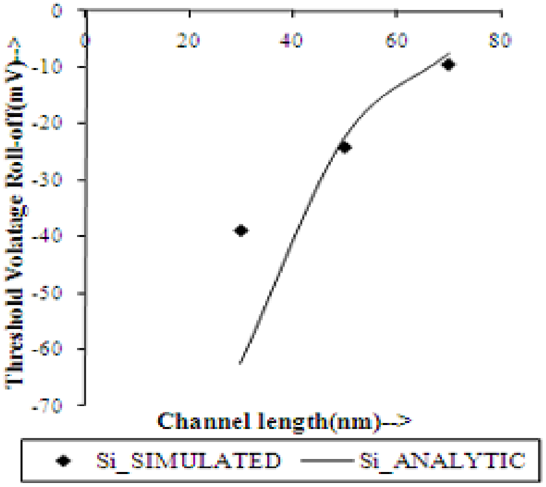

Fig. 5 Threshold Voltage roll-off in mV vs. Channel length in nm. The simulated points are for channel length 30nm, 50nm, 70nm respectively. (D=20nm, t_i=1.5nm, $N_a=10^{15}$ cm^{-3})

$$\Delta V_{t2}=\frac{kT}{q}\ln\left\{\frac{\sqrt{\pi}}{2\delta_1L}\left[erf\left(\delta_1\left(L-z_c\right)\right)+erf\left(\delta_1z_c\right)\right]\right\} \tag{27}$$

$$\Delta V_{t3}=\frac{kT}{q}\ln\left[\frac{\left(2A-\delta_1^{\,2}\right)}{e^{\left(A-\frac{\delta_1^{\,2}}{2}\right)\frac{D^2}{4}}-1}\left(\frac{D^2}{8}\right)\right] \tag{28}$$

Where erf(x) and δ_1 are defined as

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{u^2}du \tag{29}$$

$$\delta_1=\left(\frac{qk_1^2e^{-\frac{k_1L}{2}}\sqrt{\beta_1\beta_1^{/}}\sinh(k_1L)}{kT}\right)^{1/2} \tag{30}$$

IV. SUBTHRESHOLD SLOPE MODEL

To get a simplified model of subthreshold slope we approximate the subthreshold current by $I_{ds}\propto e^{q\psi_{\min}/kT}$ .Thus the inverse subthreshold slope is given by:

$$S=\left[\frac{\partial\left(\log_{10}I_{ds}\right)}{\partial V_g}\right]^{-1}\simeq\left(\frac{\partial\psi_{\min}}{\partial V_g}\right)^{-1}\times 60mv/Dec. \tag{31}$$

Using the expression for the $\psi_{\min}$ as derived in (23) we get a simplified form for the subthreshold slope as

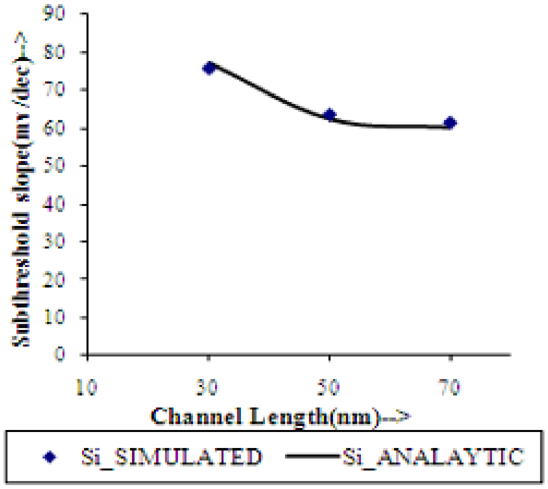

Fig. 6 Subthreshold Slope in mV/Dec. vs. Channel lenth in nm. The simulated points are for channel length30nm, 50nm, 70nm respectively. (D=20nm, t_i=1.5nm $N_a=10^{15}$ cm^{-3}).

$$\frac{\partial \psi_{\min}}{\partial V_g} = 1 - \frac{s_1 s_2 \left(\beta_1 + \beta'_2\right)}{\sqrt{\beta_1 \beta'_1}} e^{-\frac{k_1 L}{2}} \qquad (32)$$

Where s_1 and s_2 are given by

$$s_1 = 1/\left(\frac{D}{2}\right)^2 \left(\left(\frac{C_i}{k_n \in_{Si}}\right)^2 + 1\right)\left(J_0\left(k_n \frac{D}{2}\right)\right)^2 \sinh\left(k_n L\right) \qquad (33)$$

$$s_2 = \frac{1}{k_n} \frac{D}{2} J_1\left(k_n \frac{D}{2}\right) \qquad (34)$$

V. CONCLUSION

The 2-D model for the potential and current in subthreshold region for moderate channel doping has been proposed for Nanowire MOSFETs which has been validated for Silicon and Germanium. The model has been further extended by extracting the compact expressions for the short channel parameters such as threshold voltage roll off and subthreshold slope. The analytical results and the simulated results from 3-D simulations of Nanowire MOSFETs with Device3D have been plotted. The results are in close agreement and accurately predict the behavior of NW-MOS in subthreshold region.

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