3DLD Technology: Virtual Reference as a Measure of Weld Bead Surface Shape.

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Abstract

This paper summarizes the key findings of the authors' experimental and theoretical research over the past 15 years. A physico-mathematical model for the formation of a weld bead free interphase surface in fusion welding is presented. The model is based on the Laplace equation and the variational principle of minimum potential energy. This model enables the calculation of a virtual reference standard for the weld bead profile across all spatial positions (0°–360°). The results of the numerical solution of integro-differential equations implemented in the 3DLD software are provided. Examples of reference profile calculations are demonstrated.

Keywords: Weld bead surface geometry measure, reference profile, weld joint quality, numerical solution of differential equation, numerical values of reference profiles.

Introduction

The weld bead profile, including all its geometric dimensions, is a physical property that characterizes the quality of a welded joint; nevertheless, an objectively justified measure for its evaluation is not provided in the literature.

The weld bead profile, its geometric dimensions, surface defects, and methods for their inspection during welding are regulated by normative and technical documentation (NTD). The designations and classifications of various surface defects of weld beads based on NTD are widely known and have remained unchanged since the middle of the last century, as enshrined in fundamental international standards such as AWS A3.0. [1].

Existing NTD regulates weld profile defects and their evaluation criteria differentially, without connection to the weld bead surface profile, failing to account for the fact that geometric dimensions and surface defects are an inherent part of its geometry. A significant number of fundamental works, including research by leading US scientific schools, have been dedicated to defining the weld bead profile and investigating the hydrodynamics of the weld pool. The primary objective of these studies was to confirm the scientific hypothesis regarding the dominant influence of interphase surface phenomena and capillary equilibrium forces on the formation of the geometric profile of the bead during fusion welding [2-7].

Due to objective historical reasons related to the development of science and technology, researchers encountered technical difficulties when calculating the theoretical weld bead surface profile forming in various spatial positions using the proposed models. At the same time, it was impossible to measure and compare the actual bead reinforcement profile with the calculated one without mechanical destruction of the welded joint. In other words, there were neither tools for measuring the surface profile of an actual weld nor universal algorithms for calculating theoretical weld bead surface profiles in various spatial positions.

These circumstances led the authors of this paper to understand the necessity of introducing a new measure that objectively and unambiguously characterizes a welded joint in fusion welding — the measure of the weld bead surface geometry. Considering the variety of terms and definitions encountered in the scientific and technical literature and NTD that characterize the weld bead surface, such as reinforcement, undercut, reinforcement profile, profile, curve, reinforcement, free interphase surface, and external weld bead surface profile, the authors have adopted the term "weld bead surface profile."

In fusion welding, defect-free weld bead formation is of paramount importance. If a reference (defect-free) profile is taken as a measure of comparison for an actual weld bead profile, such a measure will be physically explainable and reproducible with a specified degree of accuracy.

The purpose of this paper is to propose and formalize a measure for the weld bead profile in fusion welding across various spatial positions.

Theoretical Foundations for Constructing a Virtual Reference Standard of the Weld Bead Surface Profile

The mathematical model is based on the Laplace equation, which states that the curvature of a free interphase surface is proportional to the excess pressure. In an arbitrary coordinate system, the equation has the form:

$$\kappa = -\frac{P_0 - h}{a_k^2} \quad \textbf{(1)}$$

Where:

- ***κ* -** surface curvature;
- P_0 - excess pressure at the reference point (expressed in units of liquid column height);
- ***h*** **-** height above the reference point;
- a_K **-**capillary constant.

A specific case of equation (1) in Cartesian coordinates for a long weld without surface tilt has been thoroughly investigated in the literature [2,6.7].

$$\frac{h''}{\left(1+\left(h'\right)^{2}\right)^{\frac{3}{2}}}-\frac{h}{a_K^2}+\frac{1}{R_0}=0 \qquad (2)$$

Where:

$$\frac{1}{R_0}=\frac{P_0}{a_k^2}$$

R₀-radius of curvature of the free interphase surface at the reference point. Near a wetting angle exceeding 90°, the value of **h** is not unambiguous, and the derivative ***h'*** is not continuous. Therefore, the equation in Cartesian coordinates (2) does not allow describing profiles with wetting angles of 90° or more, which can form in certain spatial positions of the weld, for instance, during horizontal welding on a vertical plane, welding a fixed pipe joint, and other spatial positions of weld beads. In their work [8,9], the authors proposed a parametric representation of the reference weld bead profile, which allows calculating the virtual reference standard of the weld bead at values of 0° ≤ φ ≤ 360.

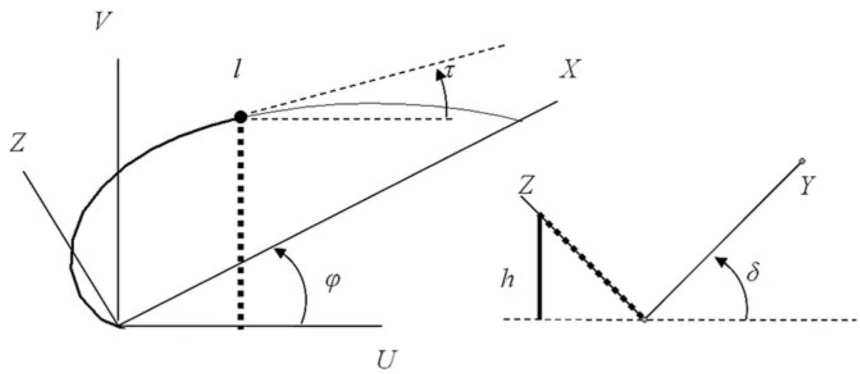


Figure 1: Analytical scheme of the free interphase surface profile in the \(l, \tau\) coordinate system.

The reference profile of a weld bead forming in any spatial position is computed by solving the integro-differential equation:

$$\frac{d\tau}{dl}=-k_0+\frac{Cos\,\delta}{a_k^2}\int_0^l Sin\,\tau\, d\lambda \qquad (3)$$

Where:

- **l** - distance from the initial point of the reference curve to the calculation point measured along the curve, with l varying from 0 to L;
- **L** - length of the reference weld bead curve;
- **dl** - differentiation step along the reference curve;
- **d**λ - integration step along the reference curve;
- $\tau(l)$ **-** angle of the tangent of the reference curve to the horizon at the calculation point;
- k_0 **-** curvature of the reference weld bead curve at the initial point, determined during the solution of the equation;
- a_k **-** capillary constant;
- δ **-** longitudinal tilt angle of the weld bead;
- φ - transverse tilt angle of the weld bead.

Z(x), the equation describing the reference curves of the weld bead, is written in parametric form $\{Z(l), x(l)\}$ and has the form:

$$\left.\begin{aligned} x &= \int_0^l \left(Cos\ \varphi\ Cos\ \tau + Sin\ \varphi\ Sin\ \tau \right) d\lambda \\ Z &= \int_0^l \left(-\,Sin\ \varphi\ Cos\ \tau + Cos\ \varphi\ Sin\ \tau \right) d\lambda \end{aligned}\right\} \quad 0 \le l \le L \qquad (4),$$

The first boundary condition is Z(e)=0, where X = e is the specified weld width:

$$\int_0^L \left(Cos\ \varphi\ Cos\ \tau + Sin\ \varphi\ Sin\ \tau \right) dl = e \qquad \int_0^L \left(-\,Sin\ \varphi\ Cos\ \tau + Cos\ \varphi\ Sin\ \tau \right) dl = 0$$

The second boundary condition is the constancy of the cross-sectional area, where S_{0} is the specified cross-sectional area of the weld:

$$S = \int_0^e Z\ dX = \int_0^L \left[\int_0^l \left(-\,Sin\ \varphi\ Cos\ \tau + Cos\ \varphi\ Sin\ \tau \right) d\lambda \right] \left(Cos\,\varphi\ Cos\,\tau + Sin\,\varphi\ Sin\,\tau \right) dl = S_0$$

[

The reference profile calculated by formulas (3-4) physically characterizes the equilibrium surface of the weld bead in the calculated cross-section, forming in any spatial position under the action of surface tension forces in a mass force field. For given values of:

- **a)** weld width \(e\) and area $\mathbf{S_0}$**;**
- **b)** capillary constant $\boldsymbol{a_k}$**;**
- **c)** tilt angles in the transverse φ and longitudinal δ directions of the weld bead in space;

there exists a unique reference weld bead profile with minimum potential energy. The reference standard quantitatively calculated by formulas (3-4) objectively and unambiguously determines the measure of the weld bead surface profile in fusion welding in any spatial position.

Software Implementation and Illustration of Calculations

Based on the mathematical model, the authors developed a computational module in the Microsoft Excel environment that allows for rapid generation of reference weld profiles. The software mathematically models the behavior of liquid metal under gravity when the spatial position of the weld bead changes.

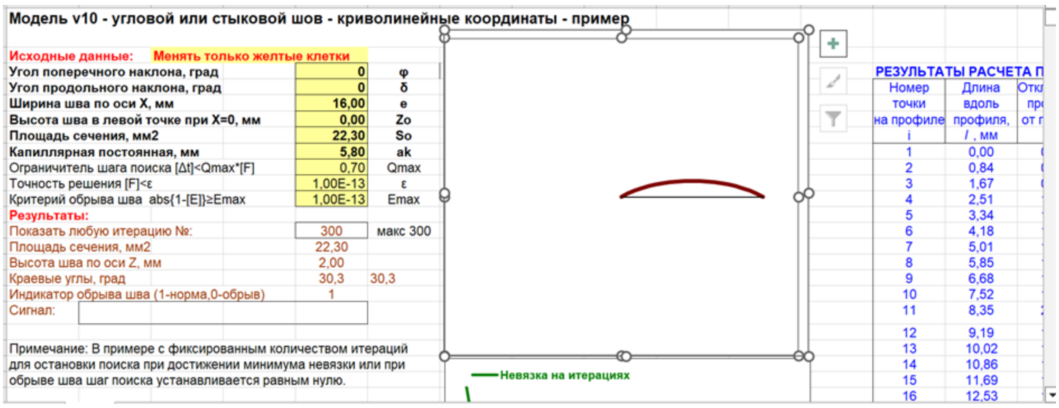


Figure 2: Software calculation window for a butt weld reference profile in the flat position with a weld width of 16 mm, cross-sectional area of 22.3 mm², and capillary constant of 5.8 mm.

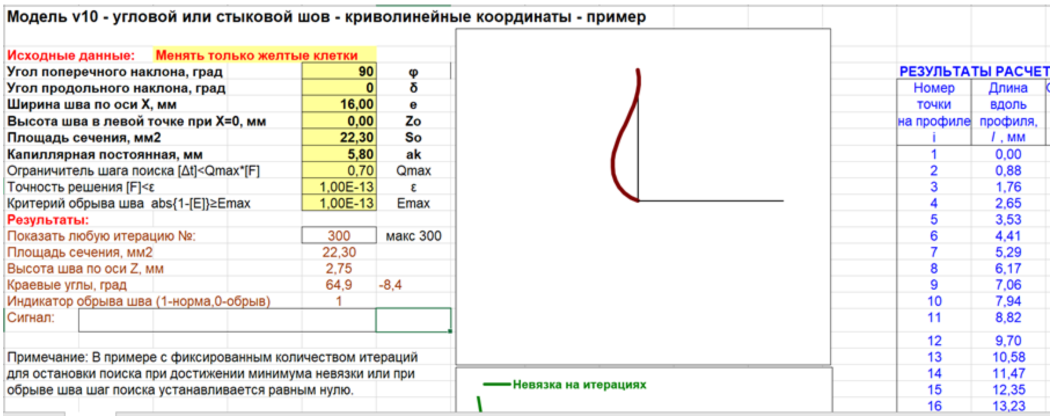


Figure 3: Software calculation window for a butt weld reference profile in the vertical position with a weld width of 16 mm, cross-sectional area of 22.3 mm², and capillary constant of 5.8 mm.

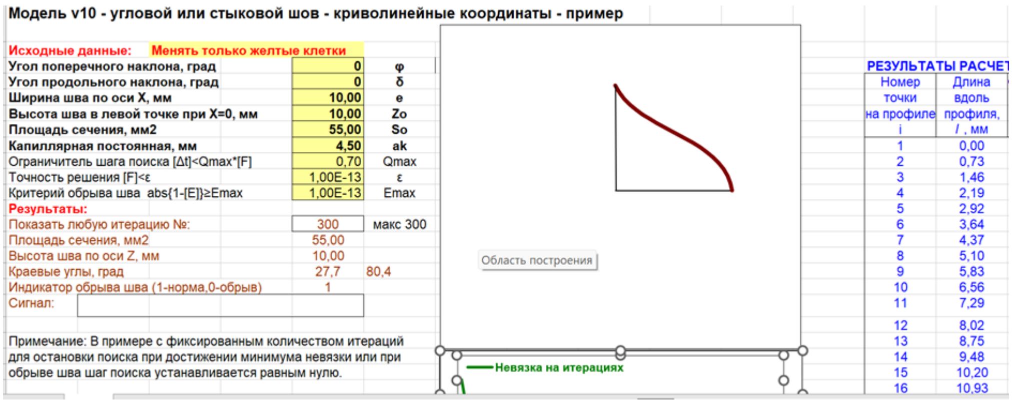


Figure 4: Software calculation window for a fillet weld reference profile in the flat position with a weld leg of 10 mm, cross-sectional area of 55 mm², and capillary constant of 4.5 mm.

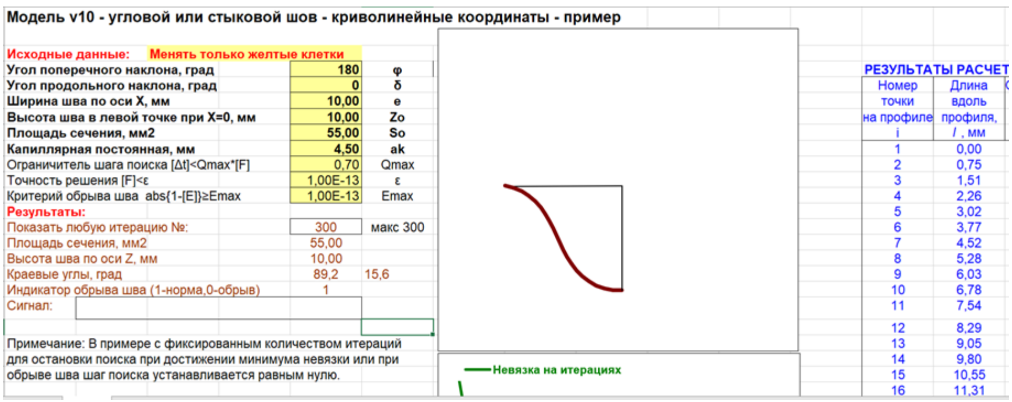


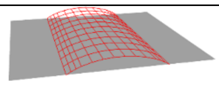
Figure 5: Software calculation window for a fillet weld reference profile in the overhead position with a weld leg of 10 mm, cross-sectional area of 55 mm², and capillary constant of 4.5 mm.

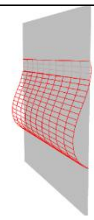
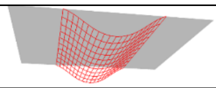
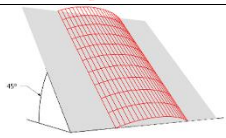
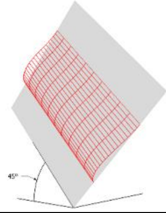
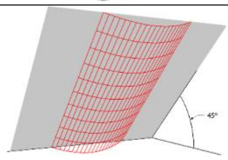
Let us illustrate the calculation results with examples:

Example 1. We calculate and graphically display the reference profiles for a butt weld according to the specified design values of width and height in accordance with API Standard 1104 (Standard for Welding Pipelines) for the weld joint designation "Single-V-groove butt joint (1G/5G/6G position)." Since weld positions other than flat are not provided in API Standard 1104, we adopt design elements based on nominal values of width $e=16$ mm and height $g=2$ mm for calculating the reference profile at a welded part thickness of $S=S_1=10$мм mm, which corresponds to a reference profile area of $S_0=22.3$ mm² and a value of a_k = 5.8) mm.

The calculation of reference profiles is performed for various values of angles φ and δ etermining the spatial position of the welded joint, while keeping their area constant. Table 1 presents the reference surface profiles of a weld bead forming in the flat, overhead positions, and on a vertical plane, when its longitudinal axis coincides with the horizontal axis $\delta=0^0$ (rows 1, 2, 3), and reference surface profiles of a weld bead when the longitudinal axis of the weld is tilted to the horizontal at an angle $\delta=45^0$ (rows 4, 5, 6). The calculated reinforcement (undercut) is determined in accordance with the requirements of AWS B1.11 (Guide for the Visual Examination of Welds).

Table 1*. Reference surface profiles of a butt weld calculated for various spatial positions of API Standard 1104, designation Single-V-groove butt joint (1G/5G/6G position).*

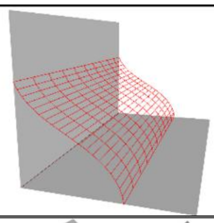
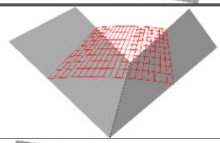
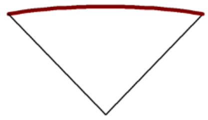
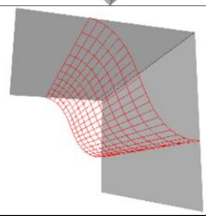
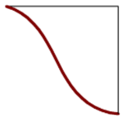
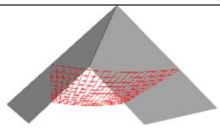
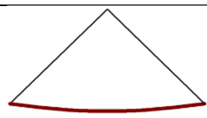
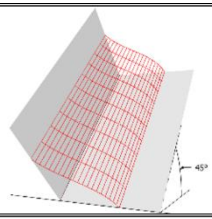
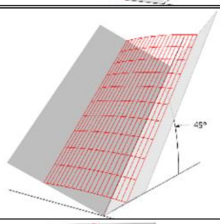
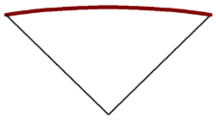
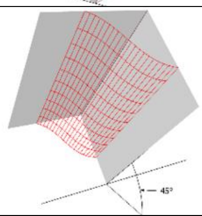
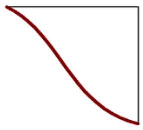
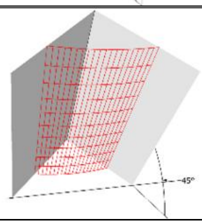
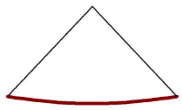
No	Weld Bead Position	δ deg	φ deg	Calculated convexity g, concavity (g3) mm	Calculation Form of the Reference Standard, a_k = 5.8 mm	
					3D	2D
1	Flat position	0	0	2.00		

2	Horizontal on a vertical plane	0	90	2.75 (-0.13)		
3	Overhead position	0	180	2.15		
4	Flat posit*ion*	45	0	2.02		
5	Horizontal on a plane tilted at 45°	45	90	2.44		
6	Overhead position	45	180	2.12		

Example 2. We calculate and graphically display the reference surface profiles of a fillet weld according to the specified design values of width and height from AWS D1.1/D1.1M (Structural Welding Code - Steel) for the weld joint designation "Fillet weld (T-joint) (2F/4F position)." We adopt leg values k = 10 mm, reference area S_0 = 55 mm², and capillary constant a_k = 4.5 mm for calculating the reference profile at a welded part thickness of $S=S_1$=40 mm.

The calculation of reference profiles is performed for various values of angles φ and δ determining the spatial position of the welded joint, while keeping the area unchanged. Table 2 presents the reference surface profiles of a fillet weld for various spatial positions when its longitudinal axis coincides with the horizontal axis $\delta=0^0$ (rows 1, 2, 3, 4) and when its longitudinal axis is tilted to the horizontal at an angle $\delta=45^0$ (rows 5, 6, 7, 8). The calculated undercut (reinforcement) of the reference standard is determined in accordance with the requirements of AWS B1.11 (Guide for the Visual Examination of Welds).

Table 2. *Reference surface profiles of a fillet weld calculated for various spatial positions of AWS D1.1/D1.1M, designation Fillet weld (T-joint) (2F/4F position).*

No	Weld Bead Position	δ deg	φ deg	Calculated convexity g, concavity (g3) mm	Calculation Form of the Reference Standard, a_k = 4.5 mm	
					3D	2D
1	Flat position	0	0	+1.08 (-0.38)		
2	Flat position (gravity-fed / "boat")	0	45	+0.51		
3	Overhead position	0	180	+1.47 (-0.76)		
4	Overhead position ("boat")	0	225	+0.56		
5	Flat posit*ion*	45	0	+0.90 (-0.20)		
6	Flat position ("boat")	45	45	+0.52		
7	Overhead position	45	180	+1.06 (-0.36)		
8	Overhead position ("boat")	45	225	+0.55		

A distinctive feature of the proposed measure described by equations (3-4) is the ability to quantitatively determine the geometric parameters of the reference profile for a butt weld (Fig. 6a) and a fillet weld (Fig. 6b)

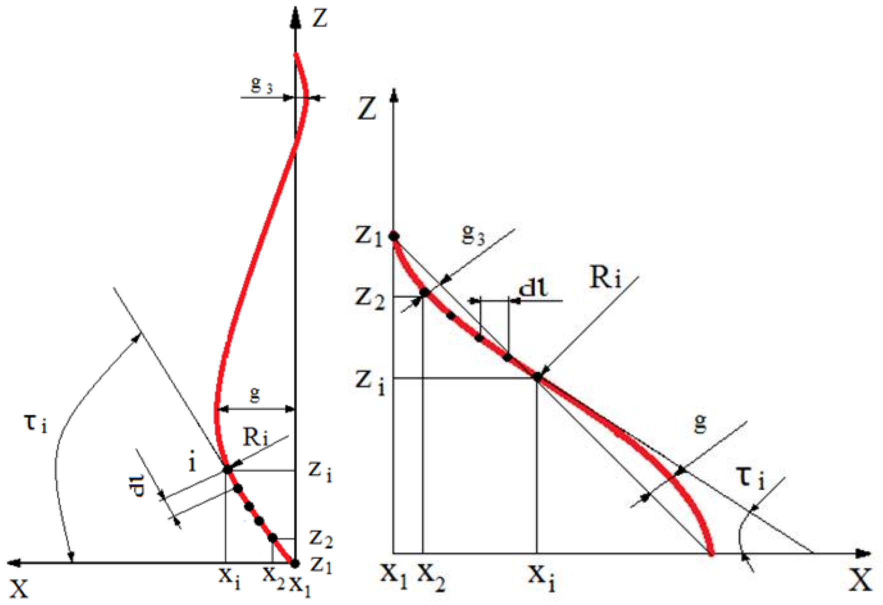


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Figure 6: Geometric parameters of the reference profile for a butt weld formed on a vertical plane (a) and a fillet weld formed in the flat position (b).

Let us the geometry of the reference profiles shown in Fig. 6 in figures.

Table 3. *Geometric parameters of the reference surface profile of a horizontal butt weld formed on a vertical plane.*

Point No. i	Length l, mm	Coord. X, mm	Height Z, mm	Tilt τ, deg.	Tilt τ, rad.	Curvature k, 1/mm	Radius R, mm
1	0.00	0.00	0.00	65	1.13	—	—
2	0.88	0.37	0.80	51	0.89	-0.272	3.7
3	1.76	0.93	1.48	38	0.67	-0.256	3.9
4	2.65	1.62	2.03	26	0.46	-0.235	4.3
5	3.53	2.41	2.42	16	0.27	-0.211	4.7
6	4.41	3.26	2.66	6	0.11	-0.186	5.4
7	5.29	4.14	2.75	-2	-0.03	-0.160	6.2
8	6.17	5.02	2.73	-9	-0.15	-0.134	7.5
9	7.06	5.89	2.59	-14	-0.25	-0.108	9.3
10	7.94	6.75	2.38	-18	-0.32	-0.083	12.1
11	8.82	7.58	2.10	-21	-0.37	-0.058	17.3
12	9.70	8.41	1.78	-23	-0.40	-0.033	30.0

13	10.58	9.22	1.44	-23	-0.41	-0.009	109.5
14	11.47	10.03	1.09	-23	-0.39	0.015	66.9
15	12.35	10.84	0.75	-21	-0.36	0.039	25.5
16	13.23	11.67	0.44	-17	-0.30	0.064	15.7
17	14.11	12.51	0.18	-13	-0.23	0.089	11.3
18	14.99	13.37	-0.02	-7	-0.12	0.114	8.8
19	15.88	14.25	-0.13	0	0.00	0.140	7.1
20	16.76	15.13	-0.13	8	0.15	0.166	6.0
21	17.64	16.00	0.00	—	—	—	—

Table 4. *Geometric parameters of the reference surface profile of a fillet weld formed in the flat position.*

Point No. i	**Length l, mm**	**Reinf. g (+) / Undercut g_{3} (-), mm**	**Coord. X, mm**	**Height Z, mm**	**Tilt τ, deg.**	**Tilt τ, rad.**	**Curv. k, 1/mm**	**Radius R, mm**
1	0.00	0.00	0.00	10.00	-62	-1.09	—	—
2	0.73	-0.22	0.34	9.35	-55	-0.95	0.182	5.5
3	1.46	-0.34	0.76	8.76	-48	-0.84	0.153	6.5
4	2.19	-0.38	1.24	8.22	-43	-0.75	0.126	7.9
5	2.92	-0.36	1.78	7.72	-39	-0.68	0.101	9.9
6	3.64	-0.28	2.34	7.26	-36	-0.62	0.079	12.7
7	4.37	-0.16	2.94	6.84	-33	-0.58	0.058	17.3
8	5.10	-0.01	3.55	6.44	-32	-0.55	0.038	26.1
9	5.83	0.16	4.17	6.06	-31	-0.54	0.019	51.5
10	6.56	0.34	4.80	5.69	-31	-0.54	0.001	950.8
11	7.29	0.52	5.42	5.31	-31	-0.55	-0.017	57.8
12	8.02	0.69	6.05	4.93	-33	-0.57	-0.036	27.7
13	8.75	0.85	6.66	4.54	-35	-0.61	-0.056	18.0
14	9.48	0.97	7.25	4.12	-38	-0.67	-0.076	13.1
15	10.20	1.05	7.82	3.66	-43	-0.74	-0.099	10.1
16	10.93	1.08	8.36	3.17	-48	-0.83	-0.123	8.1
17	11.66	1.05	8.85	2.63	-54	-0.94	-0.150	6.7
18	12.39	0.94	9.28	2.04	-61	-1.07	-0.179	5.6
19	13.12	0.73	9.63	1.40	-70	-1.22	-0.210	4.8
20	13.85	0.42	9.88	0.72	-80	-1.40	-0.244	4.1
21	14.58	0.00	10.00	0.00	—	—	—	—

Thus, the presented verification data and numerical parameters in the tables fully confirm the adequacy and stability of the developed mathematical model. The method allows for high-precision calculation of the reference surface profile of both butt welds and complex asymmetric fillet welds across any spatial position, rendering it a ready-to-use analytical tool for engineering applications and digital control systems [10-12].

Conclusions

1. Based on physical concepts of the formation of a liquid metal free interphase surface in the tail part of the weld pool during fusion welding, the reference surface profile of butt and fillet welds has been mathematically described in a parametric form.
2. The reference profile equation objectively and unambiguously defines a defect-free weld bead surface geometry in fusion welding for a given spatial position, width, cross-sectional area, and capillary constant.
3. The reference weld bead profile serves as an objective measure for evaluating the quality of welds, welding materials, and welding equipment.
4. The developed mathematical engine for calculating the reference profile forms the foundation for digital technologies in welding production.

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